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A new approach to robust finite-time H_∞ control of continuous-time Markov jump systems*

by

Shen Yan[1], Mouquan Shen[1,2], Shuimin Fei[3] and Guangming Zhang[1]

[1]College of Electrical Engineering and Control Science,
Nanjing Technology University, Nanjing, 211816, China
yanshenzdh@163.com, zgmchina@163.com
[2]Key Laboratory of Measurement and Control of Complex
Systems of Engineering,
Ministry of Education, Nanjing, 210096, China
mouquanshen@gmail.com
[3]School of Automation, Southeast University,
Nanjing, 210096, China
smfei@seu.edu.cn

Abstract: This paper studies the robust finite-time H_∞ state feedback control problem of continuous-time Markov jump systems (MJSs) subject to norm bounded uncertainties. Transition probabilities are allowed to be known, uncertain with known bounds or unknown. Based on the continuous transition probability property and the developed slack variable technique, Lyapunov variables are separated from unknown transition probabilities and system matrices. With these separations, a relaxed method for robust finite-time H_∞ controller design is proposed in terms of linear matrix inequalities (LMIs). Numerical examples are given to illustrate the effectiveness of and the benefit from the proposed method.

Keywords: Markov jump systems, partly known transition probabilities, robust finite-time $H\infty$ control, linear matrix inequality (LMI)

1. Introduction

Over the last few years, a lot of attention has been attracted to stochastic hybrid systems with Markov jump parameters, since the model can be effectively used to describe the plants whose structure is subject to random abrupt changes due to, for instance, failures or repairs, sudden environment changes, modification of the operating point of a nonlinear system, etc. The results related to this class of systems have found wide applications in various practical problems, such as

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target tracking, manufacturing processes and fault-tolerant control systems (see Mariton, 1990; Costa, Fragoso and Marques, 2005). Meanwhile, Markov jump systems (MJSs) theory has been extensively investigated and important results have been obtained, associated with problems such as stability and stabilization (see Ji, 1990; Feng, Loparo, Ji and Chizeck, 1992; Yue and Han, 2005; Xiong and Lam, 2005; Bolzern, Colaneri and De Nicolao, 2013; Huang and Shi, 2012; Ma, Boukas and Chinniah, 2010; Zhang, Boukas and Lam, 2008; Zhang and Boukas, 2009a; Shen and Yang, 2012a; Zhang, Cui, Liu and Zhao, 2011; Zhang, Gao and Kaynak, 2013; Zhang, Zhuang and Shi, 2015), H_∞ and H_2 control (see De Farias, Geromel, Do Val and Costa, 2000; Dong and Yang, 2007, 2008; Shen and Ye, 2013), or H_∞ and H_2 filtering (see He and Liu, 2010a; Zhang and Boukas, 2009b; Shen and Yang, 2012b; Wang, Zhang and Sreeram, 2010; Wu, Su and Chu, 2014; Zong and Yang, 2014); synchronization of Markovian jump neural networks with time-varying delays is discussed in Wu, Shi, Su and Chu (2013), passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time-delays is presented in Wu, Shi, Su and Chu (2011), model reduction is considered in Sun, Lam, Xu and Shu (2012), or Zhang, Boukas and Shi (2009), and so on.

As it is known, stability plays a crucial role in systems analysis, systems theory and control engineering. Concerning the above results on MJSs, most of them are devoted to the stochastic stability over an infinite time interval. While this type of stability is often sufficient for practical applications, there exist some cases where large values of the state are not acceptable, for instance in the presence of saturations. In order to avoid the unacceptable state values, finite-time stability is considered. A system is said to be finite-time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a specified time interval (see Amato, Ariola and Dorate, 2001; Amato and Ariola, 2005; Amato, Ariola and Cosentino, 2011). Recently, some appealing results related to finite-time control of MJSs have been obtained (see He and Liu, 2010b, 2012; Zuo, Li, Liu and Wang, 2012; Luan, Liu and Shi, 2010, 2011). Specifically, He and Liu (2010b) provide an observer-based finite-time control of time-delayed MJSs. Based on a fuzzy Lyapunov-Krasovskii functional approach, the finite-time H_∞ control of time-delay nonlinear MJSs via dynamic observer-based state feedback is presented (see He and Liu, 2012). Considering the transition probabilities as being partially known, the finite-time stochastic stability and stabilization problem is studied (see Zuo, Li, Liu and Wang, 2012). On the other hand, because of the difficulty of measuring various parameters, and the environmental noise in practical engineering, it is difficult to establish an exact mathematical model. The type of norm bounded uncertainties is deemed as a common and effective tool to describe system uncertainty (see Shi, Boukas and Agarwal, 1999). Regarding this topic, under the assumption that the transition probabilities are unknown, but belong to fixed finite intervals, the robust finite-time filtering and controller design for MJSs were studied (see Luan, Liu and Shi, 2010, 2011). However, when the transition probabilities are unknown, the proposed methods cannot be applied.

This paper extends the consideration of the robust finite-time H_∞ control of continuous-time MJSs with norm-bounded parameter uncertainties. Here, the transition probabilities may be known, uncertain with known lower and upper bounds, and unknown. Based on the property of continuous transition proba-bilities and a matrix transformation technique, Lyapunov variables are separated from unknown transition probabilities and system matrices, respectively. Based on these separations, sufficient conditions are established in the framework of linear matrix inequality (LMI), which guarantee that the closed-loop system is finite-time stochastic stable with the prescribed H_∞ performance index. Two numerical examples are also given to illustrate the effectiveness of and the benefits from the proposed method.

Notation: Throughout this paper, M^T represents the transpose of matrix M. The notation $X \leq Y$ ($X < Y$) means that $X - Y$ is negative semi-definite (negative definite), where X and Y are symmetric matrices. $\lambda_{min}(P)$ and $\lambda_{max}(P)$ are the minimal eigenvalue and the maximal eigenvalue of a symmetric matrix P, respectively. I and 0 represent the identity matrix and the zero matrix, respectively. $\mathcal{L}_2$ denotes the space of square integrable vector functions of a given dimension over $[0, +\infty)$ with norm $||x||_2^2 = \int_0^\infty E\{x(t)^T x(t)dt\} < \infty$, where $E\{\cdot\}$ stands for the mathematical expectation. $*$ denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions. Finally, the symbol $He(X)$ is used to represent $X + X^T$.

2. Preliminaries and problem statement

Consider the following continuous-time MJS with parameter uncertainties:

$$\left\{ \begin{array}{ll} \dot{x}(t) = & [A(r(t)) + \Delta A(r(t))]x(t) + [B(r(t)) + \Delta B(r(t))]u(t) \\ & +B_w(r(t))w(t), \\ z(t) = & C(r(t))x(t) + D(r(t))u(t) + D_w(r(t))w(t), \end{array} \right. \tag{1}$$

where $x(t)$ is the state variable, $w(t) \in R^{n_w}$ is the disturbance input, which belongs to $\mathcal{L}_2[0, +\infty)$, $z(t) \in R^p$ is the regulated output. $A(r(t))$, $B(r(t))$, $B_w(r(t))$, $C(r(t))$, $D(r(t))$ and $D_w(r(t))$ are known mode-dependent constant matrices having appropriate dimensions. $\Delta A(r(t))$ and $\Delta B(r(t))$ are the time-varying but norm bounded uncertainties satisfying

$$[\Delta A(r(t)) \quad \Delta B(r(t))] = G(r(t))F_{r(t)}(t) \left[H_1(r(t)) \quad H_2(r(t))\right],$$

where $G(r(t))$, $H_1(r(t))$ and $H_2(r(t))$ are known mode-dependent matrices having appropriate dimensions, and $F_{r(t)}(t)$ is a time-varying unknown matrix function with Lebesgue norm measurable elements satisfying $F_{r(t)}(t)^T F_{r(t)}(t) \leq I$. $r(t)$ is a time-homogeneous Markov process with right continuous trajectories, taking values on the finite set $\mathcal{I} = \{1, 2, \cdots, S\}$ with stationary transition probabilities

$$Pr\{r(t+dt) = j | r(t) = i\} = \left\{ \begin{array}{ll} \pi_{ij} dt + o(dt), & i \neq j \\ 1 + \pi_{ii} dt + o(dt), & i = j \end{array} \right.$$

where $dt > 0$ and $\lim_{dt \to 0} \frac{o(dt)}{dt} = 0$. π_{ij} is the jump rate from mode i to mode j satisfying

$$\begin{cases} \pi_{ij} \geq 0, & \forall i \neq j \in \mathcal{I}, \\ \sum_{j=1, i\neq j}^{S} \pi_{ij} = -\pi_{ii}, & i = 1, ..., S. \end{cases} \tag{2}$$

Hence, the Markov process transition probability matrix Π is given by

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1S} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{S1} & \pi_{S2} & \cdots & \pi_{SS} \end{bmatrix}.$$

In distinction from the studies to date, the transition probabilities of the jumping process $\{r(t), t \geq 0\}$ in this paper are allowed to be known, uncertain with known lower and upper bounds, or completely unknown (see Shen and Ye, 2013). For example, for system (1) with four operation modes, the transition probability matrix may be:

$$\Pi = \begin{bmatrix} \rho_{11} & ? & \rho_{13} & ? \\ ? & \rho_{22} & ? & \rho_{24} \\ \alpha & ? & \rho_{33} & ? \\ ? & ? & \beta & ? \end{bmatrix},$$

where "?" represents the unaccessible elements, α and β are uncertain with known lower and upper bounds (i.e, $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$ and $\underline{\beta} \leq \beta \leq \bar{\beta}$), and ρ_{ij} means that π_{ij} is completely known with $\pi_{ij} = \rho_{ij}$.

Therefore, the following three sets can be adopted to describe all the possible cases that the transition probabilities may belong to

$$\begin{cases} \mathcal{R}_k^i \triangleq \{j : \pi_{ij} \quad is \quad known\}, \\ \mathcal{R}_{uk1}^i \triangleq \{j : lower \;\; and \;\; upper \; bounds \; of \; \pi_{ij} \; are \; known\}, \\ \mathcal{R}_{uk2}^i \triangleq \{j : there \; is \; no \; information \; available \; for \; \pi_{ij}\}. \end{cases} \tag{3}$$

Although the elements in $\mathcal{R}_{uk1}^i$ are unknown, their upper and lower bounds can be utilized. So, we rewrite the above sets as follows:

$$\begin{cases} \mathcal{I}_k^i \triangleq \mathcal{R}_k^i \cup \mathcal{R}_{uk1}^i, \\ \mathcal{I}_{uk}^i \triangleq \mathcal{R}_{uk2}^i. \end{cases} \tag{4}$$

For any $\pi_{ij} \in \mathcal{R}_{uk1}^i$, we denote the lower and upper bounds as $\underline{\pi}_{ij}$ and $\overline{\pi}_{ij}$, respectively. For $\pi_{ij} \in \mathcal{R}_k^i$, we let $\underline{\pi}_{ij} = \overline{\pi}_{ij} = \pi_{ij}$. Meanwhile, we employ

$\mathcal{L}_k^i$ ($\mathcal{L}_{uk}^i$) to represent the index set of the known (unknown) elements in the ith row of matrix Π:

$$\mathcal{L}_k^i \triangleq \{m|m \in \mathcal{I}_k^i \text{ and } m \neq i\}, \quad \mathcal{L}_{uk}^i \triangleq \{m|m \in \mathcal{I}_{uk}^i \text{ and } m \neq i\}.$$

Moveover, the set $\mathcal{L}_k^i$ can be expressed as $\mathcal{L}_k^i = \{m_{i1}, \cdots, m_{ia}\}$, where $1 \leq m_{i1} < \cdots < m_{ia} \leq S$. Our aim is to design a state feedback controller

$$u(t) = K(r(t))x(t), \tag{5}$$

such that the resulting closed-loop system

$$\begin{cases} \dot{x}(t) &= (\bar{A}(r(t)) + \Delta\bar{A}(r(t)))x(t) + B_w(r(t))w(t) \\ z(t) &= \bar{C}(r(t))x(t) + D_w(r(t))w(t) \end{cases} \tag{6}$$

is finite-time stochastic bounded and satisfies

$$E\{\int_0^N z^T(t)z(t)dt\} \leq \gamma^2 E\{\int_0^N w^T(t)w(t)dt\} \tag{7}$$

under zero initial conditions for any non-zero $w(k)$, where

$$\begin{aligned} &\bar{A}(r(t)) = A(r(t)) + B(r(t))K(r(t)), \\ &\Delta\bar{A}(r(t)) = \Delta A(r(t)) + \Delta B(r(t))K(r(t)), \\ &\bar{C}(r(t)) = C(r(t)) + D(r(t))K(r(t)). \end{aligned}$$

For simplicity, system matrices are abbreviated as A_i, B_i, B_{wi}, C_i, D_i, D_{wi}, K_i, ΔA_i, ΔB_i, G_i, H_{1i}, H_{2i} and $F_i(t)$ when $r(t) = i$ ($i \in \mathcal{I}$), denoting also $\hat{A}_i = \bar{A}_i + \Delta\bar{A}_i$ and $\hat{C}_i = \bar{C}_i$.

Remark 1 *There exist some results related to MJSs with partly known transition probabilities in the literature. All transition probabilities π_{ij} are either assumed to be uncertain with known bounds (see Luan, Liu and Shi, 2011), or they must be known or completely unknown (see Zuo, Li, Liu and Wang, 2012). Actually, these cases may happen simultaneously in practice. Obviously, the method of Luan, Liu and Shi (2011) is not applicable to the completely unknown case, and the method of Zuo, Li, Liu and Wang (2012) may lead to conservative result since the uncertain case is treated as completely unknown.*

Before discussing the finite-time H_∞ controller design problem, some necessary assumption, definitions and lemmas are brought in as follows
Assumption 1. For any given positive number d, the external disturbance $w(t)$ is time-varying and satisfies

$$E\{\int_0^N w(t)^T w(t)dt\} \leq d. \tag{8}$$

Definition 1 *(Finite-time stochastic stability (FTSS)) For a given constant $N > 0$, the continuous-time MJSs (1) with $w(t) = 0$ is said to be FTSS with respect to (c_1, c_2, R_i, N) if*

$$E\{x^T(0)R_i x(0)\} \leq c_1 \to E\{x^T(t)R_i x(t)\} \leq c_2, \quad \forall t \in [0, N], \tag{9}$$

where $0 < c_1 < c_2$ and $R_i > 0$.

Definition 2 *(Finite-time stochastic boundedness (FTSB)) For a given constant $N > 0$, the continuous-time MJSs (1) is said to be FTSB with respect to (c_1, c_2, R_i, N, d) if (9) holds, where $0 < c_1 < c_2$, $R_i > 0$ and $w(t)$ satisfies (8).*

Lemma 1 *Let T, M, W and Y be real matrices of appropriate dimensions with $W^T W \leq I$, then for any positive scalar β, we have*

$$T + MWY + (MWY)^T \leq T + \beta MM^T + \beta^{-1}Y^T Y. \tag{10}$$

Recently, some methods for finite-time stochastic stability analysis and robust H_∞ control of continuous-time MJSs were proposed: see Zuo, Li, Liu and Wang (2012) and Luan, Liu and Shi (2011), respectively, where the transition probabilities of visited modes were also assumed to be partly known or uncertain with known lower and upper bounds. For convenience of comparison, the main results from Zuo, Li, Liu and Wang (2012) and Luan, Liu and Shi (2011) are quoted as the following lemmas.

Lemma 2 *(Zuo, Li, Liu and Wang, 2012) For a given time-constant $\mu > 0$, MJSs (1) is FTSS with respect to (c_1, c_2, R_i, N), if there exist matrices $P_i > 0$, $W_v = W_v^T (v = 1, 2, \cdots, g)$ and two positive scalars λ_1 and λ_2 such that*

$$He(A_i^T P_i) - \mu P_i + \sum_{j \in \mathcal{I}_k^i} \pi_{ij} (P_j - W_v) < 0 \tag{11}$$

$$P_j - W_v \leq 0, \forall j \in \mathcal{I}_{uk}^i, i \neq j \tag{12}$$

$$P_j - W_v \geq 0, \forall j \in \mathcal{I}_{uk}^i, i = j \tag{13}$$

$$\lambda_1 I \leq \hat{P}_i \leq \lambda_2 I \tag{14}$$

$$c_1 \lambda_2 e^{\mu T} - c_2 \lambda_1 < 0 \tag{15}$$

where $P_i = R^{\frac{1}{2}} \hat{P}_i R^{\frac{1}{2}}$.

Lemma 3 *(Luan, Liu and Shi, 2011) MJSs (1) is FTSB with respect to (c_1, c_2, R_i, N, d) and has H_∞ performance index γ via state feedback controller (5), if there exist matrices $P_i > 0$ such that*

$$\begin{bmatrix} He\left(P_i\left(\bar{A}_i + \Delta\bar{A}_i\right)\right) - (S_i - 1)\pi_{min,i}P_i + \pi_{max,i}\sum_{j\neq i} P_j & * & * \\ B_{wi}^T P_i & -\gamma^2 I & * \\ C_i & D_{wi} & -I \end{bmatrix} < 0, \tag{16}$$

$$c_1 \Omega_{max}(\hat{P}_i) + \gamma^2 d \frac{1 - e^{-\mu N}}{\mu} \leq e^{-\mu N} c_2 \Omega_{min}(\hat{P}_i), \tag{17}$$

where S_i is the number of modes visited from mode i including the mode i itself, $\pi_{min,i}$ and $\pi_{max,i}$ are known parameters for each mode or may represent the lower and upper bounds when all the jump rates are known. $\hat{P}_i = R_i^{-\frac{1}{2}} P_i R_i^{-\frac{1}{2}}$, Ω_{min} and Ω_{max} denote the minimal and maximal eigenvalues of the augment, respectively.

3. Main results

In this section, the general method for stability analysis of system (1) with general transition probabilities is first presented. Then, the finite-time robust H_∞ controller design method is also proposed in terms of LMIs.

Theorem 1 *For given (c_1, c_2, R_i, N, d) and $\mu > 0$, the closed-loop system (6) is robustly FTSB with H_∞ performance index γ if there exist $Q_i > 0$, V_i and T_i $(i = 1,\ 2,\ \cdots,\ S)$ such that the following inequalities hold:*

(i) for $\pi_{ii} \in \mathcal{I}_k^i$

$$\begin{bmatrix} He(-V_i) & * & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * & * & * \\ \bar{C}_i V_i & 0 & D_{wi} & -I & * & * & * \\ V_i & 0 & 0 & 0 & -T_i & * & * \\ \mathscr{C}_k^i & 0 & 0 & 0 & 0 & -\mathscr{D}_k^i & * \\ \sqrt{\bar{\lambda}_k^i} V_i & 0 & 0 & 0 & 0 & 0 & -Q_l \end{bmatrix} < 0, \qquad (l \in \mathcal{L}_{uk}^i) \tag{18}$$

(ii) for $\pi_{ii} \in \mathcal{I}_{uk}^i$

$$\begin{cases} \begin{bmatrix} He(-V_i) & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * & * \\ \bar{C}_i V_i & 0 & D_{wi} & -I & * & * \\ V_i & 0 & 0 & 0 & -T_i & * \\ \mathscr{C}_k^i & 0 & 0 & 0 & 0 & -\mathscr{D}_k^i \end{bmatrix} < 0, \\ Q_i \leq Q_l, \quad (l \in \mathcal{L}_{uk}^i) \end{cases} \tag{19}$$

$$c_1 \lambda_{max}(\hat{P}_i) + \gamma^2 d \frac{1 - e^{-\mu N}}{\mu} \leq e^{-\mu N} c_2 \lambda_{min}(\hat{P}_i) \tag{20}$$

where $\hat{P}_i = R_i^{-\frac{1}{2}} Q_i^{-1} R_i^{-\frac{1}{2}}$ *and*

$$\Sigma_{i21} = (\bar{A}_i + \Delta\bar{A}_i)V_i + Q_i,$$
$$\Sigma_{i22} = \begin{cases} \bar{\pi}_{ii}Q_i + T_i - 2Q_i - \mu Q_i, & i \in \mathcal{I}_k^i, \\ -\underline{\delta}_k^i Q_i + T_i - 2Q_i - \mu Q_i, & i \in \mathcal{I}_{uk}^i. \end{cases}$$
$$\mathscr{C}_k^i = \left[\begin{array}{ccc} (\sqrt{\bar{\pi}_{m_{i1}}}V_i)^T & \cdots & (\sqrt{\bar{\pi}_{m_{ia}}}V_i)^T \end{array} \right]^T,$$
$$\mathscr{D}_k^i = diag\left\{ \begin{array}{ccc} Q_{m_{i1}}, & \cdots, & Q_{m_{ia}} \end{array} \right\},$$
$$\bar{\lambda}_k^i = -\underline{\pi}_{ii} - \sum_{j \in \mathcal{L}_k^i} \underline{\pi}_{ij},$$
$$\underline{\delta}_k^i = \sum_{j \in \mathcal{L}_k^i} \underline{\pi}_{ij}.$$

Proof We first show that if (18) and (19) hold, then the following inequalities hold:

$$\begin{bmatrix} He\left(P_i\hat{A}_i\right) + \sum\limits_{j\in\mathcal{I}_k^i} \bar{\pi}_{ij}P_j - \mu P_i + \bar{\lambda}_k^i P_l & * & * \\ (P_iB_{wi})^T & -\gamma^2 I & * \\ \hat{C}_i & D_{wi} & -I \end{bmatrix} < 0 \ (i \in \mathcal{I}_k^i, l \in \mathcal{L}_{uk}^i) \tag{21}$$

$$\begin{cases} \begin{bmatrix} He\left(P_i\hat{A}_i\right) + \sum\limits_{j\in\mathcal{L}_k^i} \bar{\pi}_{ij}P_j - \underline{\delta}_k^i P_i - \mu P_i & * & * \\ (P_iB_{wi})^T & -\gamma^2 I & * \\ \hat{C}_i & D_{wi} & -I \end{bmatrix} < 0, \\ P_l \leq P_i \quad (i \in \mathcal{I}_{uk}^i,\ l \in \mathcal{L}_{uk}^i) \end{cases} \tag{22}$$

where $P_iQ_i = I$.

By pre- and post-multiplying both sides of (21) by $diag\left\{ \begin{array}{ccc} Q_i, & I, & I \end{array} \right\}$ and its transpose, respectively, we get

$$\begin{bmatrix} He(\hat{A}_iQ_i) + (\bar{\pi}_{ii} - \mu)Q_i & * & * & * \\ B_{wi}^T & -\gamma^2 I & * & * \\ \hat{C}_iQ_i & D_{wi} & -I & * \\ \mathscr{E}_k^i Q_i & 0 & 0 & -\bar{\mathscr{D}}_k^i \end{bmatrix} < 0, \tag{23}$$

where

$$\mathscr{E}_k^i = \left[\begin{array}{cccc} \sqrt{\bar{\pi}_{im_{i1}}}I & \cdots & \sqrt{\bar{\pi}_{im_{ia}}}I & \sqrt{\bar{\lambda}_k^i}I \end{array} \right]^T,$$
$$\bar{\mathscr{D}}_k^i = diag\left\{ \begin{array}{cccc} Q_{m_{i1}} & \cdots, & Q_{m_{ia}} & Q_l \end{array} \right\}.$$

From the continuity of LMI, there always exists a set of sufficiently small positive scalars ϵ_i satisfying

$$\begin{bmatrix} He(\hat{A}_iQ_i)+(\bar{\pi}_{ii}-\mu)Q_i & * & * & * \\ B_{wi}^T & -\gamma^2 I & * & * \\ \hat{C}_iQ_i & D_{wi} & -I & * \\ \mathscr{E}_k^iQ_i & 0 & 0 & -\bar{\mathscr{D}}_k^i \end{bmatrix} + \epsilon_i \begin{bmatrix} \hat{A}_i \\ 0 \\ \hat{C}_i \\ \mathscr{E}_k^i \end{bmatrix} Q_i \begin{bmatrix} \hat{A}_i \\ 0 \\ \hat{C}_i \\ \mathscr{E}_k^i \end{bmatrix}^T < 0. \tag{24}$$

After direct algebraic manipulations, (24) can be rewritten as

$$\begin{bmatrix} -\epsilon_i^{-1}Q_i+(\bar{\pi}_{ii}-\mu)Q_i & * & * & * \\ B_{wi}^T & -\gamma^2 I & * & * \\ 0 & D_{wi} & -I & * \\ 0 & 0 & 0 & -\bar{\mathscr{D}}_k^i \end{bmatrix}$$
$$+ \begin{bmatrix} (\epsilon_i\hat{A}_i+I)Q_i \\ 0 \\ \epsilon_i\hat{C}_iQ_i \\ \epsilon\mathscr{E}_k^iQ_i \end{bmatrix} (\epsilon_iQ_i)^{-1} \begin{bmatrix} (\epsilon_i\hat{A}_i+I)Q_i \\ 0 \\ \epsilon_i\hat{C}_iQ_i \\ \epsilon\mathscr{E}_k^iQ_i \end{bmatrix}^T < 0. \tag{25}$$

From the Schur complement, (25) is equivalent to

$$\begin{bmatrix} -\epsilon_iQ_i & * & * & * & * \\ (\epsilon_i\hat{A}_i+I)Q_i & -\epsilon_i^{-1}Q_i+(\bar{\pi}_{ii}-\mu)Q_i & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * \\ \epsilon_i\hat{C}_iQ_i & 0 & D_{wi} & -I & * \\ \epsilon\mathscr{E}_k^iQ_i & 0 & 0 & 0 & -\bar{\mathscr{D}}_k^i \end{bmatrix} < 0. \tag{26}$$

Let $V_i=\epsilon_iQ_i$, (26) is then further rewritten as

$$\begin{bmatrix} He(-V_i) & * & * & * & * & * \\ \hat{A}_iV_i+Q_i & -\epsilon_i^{-1}Q_i+(\bar{\pi}_{ii}-\mu)Q_i & * & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * & * \\ \hat{C}_iV_i & 0 & D_{2i} & -I & * & * \\ V_i & 0 & 0 & 0 & -\epsilon_iQ_i & * \\ \mathscr{E}_k^iV_i & 0 & 0 & 0 & 0 & -\bar{\mathscr{D}}_k^i \end{bmatrix} < 0. \tag{27}$$

Note that $\epsilon_i-2\geq-\epsilon_i^{-1}$, it is known that (27) can be guaranteed if

$$\begin{bmatrix} He(-V_i) & * & * & * & * & * \\ \hat{A}_iV_i+Q_i & (\epsilon_i-2)Q_i+(\bar{\pi}_{ii}-\mu)Q_i & * & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * & * \\ \hat{C}_iV_i & 0 & D_{wi} & -I & * & * \\ V_i & 0 & 0 & 0 & -\epsilon_iQ_i & * \\ \mathscr{E}_k^iV_i & 0 & 0 & 0 & 0 & -\bar{\mathscr{D}}_k^i \end{bmatrix} < 0, \tag{28}$$

which is just (18) by letting $T_i = \epsilon_i Q_i$. This means that if (18) holds, then (21), as well, holds. Along the lines similar to the above procedure, we can prove that if (19) holds, then (22) also holds.

Choose a candidate stochastic Lyapunov function as

$$V(x(t), r(t) = i) = V(x, i) = x^T P_i x$$

with $P_i > 0$. Along the trajectories of the system (6), the corresponding time derivative of $V(x(t), i)$ is given by

$$\begin{aligned} \varGamma V(x(t), i) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left[E\{V(x(t + \Delta t), i + \Delta t)|x(t), r\} - V(x(t), r) \right] \\ &= x^T \left(He\left(P_i \hat{A}_i\right) + \sum_{j=1}^{S} \pi_{ij} P_j \right) x + 2x^T P_i B_{wi} w. \end{aligned} \tag{29}$$

1) For the case of $\pi_{ii} \in \mathcal{I}_k^i$, one has

$$\varGamma V(x(t), i) = x^T \left(He\left(P_i \hat{A}_i\right) + \sum_{j \in \mathcal{I}_k^i} \pi_{ij} P_j + \sum_{l \in \mathcal{I}_{uk}^i} \pi_{il} P_l \right) x + 2x^T P_i B_{wi} w. \tag{30}$$

In such a situation, according to (3) and (4), there is

$$\mathcal{L}_{uk}^i = \mathcal{I}_{uk}^i \text{ and } \mathcal{L}_k^i \cup \{i\} = \mathcal{I}_k^i.$$

Note that

$$\sum_{j \in \mathcal{L}_{uk}^i} \pi_{ij} + \sum_{j \in \mathcal{L}_k^i} \pi_{ij} = 0,$$

we further have the fact that

$$\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il} = \lambda_k^i,$$

where

$$\lambda_k^i = -\pi_{ii} - \sum_{j \in \mathcal{L}_k^i} \pi_{ij}.$$

Since the transition probabilities are partly known, so $\lambda_k^i > 0$ and this yields

$$\begin{aligned} \varGamma V(x(t),i) &= x^T\left(He\left(P_i\hat{A}_i\right)+\sum_{j\in\mathcal{I}_k^i}\pi_{ij}P_j+\sum_{l\in\mathcal{I}_{uk}^i}\pi_{il}P_l\right)x+2x^TP_iB_{wi}w \\ &= x^T\left(\frac{\sum\limits_{l\in\mathcal{L}_{uk}^i}\pi_{il}}{\lambda_k^i}\left(He\left(P_i\hat{A}_i\right)+\sum_{j\in\mathcal{I}_k^i}\pi_{ij}P_j\right)+\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il}P_l\right)x+2x^TP_iB_{wi}w \\ &= \frac{1}{\lambda_k^i}\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il}\left(x^T\left(He\left(P_i\hat{A}_i\right)+\sum_{j\in\mathcal{I}_k^i}\pi_{ij}P_j+\lambda_k^iP_l\right)x+2x^TP_iB_{wi}w\right). \end{aligned} \tag{31}$$

Therefore, from (21), it one can easily see that

$$\varGamma V(x(t),i)+E\left\{z(t)^Tz(t)-\gamma^2w(t)^Tw(t)\right\}<\mu V(x(t),i). \tag{32}$$

2) For the case of $\pi_{ii}\in\mathcal{I}_{uk}^i$, there is $\mathcal{L}_k^i=\mathcal{I}_k^i$ and $\mathcal{L}_{uk}^i\cup\{i\}=\mathcal{I}_{uk}^i$. From

$$\pi_{ii}=-\sum_{j\in\mathcal{L}_k^i}\pi_{ij}-\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il},$$

(29) can be rewritten as follows

$$\begin{aligned} \varGamma V(x(t),i) &= x^T\left(He\left(P_i\hat{A}_i\right)+\sum_{j\in\mathcal{L}_k^i}\pi_{ij}P_j+\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il}P_l+\pi_{ii}P_i\right)x+2x^TP_iB_{wi}w \\ &= x^T\left(He\left(P_i\hat{A}_i\right)+\sum_{j\in\mathcal{L}_k^i}\pi_{ij}(P_j-P_i)+\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il}\left(P_l-P_i\right)\right)x+2x^TP_iB_{wi}w \\ &= x^T\left(He\left(P_i\hat{A}_i\right)+\sum_{j\in\mathcal{L}_k^i}\pi_{ij}(P_j-P_i)\right)x+2x^TP_iB_{wi}w+\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il}x^T\left(P_l-P_i\right)x. \end{aligned} \tag{33}$$

Similarly, from (22) one can derive that $\varGamma V(x(t),i)<0$ holds.
Subsequently, it is easy to see that

$$\varGamma V(x(t),i)<\mu V(x(t),i)+\gamma^2w(t)^Tw(t) \tag{34}$$

if (18) and (19) hold.
By multiplying (34) by $e^{-\mu t}$, we obtain

$$\varGamma\left[e^{-\mu t}V(x(t),i)\right]<\gamma^2w(t)^Tw(t)e^{-\mu t}. \tag{35}$$

Integration of (35) over 0 to t, gives

$$e^{-\mu t}V(x(t),i) - V(x(0),i_0) < \int_0^t \gamma^2 w(\varpi)^T w(\varpi) e^{-\mu\varpi} d\varpi. \tag{36}$$

Then, (36) is equivalent to

$$\begin{aligned} V(x(t),i) &< e^{\mu t}V(x(0),i_0) + \gamma^2 e^{\mu t}\int_0^t w(\varpi)^T w(\varpi)e^{-\mu\varpi}d\varpi \\ &< e^{\mu t}V(x(0),i_0) + \gamma^2 d e^{\mu t}\int_0^t e^{-\mu\varpi}d\varpi \\ &= e^{\mu t}\left[V(x(0),i_0) + \gamma^2 d\frac{1-e^{-\mu t}}{\mu}\right]. \end{aligned} \tag{37}$$

Moreover, by choosing $\hat{P}_i = R_i^{-\frac{1}{2}} P_i R_i^{-\frac{1}{2}}$, one has

$$V(x(t),i) < e^{\mu t}\left[c_1\lambda_{max}(\hat{P}_i) + \gamma^2 d\frac{1-e^{-\mu t}}{\mu}\right]. \tag{38}$$

On the other hand, the following condition holds

$$V(x(t),i) \geq \lambda_{min}(\hat{P}_i)x^T R_i x. \tag{39}$$

By combining (38) and (39), we obtain

$$E\{x^T R_i x\} < \frac{e^{\mu t}\left[c_1\lambda_{max}(\hat{P}_i) + \gamma^2 d\frac{1-e^{-\mu t}}{\mu}\right]}{\lambda_{min}(\hat{P}_i)}. \tag{40}$$

Thus, according to the condition (20), fulfillment of condition $E\left\{x^T R_i x\right\} \leq c_2$ can be guaranteed for $\forall t \in [0,N]$, which also signifies that the MJSs (1) with incomplete transition probabilities are FTSB with respect to (c_1, c_2, R_i, N, d).

Next, we consider the H_∞ performance problem of MJSs (1) in the framework of FTSS.

From (18) and (19), one obtains

$$\mathit{\Gamma} V(x(t),i) - \mu V(x(t),i) < E\left\{\gamma^2 w(t)^T w(t) - z(t)^T z(t)\right\}. \tag{41}$$

Upon multiplying (41) by $e^{-\mu t}$, the following is obtained:

$$\mathit{\Gamma}[e^{-\mu t}V(x(t),i)] < e^{-\mu t}E\left\{\gamma^2 w(t)^T w(t) - z(t)^T z(t)\right\}. \tag{42}$$

Integration of (42) over 0 to t with the zero initial condition, leads to the following inequality:

$$e^{-\mu t}V(x(t),i) < E\left\{\int_0^t e^{-\mu\varrho}\left(\gamma^2 w(\varrho)^T w(\varrho) - z(\varrho)^T z(\varrho)\right)d\varrho\right\}. \tag{43}$$

Therefore, for $t \in [0, N]$, the following inequality holds

$$E\left\{\int_0^N z(\varrho)^T z(\varrho)d\varrho\right\} < \bar{\gamma}^2 E\left\{\int_0^N w(\varrho)^T w(\varrho)d\varrho\right\}, \tag{44}$$

which is just (7) with $\bar{\gamma} = \sqrt{e^{-\mu N}}\gamma$. This completes the proof. □

Remark 2 *Due to the unknown transition probabilities π_{il} ($l \in \mathcal{I}_{uk}^i$), there exists a nonlinear relationship between π_{il} and P_l. In order to linearize this nonlinearity, the property of transition probabilities is made full use of in this theorem.*

Remark 3 *By employing a matrix transformation, two sets of slack variables are introduced to separate the Lyapunov variables from system matrices. With this separation, conditions (18) and (19) of Theorem 1 are expressed in the form of LMIs even if system matrices have norm bounded uncertainties. However, a nonlinear inequality (20), accompanied by (18) and (19), is difficult to be solved by means of the convex optimization methods. To overcome this difficulty, the subsequent theorem with an extra constraint on Q_i is presented below to give a controller design method.*

Based on the conditions given in Theorem 1, an LMI-based method for dealing with the robust finite-time controller design is given in the following theorem.

Theorem 2 *Considering MJSs (1) with incomplete transition probabilities, given (c_1, c_2, R_i, N), if there exist $Q_i > 0$, V_i, T_i , γ and β_i such that the following inequalities hold:*

(i) for $\pi_{ii} \in \mathcal{I}_k^i$

$$\begin{bmatrix} He(-V_i) & * & * & * & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * & * & * & * & * \\ C_iV_i + D_iL_i & 0 & D_{wi} & -I & * & * & * & * & * \\ V_i & 0 & 0 & 0 & -T_i & * & * & * & * \\ \mathscr{C}_k^i & 0 & 0 & 0 & 0 & -\mathscr{D}_k^i & * & * & * \\ \sqrt{\bar{\lambda}_k^i}V_i & 0 & 0 & 0 & 0 & 0 & -Q_l & * & * \\ 0 & \beta_i G_i^T & 0 & 0 & 0 & 0 & 0 & -\beta_i I & * \\ H_{1i}V_i + H_{2i}L_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_i I \end{bmatrix} < 0, \tag{45}$$

(ii) for $\pi_{ii} \in \mathcal{I}_{uk}^{i}$

$$\begin{cases} \begin{bmatrix} He(-V_i) & * & * & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * & * & * & * \\ C_iV_i + D_iL_i & 0 & D_{wi} & -I & * & * & * & * \\ V_i & 0 & 0 & 0 & -T_i & * & * & * \\ \mathscr{C}_k^i & 0 & 0 & 0 & 0 & -\mathscr{D}_k^i & * & * \\ 0 & \beta_i G_i^T & 0 & 0 & 0 & 0 & -\beta_i I & * \\ H_{1i}V_i + H_{2i}L_i & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_i I \end{bmatrix} < 0, \\ Q_i \leq Q_l \quad (l \in \mathcal{L}_{uk}^i), \end{cases} \tag{46}$$

$$\varepsilon_1 R_i^{-1} \leq Q_i \leq R_i^{-1}, \tag{47}$$

$$\begin{bmatrix} \gamma^2 d \frac{1-e^{\mu N}}{\mu} - c_2 e^{-\mu N} & \sqrt{c_1} \\ * & -\varepsilon_1 \end{bmatrix} < 0, \tag{48}$$

then the closed-loop system (6) is robustly FTSB with H_∞ *performance index* γ.

Moreover, the controller is given by

$$K_i = L_i V_i^{-1}. \tag{49}$$

Proof Let $L_i = K_i V_i$ and $\Delta\bar{\Sigma}_{i21} = \Delta A_i V_i + \Delta B_i L_i + Q_i$. Then, (18) is equivalent to

$$\mathcal{Z}_i = \mathcal{Z}_{1i} + \mathcal{Z}_{2i} < 0,$$

where

$$\mathcal{Z}_{1i} = \begin{bmatrix} He(-V_i) & * & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * & * \\ 0 & B_{wi}^T & -\gamma^2 I & * & * & * & * \\ C_iV_i + D_{1i}L_i & 0 & D_{wi} & -I & * & * & * \\ V_i & 0 & 0 & 0 & -T_i & * & * \\ \mathscr{C}_k^i V_i & 0 & 0 & 0 & 0 & -\mathscr{D}_k^i & * \\ \sqrt{\bar{\lambda}_k^i} V_i & 0 & 0 & 0 & 0 & 0 & -Q_l \end{bmatrix},$$

$$\mathcal{Z}_{2i} = \begin{bmatrix} 0 & * & * & * & * & * & * \\ \Delta\Sigma_{i21} & 0 & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$He\left(\begin{bmatrix} 0 \\ G_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} (H_{1i}V_i + H_{2i}L_i)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \right).$$

According to Lemma 1, one can deduce

$$\mathcal{Z}_{2i} \le \beta_i \begin{bmatrix} 0 \\ G_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ G_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \beta_i^{-1} \begin{bmatrix} (H_{1i}V_i + H_{2i}L_i)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (H_{1i}V_i + H_{2i}L_i)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T.$$

Thus, from the Schur complement, (45) is obtained, and (46) can be derived from (18) by taking the similar approach.

On the other hand, it is easy to check that condition (20) is guaranteed by imposing the conditions (47) and (48). □

Remark 4 *In order to solve the finite-time H_∞ control problem by means of Matlab LMI toolbox, an extra constraint is imposed on Q_i, which may cause some conservatism. An open problem, which is left for our future research, is the development of a procedure to reduce such potential conservatism.*

Remark 5 *A strict LMI-based method is proposed in Theorem 2 for fixed* (c_1, c_2, R_i, N). *To obtain an optimised finite-time stabilised controller, set* $\sigma = \gamma^2$ *and minimize* σ *subject to (45) -(48), namely,*

$$\min_{V_i,T_i,X_i,\beta_i \quad s.t.(45)-(48)} \sigma. \tag{50}$$

Then, the optimal H_∞ *performance index* $\gamma = \sqrt{\sigma^*}$*, as well as the corresponding controller gains, can be obtained by (49).*

4. Numerical examples

In this section, two numerical examples are provided in order to illustrate the effectiveness of the proposed method.

Example 1 *Consider the unforced system (1) with four operation modes and the following data:*

$$A_1=\begin{bmatrix} 0.8 & -2.3 \\ 1.5 & -0.9 \end{bmatrix}, A_2=\begin{bmatrix} -1.3 & 2.7 \\ -2.3 & -1.9 \end{bmatrix}, A_3=\begin{bmatrix} 0.2 & -0.8 \\ 0.7 & -0.9 \end{bmatrix}, A_4=\begin{bmatrix} -0.5 & -0.2 \\ -1 & 0.2 \end{bmatrix}$$

$$R=\begin{bmatrix} -0.5 & -0.2 \\ -1 & 0.2 \end{bmatrix}, \Pi=\begin{bmatrix} -1.3 & 0.2 & ? & ? \\ ? & ? & 0.3 & 0.3 \\ ? & ? & -1.5 & ? \\ 0.4 & ? & ? & ? \end{bmatrix}$$

$$c_1 = 2,\ c_2 = 18,\ T = 1.5,\ \mu = 1$$

where "?" denotes the completely unknown transition probabilities.

First, we use Lemma 2, Lemma 3 and Theorem 1 for finite time stochastic stability analysis. On the one hand, the LMIs in Lemma 2 are infeasible. On the other hand, from the transition probability matrix Π, it is difficult to obtain the boundary information of some unknown elements (such as in the fourth row) which leads to the conclusion that the conditions given in Lemma 3 are infeasible. However, the method proposed in this paper can give a solution to the partly known transition probabilities as follows:

$$P_1 = \begin{bmatrix} 32.3539 & -9.4587 \\ -9.4587 & 35.8116 \end{bmatrix}, P_2 = \begin{bmatrix} 32.9689 & -9.5457 \\ -9.5457 & 37.0462 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 29.1767 & -7.9654 \\ -7.9654 & 25.0810 \end{bmatrix}, P_4 = \begin{bmatrix} 34.9333 & -10.8484 \\ -10.8484 & 39.0734 \end{bmatrix}.$$

According to the above example, it can be seen that the results proposed in this paper are less conservative than the existing ones.

Another example for robust H_∞ control of MJS (1) is given below to show the effectiveness and benefit of the proposed method.

Example 2 *Consider the MJS (1) with parameters given by as in Luan, Liu and Shi (2011):*

$$A_1=\left[\text{|!}\ \begin{matrix}0 & 1\\ 1 & 2\end{matrix}\right], A_2=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}, A_3=\begin{bmatrix}0 & 1\\ 3 & 2\end{bmatrix}, G_1=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, G_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.3\end{bmatrix},$$
$$G_3=\begin{bmatrix}0.3 & 0\\ 0 & 0.2\end{bmatrix}, B_1=\begin{bmatrix}0\\ 1\end{bmatrix}, B_2=\begin{bmatrix}0\\ 2\end{bmatrix}, B_3=\begin{bmatrix}0\\ 3\end{bmatrix}, H_{11}=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix},$$
$$B_{w1}=\begin{bmatrix}0\\ 0.1\end{bmatrix}, B_{w2}=\begin{bmatrix}0\\ 0.2\end{bmatrix}, B_{w3}=\begin{bmatrix}0\\ 0.3\end{bmatrix}, H_{12}=\begin{bmatrix}0.1 & 2\\ 0 & 0.3\end{bmatrix}, H_{13}=\begin{bmatrix}0.1 & 0.2\\ 0 & 0.3\end{bmatrix},$$
$$C_1=\begin{bmatrix}1\\ 2\end{bmatrix}, C_2=\begin{bmatrix}1\\ 1\end{bmatrix}, C_3=\begin{bmatrix}1\\ 3\end{bmatrix}, H_{21}=\begin{bmatrix}0.1\\ 0\end{bmatrix}, H_{22}=\begin{bmatrix}0.2\\ 0\end{bmatrix}, H_{23}=\begin{bmatrix}0.3\\ 0\end{bmatrix},$$
$$D_1=D_{w1}=0.1, D_2=D_{w2}=0.2, D_3=D_{w3}=0.3.$$

The switching between the modes is described by

$$\Pi=\begin{bmatrix}0 & \pi_{12} & \pi_{13}\\ \pi_{21} & 0 & \pi_{23}\\ \pi_{31} & \pi_{32} & 0\end{bmatrix}.$$

The parameters π_{ij} *for all* $i, j \in \mathcal{I}$ *are assumed to satisfy* $1.3 \le \pi_{13} \le 2.8$, $1.5 \le \pi_{21} \le 2.5$ *and* $1.4 \le \pi_{32} \le 2.9$.

With introduction of the initial values for $c_1 = 0.5$, $c_2 = 4$, $N = 5$, $d = 4$ *and* $\mu = 0.5$, *and application of Lemma 3 and Theorem 2, one obtains the optimal values of* γ^*, *which are listed in Table 1.*

Table 1. γ^* values for different methods

Lemma 3	Theorem 2
2.2074	1.7458

According to Table 1, it can be seen that the proposed method is more effective than the existing result.

Furthermore, corresponding to the obtained γ^*, *the controller gains obtained from the respective solution, are given below:*

$$K_1=\begin{bmatrix}-38.4169 & -22.1939\end{bmatrix}$$
$$K_2=\begin{bmatrix}-13.8180 & -5.5900\end{bmatrix}$$
$$K_3=\begin{bmatrix}-17.8749 & -6.4380\end{bmatrix}.$$

With the obtained controller gains, along with the initial condition $x_0=\begin{bmatrix}0.5 - 0.3\end{bmatrix}^T$, *the systems state response curves and the trace of* $x(t)^T R_i x(t)$ *are shown in Fig.1 and Fig.2, respectively.*

According to these figures, it can be seen that the designed controllers render the closed-loop system finite-time stochastic stable.

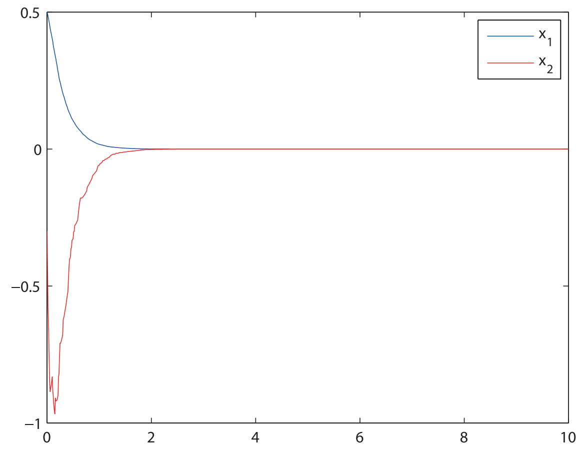

Figure 1. The state response curves of $x_1(t)$ and $x_2(t)$

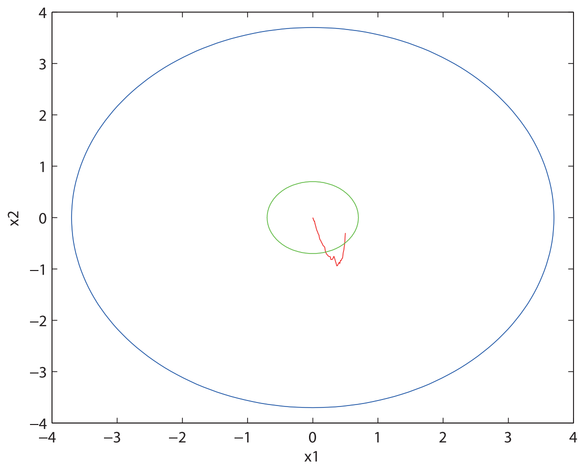

Figure 2. The dynamic trace of $x^T(t)R_ix(t)$

5. Conclusions

This paper considers the robust finite-time H_∞ control problem of continuous-time MJSs with incomplete transition probabilities. A relaxed H_∞ controller design procedure is proposed in terms of LMIs. Numerical examples are given to illustrate the effectiveness and the benefits of the proposed method.

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