Chaotic Path Planning for Grid Coverage Using a Modified Logistic-May Map

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Abstract: *A simple and efficient method for creating a motion trajectory is presented with an aim to achieve sufficient coverage of a given terrain. A chaotic map has been used in order that the motion trajectory should be unpredictable. The chaotic path generator which has been created, is used for implementing a robot's movement in four and eight directions. The path generator is tested in various scenarios and the results are discussed. After thorough examination, the proposed method shows that the motion in eight directions gives better and very satisfactory results.*

Keywords: *Logistic-May map, path planning, chaos, grid coverage*

1. Introduction

In recent years, robots are used in our lives more than ever. Especially, in industry the need for developing efficient robotic systems is increasing vastly, because they can perform tasks that for humans are unreachable [1, 2]. Some of them can be presented in space missions [3, 4], in firefighting [5–8] and more [7, 9, 10]. In military [11–13], they can be used for patrol missions, or to find explosives or other dangerous materials. In all of these missions, the robots should recognize both their initial position and also the target's position in the workspace, in order to update the workspace's map instantly. These goals could be achieved with the use of a sufficient path planning method which will create a trajectory, that gives the robot the opportunity to cover a given workspace.

However especially in patrolling missions [14–17], it is crucial for the robot to move randomly [18–20]. For that reason, nowadays, chaotic systems are used in order to control the motion of the robots. Chaotic systems have rich dynamic behavior and find a variety of applications in many fields such as engineering, cryptography, communication and many others [21–24]. Their advantages rely in the fact that they are very sensitive to initial conditions, which means that by a slight change the system will produce a completely different trajectory. This characteristic is crucial because it will be impossible for the system to produce the same motion sequence twice.

For that reason, many researchers have used chaotic systems in path planning [18, 25–33]. For the purpose of achieving randomness in the motion trajectory in discrete grids, many researchers have used chaotic random bit generators. These generators are used for moving the robot in discrete directions, four or eight, and their results are tested with appropriate statistical tests. The disadvantage of this approach is that it requires the generation of more than three times the number of iterations for the algorithm in order to obtain a statistical random motion.

In our work, a completely different approach has been used. Instead of creating a chaotic random bit generator, we divide the interval [0, 1] into equally spaced subintervals. Then, a chaotic motion command is generated based on which interval the value of the chaotic map belongs to. This is considered for motion in 4 or 8 directions. The chaotic system that is used in our method, is a modification of a Logistic and May map [34]. With the use of modulo tactics two main goals are achieved. Firstly, robot's motion is programmed in Matlab in a short and readable code. Secondly, it produces sufficient results for grid coverage and is more efficient than the methods based on chaotic random bit generation. This happens because it does not use the de-skewing method for producing random bits sequences as the chaotic random bit generators requires. So we do not have extra iterations in our code.

The rest of the paper is organized as follows. In Section 2, the chaotic path planning generator for controlling our robot as well as some simulation results by using the proposed method and its analysis are presented. Section 3 includes the conclusion of our work and a discussion of future aspects.

2. The Proposed Chaotic Path Generator

In [34], the authors proposed the following Logistic-May chaotic map

$$x_{i+1} = \left(x_i e^{(r+9)(1-x_i)} - (r+5)x_i(1-x_i)\right) \bmod 1 \quad (1)$$

where $r_i \in [0, 1]$ and $r \in [-6.8, 19.6]$. This map was constructed as a combination of the Logistics map, given by

$$x_{i+1} = r x_i (1 - x_i) \quad (2)$$

where $x_i \in [0, 1]$ and $r \in [0, 4]$, and the May map, given by

$$x_{i+1} = x_i e^{a(1-x_i)} \tag{3}$$

where $x_i \in [0, 10.9]$ and $a \in [0, 5]$.

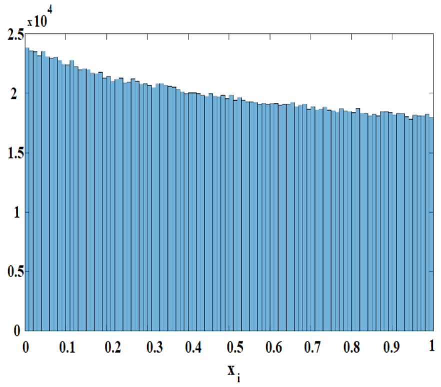

Fig. 1. Histogram for 2,000,0000 iterations of the Logistic-May map (1)

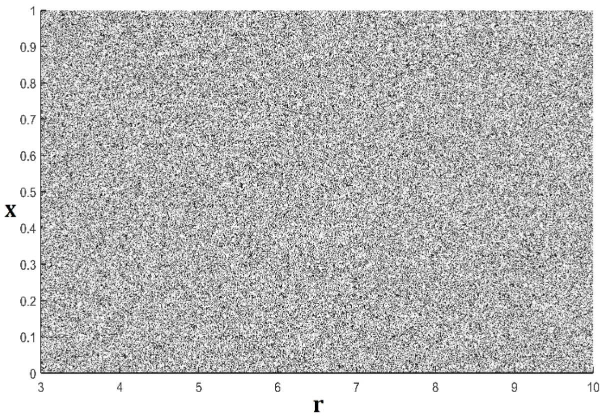

Fig. 2. Bifurcation diagram of proposed Logistic-May map

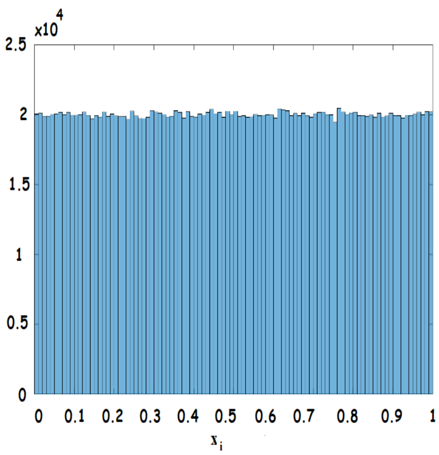

Fig. 3. Histogram for 2,000,000 iterations of the modified Logistic-May map (4)

The Histogram for 2,000,000 iterations of the Logistic-May map (1) for $r = 4$ is shown in Fig. 1. Here, an uneven distribution of the values in the integral [0, 1] is shown. This is something we want to improve, since our proposed path generator is based on dividing the interval [0, 1] to subintervals of equal length. Thus, the following modified Logistics-May map is proposed

$$x_{i+1} = \left(dx_i e^{(r+9)(1-x_i)-(r+5)x_i(1-x_i)}\right) \bmod 1 \tag{4}$$

where d is a positive parameter, chosen here as $d = 100$ and $r = 4$. The bifurcation diagram of the proposed Logistic-May map is shown in Fig. 2. The Histogram of 2,000,0000 iterations for the modified map for $r = 4$ is presented in Fig. 3. Now, an even distribution of the map values on the interval [0, 1] is observed.

2.2 Chaotic Path Planning in 4 Motion Directions

In order to generate the chaotic path for a robot moving in 4 directions (up, down, left, right), the interval [0, 1] has been divided in 4 equal subintervals. Based on this partition, the following tactic is used

$$m_i = \begin{cases} up, & x_i \in [0, 0.25) \\ right, & x_i \in [0.25, 0.5) \\ down, & x_i \in [0.5, 0.75) \\ left, & x_i \in [0.75, 1] \end{cases} \tag{5}$$

where m_i denotes the robot movement in the i-th iteration of the map.

A simulation of the proposed chaotic motion is shown in Fig. 4. Here, a 100 x 100 grid, thus having 100^2 discrete spaces (or cells) for the robot to cover is considered. In each iteration of the algorithm, a movement is generated, which the robot follows. If the generated movement is not acceptable, like moving outside the defined limits or facing obstacles, then the robot remains in its place and awaits for the next motion command. In Fig. 4, the robot starts from position $(1, 1)^T$ and performs 40,000 iterations. Also, Fig. 5 shows a color coded graph showing the number of visits in each step.

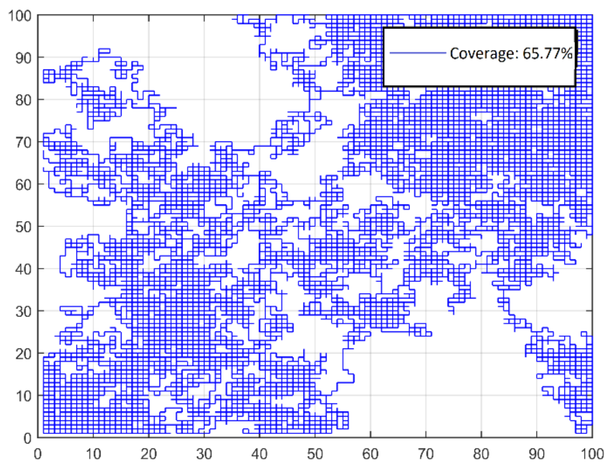

Fig. 4. Grid coverage for 40,000 steps in the case of 4 motion directions

2.2 Chaotic Path Planning in 8 Motion Directions

The use of four directions for the motion of the robot is somewhat limited. In general, we can assume that a robot can also move in eight directions, so the diagonal motions can be used in order to make the robot

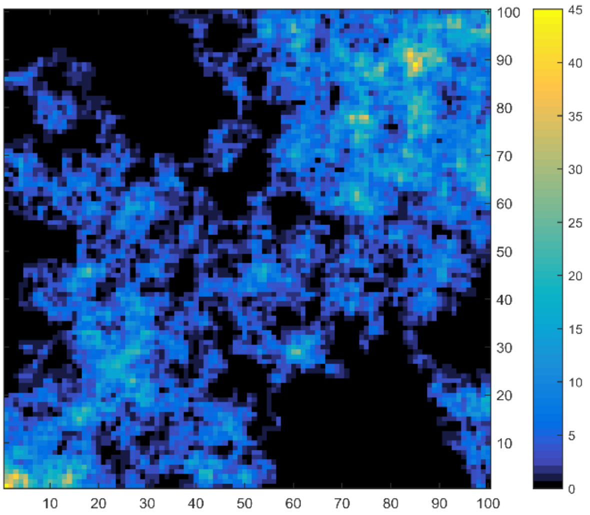

Fig. 5. Color-coded grid coverage showing the number of visits for 40,000 steps in the case of 4 motion directions

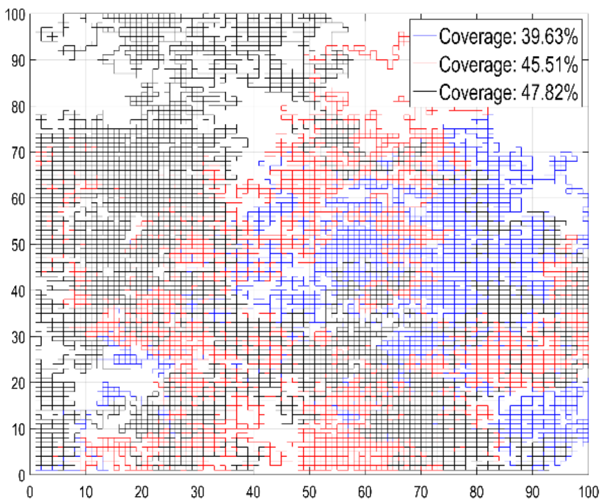

Fig. 6. Grid coverage for 20,000 steps starting from positions $(1,1)^T$ (blue), $(50, 50)^T$ (red), $(50, 100)^T$ (black), in the case of 4 motion directions

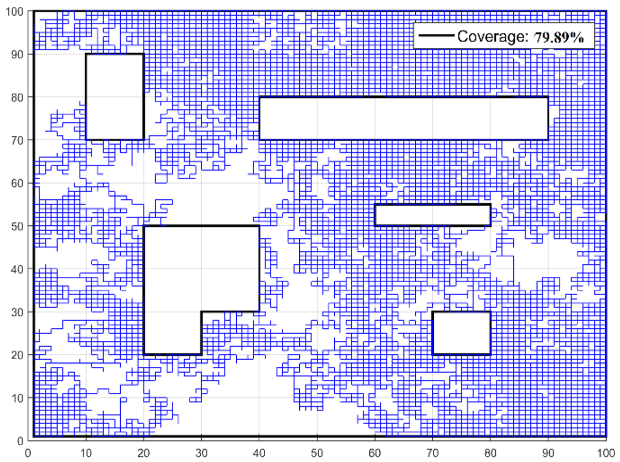

Fig. 7. Grid coverage with obstacles for 50,000 steps in the case of 4 motion directions

to move in more directions. In this case, the interval [0, 1] is divided in 8 equal subintervals and the proposed tactic is used

$$m_i = \begin{cases} up, & x_i \in [0,0.125) \\ up-right, & x_i \in [0.125,0.25) \\ right, & x_i \in [0.25,0.375) \\ down-right, & x_i \in [0.375,0.5) \\ down, & x_i \in [0.5,0.625) \\ down-left, & x_i \in [0.625,0.75) \\ left, & x_i \in [0.75,0.875) \\ up-left, & x_i \in [0.875,1] \end{cases} \quad (6)$$

where m_i denotes the robot movement in the i-th iteration of the map.

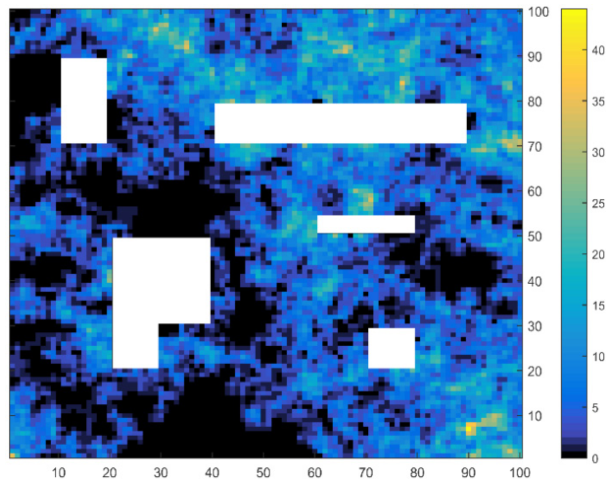

Fig. 8. Color-coded grid coverage showing the number of visits for 50,000 steps in the case of 4 motion directions

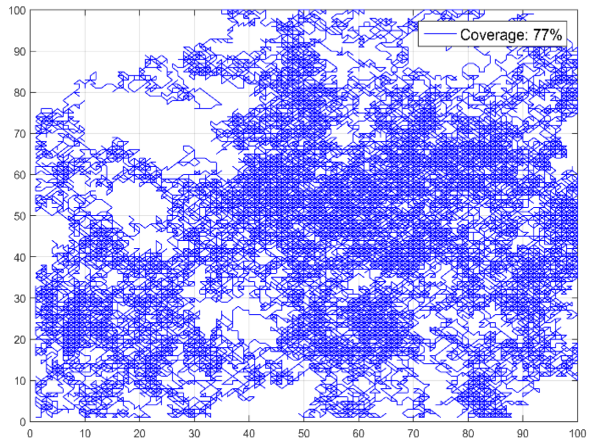

Fig. 9. Grid coverage for 40,000 steps in the case of 8 motion directions

The simulation of the proposed method is shown in Fig. 9. We consider the same grid as the previous one and also the same starting position. The behavior of the robot is studied for 40,000 iterations and the colored map in Fig. 10 shows the number of visits in each cell. The improvement in the coverage is obvious. The robot with the insertion of 4 more motions managed to visit cells that they were uncovered. The size of the black areas which represent unvisited cells is reduced and in their place shades of blue are appearing, which represent visited cells. Fig. 11 shows the grid coverage starting from different initial positions.

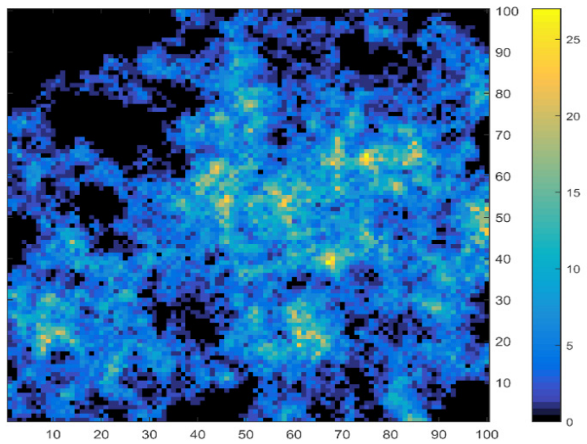

Fig. 10. Color-coded grid coverage showing the number of visits for 40,000 steps in the case of 8 motion directions

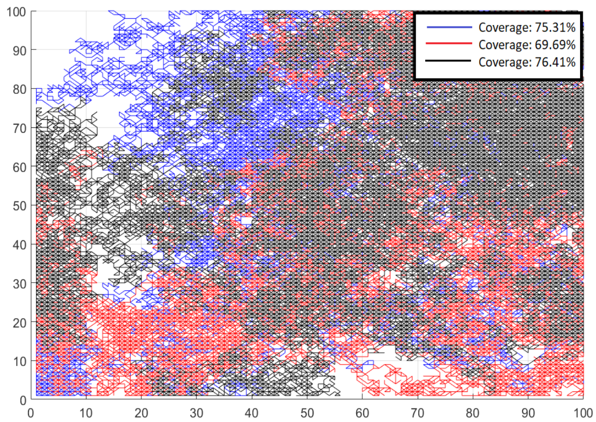

Fig. 11. Grid coverage for 40,000 steps starting from positions $(1,1)^T$ (blue), $(50, 50)^T$ (red), $(50, 100)^T$ (black) for motion in the case of 8 motion directions

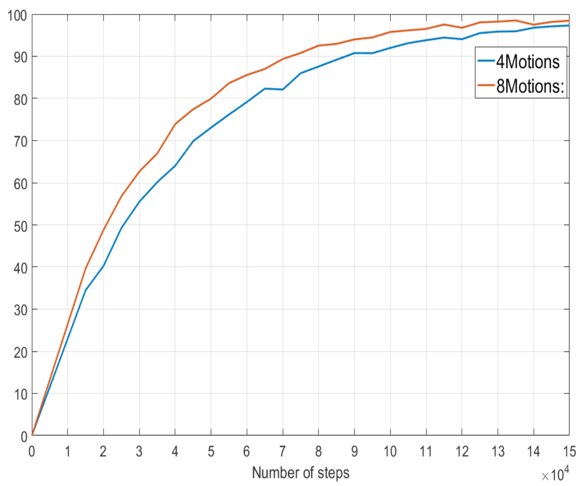

Fig. 12. Grid coverage percentage for 4 (blue) and 8 (red) motion commands

Moreover, the problem of a grid with obstacles for 50,000 movements of the robot is studied. The results are shown in Figs. 13 and 14. The case that was studied, was an 8 direction motion. Although there were 5 obstacles, the robot managed to cover large amount of the given space. Finally, Table 1 shows the average grid coverage in the case of motion in 8 directions. It can be noticed that the coverage percentage is improved compared to the 4 direction motion. Also, the mean number of visits is reduced because of the use of diagonal motions. These diagonal motions gives the opportunity to move in cells that the robot has not visited many times. The result is also plotted in Fig. 12 where it is clear, that motion in 8 directions leads to better coverage result.

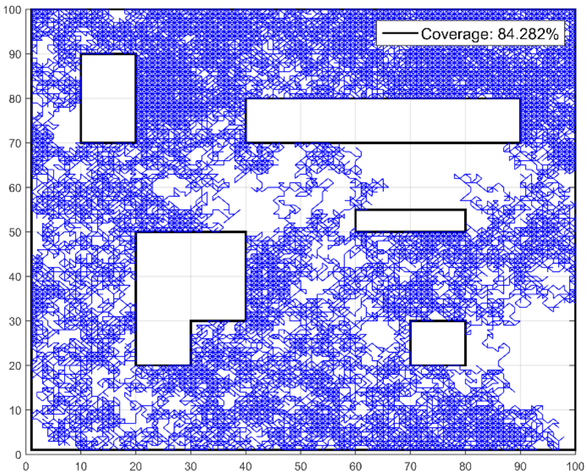

Fig. 13. Grid coverage with obstacles in the case of 8 motion directions, for 50,000 steps

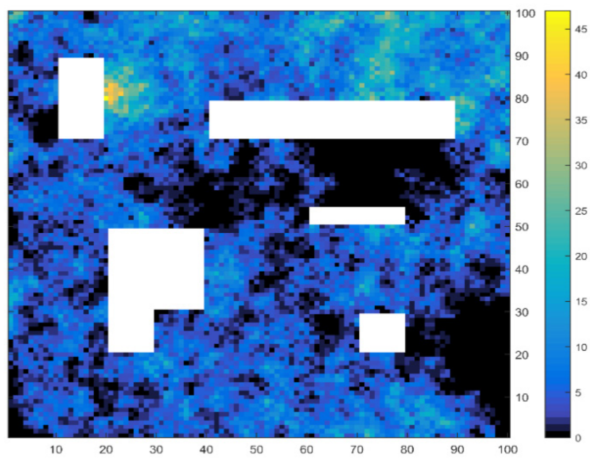

Fig. 14. Color-coded grid coverage showing the number of visits for 50,000 steps in the case of 8 motion directions

3. Conclusion

In this paper, the problem of the efficient coverage of a given space, by a mobile robot, was studied. The method that was proposed, used a modified Logistic-May map for creating a "random" motion trajectory. The method was tested in two cases. The first one produced a motion trajectory which used 4 directions and in the second one, the diagonal motions were inserted. Both cases were tested in the same environment and with the same starting positions. In Fig. 12 it can be noticed that, the use of 4 more directions can improve the behavior of the robot, both in coverage

Tab. 1. Average Grid Coverage and Mean Number of Visits in the Cases of 4 and 8 Motion Directions

Steps x10³	**Average Grid Coverage %**		**Mean number of visits**		**Steps x103**	**Average Grid Coverage %**		**Mean number of visits**	
-	**4d**	**8d**	**4d**	**8d**	**-**	**4d**	**8d**	**4d**	**8d**
15	33	41	4.6	3.7	85	88	93	9.6	9.1
20	41	49	4.9	4.1	90	89	94	10.1	9.5
25	50	56	5	4.4	95	90	95	10.5	10
30	55	63	5.5	4.7	100	92	96	10.9	10.9
35	61	70	5.8	5	105	92	96	11.5	10.5
40	66	73	6.1	5.5	110	94	96	11.7	10.9
45	69	77	6.5	5.8	115	95	96	12.1	11.4
50	73	80	6.8	6.2	120	95	98	12.6	12.1
55	77	82	7.2	6.7	125	96	98	13	12.3
60	78	86	7.7	7	130	96	98	13.6	12.7
65	80	87	8.2	7.4	135	97	98	14	13.2
70	82	90	8.6	7.8	140	97	98	14.4	13.9
75	86	91	8.7	8.2	145	97	99	14.9	14.2
80	87	92	9.2	8.7	150	98	99	15.4	14.6

percentage and in the reduction of the number of visits of same cells.

As a further improvement a pheromone method can be used for better covering of the given workspace. In future works the method could be used and tested in non-square spaces. Also, different discrete chaotic maps can be combined with the modulo tactics in order to generate the chaotic path. Finally, the implementation of the method on an actual mobile robot is crucial in order to study its behavior in real time.

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