

## Optimization of linear parametric circuits in the frequency domain

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**Abstract.** The possibility of application of the frequency symbolic method of analysis of linear parametric circuits to the decision of optimization tasks is considered. There are examples of optimization of single- and double-circuit parametric amplifier using the objective function based on the calculation of parametric transfer function of a circuit with a symbolic representation of the parameters of the parametric capacity. By the frequency symbolic method the parametric transfer functions are approximated by trigonometric polynomials of Fourier.

**Key words:** symbolic frequency method, objective function, function – characteristics.

### INTRODUCTION

In [1,2] described symbolic frequency method (FS-method), which showed high efficiency of analysis of established modes of parametric linear circuits in the frequency domain. The FS-method is based on the solution of the so-called equation of L.A.Zade [3] and approximations of a transfer function  $W(s, t)$  of a linear parametric circuit by trigonometric polynomial of Fourier which it is convenient to represent to the complex form:

$$\hat{W}(s, t) = W_0(s) + \sum_{i=1}^k [W_{+i}(s) \cdot \exp(+j \cdot i \cdot \Omega \cdot t) + W_{-i}(s) \cdot \exp(-j \cdot i \cdot \Omega \cdot t)], \quad (1)$$

where:  $s = j\omega$  - the complex variable of the Laplace transform,  $t$  - time,  $T = 2\pi/\Omega$  - period of change of parameter of parametric element,  $k$  - the number of members in approximating polynomial. The solution of the differential equation of L.A.Zade for approximation of the solution by expression (1) is translated in the solution of system of the linear algebraic equations that

are independent of a time (SLAE) [2], usually, by that,  $s$  and some or all parameters of elements of circuit are given by symbols (the variable  $t$  is present only in the exponential term of (1) and also symbolic). The result of solving SLAE is a searched fractional rational expressions  $W_0(s)$ ,  $W_{+i}(s)$ ,  $W_{-i}(s)$  of approximating polynomial (1). The value  $k$  is chosen such that provides the necessary accuracy of coincidence functions  $W(s, t)$  and  $\hat{W}(s, t)$  [1,2].

The present paper to determine the optimal values of parameters of elements of parametric circuit are used mentioned transfer functions. Calculations carried out in environment of MATLAB 7.6.0 using the program SAPC [4].

### THE FORMULATION OF OPTIMIZATION TASK

Usually in optimization tasks the objective function (optimality criterion) is the function that evaluates the quality of the optimization and implies the existence of the varied parameters whose values in process of optimizing changing and affect the value of the objective function. The solution of optimization task deem such final values of varied parameters providing the minimum (maximum) value of the objective function at specific limitations.

During optimization of characteristics of electrical circuits formation of objective function are often is done via the other two functions - the function of goal that is defines of desirable characteristics of circuit (goal of optimization), and function- characteristics of circuit by the selected values of the varied parameters. Function of goal is determined in the space of independent variables, in our case, a complex variable  $s$  and time  $t$  and does not depend on varied parameters. Function-charac-

teristics of circuit determined by in the space of the same independent variables  $s$  and  $t$ , however, depends on the varied parameters. The degree of coincidence of these two functions - the function of goals and function characteristics - is objective function which is formed on their basis by the chosen method [5]. Thus coincidence determined by for a number of specific values of the independent variables  $s_i = j\omega_i$  and  $t_j$ , which is usually in the optimization process are fixed, and thus objective function is a function only the varied parameters.

In this paper the objective function  $F$  formed on the basis transfer functions of the form (1). Let us consider that the parametric element of the circuit is parametric capacity, which periodically varying in time  $t$  according to the expression:

$$c(t) = c_0(1 + m \cdot \cos(\Omega \cdot t)). \quad (2)$$

Parameters  $c_0$  and  $m$  choose the varied, so in the calculation the transfer function of circuit they should be left in the form of symbols. By changing the latter necessary determine such optimal values of  $c_0^*$  and  $m^*$ , that are providing maximum coincidence module  $M_w$  transfer function of circuit  $W(c_0, m, \omega, t)$  with module  $M_0(\omega, t)$  given function in the frequency  $\omega_i$  and time  $t_j$  points, though, by the criterion of minimum of sum of squared deviations [5]:

$$F(c_0, m) = \sum_{i=1}^p \sum_{j=1}^q (M_w(c_0, m, \omega_i, t_j) - M_0(\omega_i, t_j))^2. \quad (3)$$

In this way, the functions  $M_0(\omega, t)$  and  $M_w(c_0, m, \omega, t)$  - is a function of goal and function-characteristics, respectively. Thus, solving of optimization task is to determine the values of  $c_0^*$  and  $m^*$  that provide minimum value of objective function  $F(c_0, m) = F_{\min}$  with restrictions  $c_0 > 0$ ,  $0 < m < 1$ .

### THE PROCEDURE OF OPTIMIZATION

The procedures of optimization are implementing by the following method:

1. The function of goal  $M_0(\omega, t)$ , as a function of two variables, set by plural of values in a discrete of points  $\omega_i, t_j$  as a surface in coordinates of frequency-time.

2. The function characteristics of circuit  $W(c_0, m, \omega, t)$  we determine by the frequency symbolic method in the form of (1) provided that the varied parameters are given in symbols. According the obtained expression we calculate the set of values of the function characteristics in the same discrete points  $\omega_i, t_j$  that the function of goal, but with unknown (the varied) parameters given in symbolic form.

3. The objective function  $F(c_0, m)$  (optimality criterion) are forming as the sum of squares of

deviations between the values of the function-characteristics and function of goal at selected discrete points  $\omega_i, t_j$ , as surface in coordinates of the varied parameters.

4. The minimum value of the objective function defined by one of the selected optimization methods determines desired values the varied parameters.

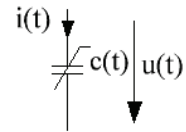
5. As a limitation, the value unknown parameters are selected based on the capabilities of their physical implementation and ensure stability established mode of circuit.

Optimization in the following examples done by the tools of MATLAB 7.6.0 functions [6,7]: «fminunc», «fminsearch» and «patternsearch».

### THE EXAMPLES OF OPTIMIZATION OF LINEAR PARAMETRIC CIRCUITS

**Example 1.** Let the parametric circuit represents a separate parametric capacity  $c(t)$  from fig.1. Necessary to determine the value of  $c_0^*$  and  $m^*$ , that the provide minimum of objective function  $F(c_0, m) = F_{\min}$  for the input resistance of circuit.

**Fig. 1.** Parametric capacity  $c(t) = c_0(1 + m \cos(\Omega \cdot t))$ ,  $\Omega = 1 \text{ rad/s}$



Parametric transfer function of the input resistance  $Z(s, t)$  of the given circuit has the exact solution in the form:

$$Z(s, t) = \frac{U(s, t)}{I(s)} = \frac{1}{s \cdot c(t)} = \frac{1}{j \cdot \omega \cdot c_0 (1 + m \cos(\Omega \cdot t))}. \quad (4)$$

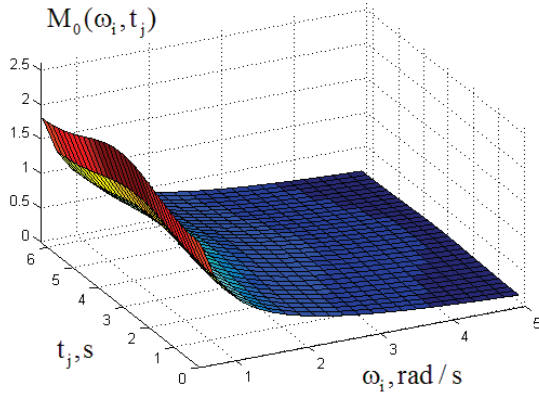
According to expression (3) we form the objective function:

$$F(c_0, m) = \sum_{i=1}^{23} \sum_{j=1}^{32} (M_z(c_0, m, \omega_i, t_j) - M_0(\omega_i, t_j))^2, \quad (5)$$

where:  $M_z(c_0, m, \omega_i, t_j)$  - function characteristics, which is a module of the transfer function  $Z(c_0, m, \omega_i, t_j) = \frac{1}{j \cdot \omega_i \cdot c_0 (1 + m \cos(1 \cdot t_j))}$  of

circuit when  $\omega = \omega_i, t = t_j$  and  $M_0(\omega_i, t_j)$  - function of goal, which is a module of the transfer function  $Z_0(\omega_i, t_j) = \frac{1}{j \cdot \omega_i \cdot 1 \cdot (1 + 0.1 \cdot \cos(1 \cdot t_j))}$  of circuit when

$c_0 = 1 \text{ F}, m = 0, 1$  in the points  $\omega = \omega_i, t = t_j$ .



**Fig. 2.** Module  $M_0(\omega_i, t_j)$  function of goal of parametric capacity

On fig.2 shown a graphic view of module  $M_0(\omega_i, t_j)$  of function of goal for values  $t_j$  and  $\omega_i$  selected accordingly within the limits 0.5 – 5 s with a step 0.2 s and 0 – 6.28 rad/s with a step 0.2 rad / s. On fig.3 shown a graphic view of the objective function  $F(c_0, m)$  for values  $c_0$  within the limits 0.5 - 1.5 F with a step 0.02 F and values  $m$  within the limits 0.05 - 0.15 with a step 0.002 for the same values of  $t_j$  and  $\omega_i$ , respectively.

Function of optimization «fminunc» when the initial values  $c_0 = 0.6$  F and  $m = 0.05$  for the 7 iterations has identified a minimum  $F_{\min}$  for  $c_0^* = 1$  F and  $m^* = 0.1$  is marked on fig.3 by symbol  $\square$ . Functions of optimization «fminsearch» and «patternsearch» with the same initial values of  $c_0 = 0.6$  F and  $m = 0.05$  have given the same values  $c_0^* = 1$  F and  $m^* = 0.1$  for 40 and 82 iterations, respectively.

**Example 2.** Determine the value of  $c_0^*$  and  $m^*$ , which provide minimum of the objective function  $F(c_0, m) = F_{\min}$  for parametric transfer function  $Z(s, t) = U_2(s, t)/I(s)$  of single-circuit parametric amplifier of fig. 4. According to FS-method [1,2,4] parametric transfer function  $Z(s, t)$  has the form:

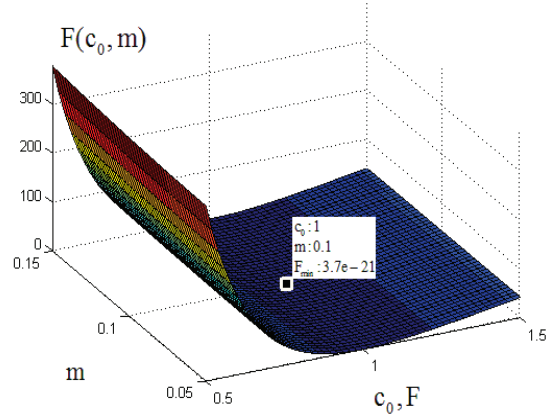
$$Z(m, c_0, s, t) = \frac{U_2(s, t)}{I(s)} = Z_0(m, c_0, s) + Z_{-1}(m, c_0, s) \cdot \exp(-j \cdot \Omega \cdot t) + Z_{+1}(m, c_0, s) \cdot \exp(j \cdot \Omega \cdot t). \quad (6)$$

Fractional-rational expressions  $Z_0(m, c_0, s)$ ,  $Z_{-1}(m, c_0, s)$ ,  $Z_{+1}(m, c_0, s)$  are not present due to their cumbersome (they are given in [4]).

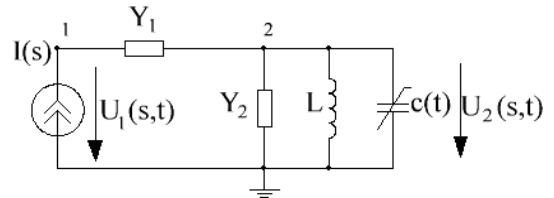
According to expression (3) we form the objective function:

$$F(c_0, m) = \sum_{i=1}^5 \sum_{j=1}^{11} (M_Z(c_0, m, \omega_i, t_j) - M_0(\omega_i, t_j))^2, \quad (7)$$

where:  $M_Z(c_0, m, \omega_i, t_j)$ - function characteristics, which is a module of the transfer function:



**Fig. 3.** The objective function  $F(c_0, m)$  in the coordinates  $c_0$  and  $m$



**Fig. 4.** Single-circuit parametric amplifier.  $c(t) = c_0(1 + \cos(4 \cdot 10^8 \pi t))$  F,  $I(s) = 10^{-4} \exp(j \cdot 2 \cdot 10^8 \pi t - \pi/4)$  A,  $L = 253.3$  nH,  $Y_1 = 0.25$  S,  $Y_2 = 0.4$  mS

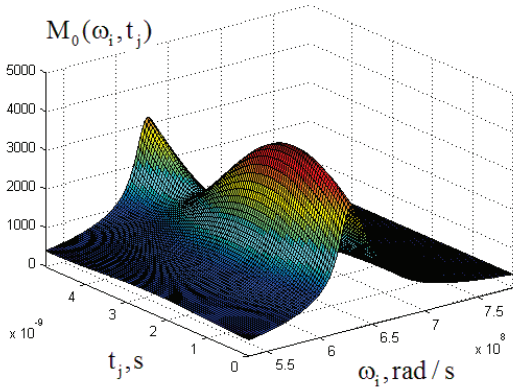
$Z(m, c_0, \omega_i, t_j) = Z_0(m, c_0, \omega_i) + Z_{-1}(m, c_0, \omega_i) \cdot \exp(-j \cdot 4\pi 10^8 \cdot t_j) + Z_{+1}(m, c_0, \omega_i) \cdot \exp(+j \cdot 4\pi 10^8 \cdot t_j)$  of circuit when  $\omega = \omega_i$ ,  $t = t_j$  and  $M_0(\omega_i, t_j)$  - function of goal, which is a module of the transfer function:

$$Z(\omega_i, t_j)_0 = Z_0(\omega_i) + Z_{-1}(\omega_i) \cdot \exp(-j \cdot 4\pi 10^8 \cdot t_j) + Z_{+1}(\omega_i) \cdot \exp(+j \cdot 4\pi 10^8 \cdot t_j),$$

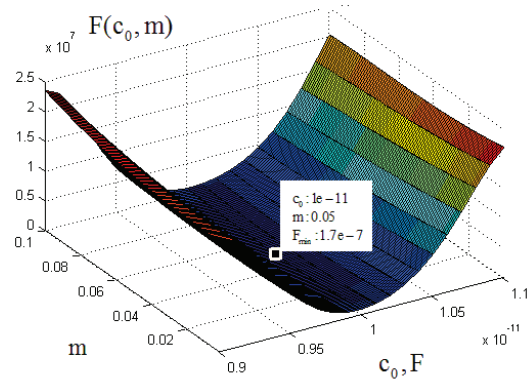
of circuit when:  $c_0 = 10$  pF,  $m = 0.05$  in the points  $\omega = \omega_i, t = t_j$ .

On fig.5 shown a graphic view of module  $M_0(\omega_i, t_j)$  of function of goal for values  $\omega_i$  and  $t_j$  selected accordingly within the limits  $1.7 \cdot \pi \cdot 10^8 - 2.5 \cdot \pi \cdot 10^8$  rad/s with a step  $0.02 \cdot \pi \cdot 10^8$  rad/s and  $0 - 5 \cdot 10^{-9}$  s with a step  $0.05 \cdot 10^{-9}$  s. On fig.6 shown a graphic view of the objective function  $F(c_0, m)$  for values  $c_0$  within the limits  $9 \cdot 10^{-12} - 11 \cdot 10^{-12}$  F with a step  $0.02 \cdot 10^{-12}$  F and values  $m$  within the limits 0.001–0.1 with a step 0.001 for the same values of  $t_j$  and  $\omega_i$ , respectively.

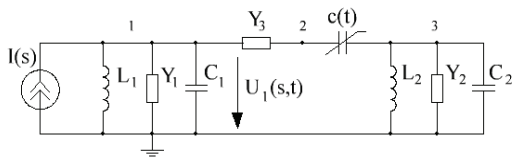
Function of optimization «fminsearch» when the initial values  $c_0 = 0.9 \cdot 10^{-11}$  F and  $m = 0.01$  for the 57 iterations has identified a minimum  $F_{\min}$  for  $c_0^* = 1 \cdot 10^{-11}$  F and  $m^* = 0.05$  is marked on fig.6 by symbol  $\square$ . Function of optimization «patternsearch» under the same initial values of  $c_0 = 0.9 \cdot 10^{-11}$  F and  $m = 0.01$  for 758 iterations has given the same values  $c_0^* = 1 \cdot 10^{-11}$  F and  $m^* = 0.05$ . By function «fminunc» result does not obtained.



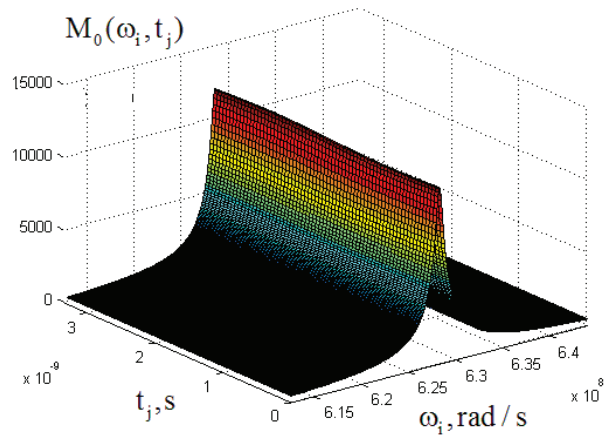
**Fig. 5.** Module  $M_0(\omega_i, t_j)$  function of goal of single-circuit parametric amplifier



**Fig. 6.** The objective function  $F(c_0, m)$  in the coordinates  $c_0$  and  $m$



**Fig. 7.** Double-circuit parametric amplifier.  $c(t)=c_0(1+m\cos(597.146\cdot 10^6\pi t))$  F,  $I(s)=10^{-4}\exp(j\cdot 2\cdot 10^8\pi t+\pi/4)$  A,  $L_1=36.70795$  nH,  $L_2=9.312609$  nH,  $Y_1=0.1$  mS,  $Y_2=0.1$  mS,  $Y_3=0.5$  S,  $C_1=68$  pF,  $C_2=68$  pF



**Fig. 7.** Module  $M_0(\omega_i, t_j)$  function of goal of double-circuit parametric amplifier

Example 3. Determine the value of  $c_0^*$  and  $m^*$ , which provide minimum of the objective function  $F(c_0, m) = F_{\min}$  for parametric transfer function  $Z(s, t) = U_1(s, t)/I(s)$  of input resistance double-circuit parametric amplifier of fig. 7. According to FS-method [1,2,4] parametric transfer function of input resistance  $Z(s, t)$  has the form:

$$Z(m, c_0, s, t) = \frac{U_1(s, t)}{I(s)} = Z_0(m, c_0, s) + Z_{-1}(m, c_0, s) \cdot \exp(-j \cdot \Omega \cdot t) + Z_{+1}(m, c_0, s) \cdot \exp(j \cdot \Omega \cdot t). \quad (8)$$

Fractional-rational expressions  $Z_0(m, c_0, s)$ ,  $Z_{-1}(m, c_0, s)$ ,  $Z_{+1}(m, c_0, s)$  they are given in [4].

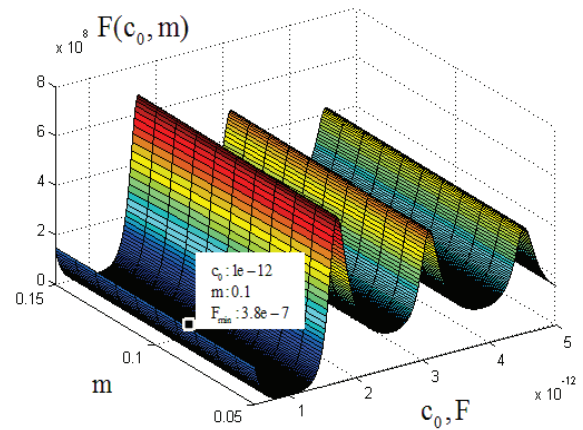
According to expression (3) we form the objective function:

$$F(c_0, m) = \sum_{i=1}^5 \sum_{j=1}^6 (M_Z(c_0, m, \omega_i, t_j) - M_0(\omega_i, t_j))^2, \quad (9)$$

where:  $M_Z(c_0, m, \omega_i, t_j)$  - function characteristics, which is a module of the transfer function

$$Z(m, c_0, \omega_i, t_j) = Z_0(m, c_0, \omega_i) + Z_{-1}(m, c_0, \omega_i) \cdot \exp(-j597.146\pi 10^6 t_j) + Z_{+1}(m, c_0, \omega_i) \cdot \exp(+j597.146\pi 10^6 t_j)$$

of circuit when  $\omega = \omega_i$ ,  $t = t_j$  and  $M_0(\omega_i, t_j)$  - function of goal, which is a module of the transfer function



**Fig. 8.** The objective function  $F(c_0, m)$  in the coordinates  $c_0$  and  $m$

$$Z_0(\omega_i, t_j) = Z_0(\omega_i) + Z_{-1}(\omega_i) \cdot \exp(-j \cdot 597.146 \cdot \pi \cdot 10^6 \cdot t_j) + Z_{+1}(\omega_i) \cdot \exp(+j \cdot 597.146 \cdot \pi \cdot 10^6 \cdot t_j)$$

of circuit when:

$$c_0=1 \text{ pF}, m=0.1 \text{ in the points } \omega = \omega_i, t = t_j.$$

On fig. 8 shown a graphic view of module  $M_0(\omega_i, t_j)$  of function of goal for values  $\omega_i$  and  $t_j$  selected accordingly within the limits  $1.95 \cdot \pi \cdot 10^8 - 2.05 \cdot \pi \cdot 10^8$  rad/s with a step  $0.0005 \cdot 10^8$  rad/s and  $0 - 3.35 \cdot 10^{-9}$  s with a step  $0.05 \cdot 10^{-9}$  s. On fig.9 shown a graphic view of the objective function  $F(c_0, m)$  for values  $c_0$  within the limits  $0.5 \cdot 10^{-12} - 5 \cdot 10^{-12}$  F with a step  $0.01 \cdot 10^{-12}$  F and values  $m$  within the limits  $0.05 - 0.15$  with a step  $0.01$  for the same values of  $t_j$  and  $\omega_i$ , respectively.

Function of optimization «fminsearch» when the initial values  $c_0 = 1.2 \cdot 10^{-12}$  F and  $m = 0.15$  for the 160 iterations has identified a minimum  $F_{\min}$  for  $c_0^* = 1 \cdot 10^{-12}$  F and  $m^* = 0.1$  is marked on fig.9 by symbol  $\square$ . Function of optimization «patternsearch» under the same initial values of  $c_0 = 1.2 \cdot 10^{-12}$  F and  $m = 0.15$  for 524 iterations has given the same values  $c_0^* = 1 \cdot 10^{-12}$  F and  $m^* = 0.1$ . By function «fminunc» result does not obtained.

#### CONCLUSIONS

1. Frequency symbolic method of analysis allows solving optimization task of designing of parametric linear circuits in the frequency domain based on use of the frequency symbolic transfer functions which are approximated by trigonometric polynomials of Fourier, particularly in complex form.

2. Surfaces of the objective function the case of linear parametric circles formed from transfer functions, which, in turn, are represented surfaces, because they contain two (not one, as in the case of linear circuits with constant parameters) of the independent variables - complex variable  $s$  and time  $t$ .

3. For all the above examples used function of optimization of MATLAB 7.6.0, in particular, «fminunc», «fminsearch» and «patternsearch» define minimum of objective functions in the same values of the variables  $s$  and  $t$ , although for different number of iterations.

4. Maximum values of variables  $i$  and  $j$  objective functions in the examples above are different, because every time were defined by practical possibilities by MATLAB 7.6.0 on a computer with a processor AMD TurionX2 Dual Core Mobile RM-76 2.30 GHz and operative memory 3.00 Gb.

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