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SAFE SPEED OF A PASSENGER CAR IN CIRCULAR MOTION

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Abstract

In this paper, selected elements of the theory of the car's motion were presented. It was emphasised that it is extremely important to know the rules of behaviour of a car in curvilinear motion, an example of which is a curve, especially when driving in adverse weather conditions resulting in the significant reduction of the car's traction to the surface. The active electronic stability programme (ESP) of the car's trajectory, its principle of operation and benefits of use were briefly discussed. The next part was to present the basics of the mechanics of a vehicle's motion, especially the forces acting on a car during its stop phase, the forces acting on the curve of the road. On this basis, the basic equilibrium equations were presented, which in turn led to defining the correlation for determining the so-called safe speed of driving in circular motion, which also guarantees the impossibility of the cars skidding. In the further part, the calculations, in accordance with the BMW 320d car's data were made. Hypothetically, it was assumed that this car might have only a front-wheel drive, only a rear-wheel drive or a four-wheel drive. The idea was to check to what extent the type of drive has an impact on the achieved value of the safe speed of driving in the circular motion with a minimum turning radius for this vehicle. The article was completed with short conclusions, in which the importance of the electronic stability programme (ESP) of the car's trajectory was emphasised.

Keywords: *driving safety, the vehicle's motion on the curvilinear track*

1. Introduction

The safe driving of a car requires not only great prudence and concentration of the vehicle's driver, but also the knowledge in the area of a car's motion. The theory of the car's motion is a more and more rapidly growing science, which primarily focuses on all issues related to the vehicle's motion and the forces acting on it [6].

The basic knowledge of the impact on the car’s motion of various factors, which can be encountered when driving, the behaviour of a car, is essential. It makes it possible for early predict the often-uncontrolled car's motions and to take appropriate preventive actions. The effects of the uncontrolled car's motions on the road may lead to very unpleasant consequences. Manoeuvring a car on the curvilinear tracks and during unfavourable road conditions, e.g. because of bad weather, is particularly dangerous. For example, driving a car in a circle requires the driver to remove the foot off the accelerator pedal at the right time and slow down to the so-called safe speed of driving. The term of safe speed of driving is very often an unknown quantity, and its value is dependent on many factors. One of such factors may be rain, which is the reason why the road surface suddenly becomes wet, and thus the friction coefficient decreases. The car is less stable and it is very easy to exceed the so-called safe speed of driving. Therefore, the adequate knowledge of how the value of the safe speed of driving a car in the circular motion may decrease on a wet surface can preserve us from the unpleasant consequences of incidents and accidents.

Modern motor vehicles, primarily including passenger cars, thanks to the knowledge in this field already possessed by the producers and designers, as well as the access to the latest technology and inventions, have special protection against uncontrolled exceeding, among others, the safe speed of driving in the circular motion. The Electronic Stability Programme (ESP) of the car's trajectory is the most popular in this field. The ESP system is the work of the Bosch group. It was introduced into the market in 1995 as the S-class Mercedes equipment. However, works on the system began over 10 years ago. Within four years since the market debut, more than one million of the ESP systems were produced. Nevertheless, due to its relatively high price, this system was reserved only for high-end cars. After some time, the production cost of ESP decreased and currently it is possible to find this system in new cars of each class.

Operation of this system is based on a number of sensors installed in a car, including the ABS (Anti-Lock Braking System) sensors, the lateral and angular acceleration sensors, the steering angle sensors. The data obtained from these sensors are sent to the ESP system, where they are analysed. The purpose of the conducted straightaway analysis is to determine whether cars keep the desired trajectory. Only if the system finds something opposite, it defines the nature of this instability by indicating the so-called oversteering or understeering (Fig. 1). As a result, the ESP sends a signal to the respective wheel in order to stop and bring back the car to the correct trajectory.

Fig. 1. Operation of ESP (electronic stability program) [3]

In modern cars, systems of this type are very important but they will not fully replace the driver. In addition, there are some single cases of their damage, and thus the knowledge of the car's behaviour during specific manoeuvres is essential. In the article, it was shown on the example of the BMW 320d car how easy it is to exceed the safe speed of driving in circular motion.

2. Mechanics of the car's circular motion

The so-called theory of the car's motion, that is the totality of the issues related to its motion and the forces acting on it during driving, is responsible for the mechanics of the car's motion [8]. This science makes it possible to learn the rules of the car's behaviour during various manoeuvres, which is necessary for the driver to drive safely. Additionally, this knowledge is required in the design of new constructions of cars, as it allows to optimally selecting particular systems, which are responsible for safe driving.

2.1. Forces acting on a car

Each car – regardless of its motion – is affected by the so-called static load, which is a result of the weight of the vehicle acting on the ground. The gravity of a car is placed in the middle of its mass, and its direction is always vertical. The reversal of gravity is directed to the surface, which causes the reaction of this surface passing through the contact points of the wheels with the surface, with the vertical direction and opposite to the gravity – Fig. 2.

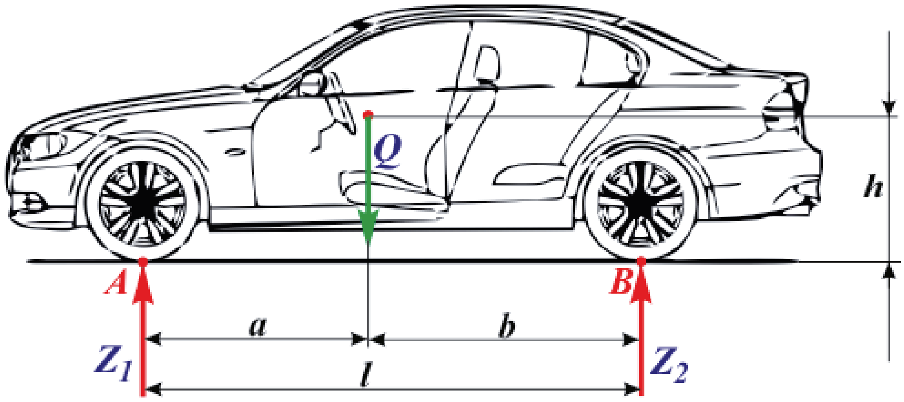

Fig. 2. Static load acting on the car during the stop phase on the horizontal road: Q – weight of the car, Z1, Z2 – ground reaction forces, l – wheel base, a, b, h – coordinates of the centre of mass of the car

According to the arrangement of forces presented in the Fig. 2, the equation of equilibrium of forces and moments can be written as:

$$\sum_{i=1}^{n} F_{z_i} = 0 , \quad \sum_{i=1}^{n} M_i = 0 , \tag{1}$$

where:
F_{z_i} – the sum of all forces acting along the vertical axis,
M_i – the sum of all moments acting in relation to the contact point of the wheel with the surface (point A or B).

Taking into account, the parameters indicated in Fig. 2 and introduced them to the equation (1), and then suitably converted; it is possible to determine the correlation on the value of the Z_1 and Z_2 reaction forces of the surface, depending on the weight of the car:

$$Z_1 = Q\frac{b}{l}, \quad Z_2 = Q\frac{a}{l}. \tag{2}$$

This method of calculating the surface reaction requires the knowledge of linear parameters of the position of the centre of the car's gravity, which usually does not exist in the technical documentation. The division of the car's mass to the front and back is more often given in percentage. Assuming marks of the car's load to the front as Q_P [%] and to the back as Q_T [%], the calculation of the reaction value can be determined from the correlation:

$$Z_1 = Q\frac{Q_P}{100}, \quad Z_2 = Q\frac{Q_T}{100}. \tag{3}$$

This kind of calculation of the reaction value will allow to – after the transformations of the correlation (2) – calculate the parameters of the position of the centre of the car's gravity:

$$a = \frac{Z_2 \cdot l}{Q}, \quad b = \frac{Z_1 \cdot l}{Q}. \tag{4}$$

2.2. Forces acting on the car on the curve of the road

During the car's circular motion, it is essential to take into account the lateral forces acting on it, and in fact the car's wheels. These forces have two origins. The first are the so-called natural ones, arising, among others, from the crosswind action, the road's tilt, etc.; the second ones come from the driver's action, which can produce them with the appropriate turn of the car's wheels. In the article (to simplify the issue), it was assumed that the lateral forces are derived only from the appropriate driver's action (Fig. 3). Therefore, it is possible for unambiguously conclude that the side reactions will only balance the centrifugal force F_Q.

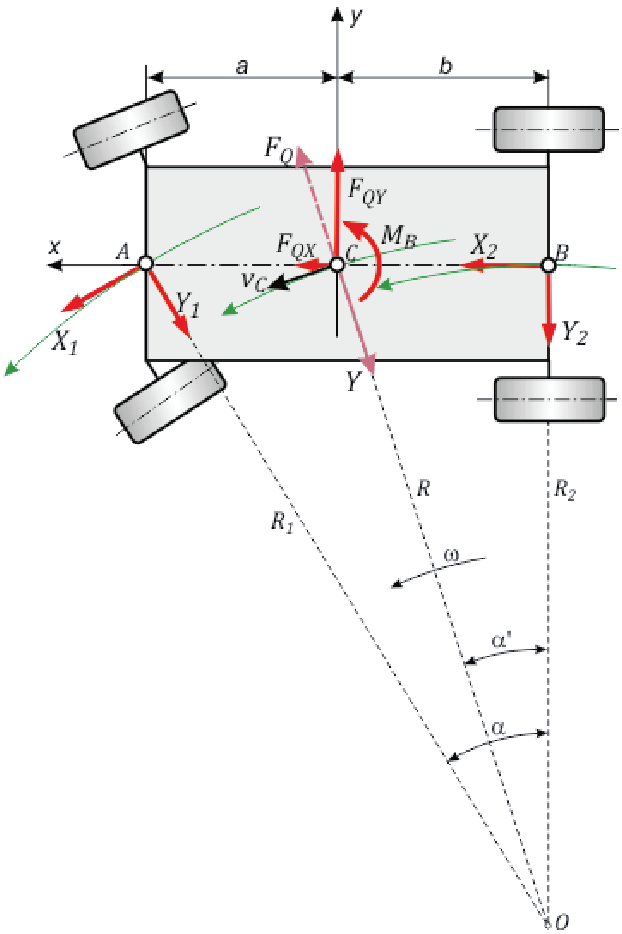

Fig. 3. Distribution of the lateral forces acting on cars in circular motion: FQ(FQX,FQY) – the centrifugal force (its components), Xi, Yi – the tangential and side reactions on the axles of wheels, MB – the moment of inertia, R, R1, R2 – the minimum turning radius, α - the angle of rotation of the steered wheels, ω – angular velocity

According to the analysis of the Fig. 3, the equilibrium equation can be composed in the form of:

$$\begin{gathered}\sum F_x = X_1 \cdot \cos(\alpha) + X_2 + F_{QX} - Y_1 \cdot \sin(\alpha) = 0\,, \\ \sum F_y = F_{QY} - X_1 \cdot sin(\alpha) - Y_1 \cdot \cos(\alpha) - Y_2 = 0, \\ \sum M_C = X_1 \cdot \mathrm{a} \cdot \sin(\alpha) + Y_1 \cdot \mathrm{a} \cdot \cos(\alpha) + M_B - Y_2 \cdot b = 0.\end{gathered} \tag{5}$$

Assuming to the calculation: $\cos(\alpha) \approx 1, \sin(\alpha) \approx 0, \alpha \approx \alpha' \approx 0, F_{QX} \approx 0, F_{QY} \approx F_Q$ we obtain:

$$\begin{aligned}&X_1 + X_2 = 0, \\ &Y_1 + Y_2 = F_Q, \\ &Y_2 \cdot b - Y_1 \cdot a = M_B.\end{aligned} \tag{6}$$

Analysing the moment of skidding, the i-th of the car's axis, it is stated that it will occur when the resultant reaction of the X_i and Y_i exceeds the level of the car's traction to the surface. Therefore, in order to avoid skidding caused by lateral forces on each axle, the following condition must be satisfied:

$$\sqrt{X_i^2 + Y_i^2} \leq \mu \cdot Z_i, \tag{7}$$

where: μ - the friction coefficient (for wet asphalt it is 0.48).

According to the correlation (7), it can be noted that the limit state can be written with the equilibrium:

$$\sqrt{X_i^2 + Y_i^2} = \mu \cdot Z_i, \tag{8}$$

which after transformation allows determining the maximum value of the lateral force that the i-th axle of wheels will move without the occurrence of skidding?

$$Y_i \rightarrow Y_{imax} = \sqrt{\mu^2 \cdot Z_i^2 - X_i^2}, \tag{9}$$

then substituting it into the 2nd equation from the correlation (6), we obtain:

$$\sqrt{\mu^2 \cdot Z_1^2 - X_1^2} + \sqrt{\mu^2 \cdot Z_2^2 - X_2^2} = F_Q. \tag{10}$$

On this basis, it will be possible to determine the safe driving speed (limit) v_{gr}, the exceeding of which causes skidding of the car.

3. Calculation of safe speed of driving

The calculations were conducted using the data obtained from the BMW 320d car, assuming (hypothetically) that it can be driven only on the front axle, only on the rear axle, or on both axles.

Its basic data (weight, linear dimensions), necessary for the calculation, are total weight Q = 15402 [N], which consists of the weight of the car amounting to 14028 [N] plus the weight of the fuel 589 [N], plus the weight of the driver 785 [N]. Moreover, the minimum turning radius of the value of R=11.9 [m] was adapted to the calculation. The lack of action of the lateral force and of the wind was assumed. The basic dimensions of this vehicle are presented on the Fig. 4.

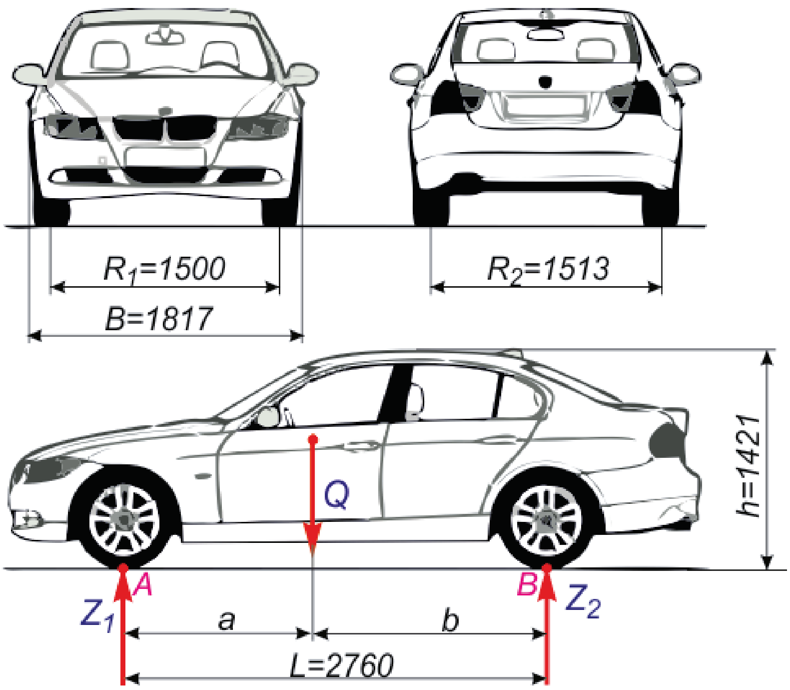

Fig. 4. Basic dimensions of the BMW 320d car [4]

3.1. A car with the front-wheel drive

Then it can be assumed that: $X_1=F_{N1}$ $X_2=0$, where F_{N1} – the driving force of the rear axle and a minimum turning radius $R_1=12.13$ [m] (calculated with the help of the Pythagorean theorem). Hence, using the correlation (10), we obtain:

$$\sqrt{\mu^2 \cdot Z_1^2 - F_{N1}^2} + \sqrt{\mu^2 \cdot Z_2^2} = F_Q\ , \tag{11}$$

and then:

$$\sqrt{\mu^2 \cdot Z_1^2 - F_{N1}^2} + \mu \cdot Z_2 = \frac{Q \cdot v^2}{g \cdot R_1}. \tag{12}$$

Taking advantage of the fact that $F_{Ni} = \gamma_{Ni} \cdot Z_i$, where $\gamma_{Ni} = 0.2\text{-}0.4$ – the unit driving force. For the analysed type of the BMW car, it is possible to adopt $\gamma_{Ni} = 0.2$, thus after transformations, we obtain:

$$v = \sqrt{\frac{g \cdot R_1}{Q}\left(Z_1\sqrt{\mu^2 - \gamma_{N1}^2} + \mu \cdot Z_2\right)}. \tag{13}$$

Accepting the data for the assumed middle-class car of the BMW 320D type and for wet asphalt ($\mu=0.48$), we obtain:

$$v \to v_{gr} = \sqrt{\frac{9.81 \cdot 12.13}{15402} \cdot \left(8040 \cdot \sqrt{0.48^2 - 0.2^2} + 0.48 \cdot 7362\right)} = 7.38 \left[\frac{m}{s}\right] \approx 26.55 \left[\frac{km}{h}\right].$$

Thus, the safe speed of the BMW 320d car on the minimum arc, on a wet surface, with the front wheel drive and without the ESP, is 26.55 [km/h].

3.2. A car with the rear-wheel drive

Then it can be assumed that: $X_1=0$, $X_2=F_{N2}$, where F_{N2} – the driving force of the rear axle and a minimum turning radius $R_2=11.81$ [m] (calculated with the help of the Pythagorean theorem). Hence, using the correlation (10), we obtain:

$$\sqrt{\mu^2 \cdot Z_1^2} + \sqrt{\mu^2 \cdot Z_2^2 - F_{N2}^2} = F_Q\ , \tag{14}$$

and then:

$$\mu \cdot Z_1 + \sqrt{\mu^2 \cdot Z_2^2 - F_{N2}^2} = \frac{Q \cdot v^2}{g \cdot R_2}. \tag{15}$$

Taking advantage of the fact that $F_{Ni} = \gamma_{Ni} \cdot Z_i$, gdzie $\gamma_{Ni} = 0.2 - 0.4$ – the unit driving force. For the analysed type of the BMW car, it is possible to adopt $\gamma_{Ni} = 0.2$ due to considerable power of engine, thus after transformations, we obtain:

$$v = \sqrt{\frac{g \cdot R_2}{Q}\left(\mu \cdot Z_1 + Z_2\sqrt{\mu^2 - \gamma_{N2}^2}\right)}\ . \tag{16}$$

Accepting the data for the assumed middle-class car of the BMW 320D type and for wet asphalt ($\mu=0.48$), we obtain:

$$v \to v_{gr} = \sqrt{\frac{9.81 \cdot 11.81}{15402} \cdot \left(0.48 \cdot 8040 + 7362 \cdot \sqrt{0.48^2 - 0.2^2}\right)} = 7.29 \left[\frac{m}{s}\right] \approx 26.26 \left[\frac{km}{h}\right].$$

Thus, the safe speed of the BMW 320d car on the minimum arc, on a wet surface, with the front wheel drive and without the ESP, is 26.60 [km/h].

3.3. A car with the four-wheel drive (4x4)

Then it can be assumed that: $X_1=F_{N1}$, $X_2=F_{N2}$ and a minimum turning radius $R=11.9$ [m]. Hence, using the correlation (10), we obtain:

$$\sqrt{\mu^2 \cdot Z_1^2 - F_{N1}^2} + \sqrt{\mu^2 \cdot Z_2^2 - F_{N2}^2} = F_Q \tag{17}$$

and then, after transformations:

$$Z_1\sqrt{\mu^2 - \gamma_{N1}^2} + Z_2\sqrt{\mu^2 - \gamma_{N2}^2} = \frac{Q \cdot v^2}{g \cdot R}, \tag{18}$$

adopting $\gamma_{N1} \approx \gamma_{N2} \approx \gamma_N$:

$$(Z_1 + Z_2) \cdot \sqrt{\mu^2 - \gamma_N^2} = \frac{Q \cdot v^2}{g \cdot R}, \tag{19}$$

we finally obtain:

$$v = \sqrt{g \cdot R \cdot \sqrt{\mu^2 - \gamma_N^2}}\,. \tag{20}$$

Accepting the data for the assumed middle-class car of the BMW 320D type and for wet asphalt ($\mu=0.48$), we obtain:

$$v \rightarrow v_{gr} = \sqrt{9.81 \cdot 11.9 \cdot \sqrt{0.48^2 - 0{,}2^2}} = 7.14 \left[\frac{m}{s}\right] \approx 25.69 \left[\frac{km}{h}\right].$$

Thus, the safe speed of the BMW 320d car on the minimum arc, on a wet surface, with the front wheel drive and without the ESP, is 25.69 [km/h].

Conclusions

The conducted research and calculations confirmed that skidding of the BMW 320d car, without the ESP system (electronic stability program), on a wet surface, is quite easy. In circular motion with a radius close to the minimum radius (according to the technical documentation of the vehicle), this speed – depending on the type of drive – fluctuates around the speed of 26 km/h. Thus, it is not possible for unambiguously indicating the specific type of the drive from this point of view, although the front-wheel drive obtained the largest value of safe driving speed in circular motion. A standard drive for the BMW 320d car is the rear drive, which results from a number of other advantages (not indicated here), and not – as it turned out – from the achieved value of safe driving speed on the minimum turning radius.

Therefore, the above analysis confirmed the need and necessity for mounting active systems to maintain traction, especially in cars which significantly increase the safe driving speed at curves in difficult weather conditions – particularly deteriorating the car's traction to the road (such as rain, snow, etc.). It is not the subject of this article to determine how much the safe driving speed in circular motion with the ESP system increases. Nevertheless, according to the information obtained from the subject literature, ESP systems are crucial. Thus, despite a significant increase in the price of vehicles with these systems, they are already a standard in modern cars.

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