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EFFECT OF CONVECTIVE HEAT AND MASS CONDITIONS IN MAGNETOHYDRODYNAMIC BOUNDARY LAYER FLOW WITH JOULE HEATING AND THERMAL RADIATION

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A free convection viscous MHD flow over a semi-infinite vertical sheet with convective heat and mass conditions has been considered. The effects of thermal radiation, chemical reaction and Joule heating on flow are also accounted. The governing nonlinear partial differential equations have been transformed into a set of highly non-linear coupled ordinary differential equations (ODEs) using appropriate similarity transformations. Numerical solutions of transformed equations are obtained by employing the 5^{th} order Runge-Kutta Fehlberg technique followed by the shooting technique. The influences of different flow parameters on the momentum, energy and mass field are discussed and shown graphically. Results reveal that temperature and concentration profiles enhance due to increasing heat and mass Biot number parameters.

Key words: convection, MHD, Joule heating, chemical reaction, thermal radiation.

1. Introduction

Convective condition at the boundary is used to characterize the convective energy transport condition throughout the surface and is directly related to the difference in fluid temperature with the ambient situation. Application of this type of condition can be seen in various manufacturing devices such as heat exchangers, gas turbines, atomic plants, thermal energy storage, and many others. Further, the use of surface convective heat along with concentration boundary conditions is a novel concept that is being inspected currently. Based on these applications, numerous investigations were carried out by various researchers. The mixed influence of natural convection, as well as heat and mass transfer on an unsteady magnetohydrodynamic motion of (Walters fluid Model-B) fluid in an asymmetrical channel under convective boundary, has been reported by Sivaraj and Rushi Kumar [1]. Thermo diffusion impact on combined convection motion of nanofluid was explored by Reddy *et al.* [2]. Hayat *et al*. [3] examined the influence of thermo-diffusion and diffusion thermo influences on Williamson liquid past an unsteady extending surface in the presence of thermal radiation, viscous dissipation and energy, and mass convective conditions. Uddin *et al.* [4] analytically examined the effect of Brownian and thermophoretic diffusions of natural convection motion of nanofluid in a Darcian permeable media in the presence of convective

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boundary conditions. Sheikholeslami *et al*. [5] have conducted the numerical investigation on magnetohydrodynamic free convection flow of Al_2O_3- water-based nanofluid with thermal radiation. Sharma *et al*. [6] have carried out the numerical study of chemical reaction impact on magnetohydrodynamic rotating liquid past a vertical surface with variable thermal conductivity. Mishra *et al.* [7] examined the magnetohydrodynamic effect on viscoelastic liquid past an extending sheet. The effect of convective boundary conditions on the three-dimensional double-diffusive flow of non-Newtonian fluid motion has been examined by Gireesha *et al*. [8] Ramzan *et al.* [9] explored the impact of viscous dissipation and convective conditions on flow of Casson nanofluid.

Nowadays the investigation of prominent magnetohydrodynamic (MHD) fluid flow has engaged several researchers due to its numerous applications in different fields. Its importance on fluid flow was described by Hartman [10] during the study of electromagnetic pump fabrication. Such type of flow was utilized in the power generator system in which heat energy is converted into electric energy. A study on the MHD motion of Maxwell liquid has been investigated by Archana *et al*. [11] and has analyzed its impact on the velocity and temperature fields. Time-dependent MHD flow of electrically conducting Newtonian fluid has been analyzed by Makinde *et al*. [12]. Some more studies on MHD flow are reported by several researchers [13] - [16][13-16] under various physical conditions.

Many researchers have carried out the theoretical as well as experimental investigations on the effect of radiation in fluid flow in which they incorporated Cogley approximation of linear type. Its impact is of considerable interest in various non-isothermal conditions [17] - [20]. Further, the cause of Joule heating due to the applied electric field around conducting fluid. This impact generates temperature gradient and leads to enhanced energy which intake helps in the decomposition of thermally labile samples and the formation of gas bubbles. The role of Joule heating in different fluid flows was studied by several authors [21] - [24] under different physical situations. Kumar *et al.* [26] investigated the nonlinear radiation effect on nanofluid due to a stretching surface with a chemical reaction.

It has been observed that the energy and mass transfer examination in the over has been frequently dispensed under the influence of the boundary condition either via a given temperature or heat flux at the sheet. Only some works in this direction are completed considering heat convective condition at the sheet in place of prearranged sheet temperature or heat flux. Though, no effort has been up till now for the convective mass condition at the sheet. This investigation establishes such a condition in the study. Therefore, the present study is concentrated on heat and mass transfer of laminar magnetohydrodynamic motion of fluid along with convective boundary conditions. The governing dimensionless nonlinear ODEs are explained numerically by using the Runge-Kutta Fehlberg-Fifth order method with the help of the shooting method. Graphs of various interesting physical parameters are plotted for the momentum, heat and concentration profiles.

2. Mathematical formulation

Two-dimensional (x^*, y^*) steady, laminar, MHD motion of an electrically conducting fluid past an extending sheet is addressed. Convective heat and mass conditions are taken into account. The magnetic field of strength B_0 has been is applied normal to the flow field (see Fig.1). The magnetic Reynolds number is chosen pretty small. As a consequence, the induced magnetic field is small in comparison to the applied magnetic field. Thus, the induced magnetic field is not considered. The influence of viscous dissipation and radiation is further considered.

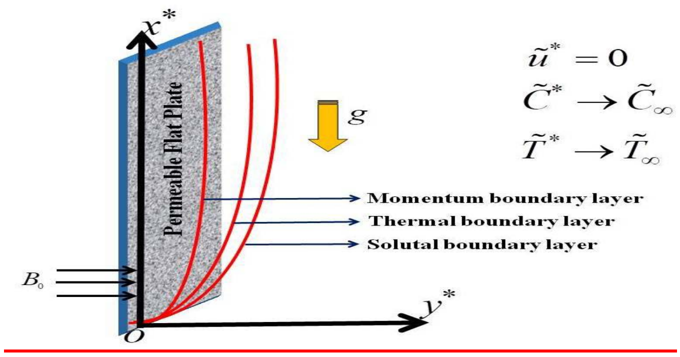

Fig.1. Flow diagram.

The two-dimensional MHD equations for formulated problem are given by

$$\tilde{v}_y{}^* = 0\,, \tag{2.1}$$

$$\tilde{v}^*\tilde{u}_{\tilde{y}}{}^* = \nu\tilde{u}^*_{\tilde{y}\tilde{y}} + g\beta_1\left(\tilde{T}^* - \tilde{T}_\infty\right) + g\beta^*\left(\tilde{C}^* - \tilde{C}_\infty\right) - \frac{\nu}{K^*}\tilde{u}^* - \frac{\sigma B_0^2\tilde{u}^*}{\rho}\,, \tag{2.2}$$

$$\tilde{v}*\tilde{T}_{\tilde{y}^*}{}^* = \frac{\kappa}{\rho C_p}\tilde{T}_{\tilde{y}^*\tilde{y}^*}{}^* + \frac{\nu}{C_p}\left(\tilde{u}^*_{\tilde{y}}\right)^2 - \frac{1}{\rho C_p}q_{r_{\tilde{y}^*}} - \frac{\sigma B_0^2}{\rho C_p}\tilde{u}^{*2}, \tag{2.3}$$

$$\tilde{v}^*\tilde{C}_{\tilde{y}^*}{}^* = D\tilde{C}^*_{\tilde{y}^*\tilde{y}^*} - k_c\tilde{C}^*\,. \tag{2.4}$$

The solution of Eq.(2.1) is

$$\tilde{v}^* = \text{constant} = -v_0 \qquad \text{and} \qquad v_0 > 0\,. \tag{2.5}$$

It is emphasized by Cogley *et al*. [25] that the rate of radiative flux in theoptically thin limit for a non-Gray gas near equilibrium is given by

$$q_{r_{\tilde{y}^*}} = 4\left(\tilde{T}^* - \tilde{T}_\infty\right)I' \tag{2.6}$$

where $$I' = \int K_\lambda \frac{\partial e_\lambda}{\partial \tilde{T}^*}d\lambda.$$

Subjected to appropriate boundary conditions

$$\begin{aligned} &\tilde{u}^* = 0, \quad k_f\tilde{T}^*_{\tilde{y}^*} = h_f\left(\tilde{T}_f - \tilde{T}\right), \quad -D\tilde{C}^*_{\tilde{y}^*} = k_m\left(\tilde{C}_f - \tilde{C}\right) \quad \text{at} \quad \tilde{y}^* = 0\,, \\ &\tilde{u}^* \to 0, \quad \tilde{T}^* \to \tilde{T}_\infty, \quad \tilde{C}^* \to \tilde{C}_\infty \quad \text{at} \quad \tilde{y}^* \to \infty\,. \end{aligned} \tag{2.7}$$

Upon using Eqs (2.5) and (2.6), Eqs (2.2), (2.3) and (2.4) will take the following form

$$-v_0\tilde{u}^*_{\tilde{y}^*} = \nu\tilde{u}^*_{\tilde{y}^*\tilde{y}^*} + g\beta_1\left(\tilde{T}^* - \tilde{T}_\infty\right) + g\beta^*\left(\tilde{C}^* - \tilde{C}_\infty\right) - \frac{\nu\tilde{u}^*}{K^*} - \frac{\sigma B_0^2\tilde{u}^*}{\rho}, \tag{2.8}$$

$$-v_0\tilde{T}^*_{\tilde{y}^*} = \frac{\kappa}{\rho C_p}\tilde{T}^*_{\tilde{y}^*\tilde{y}^*} + \frac{\upsilon}{C_p}\left(\tilde{u}^*_{\tilde{y}^*}\right)^2 - \frac{1}{\rho C_p}4\left(\tilde{T}^* - \tilde{T}_\infty\right)I' - \frac{\sigma B_0^2}{\rho C_p}\tilde{u}^{*2}, \tag{2.9}$$

$$-v_0\tilde{C}^*_{\tilde{y}^*} = D\tilde{C}^*_{\tilde{y}^*\tilde{y}^*} - kc\tilde{C}^*. \tag{2.10}$$

Consider the following non-dimensional parameters

$$f(\eta) = \frac{\tilde{u}^*}{v_0}, \quad \eta = \frac{v_0\tilde{y}^*}{\nu}, \quad \Pr = \frac{\nu\rho C_p}{\kappa}, \quad F = \frac{4\nu I'}{\kappa\nu_0^2},$$

$$\alpha = \frac{\nu_0^2 K^*}{\nu^2}, \quad \theta(\eta) = \frac{\tilde{T}^* - \tilde{T}}{\tilde{T}_f - \tilde{T}_\infty}, \quad \phi(\eta) = \frac{\tilde{C}^* - \tilde{C}}{\tilde{C}_f - \tilde{C}_\infty}. \tag{2.11}$$

Using the dimensionless quantities given by Eqs (2.11), the Eqs (2.8)-(2.10) will reduce into

$$f'' + f' - \left(\frac{1}{\alpha} + M\right)f' = -\mathrm{Gr}\theta - \mathrm{Gm}\phi, \tag{2.12}$$

$$\theta'' + \Pr\theta' - F\theta = -E\Pr f'^2 - \Pr EMf^2, \tag{2.13}$$

$$\phi'' + \mathrm{Sc}\phi' = \mathrm{Sc}K_r\phi \quad , \tag{2.14}$$

here the prime denotes differentiation w.r. to η.

$$\mathrm{Gr} = \frac{g\beta_1\left(\tilde{T}_f - \tilde{T}_\infty\right)\nu^2}{\lambda\nu_0^3}, \qquad \mathrm{Gm} = \frac{g\beta^*\left(\tilde{C}_f - \tilde{C}_\infty\right)\nu}{D\nu_0^3},$$

$$\mathrm{Sc} = \frac{\nu}{D}, \quad K_r = \frac{kc\nu}{\nu_0^2}, \quad M = \frac{\sigma\nu B_0^2}{\rho\nu_0^2}, \quad E = \frac{\nu_0^2}{C_p\left(\tilde{T}_f - \tilde{T}_\infty\right)}.$$

The corresponding boundary conditions are

$$\eta = 0, \quad f = 0, \quad \theta'(0) = -\mathrm{Bi}_1(1 - \theta(0)), \quad \phi'(0) = -\mathrm{Bi}_2(1 - \phi'(0)),$$

$$\eta \to \infty, \quad f \to 0, \quad \theta \to 0, \quad \phi \to 0 \tag{2.15}$$

where Bi_1 and Bi_2 are Biot numbers and are given by

$$\mathrm{Bi}_1 = \frac{h_f}{k_f}\sqrt{\frac{\nu}{a}}, \qquad \mathrm{Bi}_2 = \frac{k_f}{D_m}\sqrt{\frac{\nu}{a}}.$$

3. Numerical method

In the present investigation, a useful fifth-order Runge–Kutta Fehlberg scheme has been used to examine the flow. The influence of flow parameters on the dimensionless momentum, energy, and mass fields are examined. The 5th order R–K–Fehlberg technique can be established as

$$f = x_1, \quad f' = x_2, \quad \theta = x_3, \quad \theta' = x_4, \quad \phi = x_5, \quad \phi' = x_6. \tag{3.1}$$

In view of Eqs (3.1), Eqs (2.12)-(2.14) first-order ordinary differential equations as

$$x_1' = x_2, \tag{3.2}$$

$$x_2' = -x_2 + \left(\frac{1}{\alpha} + M\right)x_1 - \mathrm{Gr}\,x_3 - \mathrm{Gm}\,x_5, \tag{3.3}$$

$$x_3' = x_4, \tag{3.4}$$

$$x_4' = -\Pr x_4 + Fx_3 - E\Pr x_2^2 - \Pr EMx_1^2, \tag{3.5}$$

$$x_5' = x_6, \tag{3.6}$$

$$x_6' = -\mathrm{Sc}\,x_6 + \mathrm{Sc}\,K_r x_5. \tag{3.7}$$

Respective boundary conditions are

$$x_1(0) = 0, \quad x_2(0) = m_1, \quad x_3(0) = m_2, \quad x_4(0) = -\mathrm{Bi}_1\left(1 - x_3(0)\right),$$

$$x_5(0) = m_3, \qquad x_6(0) = -\mathrm{Bi}_2\left(1 - x_5(0)\right). \tag{3.8}$$

here m_1 to m_3 are not known and they are resolute by the shooting method.

4. Results and discussion

The explanation of the problem is gained via the shooting technique. The various effects on motion fields are presented with the help of plotted graphs [2-13]. For fixed values as revealed in figures, the impact of theporous parameter on momentum description has been examined. The enhancing values of this parameter raise the velocity as shown in Fig.2. Figure 3 displayed to view the impact of Grashof number of the mass transfer on velocity and here too the trend is similar. The impact of the Grashof number of the heat transfer on velocity increases the corresponding profile and it is observed in Fig.4. It is because Grashof number is inversely proportional to viscous force and hence when values of both Gr and Gm increase the viscous force decreases which intern increases the flow velocity. The effect of Hartman number Mreduces the velocity description because of the Lorentz force which is a resistive force exerted against flow velocity and it can be seen in Fig.5.

The effect of variation in $\mathrm{Bi}_1$is shown in Fig.6 on the temperature profile. The increasing values of Bi_1 increase the temperature description. This is because as Bi_1 enhances, heat transfer via convection takes place largely. Therefore, theheat profile increases with increasing Bi_1. In Fig.7, the impact of E on temperature has been presented. The enhance in E increases the temperature description. When the Eckert number increases viscous dissipation of the fluid increases which intern increases the temperature of the fluid. Figures 8 and 9 are designed to analyze the effect of F and Gr on temperature profiles respectively.

The increasing values of F reducing the temperature while enhancing the values of Gr enhances the temperature profile. In Fig.10, the impact of Pr on temperature can be observed. An enhance inPr reduces the temperature description because when thePrandtl number increases thermal diffusivity decreases. The effects of Bi_2, K_r and Sc on $\varphi(\eta)$ are shown in the Figs 11, 12 and 13. The increasing value of Bi_2 increases the $\varphi(\eta)$ profile. This is because of the mass convection increases the concentration of the fluid. The uplift of K_r and Sc decreases the $\varphi(\eta)$ description. In case of K_r, chemical reaction consumes the chemical in the liquid and therefore, the mass description decreases. Again, one can notice that as Sc raises, mass description declines. This is because of the truth that Scis conversely proportional to concentration diffusivity. Consequently, an enhance in values of Sc decreases the concentration diffusion and in turn decreases mass description.

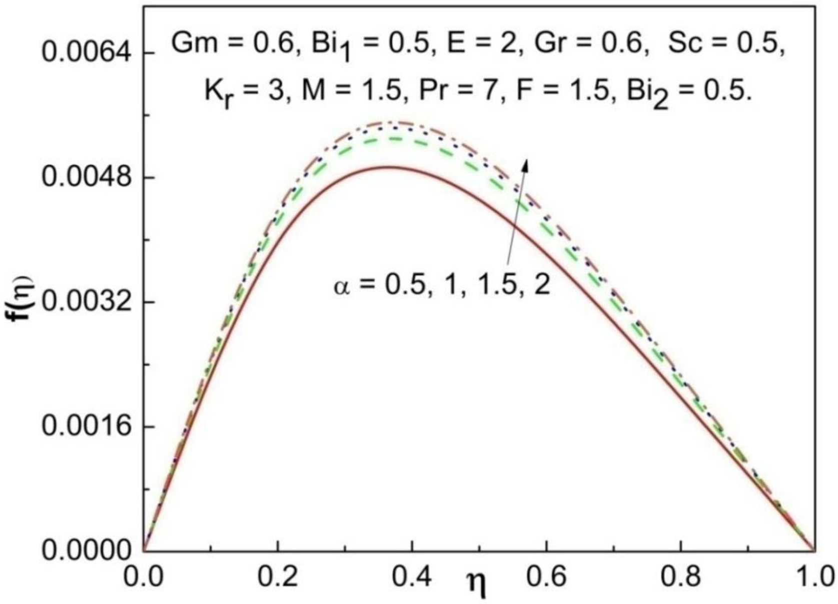

Fig.2. Consequence of α on $f(\eta)$.

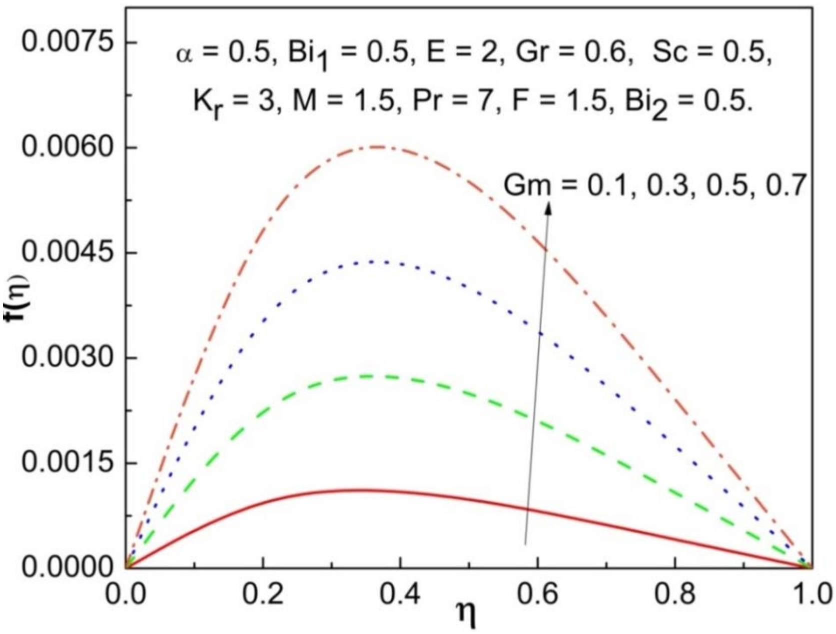

Fig.3. Consequence of Gm on $f(\eta)$.

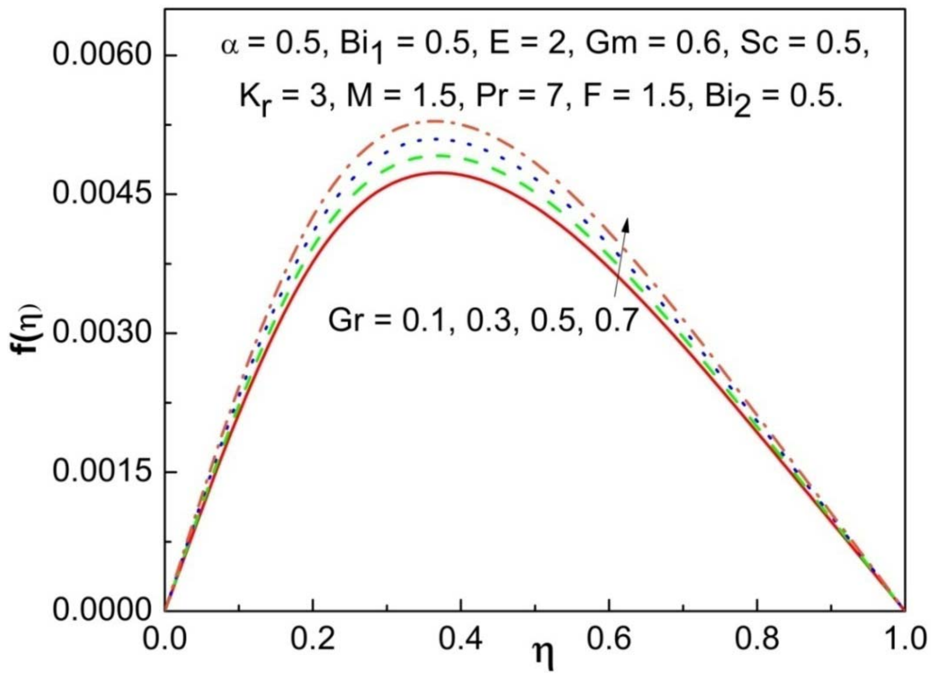

Fig.4. Consequence of Gr on $f(\eta)$.

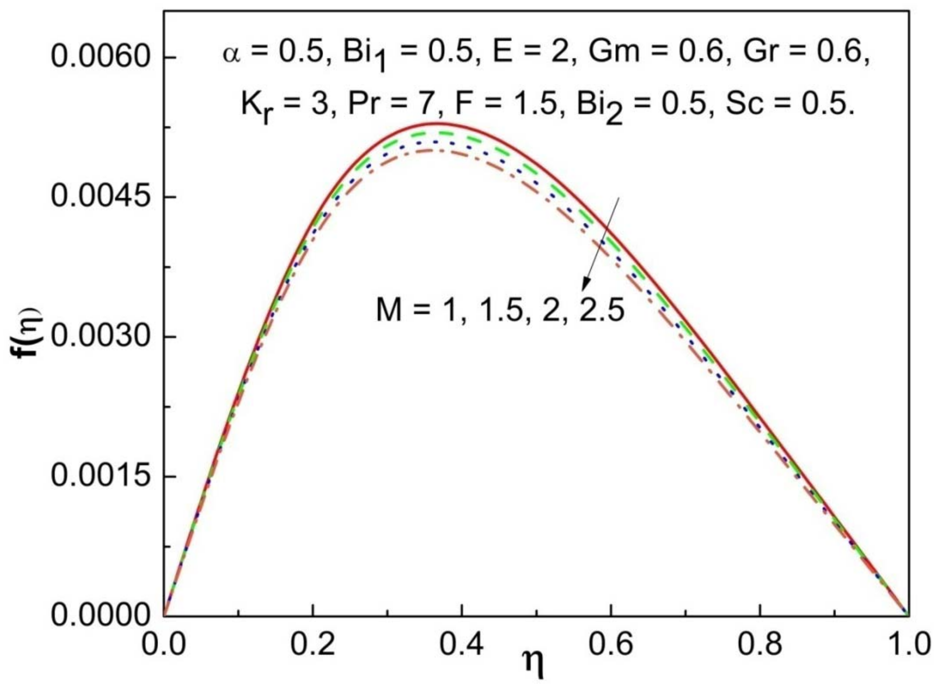

Fig.5. Consequence of M on $f(\eta)$.

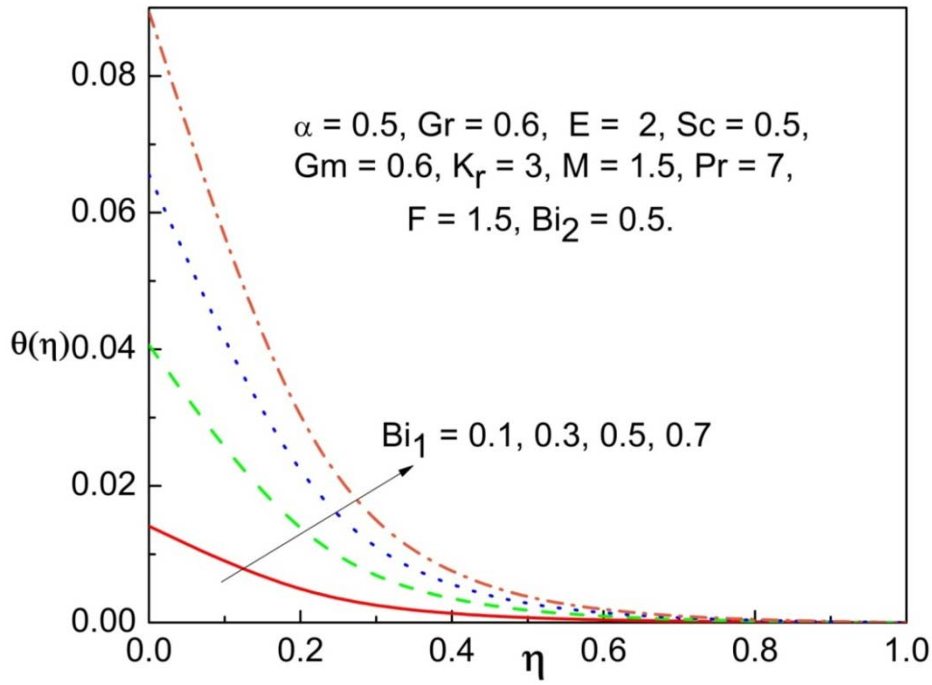

Fig.6. Consequence of Bi_1 on $\theta(\eta)$.

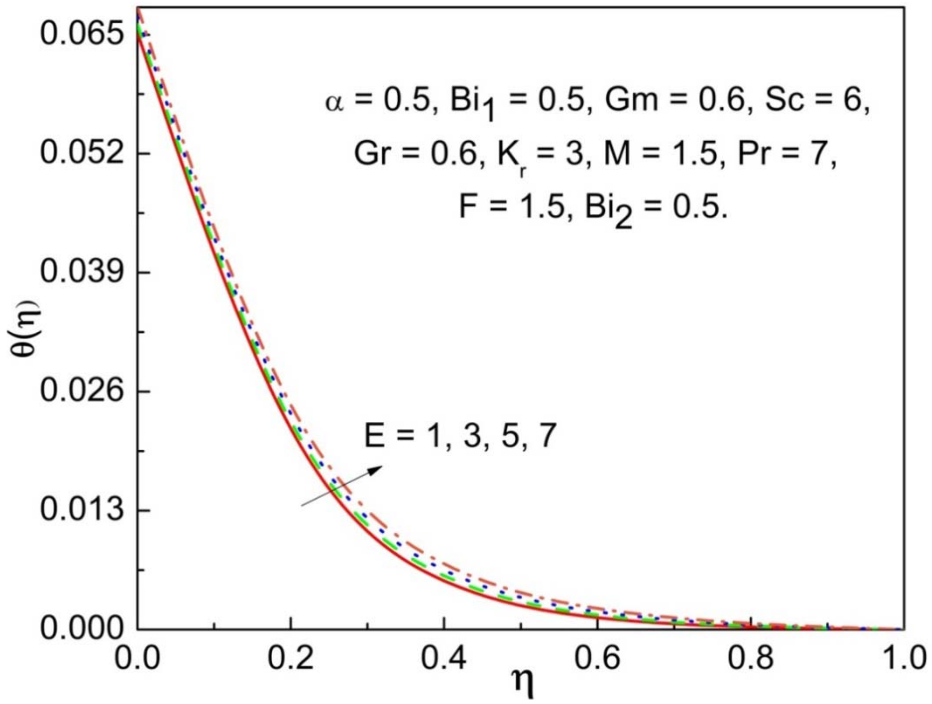

Fig.7. Consequence of E on $\theta(\eta)$.

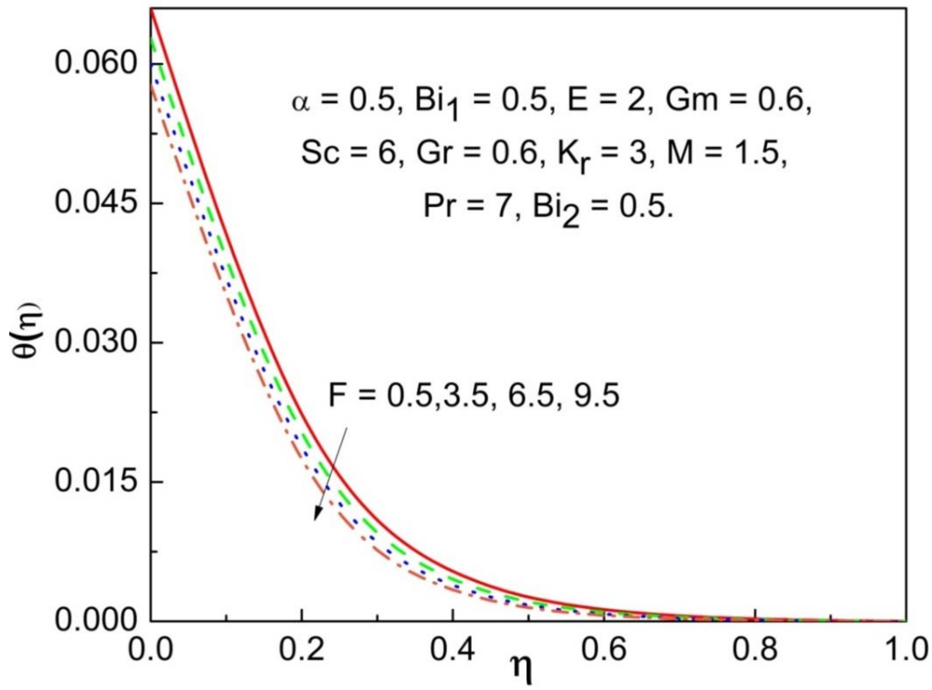

Fig.8. Consequence of F on $\theta(\eta)$.

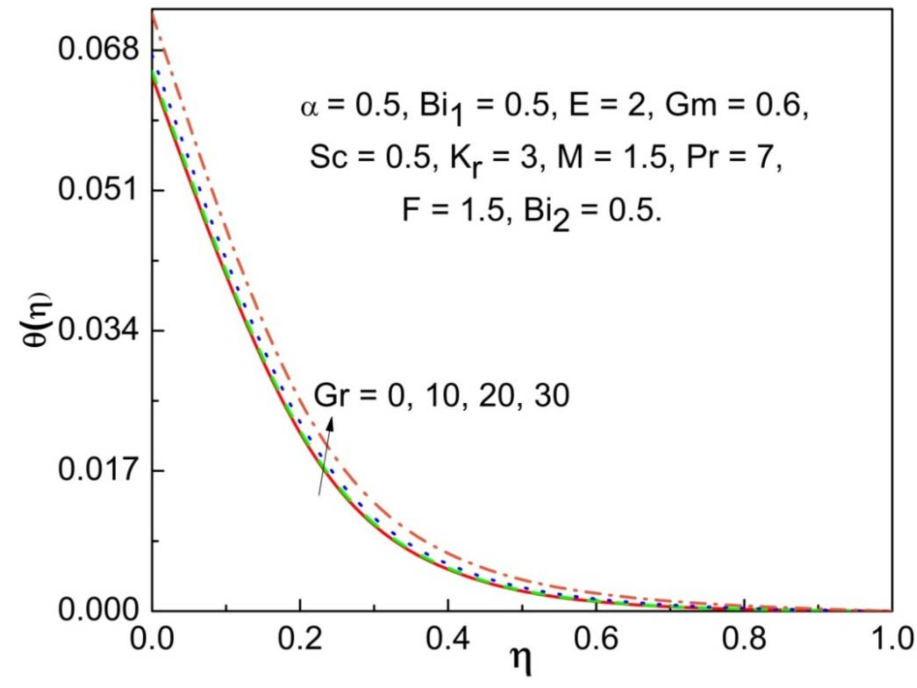

Fig.9. Consequence of Gr on $\theta(\eta)$.

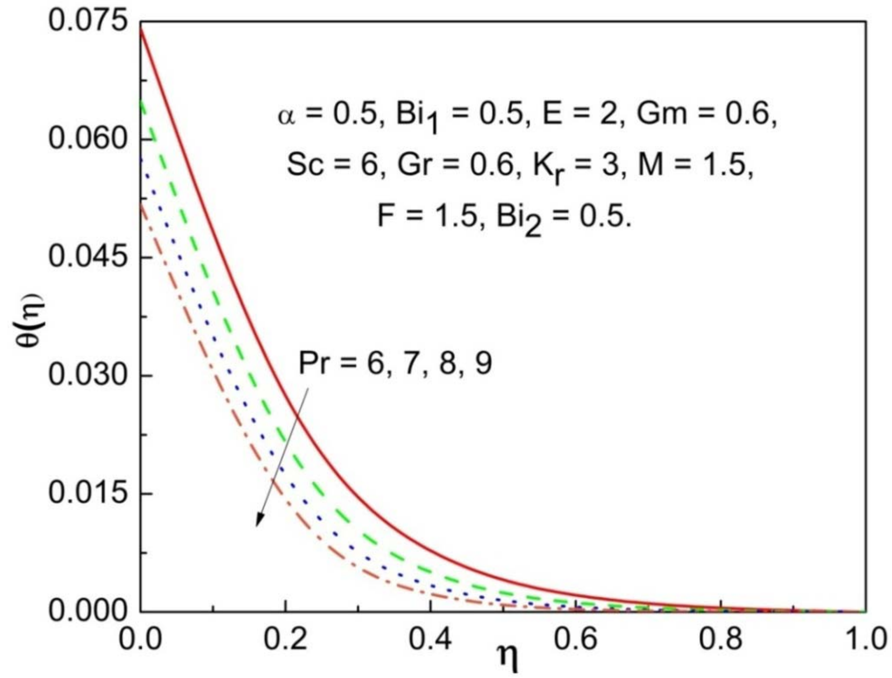

Fig.10. Consequence of Pr on $\theta(\eta)$.

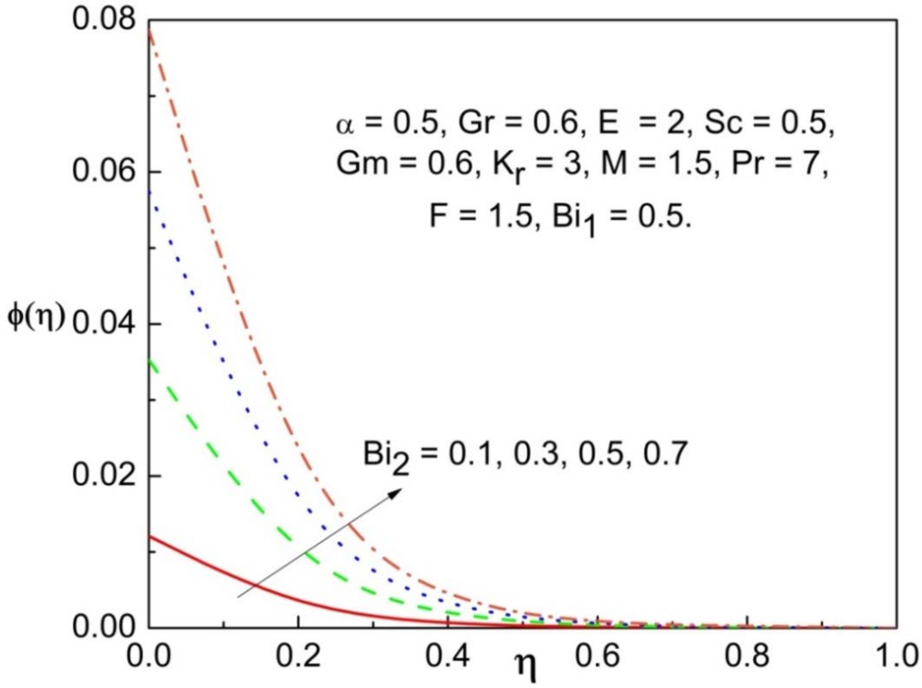

Fig.11. Consequence of Bi_2 on $\phi(\eta)$.

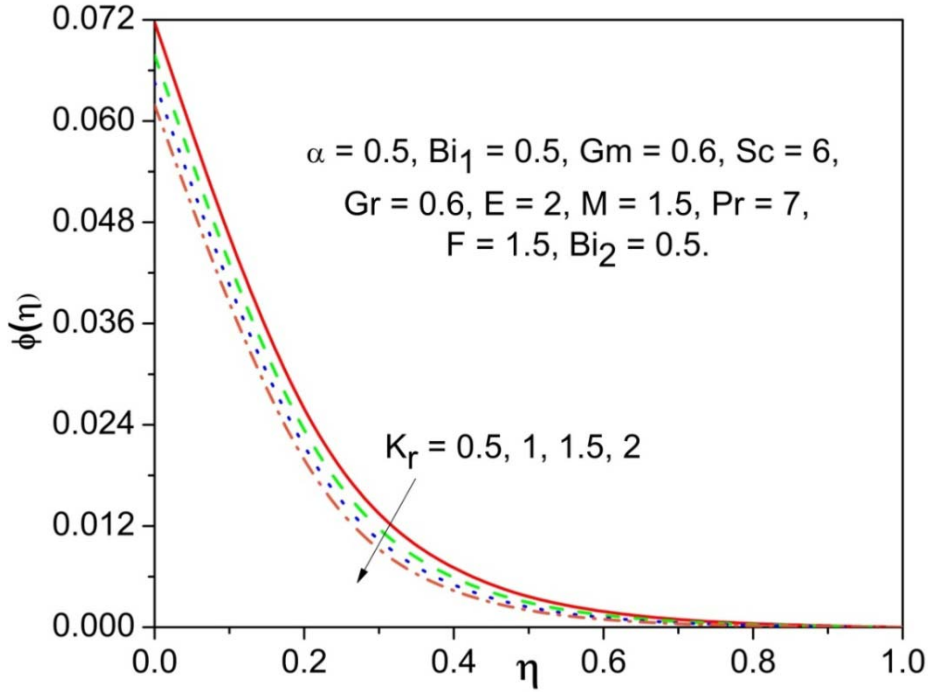

Fig.12. Consequence of *Kr* on $\phi(\eta)$.

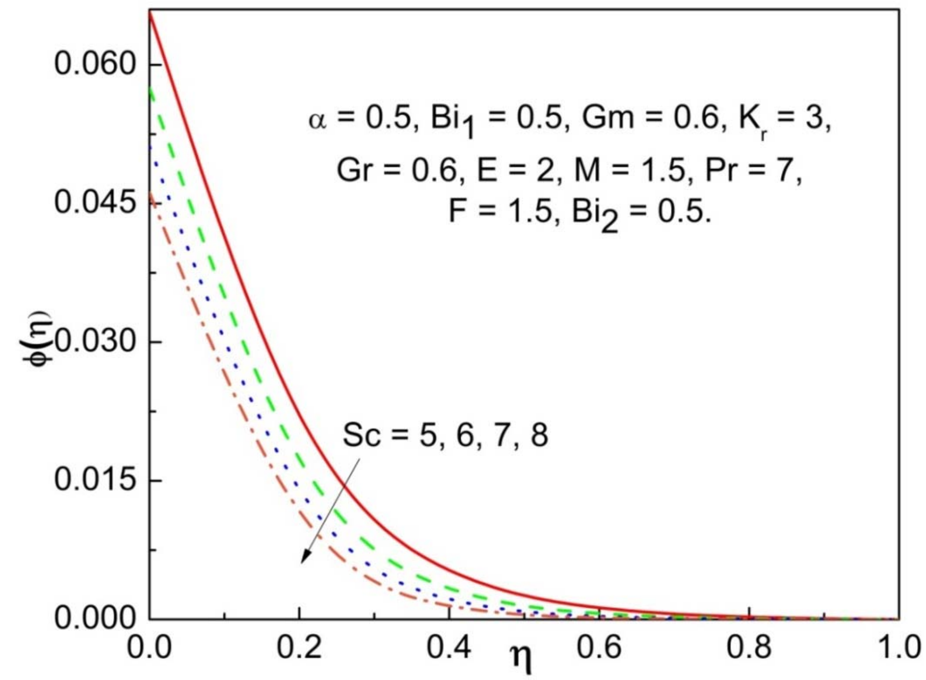

Fig.13. Consequence of Sc on $\phi(\eta)$.

4. Conclusion

The effect of thermal radiation and Joule heating on magnetohydrodynamic boundary layer motion are explored by incorporating the convective heat and mass conditions. The major results of the study are summarized as follows;

- Impact of permeability parameter, Grashoff number of heat and mass transfer increases the velocity profile and the opposite trend can be observed for the magnetic parameter.
- Impact of Biot number of heat transfer, Eckert number, Grashoff number of heat transfer enhances the temperature whereas the radiation parameter and Prandtl number reduces the corresponding description.
- The impact of the Biot number of mass transfer enhances the concentration profile whereas the chemical reaction parameter and Schmidt number reduces the profile.

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Conflict of Interest: The authors declare that they have no conflict of interest.

Nomenclature

B_0 – magnetic flux density (Tesla)
Cp – specific molecular diffusivity
$\tilde{C}$ – concentration of species ($mol\ m^{-3}$)
$\tilde{C}_\infty$ – species concentration at infinity ($mol\ m^{-3}$)
D – chemical molecular diffusivity($m^2 s^{-1}$)
e_λ – Planck's function
F – radiation parameter
Gm – expansion with species concentration
Gm – Grashof number for mass transfer
Gr – Grashof number for heat transfer
g – acceleration due to gravity (ms^{-2})
K_r – chemical reaction parameter
K_λ – absorption coefficient at the plate
M – magnetic parameter
Pr – Prandtl number
q_r – constant heat flux
Sc – Schmidt number
Sh – Sherwood number
$\tilde{T}$ – the fluid temperature (J)
$\tilde{T}_\infty$ – the fluid temperature at infinity (J)
α – permeability of porous medium (Hm^{-1})
β^* – coefficient of the volumetric expansion due to concentration ($m\ s^{-1}$)
β_1 – the volumetric coefficient expansiondue to temperature ($W\ m^{-2}\ K^{-1}$)
θ – dimensionless temperature
η – similarity variable
κ – thermal conductivity ($W\ m^{-1}\ K^{-1}$)
σ – electrical conductivity ($S\ m^{-1}$)
ρ – density of the fluid ($Kg\ m^{-3}$)

v_0 − scale of suction velocity
ν − kinematic viscosity ($m^2 s^{-1}$)
μ − dynamic viscosity ($Kg\ m^{-1} s^{-1}$)
v_0 − constant suction velocitynormal to the plate

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