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A new 3-D jerk chaotic system with two cubic nonlinearities and its adaptive backstepping control

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This paper presents a new seven-term 3-D jerk chaotic system with two cubic nonlinearities. The phase portraits of the novel jerk chaotic system are displayed and the qualitative properties of the jerk system are described. The novel jerk chaotic system has a unique equilibrium at the origin, which is a saddle-focus and unstable. The Lyapunov exponents of the novel jerk chaotic system are obtained as $L_1 = 0.2974$, $L_2 = 0$ and $L_3 = -3.8974$. Since the sum of the Lyapunov exponents of the jerk chaotic system is negative, we conclude that the chaotic system is dissipative. The Kaplan-Yorke dimension of the new jerk chaotic system is found as $D_{KY} = 2.0763$. Next, an adaptive backstepping controller is designed to globally stabilize the new jerk chaotic system with unknown parameters. Moreover, an adaptive backstepping controller is also designed to achieve global chaos synchronization of the identical jerk chaotic systems with unknown parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems. MATLAB simulations are shown to illustrate all the main results derived in this work.

Key words: chaos, chaotic systems, jerk systems, chaos control, adaptive control, backstepping control, synchronization.

1. Introduction

Modeling and applications of chaotic systems are active research areas in the literature [1, 2, 3]. The first famous chaotic system was discovered by Lorenz, when he was designing a weather model in 1963 [4]. Some well-known chaotic systems are Chen system [5], Lü system [6], Cai system [7], Tigan system [8], Sprott systems [9], etc.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [10], Hénon-Heiles system [12], Lü-Chen system [13], Liu system [14], etc. Many new chaotic systems have been also discovered like Li system [15], Sundarapandian systems [16, 17], Vaidyanathan systems [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33], Pehlivan system [34], Tacha system [35], Jafari system [36], Sampath system [37], Pham systems [38, 39, 40, 41, 42, 43, 44], Volos system [45], Akif system [46], etc.

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Chaos theory has applications in several fields of science and engineering such as oscillators [47, 48, 49, 50, 51, 52, 53, 54, 55], dynamos [56, 57, 58, 59], Tokamak systems [60, 61], chemical reactions [62, 63, 64, 65, 66, 67, 68, 69, 70, 71], neural networks [72, 73, 74, 75, 76, 77], neurology [78, 79, 80, 81, 82, 83], biology [84, 85, 86, 87, 88, 89, 90, 91, 92], electrical circuits [93, 94, 95], induction motors [96], cryptosystems [97, 98], memristors [99, 100, 101], random bit generator [102], etc.

In classical mechanics, a jerk system is expressed by an explicit third order differential equation describing the time evolution of a single scalar variable x according to the dynamics

$$\frac{d^3x}{dt^3} = f\left(\frac{d^2x}{dt^2}, \frac{dx}{dt}, x\right) \tag{1}$$

A particularly simple example of a jerk system is the famous Coullet system [103] given by

$$\frac{d^3x}{dt^3} + a\frac{d^2x}{dt^2} + \frac{dx}{dt} = g(x) \tag{2}$$

where $g(x)$ is a nonlinear function such as $g(x) = b(x^2 - 1)$. The Coullet system (2) exhibits chaos for $a = 0.6$ and $b = 0.58$.

A classical example of a cubic dissipative jerk chaotic flow was found by Sprott [104]. In this research work, we modify the dynamics of the jerk system in [104] by introducing two linear terms and taking different set values for the system parameters. Thus, we obtain a novel chaotic jerk system with two cubic nonlinearities.

In most of the synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically [105, 106, 107, 108].

In the chaos literature, an impressive variety of techniques have been proposed for chaos synchronization such as active control method [109, 110, 111, 112, 113, 114, 115], adaptive control method [116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127], backstepping control method [128, 129, 130, 131, 132, 133, 134, 135], sliding mode control method [136, 137, 138, 139, 140, 141, 142, 143, 144], etc.

All the main adaptive backstepping control results in this paper are proved using Lyapunov stability theory [145]. MATLAB simulations are depicted to illustrate the phase portraits of the novel jerk chaotic system, adaptive stabilization and synchronization results for the novel 3-D jerk chaotic system.

This research paper is organized as follows. Section 2 contains the dynamics and phase portraits of the novel chaotic jerk system. Section 3 details the qualitative properties of the novel chaotic jerk system. In Section 4, we apply adaptive backstepping control method to design an adaptive feedback control law that stabilizes the states of the novel jerk system.

In Section 5, we apply adaptive backstepping control method to design an adaptive feedback control law that achieves complete and exponential synchronization of the states of identical novel chaotic jerk systems. Finally, Section 6 contains a summary of the main results obtained in this work.

2. A new jerk chaotic system

A classical example of a cubic dissipative jerk chaotic flow was found by Sprott [104] and described by the third-order differential equation

$$\dddot{x} = -a\ddot{x} + x\dot{x}^2 - x^3 \quad (\text{with } a = 3.6) \tag{3}$$

In system form, Sprott's differential equation (3) corresponds to the jerk system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_3 + x_1x_2^2 - x_1^3 \end{cases} \tag{4}$$

where $a = 3.6$ yields a chaotic attractor.

Using Wolf's algorithm [146], the Lyapunov exponents of the Sprott system (4) for $a = 3.6$ are numerically obtained as

$$L_1 = 0.1360, \quad L_2 = 0, \quad L_3 = -3.7367 \tag{5}$$

From (5), we see that the Maximal Lyapunov Exponent (MLE) of the Sprott system (4) is $L_1 = 0.1360$. Since $L_1 > 0$, the Sprott system (4) is *chaotic*.

The Kaplan-Yorke dimension of a chaotic system of order n is defined as

$$D_{KY} = j + \frac{L_1 + L_2 + \cdots + L_j}{|L_{j+1}|} \tag{6}$$

where $L_1 \geqslant L_2 \geqslant \cdots \geqslant L_n$ are the n Lyapunov exponents of the chaotic system and j is the largest integer for which $L_1 + L_2 + \cdots + L_j \geqslant 0$. Thus, the Kaplan-Yorke dimension of the Sprott jerk system (4) is calculated as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0364 \tag{7}$$

In this work, we propose a new jerk chaotic system, which is obtained by adding two linear systems $-bx$ and $c\dot{x}$, where $b, c > 0$, to the Sprott's jerk function in the ODE (3). Thus, our new jerk chaotic flow is described by the third order ODE

$$\dddot{x} = -a\ddot{x} + x\dot{x}^2 - x^3 - bx + c\dot{x} \tag{8}$$

In system form, the third order ODE (8) corresponds to the jerk system

$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & x_3 \\ \dot{x}_3 & = & -ax_3 - bx_1 + cx_2 + x_1 x_2^2 - x_1^3 \end{cases} \tag{9}$$

where a, b and c are positive parameters.

In this paper, we shall show that the system (9) is *chaotic* when the parameters a and b take the values

$$a = 3.6, \quad b = 1.3, \quad c = 0.1 \tag{10}$$

Using Wolf's algorithm [146], the Lyapunov exponents of the novel system (9) for the parameter values (10) are numerically obtained as

$$L_1 = 0.2974, \;\; L_2 = 0, \;\; L_3 = -3.8974 \tag{11}$$

From (11), we see that the Maximal Lyapunov Exponent (MLE) of the novel system (9) is $L_1 = 0.2974$. Since $L_1 > 0$, the novel system (9) is *chaotic*. Moreover, we also note that the MLE of the novel jerk system (9) is greater than the MLE of the Sprott jerk system (4). Also, the Kaplan-Yorke dimension of the novel jerk system (9) is calculated as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0763, \tag{12}$$

which is greater than the Kaplan-Yorke dimension of the Sprott jerk system (4).

For numerical simulations, we take the initial conditions of the system (9) as

$$x_1(0) = 0.5, \;\; x_2(0) = 0.5, \;\; x_3(0) = 0.5 \tag{13}$$

The initial conditions in (13) have been chosen arbitrarily for the sake of simulations. For other initial conditions in $\mathbf{R}^3$ also, the system (9) is chaotic with a similar strange attractor.

Figure 1 depicts the chaotic attractor of the novel jerk system (9) in 3-D view. Figures 2-4 depict the 2-D projection of the strange chaotic attractor of the novel jerk chaotic system (9) on $(x_1, x_2), (x_2, x_3)$ and (x_3, x_1) planes, is shown, respectively.

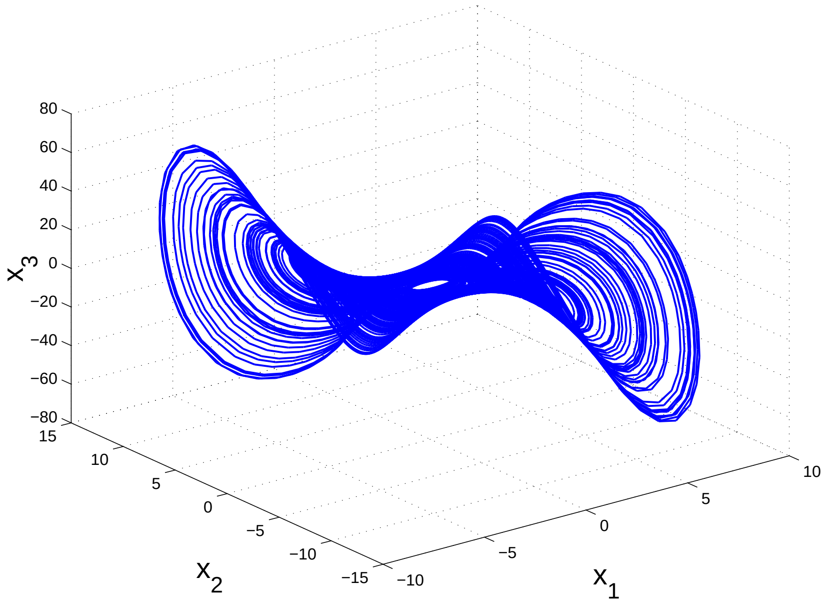

Figure 1: Strange attractor of the 3-D novel jerk chaotic System

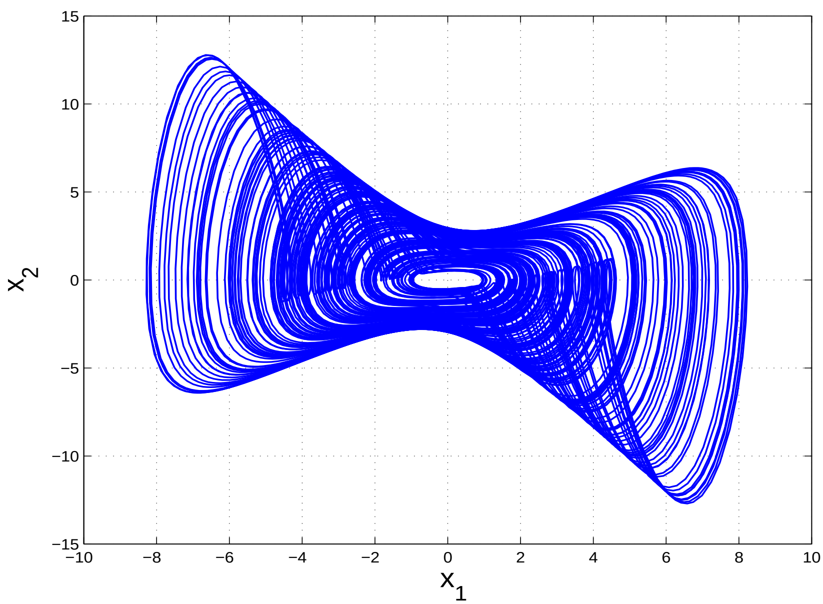

Figure 2: 2-D projection of the novel jerk chaotic system on the (x_1, x_2) plane

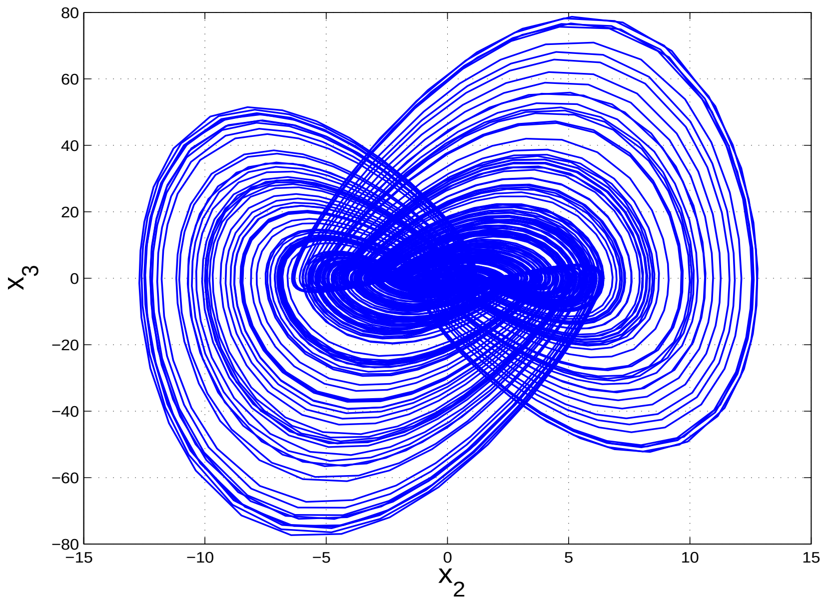

Figure 3: 2-D projection of the novel jerk chaotic system on the (x_2, x_3) plane

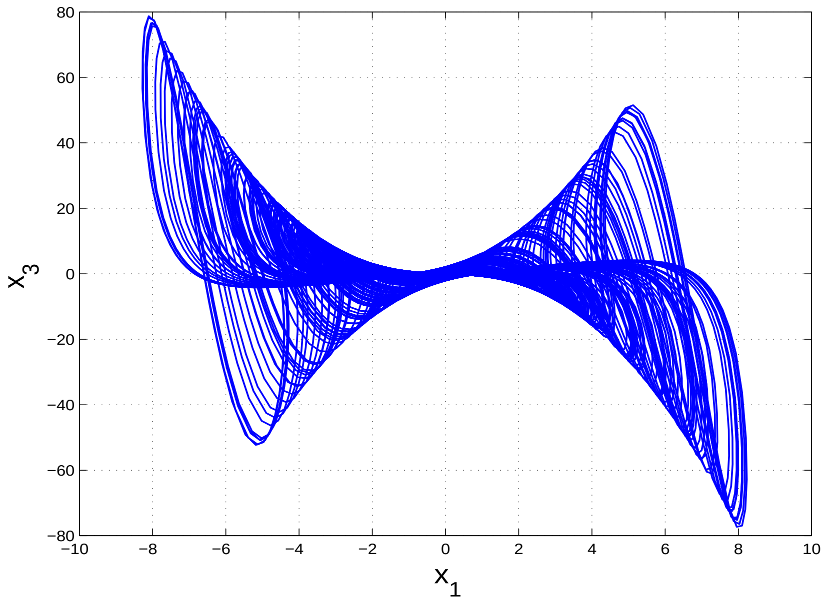

Figure 4: 2-D projection of the novel jerk chaotic system on the (x_1, x_3) plane

3. Analysis of the 3-D novel jerk chaotic system

3.1. Dissipativity

In vector notation, the new jerk system (9) can be expressed as

$$\dot{\mathbf{x}} = f(\mathbf{x}) = \begin{bmatrix} f_1(x_1,x_2,x_3) \\ f_2(x_1,x_2,x_3) \\ f_3(x_1,x_2,x_3) \end{bmatrix}, \tag{14}$$

where

$$\begin{cases} f_1(x_1,x_2,x_3) & = & x_2 \\ f_2(x_1,x_2,x_3) & = & x_3 \\ f_3(x_1,x_2,x_3) & = & -ax_3 - bx_1 + cx_2 + x_1x_2^2 - x_1^3 \end{cases} \tag{15}$$

Let Ω be any region in $\mathbf{R}^3$ with a smooth boundary and also, $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$. By Liouville's theorem, we know that

$$\dot{V}(t) = \int\limits_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3 \tag{16}$$

The divergence of the novel jerk system (14) is found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a < 0 \tag{17}$$

Inserting the value of $\nabla \cdot f$ from (17) into (16), we get

$$\dot{V}(t) = \int\limits_{\Omega(t)} (-a)\, dx_1\, dx_2\, dx_3 = -aV(t) \tag{18}$$

Integrating the first order linear differential equation (18), we get

$$V(t) = \exp(-at)V(0) \tag{19}$$

From Eq. (19), it is clear that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the novel 3-D jerk chaotic system (9) is dissipative. Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel jerk chaotic system (9) settles onto a strange attractor of the system.

3.2. Equilibrium Points

The equilibrium points of the 3-D novel jerk chaotic system (9) are obtained by solving the equations

$$\begin{cases} f_1(x_1,x_2,x_3) & = & x_2 & = & 0 \\ f_2(x_1,x_2,x_3) & = & x_3 & = & 0 \\ f_3(x_1,x_2,x_3) & = & -ax_3 - bx_1 + cx_2 + x_1x_2^2 - x_1^3 & = & 0 \end{cases} \tag{20}$$

We take the parameter values as in the chaotic case (10), *i.e.*

$$a = 3.6, \quad b = 1.3, \quad c = 0.1 \tag{21}$$

Thus, the equilibrium points of the system (9) are characterized by the equations

$$x_2 = 0, \quad x_3 = 0, \quad x_1(x_1^2 + b) = 0 \tag{22}$$

Solving the system (22), we get the equilibrium points of the system (9) as

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

The Jacobian matrix of the novel jerk chaotic system (9) at E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -b & c & -a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1.3 & 0.1 & -3.6 \end{bmatrix} \tag{24}$$

We find that J_0 has the eigenvalues

$$\lambda_1 = -3.7208, \quad \lambda_{2,3} = 0.0604 \pm 0.5880\, i \tag{25}$$

This shows that the equilibrium E_0 is a saddle-focus point, which is unstable.

3.3. Lyapunov exponents and Kaplan-Yorke dimension

We take the parameter values of the novel jerk system (9) as

$$a = 3.6, \quad b = 1.3, \quad c = 0.1 \tag{26}$$

Then the Lyapunov exponents are numerically obtained using Wolf's algorithm [146] as

$$L_1 = 0.2974, \quad L_2 = 0, \quad L_3 = -3.8974 \tag{27}$$

Thus, the maximal Lyapunov exponent (MLE) of the novel jerk system (9) is $L_1 = 0.2974 > 0$, which shows that the system (9) has chaotic behavior.

Since $L_1 + L_2 + L_3 = -3.6 = -a < 0$, it follows that the novel jerk chaotic system (9) is dissipative. Also, the Kaplan-Yorke dimension of the novel jerk chaotic system (9) is obtained as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0763, \tag{28}$$

which is fractional.

4. Adaptive control of the 3-D novel jerk chaotic system

In this section, we use backstepping control method to derive an adaptive feedback control law for globally stabilizing the 3-D novel jerk chaotic system with unknown parameters. Thus, we consider the 3-D novel jerk chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_3 - bx_1 + cx_2 + x_1x_2^2 - x_1^3 + u \end{cases} \tag{29}$$

where a, b, c are unknown constant parameters, and u is a backstepping control law to be determined using estimates of the unknown system parameters.

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{30}$$

Differentiating (30) with respect to t, we obtain the following equations:

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{31}$$

Next, we shall state and prove the main result of this section.

Theorem 1 *The 3-D novel jerk chaotic system* (29)*, with unknown parameters a, b and c, is globally and exponentially stabilized by the adaptive feedback control law,*

$$u(t) = -[3 - \hat{b}(t)]x_1 - [5 + \hat{c}(t)]x_2 - [3 - \hat{a}(t)]x_3 - x_1x_2^2 + x_1^3 - kz_3 \tag{32}$$

where $k > 0$ is a gain constant,

$$z_3 = 2x_1 + 2x_2 + x_3, \tag{33}$$

and the update law for the parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$ is given by

$$\begin{cases} \dot{\hat{a}}(t) = -z_3x_3 \\ \dot{\hat{b}}(t) = -z_3x_1 \\ \dot{\hat{c}}(t) = z_3x_2 \end{cases} \tag{34}$$

Proof We prove this result via Lyapunov stability theory [145].
First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{35}$$

where

$$z_1 = x_1 \tag{36}$$

Differentiating V_1 along the dynamics (29), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2) \tag{37}$$

Now, we define

$$z_2 = x_1 + x_2 \tag{38}$$

Using (38), we can simplify the equation (37) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{39}$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{40}$$

Differentiating V_2 along the dynamics (29), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3) \tag{41}$$

Now, we define

$$z_3 = 2x_1 + 2x_2 + x_3 \tag{42}$$

Using (42), we can simplify the equation (41) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{43}$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, e_a, e_b, e_c) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2}(e_a^2 + e_b^2 + e_c^2) \tag{44}$$

which is a positive definite function on $\mathbf{R}^6$. Differentiating V along the dynamics (29), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3(z_3 + z_2 + \dot{z}_3) - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{45}$$

Eq. (45) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{46}$$

where

$$S = z_3 + z_2 + \dot{z}_3 = z_3 + z_2 + 2\dot{x}_1 + 2\dot{x}_2 + \dot{x}_3 \tag{47}$$

A simple calculation gives

$$S = (3-b)x_1 + (5+c)x_2 + (3-a)x_3 + x_1x_2^2 - x_1^3 + u \tag{48}$$

Substituting the adaptive control law (32) into (48), we obtain

$$S = -[b-\hat{b}(t)]x_1 + [c-\hat{c}(t)]x_2 - [a-\hat{a}(t)]x_3 - kz_3 \tag{49}$$

Using the definitions (31), we can simplify (49) as

$$S = -e_b x_1 + e_c x_2 - e_a x_3 - kz_3 \tag{50}$$

Substituting the value of S from (50) into (46), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2 + e_a\left(-z_3x_3 - \dot{\hat{a}}\right) + e_b\left(-z_3x_1 - \dot{\hat{b}}\right) + e_c\left(z_3x_2 - \dot{\hat{c}}\right) \tag{51}$$

Substituting the update law (34) into (51), we get

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2, \tag{52}$$

which is a negative semi-definite function on $\mathbf{R}^6$. From (52), it follows that the vector $\mathbf{z}(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t)))$ are globally bounded, i.e.

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix} \in \mathbf{L}_i nfty \tag{53}$$

Also, it follows from (52) that

$$\dot{V} \leqslant -z_1^2 - z_2^2 - z_3^2 = -\|\mathbf{z}\|^2 \tag{54}$$

That is,

$$\|\mathbf{z}\|^2 \leqslant -\dot{V} \tag{55}$$

Integrating the inequality (55) from 0 to t, we get

$$\int_0^t |\mathbf{z}(\tau)|^2 \, d\tau \leqslant V(0) - V(t) \tag{56}$$

From (56), it follows that $\mathbf{z}(t) \in \mathbf{L}_2$. From Eq. (29), it can be deduced that $\dot{\mathbf{z}}(t) \in \mathbf{L}_\infty$. Thus, using Barbalat's lemma [145], we conclude that $\mathbf{z}(t) \to \mathbf{0}$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{z}(0) \in \mathbf{R}^3$. Hence, it is immediate that $\mathbf{x}(t) \to \mathbf{0}$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{x}(0) \in \mathbf{R}^3$. This completes the proof. □

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (29) and (34), when the adaptive control law (32) is applied.

The parameter values of the novel jerk chaotic system (29) are taken as in the chaotic case (10), *i.e.*

$$a = 3.6, \quad b = 1.3, \quad c = 0.1 \tag{57}$$

The positive gain constant k is taken as $k = 10$. As initial conditions of the novel jerk chaotic system (29), we take

$$x_1(0) = 7.5, \quad x_2(0) = 12.1, \quad x_3(0) = 15.4 \tag{58}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 3.1, \quad \hat{b}(0) = 6.8, \quad \hat{c}(0) = 9.2 \tag{59}$$

In Figure 5, the exponential convergence of the controlled states is depicted, when the adaptive control law (32) and parameter update law (34) are implemented.

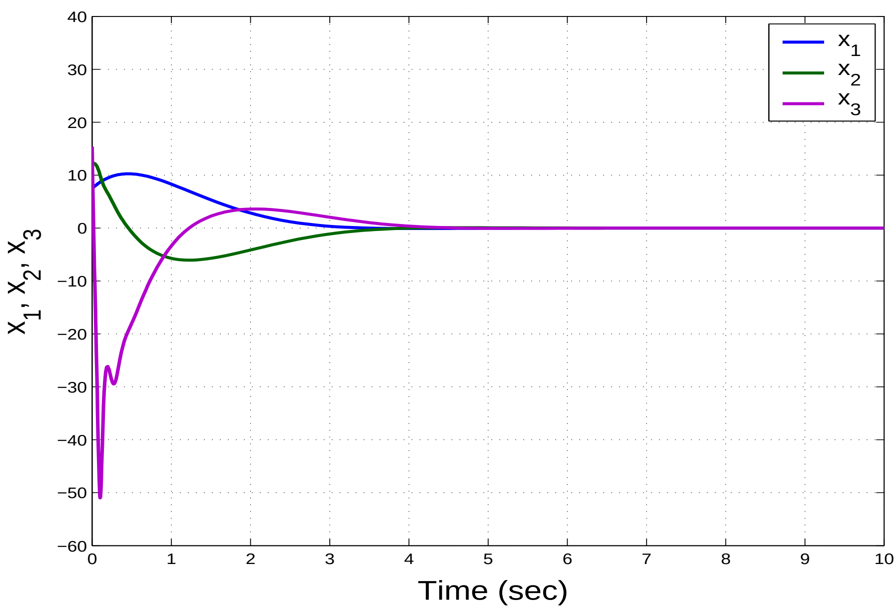

Figure 5: Time-history of the controlled states $x_1(t), x_2(t), x_3(t)$

5. Adaptive synchronization of the identical 3-D novel jerk chaotic systems

In this section, we use backstepping control method to derive an adaptive control law for globally and exponentially synchronizing the identical 3-D novel jerk chaotic systems with unknown parameters.

As the master system, we consider the 3-D novel jerk chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_3 - bx_1 + cx_2 + x_1x_2^2 - x_1^3 \end{cases} \tag{60}$$

where x_1, x_2, x_3 are the states of the system, and a, b, c are unknown constant parameters.

As the slave system, we consider the 3-D novel jerk chaotic system given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -ay_3 - by_1 + cy_2 + y_1y_2^2 - y_1^3 + u \end{cases} \tag{61}$$

where y_1, y_2, y_3 are the states of the system, and u is a backstepping control to be determined using estimates of the unknown system parameters.

We define the synchronization errors between the states of the master system (60) and the slave system (61) as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{62}$$

Then the error dynamics is easily obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -ae_3 - be_1 + ce_2 + y_1y_2^2 - x_1x_2^2 - y_1^3 + x_1^3 + u \end{cases} \tag{63}$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{64}$$

Differentiating (64) with respect to t, we obtain the following equations:

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{65}$$

Next, we shall state and prove the main result of this section.

Theorem 2 *The identical 3-D novel jerk chaotic systems* (60) *and* (61) *with unknown parameters a, b and c are globally and exponentially synchronized by the adaptive control law*

$$\left\{ \begin{array}{rcl} u(t) & = & -[3-\hat{b}(t)]e_1 - [5+\hat{c}(t)]e_2 - [3-\hat{a}(t)]e_3 \\ & & -y_1 y_2^2 + x_1 x_2^2 + y_1^3 - x_1^3 - k z_3 \end{array} \right. \tag{66}$$

where $k > 0$ is a gain constant,

$$z_3 = 2e_1 + 2e_2 + e_3, \tag{67}$$

and the update law for the parameter estimates $\hat{a}(t), \hat{b}(t)$ is given by

$$\left\{ \begin{array}{rcl} \dot{\hat{a}}(t) & = & -z_3 e_3 \\ \dot{\hat{b}}(t) & = & -z_3 e_1 \\ \dot{\hat{c}}(t) & = & z_3 e_2 \end{array} \right. \tag{68}$$

Proof We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{69}$$

where

$$z_1 = e_1 \tag{70}$$

Differentiating V_1 along the error dynamics (63), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{71}$$

Now, we define

$$z_2 = e_1 + e_2 \tag{72}$$

Using (72), we can simplify the equation (71) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{73}$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{74}$$

Differentiating V_2 along the error dynamics (63), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{75}$$

Now, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{76}$$

Using (76), we can simplify the equation (75) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{77}$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, e_a, e_b, e_c, e_p) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2\right) \tag{78}$$

which is a positive definite function on $\mathbf{R}^6$. Differentiating V along the error dynamics (63), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3(z_3 + z_2 + \dot{z}_3) - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{79}$$

Eq. (79) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{80}$$

where

$$S = z_3 + z_2 + \dot{z}_3 = z_3 + z_2 + 2\dot{e}_1 + 2\dot{e}_2 + \dot{e}_3 \tag{81}$$

A simple calculation gives

$$S = (3-b)e_1 + (5+c)e_2 + (3-a)e_3 + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \tag{82}$$

Substituting the adaptive control law (66) into (48), we obtain

$$S = -[b - \hat{b}(t)]e_1 + [c - \hat{c}(t)]e_2 - [a - \hat{a}(t)]e_3 - kz_3 \tag{83}$$

Using the definitions (65), we can simplify (83) as

$$S = -e_b e_1 + e_c e_2 - e_a e_3 - kz_3 \tag{84}$$

Substituting the value of S from (84) into (80), we obtain

$$\left\{ \begin{array}{rcl} \dot{V} & = & -z_1 - z_2 - (1+k)z_3^2 + e_a[-z_3 e_3 - \dot{\hat{a}}] + e_b[-z_3 e_1 - \dot{\hat{b}}] \\ & & +e_c[z_3 e_2 - \dot{\hat{c}}] \end{array} \right. \tag{85}$$

Substituting the update law (68) into (85), we get

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2, \tag{86}$$

which is a negative semi-definite function on $\mathbf{R}^6$. From (86), it follows that the vector $\mathbf{z}(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t))$ are globally bounded, i.e.

$$\left[\begin{array}{cccccc} z_1(t) & z_2(t) & z_3(t) & e_a(t) & e_b(t) & e_c(t) \end{array}\right] \in \mathbf{L}_i nfty \tag{87}$$

Also, it follows from (86) that

$$\dot{V} \leqslant -z_1^2 - z_2^2 - z_3^2 = -\|\mathbf{z}\|^2 \tag{88}$$

That is,

$$\|\mathbf{z}\|^2 \leqslant -\dot{V} \tag{89}$$

Integrating the inequality (89) from 0 to t, we get

$$\int_0^t |\mathbf{z}(\tau)|^2 \, d\tau \leqslant V(0) - V(t) \tag{90}$$

From (90), it follows that $\mathbf{z}(t) \in \mathbf{L}_2$. From Eq. (63), it can be deduced that $\dot{\mathbf{z}}(t) \in \mathbf{L}_\infty$. Thus, using Barbalat's lemma, we conclude that $\mathbf{z}(t) \to \mathbf{0}$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{z}(0) \in \mathbf{R}^3$. Hence, it is immediate that $\mathbf{e}(t) \to \mathbf{0}$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{e}(0) \in \mathbf{R}^3$. This completes the proof. □

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (60) and (61).

The parameter values of the novel jerk chaotic systems are taken as in the chaotic case, (10), i.e.

$$a = 3.6, \;\; b = 1.3, \;\; c = 0.1 \tag{91}$$

The positive gain constant is taken as $k = 10$. As initial conditions of the master chaotic system (60), we take

$$x_1(0) = -5.8, \quad x_2(0) = 3.7, \quad x_3(0) = -4.9 \tag{92}$$

As initial conditions of the slave chaotic system (61), we take

$$y_1(0) = 4.5, \quad y_2(0) = 8.4, \quad y_3(0) = -8.5 \tag{93}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 11.2, \quad \hat{b}(0) = 6.1, \;\; \hat{c}(0) = 12.6 \tag{94}$$

In Figs. 6-8, the complete synchronization of the identical 3-D jerk chaotic systems (60) and (61) is shown, when the adaptive control law (66) and the parameter update law (68) are implemented.

Also, in Fig. 9, the time-history of the synchronization errors $e_1(t), e_2(t), e_3(t)$, is shown.

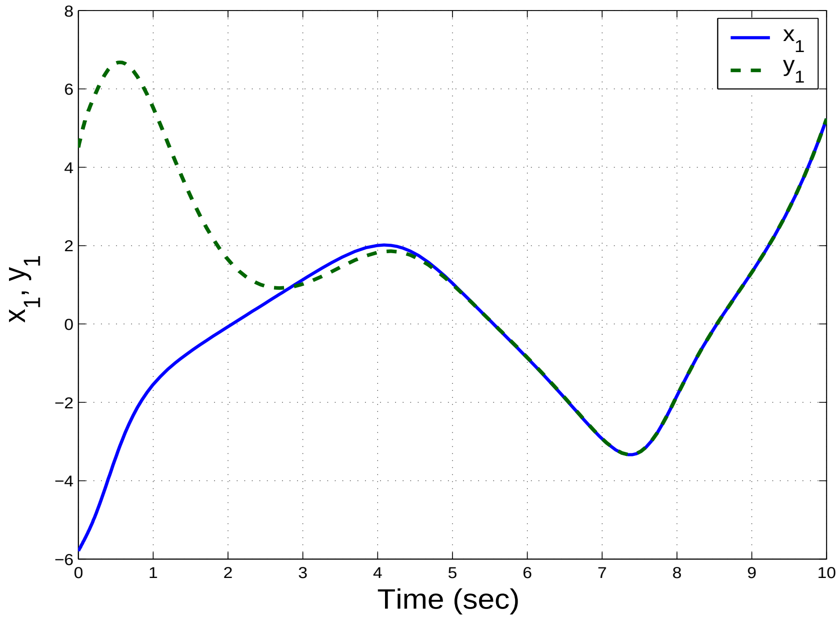

Figure 6: Synchronization of the states $x_1(t)$ and $y_1(t)$

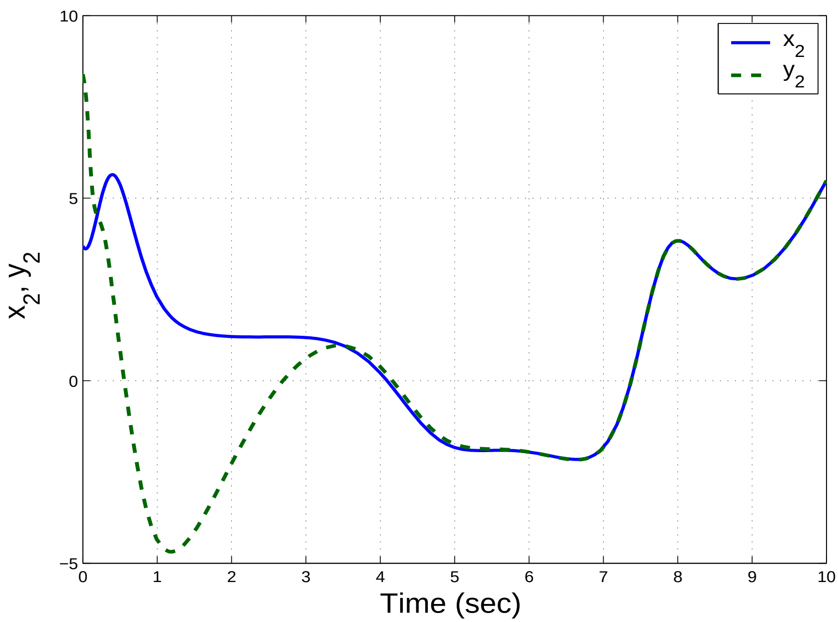

Figure 7: Synchronization of the states $x_2(t)$ and $y_2(t)$

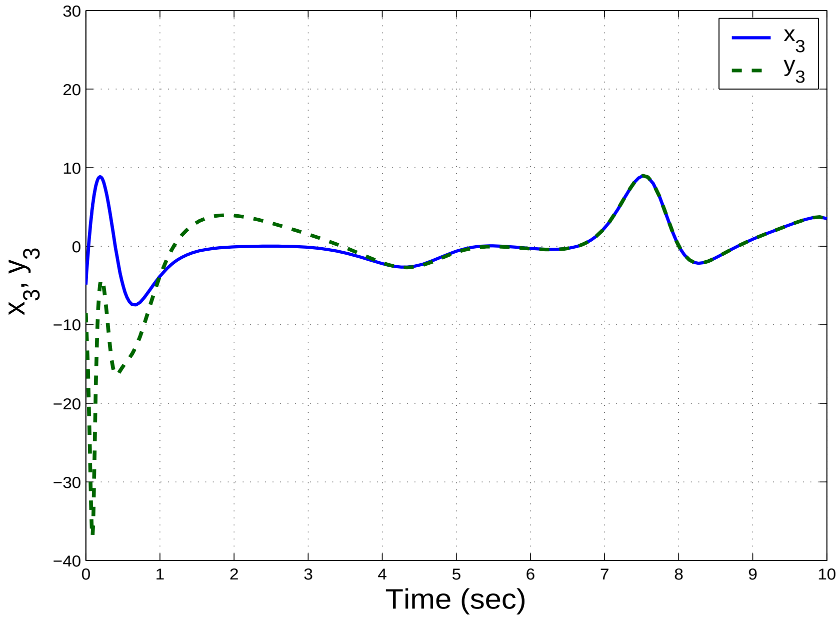

Figure 8: Synchronization of the states $x_3(t)$ and $y_3(t)$

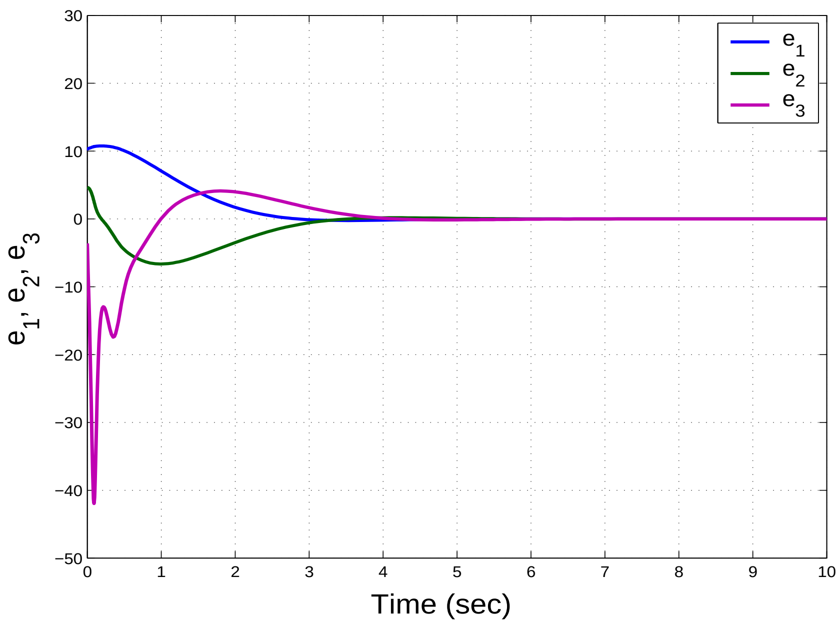

Figure 9: Time-history of the synchronization errors $e_1(t), e_2(t), e_3(t)$

6. Conclusions

In this paper, we announced a seven-term novel 3-D jerk chaotic system with two cubic nonlinearities. The phase portraits of the novel jerk chaotic system were displayed and the qualitative properties were discussed. Next, an adaptive backstepping controller was designed to globally stabilize the novel jerk chaotic system with unknown parameters. Moreover, an adaptive backstepping controller was also designed to achieve global chaos synchronization of the identical jerk chaotic systems with unknown parameters. MATLAB simulations were depicted to illustrate the phase portraits of the novel jerk chaotic system and also the adaptive backstepping control results.

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