Multi-aspect fuzzy sets in modelling the decision support systems

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The paper presents the new mathematical modelling concept using the so-called multi-aspect fuzzy sets. The paper contains definitions of the most important characteristics of multi-aspect fuzzy sets in the context of their application in decision support algorithms. These include characteristics such as the image of the multi-aspect fuzzy set, the carrier and core, the bottom and top fronts of the fuzzy set, and many other characteristics derived from multi-criteria optimization. These concepts are illustrated with numerical examples.

Keywords: multi-aspect fuzzy set, global membership function, image of fuzzy set, lower and upper front of fuzzy set.

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1. Introduction

The classic definitions related to fuzzy sets are not sufficient in many mathematical modelling situations. We are faced very often with the situation where we are forced to define a set of objects that have many features (more than one), and each of which "in its own way" determines the degree of membership in the defined fuzzy set (somewhat separately). For example, the education degree of a candidate for a given managerial position, his/her to-date seniority at managerial positions, appearance, communication with the environment, age, mobility, etc. In such cases, fuzzy sets (in the classic version) can be defined as the products [10, 13, 28, 33, 36] of the corresponding (classic "one-aspect") fuzzy sets. The need to define "aggregated" function of membership (usually t-norm) in a set that is the product of multiple fuzzy sets then appears [10, 13, 33]. In such situations, however, doubts may arise, supported by many analyses and practical studies, as to whether these functions [33, 36] "properly" describe the degree of membership of elements in the fuzzy set. The product of the two fuzzy sets A and B is the fuzzy set C of elements, each of which "to some extent" is simultaneously the member of the both sets. The total "resultant" degree of membership is expressed by the new function of membership of the elements in the set C. The construction (definition) of this function is not obvious [10, 33, 36]. Such a function in decision-making models should guarantee the execution of certain practical postulates [3, 4, 10, 13]. These postulates generally result from intuitive decision-making expectations in decision support systems. The alternative and more natural approach in such a situation may turn out to be an attempt to use the concept of a multi-faceted fuzzy set. Below, we will provide definitions of the most important concepts, concerning multi-aspect sets, useful for mathematical modelling.

2. Multi-aspect fuzzy sets – the new approach to modelling the fuzzy decision-making problems

The classic definition of a fuzzy set [10, 13, 33, 36] can be easily extended to multi-aspect fuzzy sets. Membership in a set in this case shall be understood as the membership in a set in the sense that an element has multiple facets (features) on various degrees. Let us further assume that X is the finite set of elements (objects) with N features (aspects), numbered with the index $n \in \boldsymbol{N} = \{1, \ldots, N\}$.

Definition 1
The multi-aspect fuzzy set A is a set of ordered pairs having the form of:

$$A = \{(x, \mu_A(x)) | x \in X\}, \tag{1}$$

where $\mu_A(x),\ x\in X$ means the multi-aspect degree of the membership of the element x in the set A. The function μ_A is *the multi-aspect membership function* that takes the normalized values from the area $[0,1]x...x[0,1]=$ $=[0,1]^N \subset \mathcal{R}^N$. This is the vector function having the form of:

$$\mu_A(x)=\left(\mu_A^1(x),...,\mu_A^n(x),...,\mu_A^N(x)\right)\in\mathcal{R}^N,\ x\in X \tag{2}$$

where $\mu_A^n(x)$ – is the degree of membership of the element x in the set A from the point of view of the feature (in terms of the feature) number $n\in\mathcal{N}$. The multi-aspect (N-aspect) fuzzy set A in space X is empty if and only if for each $x\in X$ there is $n\in\mathcal{N}$, that $\mu_A^n(x)=0$. The characteristics of a multi-aspect fuzzy set as well as operations on such sets are constructed in the same way as for a classic fuzzy sets.

Definition 2

The carrier of the fuzzy set A shall be called the classical (sharp) set in the form of:

$$\operatorname{supp}(A)=\left\{x\in X \middle| \text{exists } n\in\mathcal{N}\text{, that } \mu_A^n(x)>0\right\}\subset X \tag{3}$$

The below set shall be called the *essential carrier* or the *super carrier* of the fuzzy set A:

$$\overline{\operatorname{supp}}(A)=\left\{x\in X \middle| \mu_A^n(x)>0, n\in\mathcal{N}\right\}\subset X$$

Definition 3

The below set shall be called *the image* of the fuzzy set A in the space X

$$O_A(X)=\left\{\mu_A(x) \middle| x\in\operatorname{supp}(A)\right\}\subset\mathcal{R}^N \tag{4}$$

The below set shall be *the counterimage* to set $C\subset O_A(X)$

$$\mu_A^{-1}(C)=\left\{x\in X \middle| \mu_A(x)\in C\right\}\subset X \tag{5}$$

Definition 5

The upper pole $hgt(A)$ (or height) of the fuzzy set A shall be called the element in the form of:

$$\operatorname{hgt}(A)=\overset{*}{y}(A)= =\left(\overset{*}{y}_1(A),...,\overset{*}{y}_n(A),...,\overset{*}{y}_N(A)\right)\in\mathcal{R}^N \tag{7}$$

where $\overset{*}{y}_n(A)=\max_{x\in X}\mu_A^n(x)=\overset{*}{y}_n, n\in\mathcal{N}$

In case of the fuzzy normalized (normal) sets [33, 36] we have:

$$\operatorname{hgt}(A)=\overset{*}{y}(A)=(1,...,1)\in\mathcal{R}^N,$$

that is $\overset{*}{y}_n(A)=1$, $n\in\mathcal{N}$. (7a)

The below number shall be called *the normative* (scalar) *height* of the fuzzy set:

$$\overline{\operatorname{hgt}}(A)=\frac{1}{\sqrt{N}}\left\|\overset{*}{y}(A)\right\|$$

Definition 6

The lower pole (threshold) of the fuzzy set A is called the element in the form of:

$$\operatorname{thres}(A)= =\overset{\circ}{y}(A)=\left(\overset{\circ}{y}_1(A),...,\overset{\circ}{y}_n(A),...,\overset{\circ}{y}_N(A)\right)\in\mathcal{R}^N \tag{8}$$

where $\overset{\circ}{y}_n(A)=\min_{x\in X}\mu_A^n(x)=\overset{\circ}{y}_n, n\in\mathcal{N}$.

Often, in practical terms, we have:

$$\overset{\circ}{y}_n(A)=0,\ n\in\mathcal{N}. \tag{8a}$$

The below number shall be called *the normative* (scalar) *threshold* of the fuzzy set:

$$\overline{\operatorname{thres}}(A)=\frac{1}{\sqrt{N}}\left\|\overset{\circ}{y}(A)\right\|$$

Definition 7

The extension (span) of a multi-aspect fuzzy set A is called the element in the form of:

$$\operatorname{exten}(A)=\operatorname{hgt}(A)-\operatorname{thres}(A)=\overset{*}{y}(A)-\overset{\circ}{y}(A) \tag{9}$$

The below number shall be called the *normative* (scalar) *span* of the fuzzy set:

$$\overline{\operatorname{exten}}(A)=\frac{1}{\sqrt{N}}\left(\left\|\overset{*}{y}(A)\right\|-\left\|\overset{\circ}{y}(A)\right\|\right)$$

The multi-aspect *expressiveness* (*expressiveness factor*) of a fuzzy set in the decision-making context [1, 4, 8, 10, 16] is the important characteristic of such a fuzzy set.

Definition 8

The expressiveness of the multi-aspect fuzzy set is defined as follows:

$$\mathrm{sharp}(A)= =\left(\mathrm{sharp}_1(A),...,\mathrm{sharp}_n(A),...,\mathrm{sharp}_N(A)\right)\in\mathcal{R}^N \quad (10)$$

where $\mathrm{sharp}_n(A)=\dfrac{\sum_{x\in\mathrm{supp}(X)}\mu_A^n(x)}{\left|\mathrm{supp}(X)\right|}$

The below number shall be called the *normative* (scalar) *distinctiveness* of the fuzzy set:

$$\overline{\mathrm{sharp}}(A)=\frac{\sum_{n\in\mathcal{N}}\mathrm{sharp}_n(A)}{N}$$

or

$$\overline{\mathrm{sharp}}(A)= =\min\left(\mathrm{sharp}_1(A),...,\mathrm{sharp}_n(A),...,\mathrm{sharp}_N(A)\right)$$

Definition 9

The fuzzification of a multi-aspect fuzzy collection is defined as follows:

$$\mathrm{fuzze}(A)= =\left(\mathrm{fuzze}_1(A),...,\mathrm{fuzze}_n(A),...,\mathrm{fuzze}_N(A)\right)\in\mathcal{R}^N \quad (11)$$

where $\mathrm{fuzze}_n(A)=1-\dfrac{\sum_{x\in\mathrm{supp}(X)}\mu_A^n(x)}{\left|\mathrm{supp}(X)\right|}$

The number $\overline{\mathrm{fuzze}}(A)=\dfrac{\sum_{n\in\mathcal{N}}\mathrm{fuzze}_n(A)}{N}$

or

$$\overline{\mathrm{fuzze}}(A)=\max\left(\mathrm{fuzze}_1(A),...,\mathrm{fuzze}_N(A)\right) \quad (12)$$

shall be called the *normative* (scalar) *fuzzification* of the fuzzy set.

Definition 10

The below set shall be called *the upper front (the ceiling)* of the fuzzy set A.

$$\mathrm{roof}(A)= =\left\{\begin{array}{l}x\in\mathrm{supp}(A)\,|\,\text{does not exist } y\in\mathrm{supp}(A),\\ \text{that } \mu_A(y)\ge\mu_A(x) \text{ and } \mu_A(y)\ne\mu_A(x)\end{array}\right\} \quad (13)$$

An element belonging to the top front (ceiling) of a fuzzy set is such an element belonging to the carrier of the set that, among the other elements of the carrier, there is no element that "has larger membership" in this set in the sense of all facets.

Definition 11

The below set shall be called *the bottom front (the floor)* of the fuzzy set A.

$$\mathrm{floor}(A)= =\left\{\begin{array}{l}x\in\mathrm{supp}(A)\,|\,\text{does not exist } y\in\mathrm{supp}(A),\\ \text{that } \mu_A(y)\le\mu_A(x) \text{ and } \mu_A(y)\ne\mu_A(x)\end{array}\right\} \quad (14)$$

An element that is the member of the lower front of the fuzzy set is such element being member of the carrier of the set that among the other elements there is no element that would be member even less in this set. The elements belonging to the fronts of the fuzzy set are called *the front elements* (upper or lower respectively). They play an important role in decision support (optimization) processes [5, 10].

Below, the example of a multi-aspect fuzzy set shall be considered, for which all previously defined characteristics shall be determined. Due to the possibility of graphical interpretation, the number of considered features (facets) of the elements of the set shall be reduced to two elements.

The example

Let X be the set of thirty persons. Let it be the set of potential candidates for the CEO position with a large company. The features of the candidates that are taken into account as professional predispositions are:

1) education in the scope of organizational and management skills
2) experience on managerial positions
3) appearance of the candidate
4) relevance of age of the candidate
5) communicativeness with other people.

This set can be defined as the multi-aspect fuzzy set *A* having the following form:

$$A=\left\{\left(x,\mu_A(x)\right)\,|\,x\in X\right\}$$

where

$\mu_A(x)=\left(\mu_A^1(x),...,\mu_A^5(x)\right),\ x\in X$

is the vector function of membership of individual persons in the set of candidates for the CEO position. The set $X=\{1,2,...,30\}$ is the set of numbers of persons aspiring to be a candidate for CEO (vide Fig.1). The number

$\mu_A^n(x)$ – is the value of the *n*-th feature of a person $x \in X$, which such value is determined in an appropriate way on a scale of [0,1] by the competition experts, based on data on the candidates. In this situation, we can talk about the degree, in which a given person is member of the set of candidates for CEO in the aspect of the five features.

We shall say about the person $x \in X$ that he/she fits more (belongs more) to the set *A* of candidates for CEO position than person $y \in X$ if there is $\mu_A(x) \geq \mu_A(y)$ and $\mu_A(x) \neq \mu_A(y)$. The image of the fuzzy set *A* is

$O_A(X) =$

$= \mu_A(\text{supp}(A)) = \{\mu_A(x) | x \in \text{supp}(A)\} \subset \mathcal{R}^5$

it is the "cloud of thirty points" in the five-dimensional space. To simplify the example and its graphical interpretation, the number of features shall be reduced from 5 to 2 (for example, education and professional experience). The data are presented in the table next to Figure 1, which shows an image of the fuzzy set $O_A(X) = \{\mu_A(x) | x \in \text{supp}(A)\}$. Then we have the situation that in case of persons with numbers $x \in \{3,5\}$ $\mu_A(3) = \mu_A(5) = (0.8,\ 0.9)$ occurs. Hence $|O_A(X)| = 29$.

Below you can find some of the characteristics of the set *A*:

Let's use the following example $C = \{(1,\ 0.6),(0.9,\ 0.8),(0.8,\ 0.9),(0.4,\ 1)\} \subset A$

The counter-image of the set $C \subset O_A(X)$ is the set $\mu_A^{-1}(C) = \{3,\ 5,\ 7,\ 9,\ 16\}$. The *carrier* of the fuzzy set *A* is the set $\text{supp}(A) = X$. The *essential carrier* of the set *A* (the *super carrier*) is the set $\overline{\text{supp}}(A) = X - \{20\}$. The core of the fuzzy set *A* is the set $\text{core}(A) = \{2,\ 9,\ 10,\ 16\}$. *The essential core* (*the super core*) of the fuzzy set *A* is the empty set. The fuzzy set *A* therefore does not have *the essential core* (*the super-core*).

The further characteristics of the fuzzy set:

a) the height the fuzzy set

$$\text{hgt}(A) = \overset{*}{y}(A) = (1,\ 1) \in \mathcal{R}^2$$

b) the *ceiling* of the fuzzy set *A* (*the upper front of the fuzzy set*) is the set $\text{roof}(A) = \{3,\ 5,\ 7,\ 9,\ 16\}$,

c) the *floor* of the fuzzy set *A* (*the lower front of the fuzzy set*) is the set $\text{floor}(A) = \{11,\ 13,\ 20\}$.

The sets $\text{roof}(A) = \{3,\ 5,\ 7,\ 9,\ 16\}$ and $\text{floor}(A) = \{11,\ 13,\ 20\}$ are marked in Figure 1.

The other characteristics are:

$$\overline{\text{hgt}}(A) = \frac{1}{\sqrt{2}} \left\| \overset{*}{y}(A) \right\|_2 = \frac{1}{\sqrt{2}} \|(1,1)\|_2 = 1,$$

$$\text{thres}(A) = \overset{\circ}{y}(A) = (0,0.1) \in \mathcal{R}^2,$$

$$\overline{\text{thres}}(A) = \frac{1}{\sqrt{2}} \left\| \overset{\circ}{y}(A) \right\|_2 = \frac{1}{\sqrt{2}} \|(0,0.1)\|_2 = \frac{1}{10\sqrt{2}},$$

$$\text{exten}(A) = \overset{*}{y}(A) - \overset{\circ}{y}(A) = (1,0.9),$$

$$\overline{\text{exten}}(A) = \frac{1}{\sqrt{2}} \left(\left\| \overset{*}{y}(A) \right\|_2 - \left\| \overset{\circ}{y}(A) \right\|_2 \right) \simeq 0.953$$

$\text{sharp}(A) = (0.52,\ 0.54)$, $\overline{\text{sharp}}(A) = 0.53$.

Figure 1 illustrates the image $O_A(X) = \{\mu_A(x) | x \in \text{supp}(A)\}$ of the fuzzy set *A* in the space *X*. The circles with a number in the centre represent the images of the elements.

In many decision-making situations using fuzzy sets, there may be a need to "scalar the sets". This procedure is meant to be the process of determining the global, total function of membership in a fuzzy set based on the original, multi-aspect membership function.

X	$\mu_A^1(x)$	$\mu_A^2(x)$
1	0,50	0,20
2	1,00	0,50
3	0,80	0,90
4	0,50	0,50
5	0,80	0,90
6	0,30	0,40
7	0,90	0,80
8	0,70	0,10
9	1,00	0,60
10	0,20	1,00
11	0,30	0,10
12	0,60	0,40
13	0,10	0,30
14	0,30	0,80
15	0,90	0,30
16	0,40	1,00
17	0,90	0,60
18	0,40	0,30
19	0,70	0,70
20	0,00	0,50
21	0,50	0,10
22	0,70	0,30
23	0,50	0,80
24	0,20	0,60
25	0,80	0,50
26	0,60	0,70
27	0,10	0,70
28	0,80	0,20
29	0,40	0,70
30	0,70	0,60

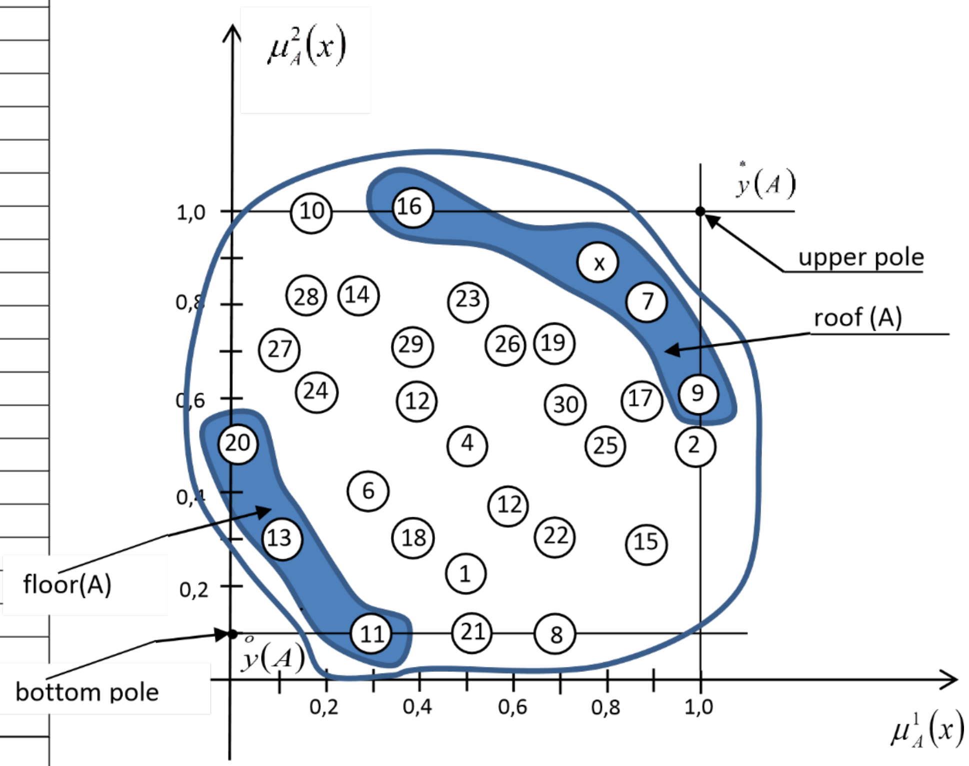

Fig. 1. Image of multi-aspects fuzzy set

3. Global membership functions – decision-making postulates

The selection of an appropriate membership function, especially in the decision-making usage of fuzzy sets being the result of various additional operations (for example, the product of fuzzy sets or multi-aspect sets) is very essential. For the classic product of two fuzzy sets A and B, the so-called *t-norms* (vide [10, 13, 33, 36]) are used most often, in particular the following membership functions:

1) $\mu_\cap^1(x) = \min\{\mu_A(x), \mu_B(x)\},\ x \in X$ (15)

2) $\mu_\cap^2(x) = \mu_A(x)\mu_B(x),\ x \in X$ (16)

3) $\mu_\cap^3(x) = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$ (17)

From the point of view of practical applications of fuzzy sets in the decision support process, the membership functions should fulfil a number of postulates.

Let μ_A be the multi-aspect function of membership of elements $x \in X$ in the non-empty, multi-aspect fuzzy set A. Let $\eta_A : X \to [0,1]$ be a certain scalar function, called the global (aggregate) function of membership of the elements $x \in X$ in the set A.

Definition 12 (the postulate for monotonicity)

The global membership function $\eta_A(x),\ x \in X$ meets the monotonicity postulate, if for $x, y \in X$ such as that $\mu_A(x) \geq \mu_A(y)$ is $\eta_A(x) \geq \eta_A(y)$. Moreover, the global membership functions $\eta_A(x),\ x \in X$ should guarantee the realization of some additional practical postulates resulting from intuitive decision-making expectations when used in the mathematical modelling of decision support systems [3, 10, 13]. The most important of them are the following.

Definition 13 (the postulate for discrimination)

The global membership function $\eta_A(x),\ x \in X$ meets the postulate for discrimination if for each ones $x, y \in X$, such as that $\mu_A(x) \geq \mu_A(y)$ and $\mu_A(x) \neq \mu_A(y)$, $\eta_A(x) > \eta_A(y)$ occurs.

Definition 14 (the postulate for lack of internal contradiction)

The global membership function $\eta_A(x),\ x \in X$ meets the postulate for lack of internal contradiction if for each ones $x, y \in X$, such as $\eta_A(x) = \eta_A(y)$ and $\mu_A(x) \neq \mu_A(y)$, no $\mu_A(x) \geq \mu_A(y)$ and no $\mu_A(y) \geq \mu_A(x)$ occurs.

Definition 15 (the postulate for continuation)

The global membership function $\eta_A(x),\ x \in X$ meets the postulate for continuation, if for each one $x \in X$, such as that $\mu_A^n(x) \neq 0, n \in \mathcal{N}$ is $\eta_A(x) \neq 0$.

In addition to the above postulates, other decision-making postulates are sometimes formulated according to the class of decision-making problems. Unfortunately, the product membership functions (for example, (15), (16), (17)) do not always meet these postulates. Hence, their usage as global membership functions in decision support systems may be questionable.

4. Final conclusions

The paper presents the concept of a multi-aspect fuzzy set and its characteristics in the context of applications of decision support systems in the mathematical modelling. Operations on multi-aspect fuzzy sets are defined in the same way as operations on classic fuzzy sets. The modelling of multi-aspect fuzzy objects (phenomena) seems to be more natural than trying to represent them as a product of many classical ("single-aspect") fuzzy sets. This conclusion is clearly confirmed by the results derived for multi-aspect medical diagnoses [4, 6, 7, 8, 9, 10, 13]. Many of the characteristics of the multi-aspect fuzzy set, which such characteristics are defined in item 2 of this paper, such as carrier, core, height, span, fuzzification or clarity, have a very concrete, practical decision-making interpretation (see fuzzy medical diagnosis [4, 8, 10, 13]). A particularly interesting decision interpretation is undoubtedly *the upper front* concept of the multi-aspect fuzzy set $roof(A)$ and *the upper pole* concept of the fuzzy set $\mathrm{hgt}(A) \in \mathcal{R}^N$ (see Figure 1). *The upper front* of a fuzzy set is the set of such elements *x* belonging to the carrier of the set *A*, that, in comparison to these elements, another element, "which has larger membership" in the fuzzy set *A* than the element *x* (analogously the lower front), does not exist in the set $\mathrm{supp}(A)$. The paper formulates four basic postulates that should be met by global membership functions, including product functions used in decision support systems.

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Wieloaspektowe zbiory rozmyte w modelowaniu systemów wspomagania decyzji

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W artykule przedstawiono nową koncepcję modelowania matematycznego wykorzystującą tzw. wieloaspektowe zbiory rozmyte.W pracy zawarto definicje najważniejszych charakterystyk wieloaspektowych zbiorów rozmytych w kontekście ich zastosowania w algorytmach wspomagania decyzji. Należą do nich takie cechy, jak obraz wieloaspektowego zbioru rozmytego, nośnik i rdzeń, dolny i górny front zbioru rozmytego oraz wiele innych cech pochodzących z optymalizacji wielokryterialnej. Koncepcje te zilustrowano przykładami numerycznymi.

Słowa kluczowe: wieloaspektowy zbiór rozmyty, globalna funkcja przynależności, obraz zbioru rozmytego, dolny i górny front zbioru rozmytego.