Mariusz KRAJEWSKI
INSTITUTE OF METROLOGY, ELECTRONICS AND COMPUTER SCIENCE, UNIVERSITY OF ZIELONA GÓRA,
2 prof. Z. Szafran St., 65-246 Zielona Góra, Poland

Frequency synchronization error correction by using DFT in sinusoidal voltage measurements

Abstract

The paper presents a method of correcting a frequency synchronization error in the sinusoidal voltage measurement. Correction is performed by changing a frequency of the measured signal on the basis of the respectively determined synchronization error by using DFT. Carried out simulation experiments indicate that the correction can theoretically improve the accuracy of measurement of the RMS voltage to the level below 1 ppm. The results of measurements taken by a highly accurate voltmeter Agilent 3458 confirm the conclusions of the simulation, that it is possible to repeatedly reduce the frequency synchronization error by applying the developed correction method in the voltage measurement.

Keywords: correction of synchronization error, accurate RMS voltage measurement, DFT algorithm.

1. Introduction

The paper presents the developed method of correcting a synchronization error by using a discrete Fourier transform (DFT) applied in the sinusoidal voltage digital measurement. The error occurs in the case when the following condition of synchronization of a frequency f_s of the measured voltage with a sampling rate f_p is not fulfilled:

$$k = f_s \cdot M / f_p \,, \tag{1}$$

where M is the number of voltage samples, and k is the coefficient of synchronization which in the case of synchronization takes values of positive integers. When desynchronization occurs, accordingly with the relationship (1), the frequency synchronization relative error δ_k is the difference of the signal frequency relative error δ_{fs} and the sampling frequency error δ_{fp}, namely:

$$\delta_k = \delta_{fs} - \delta_{fp} \,. \tag{2}$$

The described method of correction can be applied in the most accurate measurements of voltage presented e.g. in [1, 2, 3, 4] with assumption that it should allow the reduction of the sine wave voltage measurement error, arising from δ_k, to the level of 1 ppm. In addition, with regard to the accuracy of the crystal clocks used in measuring instruments, it has been assumed that the modulus of the corrected error δ_k would not be greater than 0.02%.

2. The concept of frequency synchronization error correction

The concept of frequency synchronization error correction is based on the finding that in the case when quantities f_s and f_p are burdened with errors δ_{fs} and δ_{fp} , respectively, a correction p for the signal frequency can be found to provide the required synchronization. This follows from the formula:

$$k = \frac{\left(f_s\left(1+\delta_{fs}\right)+p\right)\cdot M}{f_p\left(1+\delta_{fp}\right)} \,, \tag{3}$$

which transforms into the equation (1) for $p = -\delta_k f_s$.

Let us consider how the correction p can be determined in practice and applied to correct measurement results. Let $u(t, \delta_k)$ be the sinusoidal voltage with the amplitude U_m, the initial phase $\varphi = 0$ and the frequency f_s with assumption that these values depend on the time t and are burdened by synchronization error δ_k. If the voltage is sampled uniformly with the frequency f_p and the number of samples equals $2M$, then the sequence of samples described by the formula:

$$\begin{aligned} u(n,\delta_k) &= u(n,\frac{\Delta_k}{f_s}) = U_m \sin\left(2\pi \frac{f_s+\Delta_k}{f_p} n\right) = \\ &= U_m \sin\left(2\pi\left(\frac{f_s}{f_p}+\frac{\delta_k k}{M}\right) n\right), \end{aligned} \tag{4}$$

is obtained, where $n = 0,1,..,2M-1$, and $\Delta_k = \delta_k f_s$ is the absolute frequency synchronization error.

The samples $u(n,\delta_k)$ can be divided into two equal parts being two sequences of samples: $u_1(n,\delta_k)$ for $n = 0,1,.., M-1$ and $u_2(n,\delta_k)$ for $n = M, M+1,.., 2M-1$. After applying the discrete Fourier transform (DFT) for both sequences, the following formulas are obtained:

$$\begin{aligned} \overline{X}_{u1}(k,\delta_k) &= \sum_{n=0}^{M-1} u(n,\delta_k)\cdot e^{-\mathrm{j}2\pi nk\frac{1}{M}}, \\ \overline{X}_{u2}(k,\delta_k) &= \sum_{n=M}^{2M-1} u(n,\delta_k)\cdot e^{-\mathrm{j}2\pi nk\frac{1}{M}} . \end{aligned} \tag{5}$$

On the basis of (5), the complex values of fundamental harmonics, determined by number k from the equation (1), are calculated. Determining the difference of arguments of the complex functions (5), we obtain relation describing the phase shift between voltage $u_1(n, \delta_k)$ and $u_2(n, \delta_k)$ as:

$$\begin{aligned} \phi(k,\delta_k) &= \arg\left(\overline{X}_{u2}(k,\delta_k)\right) - \arg\left(\overline{X}_{u1}(k,\delta_k)\right) = \\ &= \arg\left(\overline{X}_{u2}(k,\delta_k) / \overline{X}_{u1}(k,\delta_k)\right) \end{aligned} , \tag{6}$$

the value of which depends on the k value and the δ_k error. Determining the inverse function $\delta_k = \phi^{-1}(k,\delta_f)$ of the function (6), the formula describing the frequency synchronization absolute error is obtained in the form:

$$\Delta_k = \delta_k f_s = \phi^{-1}(k,\delta_k) f_s \,. \tag{7}$$

The searched correction $p = -\Delta_k$. After introducing correction of the frequency into the measured voltage, one obtains the following formula describing re-sampling the signal:

$$u'(n,\frac{\Delta_k + p}{f_s}) = u'(n,\frac{\Delta_k - \Delta_k}{f_s}) = U_\mathrm{m}\sin\left(2\pi\frac{f_s}{f_p}n\right) \tag{8}$$

where $n = 0,1,.., M-1$.

For the samples obtained by using Eq. (8), the RMS value of the measured sinusoidal voltage can be determined on the basis of the module of the complex value obtained by means of DFT. It is:

$$U_{rms} = \mathrm{mod}\left(\overline{X}(k, \frac{\Delta_k + p}{f_s}) \right) \cdot \frac{\sqrt{2}}{M} = \\ = \mathrm{mod}\left(\sum_{n=0}^{M-1} u'(n, \frac{\Delta_k + p}{f_s}) e^{-\mathrm{j}2\pi nk \frac{1}{M}} \right) \cdot \frac{\sqrt{2}}{M} . \tag{9}$$

3. A method of determining the frequency synchronization error

The method of determining the relative frequency synchronization error δ_k is presented below on the example of the simplest case where $k = 1$ and the number of samples is small and equals $M = 3$. In such a case, the equation (6) for a sequence of samples (4) takes the form:

$$\phi(1, \delta_k) = \arg\left(\frac{\sin(6\alpha) - \sin(8\alpha)\cdot\beta + \sin(10\alpha)\cdot\beta}{-\sin(2\alpha)\cdot\beta + \sin(4\alpha)\cdot\beta} \right) \tag{10}$$

where $\alpha = \pi\left(\frac{\delta_k}{3} + \frac{1}{3}\right)$, $\beta = \left(\frac{1}{2} + \frac{\mathrm{j}\sqrt{3}}{2}\right)$.

Determination of the inverse function $\phi^{-1}(1, \delta_k)$ from the above formula, which allows calculating the δ_k, is not an easy task. A simpler way to solve this problem is to find an equation allowing the determination of the synchronization error on the basis of the equation (10) by means of the least squares method. In the considered example, it is assumed that the searched equation takes the form of a straight line $\tilde{\delta}_k = a \cdot \phi(k, \delta_k) + b$. In this case, the following matrix equation can be built as:

$$\boldsymbol{\delta} = \boldsymbol{\Phi}\cdot\mathbf{a}\,,\ \boldsymbol{\delta} = \begin{bmatrix} \delta_{k(1)} \\ \delta_{k(2)} \\ \vdots \\ \delta_{k(N)} \end{bmatrix},\ \boldsymbol{\Phi} = \begin{bmatrix} 1 & \phi(1, \delta_{k(1)}) \\ 1 & \phi(1, \delta_{k(2)}) \\ \vdots & \vdots \\ 1 & \phi(1, \delta_{k(N)}) \end{bmatrix},\ \mathbf{a} = \begin{bmatrix} b \\ a \end{bmatrix}, \tag{11}$$

where $\boldsymbol{\delta}$ is the matrix of synchronization errors, $\boldsymbol{\Phi}$ - phase shifts matrix, $\mathbf{a}$ - the matrix of searched parameters of line function, N - number of points of approximated characteristics.

The values for the required parameters can be obtained by means of the equation:

$$\mathbf{a} = (\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\delta}\,. \tag{12}$$

Assuming that $|\delta_k| \le 0.02\%$, the elements of the error matrix achieve the values: $\delta_{k(i)} = -0.0002 + 0.0004i / N$, where $i = 0,1,..N$, $N = 1000$. Then, by determining the phase shifts matrix on the basis of (10) and making calculation according to the parameter (12), $\tilde{\delta}_k = 0.15915 \cdot \phi(1, \delta_k) + 1.6 \cdot 10^{-8}$ is obtained.

Tab. 1. Formulas for calculating the synchronization error $\tilde{\delta}_k$ for different values of the synchronization coefficient k and the number of samples M

k	M	$\tilde{\delta}_k$
1	3	$0.15915\phi(1,\delta_k)$–1.6E–8
10	30	$0.015915\phi(10,\delta_k)$–1.6E–8
100	300	$0.0015915\phi(100,\delta_k)$–1.6E–8
1	20	$0.15915\phi(1,\delta_k)$–1.3E–8
10	200	$0.015915\phi(10,\delta_k)$–1.3E–8
100	2000	$0.0015915\phi(100,\delta_k)$–1.3E–8
10	600	$0.015915\phi(10,\delta_k)$–1.3E–8

The presented method of determining the frequency synchronization error may be used for larger values of k and M. Table 1 shows exemplary equations for the relative error for selected values of k and M. It is interesting that the signal frequency and the sampling frequency are absent in the formulas. This means that for a given value of k and M, these frequencies can be freely selected according to the equation (1).

4. Results of simulation experiments

Simulation experiments were performed in order to determine the RMS value of sinusoidal voltage. The calculation of the effective value was carried out on the basis of formulas (4-6, 8, 9), and the correction was calculated on the basis of the frequency synchronization error which was determined by using the method described in Section 3.

On the basis of the performed experiments, the results of RMS value measurement relative error (relative to a theoretical RMS value) resulting from the influence of a synchronization error and the additive noise of sinusoidal voltage samples were analyzed. These tests were carried out for different values of U_{m}, f_s, f_p and M satisfying the equation (1). The obtained results proved that for the particular values of k and M the relative errors of RMS values are the same regardless of the assumed value of f_s and f_p. For this reason, in Figs. 1 and 2 there are shown exemplary graphs with the relative values on both axes, without giving a specific values of f_s and f_p. Fig. 1 shows the effect of the error δ_k impact on the relative error of RMS voltage estimation, whereas in Fig. 2, the cumulative influence Gaussian noise of standard deviation σ disturbing analyzed signal and error δ_k to the maximum relative error of estimation voltage is shown. For comparison, two figures show additionally the results of simulation calculations for two algorithms, which do not apply frequency error correction, i.e. DFT and DFT with exemplary Hanning's window. These algorithms are used in [5].

a)

b)

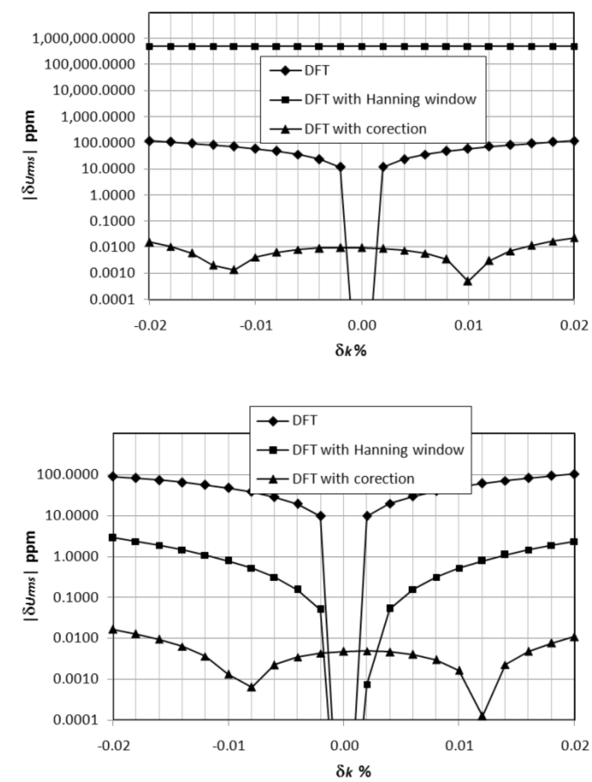
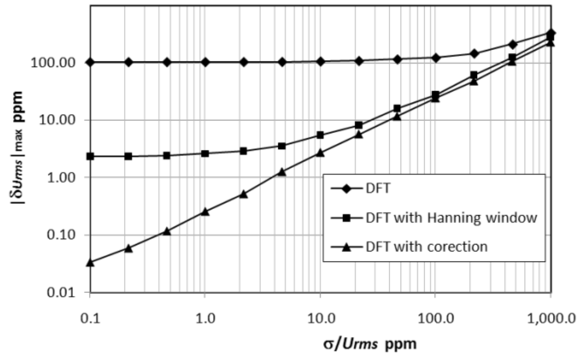


Fig. 1. The RMS voltage estimation errors: a) module of error $|\delta_{Urms}|$ as a function of error δ_k for k=1, M=3; b) module of error $|\delta_{Urms}|$ as a function of error δ_k for k=10, M=200

a)



b)

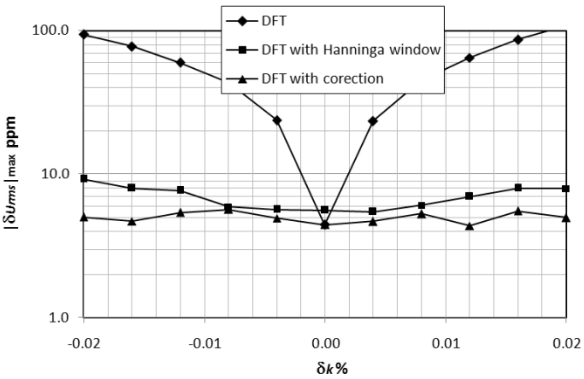


Fig. 2. The RMS voltage estimation errors: a) maximum error $|\delta_{Urms}|_{max}$ as a function of the relative standard deviation of the Gaussian noise σ/U_{rms} for δ_k =0.02%, k=10, M=200; b) maximum error $|\delta_{Urms}|_{max}$ as a function of error δ_k for σ/U_{rms}=10 ppm, k=10, M=200

5. Results of measurements

Measurements of samples of a sinusoidal voltage were made by means of a highly accurate Agilent 3458A sampling voltmeter activated by Agilent 33220 generator (Figure 3). The voltmeter was connected to a PC, in which the developed measurement application in LabWindows programming environment had been installed. This application enabled multimeter programming, reading samples of the measured voltage from the multimeter and performing calculations of the phase shift ϕ on the basis of (6) as well as the synchronization error and correction on the basis of the formulas from Table 1. The measurements were made for a sinusoidal voltage of two selected frequencies f_s = 1000 Hz and 73.42 Hz. Frequencies f_{gen} set on the generator and measurement results are shown in Tables 2 and 3. The sampling frequency before realization of the error correction was given as $f_{gen} = f_s$, while after the correction it was determined from the formula: $f_{gen} = f_s + p$. The phase shift was measured 10 times. Then, the average values of the phase $\bar{\phi}$ and standard uncertainty $u_{\bar{\phi}}$ were calculated. On the basis of phase $\bar{\phi}$, the synchronization error $\tilde{\delta}_k$ and the correction p were calculated.

6. Conclusions

The obtained results of the simulation experiments show that the proposed method of frequency synchronization error correction fulfills all the working assumption regarding the possibility to improve the accuracy of calculating the RMS value of sinusoidal voltage with an error less than 1ppm. These results indicate also that in the narrow range of δ_k the errors of this method may be greater than those obtained for the DFT algorithm with a Hanning's window. However, the additional influence in the form of noise in practice makes it possible to expect better results by means of the method of correcting synchronization error in the case of the most accurate voltage measurements. During experimental studies it was confirmed that the developed method of synchronization error correction can be useful because it allows the multiple reduction of the synchronization error.

In his further work, the author intends to show the properties of the frequency synchronization error correction method basing on the RMS voltage real measurement. This will require additional verification of voltage measurements at a very high accuracy level.

Fig. 3. View of the instruments used in the measurement of voltages

Tab. 2. Measurement results for f_s = 1 kHz, f_p = 20 kHz, k = 10, M = 200

Case	f_{gen} Hz	$\bar{\phi}$ µrad	$u_{\bar{\phi}}$ µrad	$\tilde{\delta}_k$ ppm	p Hz
Before correction of error δ_k	1000	−1400	7	−22.86	0.02286
After correction of error δ_k	1000.0229	−14	4	−0.23	0.00023

Tab. 3. Measurement results for f_s = 73.42 Hz, f_p = 4405.28 Hz, k = 10, M = 600

Case	f_{gen} Hz	$\bar{\phi}$ µrad	$u_{\bar{\phi}}$ µrad	$\tilde{\delta}_k$ ppm	p Hz
Before correction of error δ_k	73.42	−2684	3	−42.73	0.003137
After correction of error δ_k	73.423137	4.8	4	0.07	−0.000005

7. References

[1] Kampik M., Laiz H., Klonz M.: Comparison of Three Accurate Methods to Measure AC Voltage at Low Frequencies. IEEE Trans. Instr. Meas., 49(2), 2000.

[2] Kyriazis G. A.: An Algorithm for Accurately Estimating the Harmonic Magnitudes of Periodic Arbitrary Signals Using Asynchronous Sampling. Symposio de Metrologia 2004.

[3] Krajewski M.: The properties comparison of DFT and classical algorithm in voltage measurement using a sampling voltmeter. Przegląd Elektrotechniczny (Electrical Review), no 11, 2014.

[4] Rybski R., Kaczmarek J., Kozioł M.: Amplifier-aided isolation voltage transformer for use in accurate measurements in the audio frequency range. PAK, no 4, 2013.

[5] Krajewski M.: Construction of uncertainty budget using the Monte Carlo method for analysis of digital signal algorithm properties. PAK, no 9, 2012.

Received: 25.04.2016 Paper reviewed Accepted: 01.06.2016

Mariusz KRAJEWSKI, PhD, eng.

He graduated with a masters degree (2001) with specialization in digital measuring systems from the Technical University of Zielona Góra. Doctoral thesis defended (2009) in the discipline of electrical engineering at the Faculty of Electrical Engineering, Computer Science and Telecommunications at the University of Zielona Gora. Research interests focus on issues relating to accurate measurements of voltage, digital signal processing and determination of measurement uncertainty.

e-mail: M.Krajewski@imei.uz.zgora.pl