# Effect of Confinement Stress on Rock Mass Stability

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## Abstract

*A huge number of factors controls rock mass failure, but it is mainly influenced by the state of stress and in particular on the bearing capacity and failure mechanism of the massif. The evaluation of rock mass strength in confined and unconfined compression, as well as its tension strength, are key issues to understand rock mass behaviour prior to failure. A connection between the laboratory analyses of the rock mass and the practical use of the obtained data is presented in the current work. The strength properties, confinement effect and failure mechanisms are successfully studied in volcanic rock specimens from an underground mine. In order to estimate the confinement effect on rock mass strength properties, different confined compression stresses on rock specimens are applied. In addition, the crack initiation and propagation in rock samples are observed and rock mass failure mechanisms are studied. The obtained data is used for stability analyses of an underground openings through determination of the safety factor. The obtained results of the safety factors underlined the influence of the confining stress on the rock mass. The tendency of increasing values of the shear safety factor and decreasing values of the tensile safety factor as confinement increases is found. This is an important observation that would allowed more accurate predictions of the stable and unstable zones of the underground openings to be carried out, and thus the stability of the rock mass to be improved.*

*Keywords: effect of confinement stress, rock mass stability, rock strength, safety factor*

**Introduction**

Rock mass failure is controlled by a huge number of factors, but it is mainly influenced by the state of stress. It has a significant influence on the rock mass behaviour, in particular on the bearing capacity and failure mechanism. Under different stress conditions, the rock mass can fail in a certain mode. Therefore, the in-situ and mining-induced stresses are among the most important parameters that have to be determined in understanding and predicting the rock mass behaviour [1].

The determination of compression rock mass strength in uniaxial and confined compression and the tensile strength is a common laboratory process. However, especially when the rock mass in underground mines presents a much-expressed brittleness behaviour, these kinds of loading systems provide essentially important data for the rock mass strength. Significant work has been done by several researchers in order to study rock behaviour under confined and unconfined states of stresses [2]. The confining pressure has an important role in the damage and failure of rocks. Many studies indicate that, especially under unconfined tests and at low confining pressure, the rock specimens show brittle behaviour [3, 4]. Its effect is widely studied and it is accepted that increasing confining pressure leads to fracture pattern changes. Thus, under low confining pressure, a splitting failure type caused by tensile cracks occurs and, as the confining pressure increases, growth of tensile cracks is inhibited. Also, echelon arrays of tensile cracks form with the overall effect of producing, first, single tensile induced shear fractures across the specimen and then multiple tensile induced shear fractures [5]. Based on experimental evidence, other researchers indicate that rocks are significantly strengthened by confinement which also influences the failure mode of the rock mass and thus, there develops a brittle-ductile transition zone as the confining pressure is increased [6, 7]. Numerical results indicate that the confining stress influences the brittleness of the specimens in a way that, for higher confining stresses brittleness behaviour decreases, and when there is low or no confinement the specimen fails within a small strain range, demonstrating brittle behaviour.

Another factor, controlling many failure processes is the tensile capacity of the rock mass, which is considerably important property when assessing the resistance of failure [8]. It is generally known, that rock in its natural state is weak in tension due to the discontinuities and fractures a rock may contain [9]. Experimental data shows, that most of the rocks in biaxial stress fields fail because of tensile stresses. This is valid when one principal stress is in tension and the other is in compression with a magnitude that does not exceed three times that of the tensile stresses [8, 9]. Nevertheless, tensile strength determination is often overlooked because of the difficulties with obtaining reliable results, especially the direct tensile strength. Therefore, indirect methods, such as the Brazilian tensile test, are typically applied in the engineering practice.

Working in complex geomechanical conditions leads to an increase in the requirements of safety and sustainability of the underground mining works. Commonly, rock mass strength and stability prediction methods are applied to a wide range of analytical, empirical, and numerical approaches [8]. In similar analyses, different failure criteria, based on rock mechanical properties, such as Tresca's, Coulomb's, Mohr's, Griffith's, Johnstone's, Drucker-Prager's, Mohr-Coulomb's, and Hoek-Brown's, are applied [10, 11, 12, 13]. Safety analysis is often linked to safety factor determination, which is usually interpreted within the framework of an accepted scale and is founded on previous experience [14].

This paper presents a case study of the confinement effect and its influence on the rock mass in an underground mine, where the major principal stress is horizontal and the minor principal stress is vertical. In addition, the tension strength of the rock mass and the impact on its behaviour are studied. For this purpose, laboratory analyses were conducted in the Geomechanics Laboratory of the Center for Natural Resources and Environment (CERENA) Universidade de Lisboa, Portugal. The main objective of the laboratory tests is to determine the mechanical properties of the rock samples. The obtained results are used for further analysis of the rock mass stability by assessing the safety factors under shear (Fs) and tension (Ft) failure.

## Material and Methods

The tested material refer to volcano-copper-pyrite formation, which has been developed during the Upper Cretacious volcanic activity. The rock mass structure is a result of tectonically fractured zones and has been hydrothermally altered. As a result, the state of stress in the rock mass is inferred to be variable due to the presence of structural features and variations in the stress orientation, such as the vertical normal stress component being less than the value calculated according to the depth and the unit weight of the rock. This may be an indication of heterogeneity of the stress field [15]. Most probably, the stress path defined by the geological history of the rock mass causes anomalies to its state of stress. Thus, the in-situ stresses are not subjected to the widely accepted theory where the maximum principal stress is due to the vertical weight component but rather the minor principal stress, $\sigma_3$, is the vertical one.

The principal stresses and the strength properties of the rock mass can significantly influence the rock mass stability and to have an even greater influence when working in complex geomechanical conditions. To improve the safety conditions during mining work, analyses of the rock mass stability is needed. In the mining practice, the safety factor is commonly used for a similar study. This requires the rock mass properties to be determined.

### Material for experimental work

An experimental programme for geomechanical characterization of the material has been established. It consists of Uniaxial Compression Strength test, Brazilian test, and Triaxial compression test performed on cylindrical specimens. The testing procedures recommended by the ISRM suggested methods have been followed. The dimensions of the core samples for the Uniaxial Compression Strength (UCS) tests are 36.40 mm in diameter and from 100 to 110 mm in length while for the Brazilian tests the sample size is 63.50 mm in diameter and 34 mm to 38 mm thickness. For the triaxial tests, cylinders with diameter of 40 mm and height of 90 mm to 100 mm, with length to diameter ratio of between 2.5 and 3.0, have been tested. The ends of the specimens were smoothed and perpendicular to the core axis.

### Methods for mechanical characterisation

The laterally unconfined test is commonly the most rigorous method for studying the mechanical properties of rocks [16]. The testing equipment complies with the ASTM D2938-95 [17] and ISRM [16] suggested methods for this analysis where a Servo-controlled testing machine "FORM+TEST" model "506/1000/200 D" with a maximum vertical loading capacity of 1000/200 kN for applying the load on the cylindrical specimens. The load was applied continuously at a rate of 0.70-0.80 MPa/s. The peak loading of the rock specimens were measured. The maximum loading capacity was recorded for each specimen.

The most commonly used laboratory method for tensile strength determination is the Brazilian tensile test. The used apparatus complies with ISRM [9] suggested methods where the specimen is placed between two steel loading jaws. The same loading machine as for UCS and Triaxial tests was used for applying the compressive loads to the specimen. It is applied continuously at a rate of 0.20-0.30 MPa/s. To be valid, the fracture on the specimen should start and pass from the central region of the specimen out towards the loading platens [8]. Invalid tests often occur due to deviations in the failure plane (fractures along the fabric plane) or when fracturing begins in the platen area. The orientation of the discontinuities, when observed in the samples, are considered when placing the specimens into the apparatus for performing the Brazilian test.

To determine the relationship between axial and confinement compressive strength triaxial tests have been performed [5]. The behaviour and failure mode of cylindrical specimens under triaxial stress are observed. The measurement and recording of the axial load and confining pressure during the tests were conducted using a system, which includes: (a) control unit for continuous applying and controlling the axial load; (b) equipment for generating and controlling the confining pressure with a maximum capacity up to 70 MPa. An axial force is applied according to the ISRM recommendations until the specimen fails. The magnitude of the confinement stress is applied constantly during the experiments, and it is considered in advance in line with the in situ stress in the rock mass. Thus, a series of confining pressures at 5, 8.8, 15, and 20 MPa were applied in this study and the maximum strength of the rock was determined at those rates.

### Safety factor

Nowadays, many empirical and numerical methods are widely used for rock mass stability analyses. The classical approach used in designing engineering structures is to consider the relationship between the capacity of the element and the load or stress that are expected [10]. This ratio is expressed as the Safety Factor (SF), which is one of the most used parameters to study rock mass stability. Typically, the safety factor can be used to analyze rock mass stability in tunneling, civil engineering, and mining activities [10, 14, 18, 19].

In many empirical approaches for the determination of the safety factor (i.e. formulas, failure criterion, software), the main required parameters are the deformation properties of rock mass such as Young's modulus, Poisson's ratio, cohesion, and internal friction angle [20]. For instance, in computational geomechanics, these parameters are generally computed by the strength-reduction technique by reducing the shear strength parameters monotonically until convergence of the boundary-value problem is no longer achieved [21].

The understanding of the rock mass behavior is of fundamental significance in engineering fields. Despite many years of research through different approaches, there is no universally accepted failure criterion for the general case of stressed rock sample [7]. Based on the Mohr-Coulomb failure criterion, the safety factor under shear failure $F_s$ and tension failure $F_t$ can be obtained, respectively, with the following equations [22]:

$$F_s = \frac{c_m \cos\varphi_m + \frac{\sigma'_1 - \sigma'_3}{2} \sin\varphi_m}{\frac{\sigma'_1 + \sigma'_3}{2}} \tag{1}$$

$$F_t = \frac{\sigma_{tm}}{\sigma'_3} \tag{2}$$

where:
$c_m$ - cohesion;
$\varphi_m$ - internal friction angle;
$\sigma'_1$ and $\sigma'_3$ - maximum and minimum effective principal stress at failure;
$\sigma_{tm}$ - tensile strength.

According to the widely accepted understanding of the safety factor limitations, if they have values lower than 1, then shear failure or tension failure occurs in the corresponding rock mass. This understanding is more often perceived in numerical modelling [22]. Also, the time dependency of the openings should be considered, and for a temporary mine opening, a SF of 1.3 would generally be considered adequate, while a value of 1.5 to 2.0 may be required for a permanent excavation. In all cases, the minimum values of the safety factors should be considered in regards to the certain stress state conditions of the case study. The numerical value of the factor of safety chosen for a particular design depends upon the level of confidence that is used for the required input parameters [23]. Furthermore, the reliability of the results will be in accordance with the same parameters.

## Mechanical Characterisation

The results from the geomechanical tests and the observed behaviour of the tested specimens are presented hereafter. Two possible failure mechanisms should be considered. This includes the failure to occur at an existing discontinuity, or at newly formed one as a result of the applied stresses during the experiments. As the rocks are heterogeneous, they contain discontinuities of different scale, orientation, and fillings, which plays a critical role to understand physico-mechanical characteristics [24, 25], and this can also greatly influence rock strength parameters and their failure mode.

### Uniaxial Compressive Strength

The Uniaxial Compressive Strength of the rock samples was characterized by the load under which they failed. At an interval of 5 kN of the vertical load, the values for vertical and horizontal displacements recorded by LVDT's were registered. In all experiments, two sensors were used to detect horizontal displacements and one for vertical displacements. As a result, the stress-strain curves were recorded. After a given vertical stress value oscillations in the deformations have been observed. This is due to the initial stage of failure of the sample, which was observed during the experiment. It is associated with the destruction of smaller pieces of rock core and in this process there is an "unloading" of the sample, which in turn causes fluctuations in the curve.

Based on the maximum loading force (P), the peak value of the uniaxial compressive stress $\sigma_c$ was determined for each specimen, and the results are reported in Table 1. According to the obtained strength results, the specimens can be separated into two sets. In the first set (samples Nr. from 1 to 7) the strength values are in the range from 91.3 MPa to 127.8 MPa. The second set (samples Nr. 8, 9, 10) is characterized by lower strength values varying from 62.7 MPa to 81.7 MPa. The wide range of the strength properties obtained for the first set of samples probably was due to the irregular distribution of ore mineralization. Thus, the lower values are typical for less mineralized rocks. In addition, macroscopic fractures or other similar characteristics, that could affect the strength properties of the specimens, were not observed.

Tab. 1. Results from the Uniaxial Compressive Strength tests on cylindrical samples.

| Sample Nr. | $\sigma_c$ (MPa) |
|---|---|
| 1 | 91.3 |
| 2 | 127.8 |
| 3 | 111.5 |
| 4 | 121.2 |
| 5 | 117.0 |
| 6 | 106.7 |
| 7 | 94.0 |
| 8 | 81.7 |
| 9 | 62.5 |
| 10 | 77.9 |

The strain values were used to determine the Elastic modulus and Poisson's ratio of the tested rock material. The values obtained for the Elastic modulus range between 54 GPa and 72 GPa. These variations are rather due to the degree of mineralization of the rock samples as logically those with a lower degree of mineralization have lower values of the Elastic modulus. The Poisson's ratio the results vary in a relatively narrow range from 0.18 to 0.26. The comparison of the results between the two characteristics demonstrates a certain correlation between them. In almost all studied samples, there is a clear pattern between the Elastic modulus and the Poisson's ratio, according to which samples with lower values for the Elastic modulus have higher values for the Poisson's ratio.

The failure type of all tested specimens under uniaxial compression was observed. During the testing at a large number of the samples, after certain levels of load, the damage initiation process was observed. Mostly, it started from the specimen edges close to the contact between the rock samples and the metal plates of the testing machine. This process can be explained by the difference of the modulus of elasticity of these two materials as the stress distribution results differ, depending on the contact conditions between the rock samples and the platens [7]. With the continued loading of the specimens, gradual development of the process was not observed. Under these conditions, the propagation of this crack resulted in tensile rupture of the specimen. It is important to note that the final failure occurred suddenly and with a high energy release. Most of the specimens were disrupted in more than two pieces, while others disintegrated completely. The failure process, as explained, largely resembled to the rock burst phenomenon but observed under laboratory conditions. The failure patterns are illustrated in Figure 1, where an irregular, longitudinal brittle splitting of a rock specimen with rough failure planes was observed. By combining the data about the peak strength and the failure pattern of the specimens after the tests, it is assumed that the rupture is caused by tension stress resulting in vertical tension cracks.

(a) (b) (c)

Fig. 1. Mechanism of failure of rock specimen under Uniaxial compressive test.

**Brazilian Tensile Strength**

The Brazilian Tensile Strength is determined by an indirect testing method, where the stress at failure ($\sigma_t$) is a function of the applied load (P), the diameter (D), and the thickness (t) at the center of the specimen [8]. As valid tests are accepted those in which the failure plane passes through the middle of the specimen and is parallel to the direction of the applied load, as is shown in Figure 2 (a, b). Another sample, where the failure occurs on an already existing crack is demonstrated in Figure 2 (c). The tensile strength for each tested specimen was determined, and the results are presented in Table 2.

Tab. 2. Results from the Brazilian Tensile Strength tests on cylindrical samples.

| Sample Nr. | $\sigma_t$ (MPa) |
|---|---|
| 11 | 22.1 |
| 12 | 19.4 |
| 13 | 21.7 |
| 14 | 13.6 |
| 15 | 5.2 |
| 16 | 12.5 |
| 17 | 8.2 |
| 18 | 7.2 |

As can be seen from the obtained data, the values for $\sigma_t$ vary considerably. On this base, the specimens can be separated into two sets: The first set is differentiated with all samples from 11 to 14 where the average tensile strength value is about 21.0 MPa and vary in a very small range. Among them, an exception is only a sample Nr. 14. Although, no any discontinuities were observed macroscopically, the value obtained for the tensile strength is approximately 30 % lower than that of the other specimens from this set. The second set (Sample Nr. From 15 to 18) is characterized by significantly lower tensile strength values than the first one. The strength values vary from 5.2 MPa to 12.5 MPa. Similarity of the results for three of the tested specimens was highlighted, in this set, as the average value for $\sigma_t$ is slightly lower than 7.0 MPa. Before testing, discontinuities and other fractures were observed in these samples, and their orientation was considered when placing the specimens into the apparatus. The results show that the failure occur along the existed fractures. An exception is sample Nr. 16, as the value obtained was almost two times higher than that of the other in the set. The probable reason for the higher value could be the failure mechanism of the specimen because the observed failure plane was orientated approximately normal to the direction of applied load, and crossed the specimen through the middle. The above-described features of the Brazilian tensile strength data were analyzed in detail and taken into account in their subsequent use. Due to the observed failure mechanism and according to ISRM suggested method [9] the samples Nr. 15, 17, and 18 were assumed as invalid. Their values were not used in the following analysis.


(a) (b) (c)

Fig. 2. Mechanism of failure of rock specimen under Brazilian tensile test.

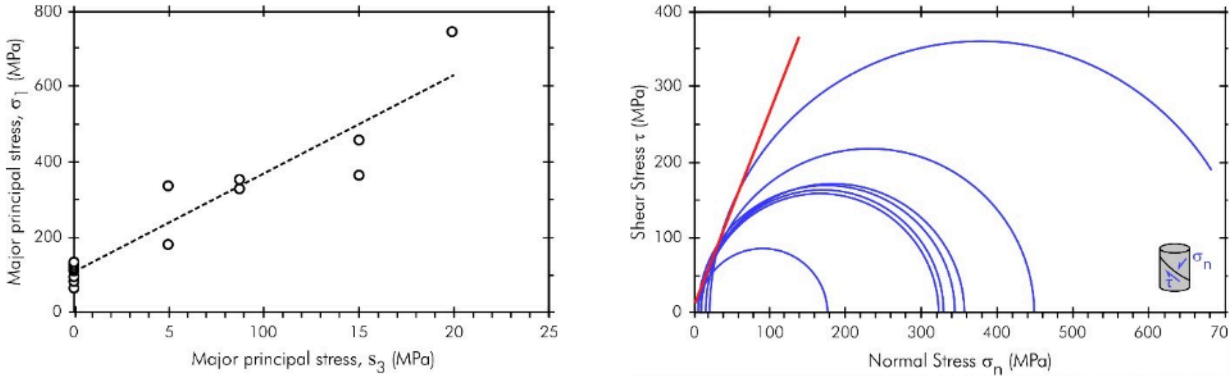
### Triaxial Compressive Strength

Under triaxial compressive test, the specimens were subjected to major compressive stress applied along their vertical axis and lateral confining pressure. In performing the calculation of the maximum ($\sigma_1$) and minimum ($\sigma_3$) principal stresses, the data obtained for the peak strength and the corresponding confining pressure for each specimen as well as the area of the specimens were used. The results are represented in Table 3.

Tab. 3. Results from the Brazilian Tensile Strength tests on cylindrical samples.

| Sample Nr. | $\sigma_1$ (MPa) | $\sigma_3$ (MPa) |
|---|---|---|
| 19 | 331.4 | 5.0 |
| 20 | 177.3 | 5.0 |
| 21 | 324.1 | 8.8 |
| 22 | 347.0 | 8.8 |
| 23 | 450.8 | 15.0 |
| 24 | 359.4 | 15.0 |
| 25 | 729.3 | 20.0 |

The reported results were used as input data for the widely used software *RocData* (version 3.0), developed by *RocScience* [26]. The obtained principal stresses are used to plot the relationship between the major and minor principal stress for each specimen (Figure 3a). Their linear relationship is shown as a straight line on the diagram. The coefficient of determination ($R^2$) was found to be sufficiently high with a value equal to 0.91. As expected, the plotted results show that the compressive strength of the samples increases with increasing confining pressure. The data represented in Table 3 was used to draw the Mohr's circles on Normal vs. Shear stresses diagram and the strength envelope plotted on the graph (Figure 3b). The strength envelope is distinctly steep, which corresponds to a higher value of the friction angle of the tested rock samples. These facts also influence the values of the tensile strength, as they were expected to be relatively low. The widely used Mohr-Coulomb strength parameters (the internal friction angle $\varphi$, and the cohesion C) were obtained using *RocData* software. They were used for further analyses of safety factor in this study. Comparison of the average UCS value, obtained from the unconfined compression tests and those performed using the data from the Triaxial tests, showed that the results are very similar. This confirms that both test types were accurately performed and underlines the reliability of the obtained results.




(a) (b)

Fig. 3. Results from Triaxial tests: (a) Plot of the Principal stresses obtained from laboratory tests (Major vs. Minor principal stresses); (b) Failure envelope in Normal stress vs. Shear stress.

A change in failure mechanism under triaxial tests was observed. The applied triaxial compression closed the existing micro fissures and increase the compactness of the samples. Specimens under different confining pressure after the failure are shown in Figure 4. On the base of the failure plane, it can be assumed that, with increasing confining pressure, the brittleness of the rocks decreases. This results in a change of the failure mode from spalling to shear, where the failure occurs on one main plane across the specimen.

(a) (b) (c)

Fig. 4. Mechanism of failure of rock specimen under Triaxial compression strength test.

The loading scheme represents a combination of axial load and confinement stress, which has a significant role in a change of a load of specimens and their behaviour. This loading configuration leads the samples to fail as a result of the tensile and shear stresses. The observed specimens failed by one main transverse plane (Figure 4). The crack initiation under the confinement is a combination of the tensile and shear stresses. At the same time, the applied confinement stress decreases the effect of the tensile stress and the failure is controlled by the induced shear stresses. Thus, the rupture occurs in a plane on which the shear stress exceeds the shear strength of the tested rock mass. As a result, failure caused by shear force was observed and brittle behavior of the specimens was dissipated.

**Safety Factor**

The proposed equations for the determination of the shear and tensile safety factors were based on the rock mass properties obtained by the laboratory tests described above. Using the summarized results of uniaxial and triaxial tests, as well as those of tensile strength, $F_s$ and $F_t$ were calculated according to Eq. (1) and (2) respectively.

The results concerning the safety factor under shear failure clearly underline a direct-proportional correlation between the confinement stress and $F_s$ (Figure 5a) such that the increase of the confinement stress increases the safety factor. This would lead to an enhancement of the rock mass stability as well.

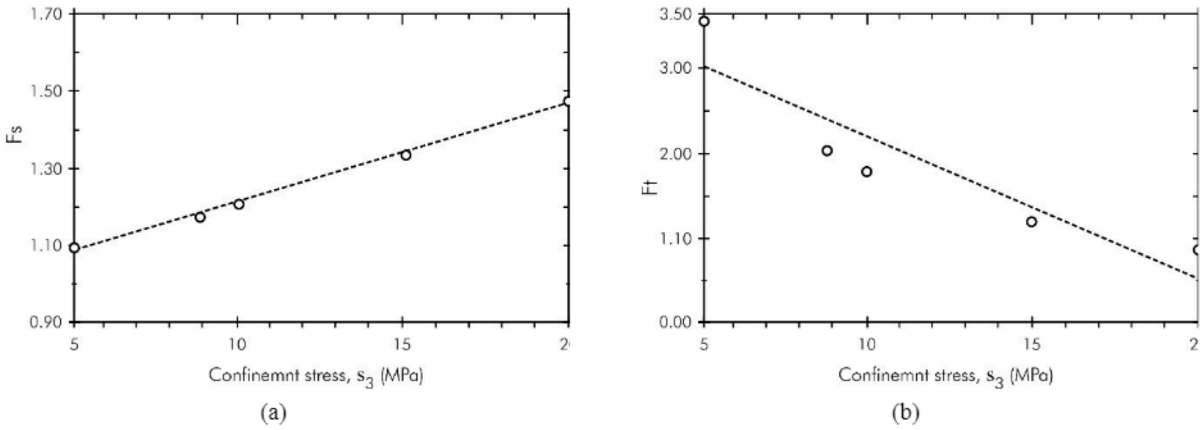



Fig. 5. Plot of the relationship between the confinement stress and: (a) the Shear Safety Factor ($F_s$), and (b) the Tension Safety Factor ($F_t$).

It is generally known that the tensile strength of the rocks is significantly less than its compressive strength, frequently considered as about 10 % of it [9]. Rocks in their natural state are relatively weak in tension. According to the maximum tensile stress criterion of rock, the rock material is assumed to fail by a brittle fracture in tension when the applied least principal stress ($\sigma_3$) to the rock is equal to its uniaxial tensile strength. Subjected to a similar understanding of the rock mass behavior under tensile stress state, the proposed Eq. 2 was used in this study to calculate the Safety factor (SF) under tension. The data about the tensile rock mass strength was obtained under Brazilian tensile tests in laboratory conditions. Only the valid tests are used in this analysis. Thus, after performing the calculations, the values about the $F_t$ and the confinement stress showed that they are inversely related (Figure 5b). This tendency shows that the higher the confinement, the lower the Safety factor under tensile is.

Figure 5 shows that confinement stress significantly influences the stability of the rock mass since the confining pressure was used for calculating the $F_s$ and $F_t$. Analyzed results highlighted that the Safety factor increases under shear with the increasing confining pressure whilst it decreases under tension for the same states of stress.

**Conclusions**

In this study, the mechanical properties of rock specimens under different confining pressure were investigated. Based on the obtained data and observations, the following conclusions can be drawn: (a) under unconfined conditions, the rock samples fail in spalling; (b) the confining pressure has an influence on the rock failure mechanism such as a higher confining pressure results in a shear failure; (c) under both loading conditions (unconfined and confined), the rock exhibits brittle behavior.

Preliminary results on crack initiation and propagation in uniaxial and confined compression testing indicate the key role of tensile stress. The observed failure process is typical for brittle rocks, especially when rock specimens have low tensile strength. In addition, the rock mass behavior and failure mode under triaxial conditions showed that the confining pressure results in a shear failure mode of the samples. When confinement stress is applied, the rock mass failure mechanism is controlled mainly by the shear stress. This effect can be studied in detail by applying numerical methods of rock mechanics. Additional analyses on the confinement effect in tensile stress state would contribute to better understand the rock mass behavior, in particular around the free surfaces where the confinement stress is equal to zero tension cracks and failure can be expected.

Calculations of the safety factors under shear and tensile failure were performed. The tendency of increasing values of the shear safety factor and decreasing values of the tensile safety factor when the confinement increases results underlined the influence of the confining stress on the rock mass. Such calculations would allow the stable and unstable zones of the underground openings to be determined in advance, and thus the stability of the rock mass to be improved.

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