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DISCOUNTING AND IDEAS OF INTERGENERATIONAL EQUITY AND SUSTAINABILITY

Some aspects of discounting of infinite streams of economic quantities (consumption, saving, utility) are considered. The main points of the paper may be summarized as follows. The philosophy of discounting concerns methods of evaluation and/or comparison of multi-period phenomena and processes. The techniques applied (numerical, analytical) may equalize the gravity of subsequent generations or privilege certain generations rather than other ones. They can – to some extent – influence quality of life and the chances of survival in future generations. These (quite natural) connections between discounting and intergenerational justice and sustainability have been pointed out. Systems of axioms for "ethical" and "sustainable" preferences are presented, together with some related (sub)disciplines. Inspirations derived mainly from psychology suggest the modification of the geometric discounting paradigm by slowing down the convergence of discount factors to zero. It should be pointed out that such "heavy tailed" discounting has proved to be useful (and become strongly recommended) for evaluation of a variety of long-time horizon undertakings. Some examples of non-classical discounting rules are given. Finally, the author's proposal of modeling the duration of the world as a Poisson random variable has been formulated.

Keywords: *discounting, evaluation, preferences, equity, efficiency, sustainable development, present, future*

With very little exaggeration or cynicism, an "expert" here might be defined as an economist who knows the literature well enough to be able to justify any reasonable social discount rate by some internally consistent story

M. Weitzman [72]

1. Introduction

We consider some aspects of the discounting of infinite (in time) streams of economic quantities (consumption, saving, investment, wealth, utility) – in connection with requirements concerning the "justice and efficiency" of underlying preferences (in the space of such streams). Special attention will be given to the genuine relations between various forms of discounting and conditions for intergenerational equity.

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At the level of an "operational plan" one may ask which evaluation techniques should be recommended in a long time horizon undertaking, taking into account not only mathematical elegance and/or tractability, as well as economic criteria, but also questions of an ethical nature. The acceptation of a specific philosophy of discounting involves determining the method of evaluation, which in turn influences multi-stage (investment) strategies and may increase the chances of survival in a possibly large number of future generations. However, a somewhat perverse and provocative Weitzman's opinion (from the above quotation) reveals that there is clearly no consensus on which approach to use.

Inspirations derived from psychology suggest the modification of Samuelson's geometric (exponential) discounting paradigm by slowing down the convergence of discount factors (functions) to zero. At the price of losing the dynamic coherency (of preferences, as well as sequences of decision rules and, consequently, choices), one can obtain a more accurate model of the real behavior of economic agents and aggregates (which are actually themselves myopic and may well change their own preferences!).

Contemporary systems of axioms protecting evaluations of development paths from the so called dictatorship of the present, as well as of the future, are considered. The author also suggests a proposition for modeling preferences (with the use of classical discount functionals) in the case that the world has a random life-time (i.e. the date of the end of the world is uncertain), assuming that this (exogenic) variable has the Poisson distribution. This is intended as an introduction to this approach (the author will elaborate some questions in future research).

The author prefers to present his thoughts in the form of an essay (not avoiding mathematics when necessary). The goal of this paper to provoke discussion is in line with such an approach. Hence, the paper will be presented in a "mixed style". Some social, heuristic and historical considerations will be interwoven with more formal statements. The first part of the essay will use arguments from the social sciences and humanities, whereas quantitative reasoning and formulas turn out to be necessary to describe the constructions presented in the later sections (it is commonly known that contemporary topics in the field of economics must be formulated and elaborated in mathematical language). These concepts in turn need to be conveyed at an intuitive level. The bibliography provided plays a significant role as a part of the communication intended to be shared with the reader. It is a fairly extensive list – a kind of guide to the history and present state of the subject of discussion. Nevertheless, it is far from complete (inevitably, it must be a subjective selection). It should be noted that not all the sources in the bibliography are quoted in the main text of the paper, some of them have been consulted but not cited in the text, and some are listed for their historical significance. The author wishes to thank Prof. Colin Price for his valuable suggestions for the bibliography, especially for pointing out his comprehensive book concerning many aspects of problems involving discounting, additionally revealing (in a fairly

natural way) their links with the economics of sustainability and the management of natural resources.

Let us begin by invoking some quotations, which seem to be the best introduction to the climate of the debate. In his famous philosophical treaty (first edition in 1874) Sidgwick [65] wrote: *The time of which a man exists cannot affect the value of his happiness from the universal point* […] *the interest of posterity must concern a Utilitarian as much as those of his contemporaries*. These phrases sound like an imperative of an ethical nature. The same may be said of the declaration (in 1928) of Ramsey contained in his pioneering paper on economic growth *The mathematical theory of saving*: *practice of discounting later enjoyment vis-à-vis earlier ones is ethically indefensible* […] *arises merely from the weakness of imagination* [51]. The above opinions may be complemented by Rawls' requirement regarding a decision regime: one has to pose himself to make choices *from behind of the veil of ignorance* (i.e. being deprived of any awareness of one's position in society, time, space or finding oneself at the "primal circumstances" [51]). These thoughts evidently refer to Kant's ideas. Similar – in spirit – was the point of view of Pigou (1920), who laid the foundations for welfare economics [37]. In sharp opposition to the above arguments for "equalizing" were the views of many other prominent philosophers and scientists: von Böhm-Bawerk [1889], Fisher (1930) [7], [17], who argued for the necessity for the use of a non-zero discount rate, as dictated by the principles of interest rate theory. They are shared (to some extent) by Bergson (1938 [5]) and Samuelson (1937 [56]), who dictated for years the philosophy and "technology" behind discounted sum of utilities (DU). This modified utilitarian tradition made a clear, convincing and universal benchmark for evaluation and comparison in problems appearing in finance and – generally – in calculations concerning social welfare. However, the real breakthrough in this field of research occurred in the 60ties of the XX century, thanks to the works of Koopmans [25] and Diamond [14]. They initiated an entirely new approach to the fundamental questions considered here: the modern pre-order framework with topological elements. Such a methodology enabled them to more accurately model behavior while, at the same time, somewhat complicating the answers (although more refined tools from mathematical economics a'la G. Debreu, later became available). Koopmans introduced the notion of stationarity and impatience of preferences in the space of infinite streams (i.e. paths of economic growth). The "giants of economic thought" differed significantly in the crucial question of interpreting intergenerational equity (and thus – discounting, and "sharing the weights" of generations in consumption). Koopmans argued in the following way [25]: *There seems to be no way in an indefinitely growing population, to give equal weights to all individuals living at any time in the future*.

We close the current section with a quotation which will reveal additional aspects of questions regarding "the proper treatment" of all generations (tacitly – discounting is in mind). Lauwers and Van Liedekerke [29] invoked the suggestive hyperbole: *measures which caused misery and death to tens of millions today would result in saving from greater misery and from death hundreds of millions in the future (can be recommended). The case is surely no different in principle from that of the battalion commander who sacrifices a patrol to save a company* [...] *from a god-like point of view is all very well, but what if the present generation happens to be the patrol? Will we go into battle?*" So it is no wonder that Rubinstein called the problem of discounting *one of the fundamental dilemmas of economic theorists at the beginning of the XXI century* [49].

2. The state of research in the field of discounting, intergenerational equity and sustainable development

Again we start with a quotation. The Report of the World Commission on the Environment and Development, the famous *Our Common Future*, published by Oxford University Press [9] states its crucial recommendation as a sentence: *Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs*. This *Brundtland Report* calls for making efforts to achieve a balance (based on fairness) between the interests of the present generation and those of their descendants.

At a first glance, it seems that postulates, such as impartiality (anonymity), addressed to techniques of comparing multigenerational streams reflect these somewhat idealistic demands a'la Kant–Rawls. However, the above properties have well defined (formal) meaning in the framework of pre-orders in spaces of sequences [14], [2]. But it is not easy to fulfil them at a satisfactory level. First of all, some mathematical problems appear (for instance: finite versus infinite permutational invariance (symmetry) [29], [25]. Secondly: when one wants to couple such – mutually contradictory – requirements as equity and efficiency, fundamental difficulties arise: actually there are many more "impossibility theorems" than conclusions in the affirmative [14], [52]. Elements of relativity (and ambiguity) also appear when considering questions of various kinds of continuity [14], [52]. We do not follow this approach but concentrate on some praxeological, economic and ethical threads. One can distinguish two stages when operating in terms of infinite sequences of periods ("selves", generations). Firstly, we "translate" some primary postulates (concerning evaluation and policies in multiperiod, multigenerational undertakings) into the mathematical language of axioms (properties) of preferences in spaces of the realizations of the phenomena described. Next, we operationalize "starting" postulates by means of mathematical representations of such beings (somewhat abstract and unmanageable), as utilities (functions), social welfare functionals and discount functionals. The latter are intended to satisfy the initial (demands and expectations. The crucial question is avoiding the two extremes: the "dictatorship" of the present or "sacrificing the present for the future". Thus one searches for some compromise, "golden rules" for evaluations and/or discounting.

Let us look for the moment at two such extreme situations (both in methodology and consequences). If one discounts future "flows" (of a great variety) in a traditional, geometric fashion, even at the rate close to zero, the significance of "far distant" (future) periods diminishes drastically rapidly (exponentially!). The "canonical" (supported by Samuelson's authority) formula for evaluation using a geometrically discounted sum of utilities has been extensively criticized by economists, philosophers and psychologists (for its "unjust" treatment of generations – their weights depend on when they were born). When applied to some models of economic growth (such as Dasgupta and Heal [13], as well as Solow [59]), the logic of discounted utilities leads to highly undesirable consequences (remember that they considered capital accumulation and resource depletion simultaneously). It turns out that the application of classical discounting forces consumption to approach zero as time goes to infinity, even though sustainable streams with constant or increasing consumption are feasible [2]. *When applied to the above model, the use of discounted utility procedures undermines the livelihood of generations in the future*... [2].

On the other hand, at the opposite pole there are investment strategies which totally sacrifice the quality of life of the present generation, reducing contemporary consumption in favor of future benefits. This is also ethically indefensible: in the case of long-term projects the real investors cannot, *ex definitione*, consume the "fruits" of their refraining from current consumption, which is in fact postponed by decades or several generations. An interesting analysis of the paradoxes generated by such defective optimization rules was done by Czerwiński in his essay *How to compare tomorrow's welfare with today's poverty* [12]. He demonstrated (by reasoning in the spirit of dynamic programming) that uncritical efforts to maintain a certain level of consumption in the future lead to restricting current consumption to zero – in favor of investments. Such a requirement by no means can be accepted by societies from a life-cycle perspective, and especially, in an intergenerational setting: would we like *to play the role of a patrol*?

The following remarks have to be taken into account. The future is "physically" non-verifiable by the present generation (only "short-term" economic phenomena can really be studied seriously – according to Keynes). There is intrinsic uncertainty about far distant periods. Thirdly, it is impossible to reallocate future benefits to compensate the losses suffered by earlier generations. The above observation actually concerns the effects of future generations suffering from injustice: *the differentia specifica* of problems concerning the evaluation (and politics) of processes running over long periods, lies in the irreversibility of their courses and results. It is a rule rather than discretion that none of the participants of multigenerational games can repair certain kinds of their own mistakes – such as erroneous valuations of processes. It is also worth noting that in calculations of the "correct" discount rate, "wealth effects" are usually invoked:

future generations would be characterized by greater wealth, faster growth and lower marginal utility. Thus one should discount not merely time, but also other factors, including risk (uncertainty about the magnitude of future rates of interest, as well as about the existence of mankind as a whole or markets as institutions). These issues make problems involving discounting more complex.

The XX century provided some new, unprecedented (global) problems and challenges. As well as realizing that we, mankind, are in danger from the accelerating impoverishment of our environment (in the widest sense): it has been observed that the depletion of natural resources, the rapidly diminishing diversity of species and their downgrading, as well as increasing pollution (plus the green-house effect, problems with carbon and sulphur dioxide, etc.). Beside the undoubted benefits brought by the information and technological revolution, negative effects have appeared too. This problem was noticed at the beginning of the XX century (the work of Sidgwick and Pigou has already been mentioned, one should also mention the contribution of H. Hotelling – Hotelling's famous rule determining the desired relation between the intensity of the depletion of resources and their regeneration. At this point we merely signal some concluding remarks: for at least 50 years, the scientific community, philosophers, politicians and managers of large, long-term projects have become aware of a "new dimension" of the concept of scarcity, and the vanishing of free-goods (in the classic sense a'la Smith, Ricardo and Say). The opposite to Hotelling's rule seemed to be true: the presentiments of pastor Malthus turned out to be close to fulfillment. Comparisons and calculations carried out in a "horizontal" manner became necessary. Unorthodox, "new" methods of optimization and interpretations of utility had to replace the "old" classic ones. But the imperative of intergenerational equity appeared not only from "idealistic" reflection, but rather for "biological" reasons: one has to choose strategies supporting the survival of mankind as a whole (as thus also supporting the survival of animals and flora...). All the above remarks and statements are reflected in the techniques of discounting infinite utility streams and algorithmic procedures recommended. Economists, sociologists and psychologists definitely differentiate studying the "lifetimes" of bulbs or calculation the present values (more generally: accounting and valuation of streams of "classical" financial flows) from the determination of intergenerational (social) discount rates.

We close this section by presenting a kind of a brief catalogue of the branches (or scientific subdisciplines) related to the subject of intergenerational equity and the question of choosing the "correct" method of discounting. These connections appear at the "operational level" (tools and methods), as well as in "philosophical plans" (the essence of some ever-green themes not only in the economic sphere). So we should consider the following (sub)disciplines.

A. Theories of economic growth.

B. The problem of sustainability.

C. Dilemmas regarding intergenerational equity.

D. Sequential decision problems.
E. The general problem of discounting.
F. The "dynamics" of stochastic finance.
G. Theories of saving and investment (life-cycle, permanent income, etc.).
H. Taxation theory.
I. Vertical and horizontal (bequeathing) transfers.
J. The economics of gifts.
K. The theory of altruism in economics and the social sciences.
L. Theories of public debt (the Ricardo–Barro theorem).
M. Welfare economics, common goods, "non-market management", externalities.
N. The consistency of optimum plans and dynamic (sequential) choices.
O. Operational research (in its widest sense):
   a) dynamic programming,
   b) optimal control theory,
   c) (intergenerational) "dynamic" games.
P. Psychology and biology.

3. Milestones in the subject and selected contributors

In this section, we note some important facts (and trends) from the history of scientific efforts aimed at determining the principles of the just evaluation of processes. We also mention some prominent people, who significantly contributed to the development of ideas of intergenerational equity, and searched for "appropriate" discounting rules, observing the profound differences between short-term finance and the long-term perspective of the continued existence of the world. So, in fact, all of them aim "to manage" the quality of life in the far distant future, by means of the appropriate evaluation – ex ante – of the expected costs and benefits brought by such processes.

Restricting our retrospection to the post-medieval period, we may find the roots of these thoughts in the principles (and recommendations) of the Utilitarians, who earned their place in the history of the subject (Bentham [4], Mill [34]), and whose terminology and ideas have survived – in a modified form – to the present day. An interesting reflection about the generality of their framework can be made: it agrees with opposing approaches to the methodology of evaluation, by adapting the systems of weights applied in simple, additive formulas [67]! As has already been mentioned, at the turn of the XIX and XX centuries, von Böhm–Bawerk stated the need for appropriate discounting in evaluation. The opinion of the philosopher Sidgwick – appealing to the equal treatment of generations – was also quoted. We should also consider the thoughts of C. Pigou, who initiated complex, theoretical studies (and practical recommendations) in the field of protecting the quality of the environment for future generations. As well, he introduced the notion of externalities (the famous Pigou tax). Then came a pair of great opponents: the theories of Ramsey and Fisher (also mentioned above) who dominated the debate on the question of "correct" discounting for several decades. The works of Bergson and Samuelson stated the general form of formulas for so called functionals of social welfare. Questions regarding the quantification of the role of "natural driving forces and constraints" in defining economic growth were extensively studied by Hotelling (1931) (in the previously mentioned paper [23]). Environmental problems were in turn widely and deeply analyzed by Baumol [3] who began treating the intergenerational social discount rate as an object entirely different from the "routine, financial" rate of interest.

Now we should point out the role of the discovery made by Strotz. In his seminal paper [69], the author showed that the subjects observed – in the face of problems involving inter-temporal sequences of decisions – acted at variance with the rules of consistency implied by geometric discounting (which, in turn, are implied by the requirement of consistency!). The most disturbing observation was the common violation of the so called rule of the stationarity of impatience, "hidden behind" the constant discount rate. Paradoxically, such inconsistency does not prove the irrationality of subjects (due to the "non-scientific, myopic character" of their behavior). Just the opposite: they do act rationally because they instinctively adapt to changing circumstances. The last statement may seem controversial – because objective circumstances may not, in fact, change. But the author has in mind changes in the whole time-space system, consisting of the (relative) positions in time of certain events and possibilities – as well as the subjects observed themselves: time passing influences the perception of events by these subjects! They (people, as well as animals) have learnt such flexibility over generations or, at least, during an individual's lifetime (many especially instructive experiments carried out have involved observing pigeons' behavior by Rachlin, Green [50] or Mazur [36]. Strotz's investigations began a genuine revolution in the philosophy of discounting.

A parallel "revolution" was initiated by Koopmans in 1960 (and accomplished by Diamond five years later) in defining an axiomatic approach to the problems of evaluating and (pre)ordering infinite streams of economic quantities (we have already mentioned their contributions). So fully justified are their formulations concerning the development of these fields that the terms "pre-Strotz and post-Strotz", as well as, "pre-Koopmans and post-Koopmans" are used.

The "paths" of Strotz and Koopmans differ in some aspects and converge in others. Their common influence has been strengthened by impulses coming from the field of sustainable development.

Now we present a list of prominent persons involved in the subject during the last half century. It should be stressed that this list is subject to extremely severe selection – many eminent scientists have not been included. It may be, somewhat ironically, stated that only a few Nobel prize winners remain. However, they are supplemented by some creators of new views and ideas: K.J. Arrow, P. Samuelson, J. Stiglitz, R. Solow, J. Heal,

P. Dasgupta, T. Mitra, G. Chichilnisky, G. Asheim, J. Parfitt, J. Pezzey, C. P. Hammond, C. Price and W. Bucholtz may be counted among those in the “sustainability stream”. The following have been “animators of the theory of discount paths”: E. Phelps, R. Pollak, F. Kydland, E. Prescot, D. Prelec, G. Loewenstein, D. Laibson, C. Harvey, M. Weitzman, T. Cowen, C. Price, Ch. Gollier and G. Chichilnisky. Some seminal (representative) papers of these authors have been included in the bibliography. It is impossible (and inappropriate) to discuss the topics of these papers in detail in this (not too long) paper (or even – to present the subdisciplines they worked in). Some of them have been presented in previous (review) papers of the author [59], [62]. The crucial questions (fundamental controversies) arising at the philosophical level and accompanying these methodological breakthroughs have already been signaled in the paper. So I merely mention the (commonly known) fact of their diverse fields of application: from microeconomics, decision theory, finance, psychology, welfare economics and the theory of economic growth, detailed problems regarding sustainable development, the management of renewable and non-renewable resources and even topology.

Now we recount some facts of particular (great, global) social, economic and political significance. They express the main worries of the whole community of scientists, politicians, strategic investors and “ordinary people”. Generally, these acts were aimed at stimulating the consciousness of “an obligation to the future”: stopping the voracious exploitation of resources, damage to the environment and instead promoting “sustainable thinking”. In the sphere of planning and calculation, their recommendations can be translated as a refreshing of the old-fashioned technique of evaluating the flows of everything.

The modern age of these attempts can be dated from “Our Common Future” – the “Brundtland Report” of the U.N. World Commission on the Environment and Development (1987). Exactly 20 years after the famous Rome Club Report in 1992 the so called (first) Earth Summit (U.N. Conference on Environment and Development) took place in Rio de Janeiro. The main resolution (UN Agenda 21) obliged governments to “unite countries to pursue sustainable development”. On the 10th anniversary of the first Earth Summit, the so called “Rio+10” World Summit on Sustainable Development was organized in Johannesburg (WSSD 2002) which summarized, continued and developed the resolutions of the Rio Conference.

The last international undertaking that we mention here is The United Framework Convention on Climate Change and International Agreement to Prevent Global Warming, defined by the so called Kyoto Protocol (Kyoto 1997, ratified by Russia in 2002). We should also mention the role of the excellent scientist, professor of economics and mathematics, Graciela Chichilnisky, who was a coauthor of the above document (concerning carbon emissions and the market).

At the end of this section, we present the titles of several volumes which appeared over the last three decades and show the common interests of economists, sociologists, statisticians and mathematicians in various aspects of care for the existence of our descendents, as well as documenting the "maturity" of works in related fields*.

In the next three sections being more formalized, we will omit such (important, but "technical") questions as, e.g. the convergence of the resulting series.

4. Towards "just" (ethical, sustainable) preferences

Let **X** be the space of infinite sequences of non negative real numbers (e.g. "streams of consumption" or other economic quantities). So, $x \in \mathbf{X}$ iff $x = \{x_t,\ t \in N\}$, (in fact $\mathbf{X} \subset l_+^\infty$). Similarly, let **U** be the space of sequences of "instantaneous" (real) utility functions: $u \in \mathbf{U}$ iff $u = \{u_t,\ t \in N\}$. The rule of "classical" discounting is given by the additive functional $U : \mathbf{X} \to \mathbf{R}$

$$U(x) = \sum_{t=0}^{\infty} (1+\rho)^{-t} u(x_t) \qquad (u_t \equiv u) \tag{1}$$

where $\rho \geq 0$ describes time preferences (impatience, discount rate), $d(t) = (1 + p)^{-1}$ is then the discount factor (function) and u is a "standard" utility function (increasing, concave, bounded), the same for each period t, $u : R \to R$. The discount functional $U(1)$ makes a tool for evaluating and comparing streams. Note Ramsey's "ethical" postulate: $\rho = 0$.

Three definitions follow. The first two have become standard. They were formulated during the period of Koopmans' work (1960), while the third was introduced by Asheim (1996).

Definition 1. The preference relation $\preceq$ in l_+^∞ is called intergenerational finitely impartial, if for all permutations $\pi_f : l_+^\infty \to l_+^\infty$, leaving unchanged almost all coordinates

$$\forall x, y \in l_+^\infty \quad x \preceq y \Leftrightarrow \pi_f(x) \preceq \pi_f(y) \tag{2}$$

Definition 2. The preference relation $\preceq$ in l_+^∞ is called weak monotone (Pareto, efficient) if the following implication holds

*SIKORA R.J., BARRY B., *Obligations to Future Generations*, Temple, Philadelphia 1978. *Justice between Age Groups and Generations*, P. Laslett, J.S. Fishkin (Eds.), New Haven, Yale University Press, 1992. PRICE C., *Time, Discounting and Value*, Blackwell, Oxford 1993, *Discounting and Intergenerational Equity*, P.R. Portney, J.P. Weyant (Eds.), Resources for the Future, Washingon 1999. ROEMER J., SUZUMURA K., *Intergenerational Equity and Sustainability*, Palgrave Macmillan, Houndmils, 2007. Journal of Risk and Uncertainty, 37, 2/3, 2008, Special issue on discounting dilemmas. Mathematical Social Sciences, 2010, 59, Special issue on sustainability.

$$\forall t \in \mathbf{N} \quad x_t > y_t \Rightarrow x \succ y \tag{3}$$

Definition 3. Preference relations satisfying the postulates of the two definitions above are named ethical preferences [2].

During the last fifty years many authors contributed to the theory of intergenerational equity in the above setting. Thus numerous very interesting, important and mathematically refined – sometimes, somewhat shocking – topics appeared (see e.g. the review paper [59]). At this point we will sketch the main ideas and theses contained in the paper of Chichilnisky [11], which may be thought of as a kind of mile-stone in the subject: it summarizes previous experience and opens a new perspective for research, as well as relating studies of the properties of preferences with recommendations regarding the method of discounting. In 1996, Chichilnisky proposed two axioms, which would guarantee a form of intergenerational justice in the theory and practice of evaluating and comparing streams. "Her" basic space was slightly more general. **Y** is taken to be the set of vector-valued sequences: $y \in Y$ where $y = \{y_g, \quad g = 1, 2, ...\}$, $y_g \in R^n$ (the subscripts g denote the generations). Consequently, utility functions act according to the rule $u_g : R^n \to R$, $g \in \mathbf{N}_+$. Now we omit (without neglecting!) the regularity conditions and pass to the "core" of the concept. Let us denote the space of all "feasible utility streams" by $\mathbf{\Omega}$

$$\mathbf{\Omega} = \left\{ \mathbf{u} : \mathbf{u} = \left\{ u_g, g \in \mathbf{N}_+ \right\}, \quad u_g = u_g\left(y_g\right), \quad g \in \mathbf{N}_+, \quad y \in \mathbf{Y} \right\} \tag{4}$$

and the valuation (discount) operator by W (W is assumed – as usual – to generate the preferences in $\mathbf{\Omega}$) $W : l_\infty \to R$. For the natural, positive index $K \in \mathbf{N}_+$ we introduce the notions of K-head and K-tail, respectively, of an element $\mathbf{u} \in \mathbf{\Omega}$ by

$$\mathbf{u}^K = \left(u_1, ..., u_K, 0, 0, ..., 0\right), \qquad \mathbf{u}_K = \left(0, ..., 0, u_{K+1}, u_{K+2}, ...\right). \tag{5}$$

The "present" (**P)** and the "future" (**F**) (of the "world" $\mathbf{\Omega}$) is defined by

$$\mathbf{P} = \left\{ \mathbf{u}^K, \ K \in N_+ \right\}, \quad \mathbf{F} = \left\{ \mathbf{u}_K, \ K \in N_+ \right\}, \qquad u \in \mathbf{\Omega} \tag{6}$$

Let us take two (arbitrary) sequences $\mathbf{u}, \mathbf{v} \in l_\infty$ and define the "combined sequence" as the pair $\left(\mathbf{u}^K, \mathbf{v}_K\right)$ joining the head of u to the tail of v, according to the following definition

$$\left(\mathbf{u}^K, \mathbf{v}_K\right) = \left(u_1, ..., u_K, v_{K+1}, v_{K2}, ...\right) \tag{7}$$

Definition 4. A. The functional W is said to represent the dictatorship of the present (in short: W is DP), if $\forall u, v \in l_\infty$

$$W(u) > W(v) \Leftrightarrow \underset{M \in \mathbf{N}_+}{\exists} M = M(u,v)$$
$$\times\left[(K > M) \Rightarrow \left(W\left(u^K, r_K\right) > W\left(v^K, s_K\right)\right)\right] \quad \forall r, s \in l_\infty \tag{8}$$

B. The functional W is said to represent the dictatorship of the future (in short: W is DF), if $\forall u, v \in l_\infty$

$$W(u) > W(v) \Leftrightarrow \underset{M \in \mathbf{N}_+}{\exists} M = M(u,v)$$
$$\left[(K > M) \Rightarrow \left(W\left(r^K, u_K\right) > W\left(s^K, v_K\right)\right)\right] \quad \forall r, s \in l_\infty \tag{9}$$

It seems useful to comment briefly on the formal definitions given above. Roughly speaking, the dictatorship of the present requires (from the discount functional W) the optimum behaviour of the present generation not to depend on the utility u_g, $\mathbf{u} \in \mathbf{\Omega}$ gained by almost all generations.. Analogously, the dictatorship of the future can be described as "insensibility" to initial (finite) segments of the utility sequence.

The two ethical postulates of G. Chichilnisky require that the functional W does not support either the *dictatorial role of the present or of the future*. This is formulated as the following two axioms, A1: W is not DP, A2: W is not DF.

Definition 5. The preference relation $\preceq$ in the space $\mathbf{\Omega}$ is said to be support sustainable preferences (is SP) iff

(*) $\preceq$ fulfils axioms A1 and A2 (expressed in terms of "its" W),

(**) $W_{\preceq}$ is an increasing function: $W_{\preceq} : l_\infty \to R$, where the space l_∞ is equipped with the Pareto order.

Now we are able to formulate (a part of) Chichilnisky's theorem, concerning the existence of SP, supplemented by the formulas defining the analytical form of the corresponding function W, as well as deriving from this the famous Chichilnisky criterion.

Theorem 1 (Chichilnisky [11]). There exists a functional W_{SP} which is SP, i.e. fulfills postulates (*) and (**), defined by the formula

$$W_{\mathrm{SP}}(u) = \sum_{s=1}^{\infty} \lambda_g u_g + \Phi(u) \tag{10}$$

where $\forall g \in N,\ \ \lambda_g > 0,\ \ \sum \lambda_y < \infty$ and $\Phi(u)$ denote the extension of the limit functional $\Phi(u) = \lim_g u_g$ to the whole space l_∞.

The Chichilnisky Criterion described above (a discounted sum of instantaneous utilities plus a generalized limit of the utilities of future generations) exploits the following expression

$$W_{\mathrm{SP(CH)}}(u) = \alpha \sum_{g=0}^{\infty} \frac{u_g}{(1+r)^g} + (1-\alpha) \lim_{g\to\infty} u_g \tag{11}$$

for some (arbitrary) $\alpha \in \langle 0, 1 \rangle$. The above expression may be described as a "mixture" of Samuelsian DU utilitarianism together with the "noise of the future" (represented by the generalized limit Φ).

5. Searching for "correct" discounting rules

The time has come to take up the subject of discounting *sensu stricte*. Note that the main purpose of discounting is to enable comparison of (the values of) future capital flows or sequences of flows (in time), from the same (earlier, for instance: the present day's) perspective. Recommended valuation rules take into account the changing value of financial flows over time, and assess future delayed costs and benefits fairly – including various kinds of risks, everything to be faced – in the future. Thus, we (investors, theorists) are forced to practice the "time-values-risk" trade-off. At this point, we consider selected parts of the development of the search for the above mentioned "common denominator" for comparing and evaluating phenomena proceeding ("dynamically") in time. We also note an attempt to describe and quantify the (apparent) anomalies of choice in time, observed by Strotz and his followers and concerning mainly preference reversals and the variability of impatience [69] [50] [70] [27] [73] [33].

Let us consider "time-ouput" streams of pairs of the form

$$(\mathbf{x}, \mathbf{t}) = \{(x_0, t_0), (x_1, t_1), \ldots\}$$

and the (generalized) utilitarian evaluation rule

$$U(\boldsymbol{x}, \boldsymbol{t}) = \sum_{t=0}^{\infty} d(t)\, u(x_t) \tag{12}$$

(with the usual meanings of the symbols d and u).

In such a general setting, underlying preferences characterize subjects as being timing averse if the function d is decreasing, timing neutral (stationary) if d is constant and "timing-prone" for increasing discount functions. The above classification corresponds (roughly speaking) to the following properties: impatience, patience and altruism toward descendents, respectively. The classical example of so called stationary preferences (as well as constant coefficients of impatience and inter-temporal altruism) is provided by "Samuelsian discounting", where $d(t) = d^t$ (or $d(t) = \mathrm{e}^{\delta t}$ – in the continuous case). These are the only discount functions guaranteeing the time consistency of preferences (which may be deduced simply from Euler's formula)

$$\forall s, t \in (0, \infty) \qquad d^t d^s = d^{s+t}, \qquad \mathrm{e}^{-\delta s}\mathrm{e}^{\delta t} = \mathrm{e}^{-\delta(t+s)} \tag{13}$$

Additionally, the „force of discounting" depends merely on the length of intervals, not on their „location" on the time axis. Slightly more general conditions are provided by the principles of consistency for the accumulation of capital in multi-period capitalization (also classical)

$$r < s < t \to A(r, s)A(s, t) = A(r, t) \tag{14}$$

It is commonly known that the above equality (for accumulation factors – in the "Markov style of moving") constitutes the foundation of the methodology of finance.

The stationarity conditions, formulated in the "language of preferences" (in the space of sequences of pairs), may be written as follows

$$(\mathbf{x}, \mathbf{t}) \sim (\mathbf{y}, \mathbf{t}) \Rightarrow (\mathbf{x}, \mathbf{t} + h) \sim (\mathbf{y}, \mathbf{t} + h) \qquad \forall h > 0 \tag{15}$$

Harvey [23] introduced the notion of relative time neutrality (of the above preferences) in the form

$$\exists b > 0: \; (x, s) \sim (y, t) \Rightarrow \forall m > 0 \left(x, s + m(b + s)\right) \sim \left(y, t + m(b + t)\right) \tag{16}$$

where the indifference relation (~) between pairs of numbers ("output, time") is generated by the appropriate (logically – earlier) relation between streams. Harvey characterized the above property by requiring the analytical form of discount functions to have the following form:

$$d(t) = \left(\frac{b}{b + t}\right)^r, \qquad b > 0, \qquad -\infty < r < \infty \tag{17}$$

It should be stressed that for any $r > 0$, the function (17) discounts much more slowly than the geometrical one. Harvey concentrated on so called "proportional" (with respect to time) discounting (as a special case of property (16))

$$\begin{gathered} \forall x, y > 0, \quad \forall s_0, t_0 \\ \left(x, s_0\right) \sim \left(y, t_0\right), \quad \Delta x > 0, \quad \Delta t > 0, \\ \frac{\Delta t}{\Delta s} = \frac{y}{x} \Rightarrow \left(x, s_0 + \Delta s\right) \sim \left(y, t_0 + \Delta t\right) \end{gathered} \tag{18}$$

The main topic of [23] consists of identifying the functional form of the discount factors for postulates (18):

$$d(t) = \frac{b}{b+t} \tag{19}$$

In this way, Harvey's reasoning led to hyperbolic discounting, suggested earlier by psychologists and biologists [27], [36], [50], [70] and then appearing in the well known papers of Prelec [44], Loewenstein [34] and Laibson [31], who proposed the following forms for the function *d*:

$$d(t) = (1+\mu t)^{-\gamma/\delta}, \qquad \delta, \mu, \gamma > 0 \tag{20}$$

$$d(t) = (1+\mu t)^{-\gamma}, \qquad \mu, \gamma > 0 \tag{21}$$

which fit observations better than the "classical" one.

The early idea of Phelps and Pollak [39] (see also [63], [6]), so-called quasi--geometric (or quasi-hyperbolic) discounting should also be mentioned

$$d(t) = \begin{cases} 1 & \text{for} \quad t = 1 \\ \beta\delta^t & \text{for} \quad t > 0, \quad \left(\beta \in \langle 0;1 \rangle\right) \end{cases} \tag{22}$$

The „tricky" translation by one unit and then multiplying by the positive factor β removes some of the failures of classical discounting. The majority of discounting rules proposed over the last decades attempt to describe the commonly observed phenomenon of decreasing aversion with respect to time. We quote the definition of so called absolute decreasing aversion with respect to time. For $s < t$ and $0 < x < y$

$$(x, s) \sim (y, t) \Rightarrow (x, s+h) < (y, t+h) \quad \forall h > 0\,. \tag{23}$$

Condition (23) holds if the discount function $d(t)$ representing preferences is increasing and lg$d(t)$ is strictly convex [23].

At the end of this section, we outline some additional information on this theme. The idea of "lying down" the space of states and attributing to it the role of the time axis is not new, [18], [40], [43]. In such a manner, we can formulate "the horizontal" (with respect to time) version of the St. Petersburg paradox, additive separability of the discounted utilitarian sum corresponds to an (additive) expected utility functional. Acting in this spirit, Bleichord et al. [6] proposed a classification of "postures towards time" analogous to the classical [17] attitudes to risk (CARA, CCRA etc.) referring, in turn, to the Arrow–Pratt tradition. They formulated the concept of "constant absolute decreasing impatience" (CADI) etc., – generally: impatience is a counterpart of risk. The authors obtained non-hyperbolic discounting rules, reflecting various kinds of time inconsistency. Other trials were performed by Gollier [20] regarding so called ecological discounting, linking the ideas of "environmental and financial economics". The subsequent proposition due to Read [53] showed that "discounting in instalments" (after dividing intervals into subintervals) has a "stronger" effect than "one-time discounting" – for the whole interval (subadditivity of real discounting). One very interesting experiment performed Weitzmann [72], who asked groups of eminent economists about their opinion on the "proper" (classical) discount rate. The proposed quantities varied according to the gamma distribution! A second, typical, observation was their common diversity from the Stern Report [71]!

6. Discounting in the case of the stochastic end of the world

In the last section of the paper, we give some thoughts on the consequences (for discounting functionals) of the assumption of uncertainty about the duration of the world (economy). At the beginning, we will follow the framework of the paper by Llavador et al. [33]. The space $\mathbf{\Omega}$ of utility streams (*u* a'la Chichilnisky) is defined and the so called "central planner" (or "ethical observer", EO) is introduced, being exogenic and "neutral" with respect to the mechanics generating utilities. His aim is to maximize the (evaluation, discount) functional $W: \mathbf{\Omega} \to R$. Let us consider two "extreme" evaluation rules (at the disposal of the hypothetical EO: (a) a "Utilitarian EO" has to maximize the sum $\sum u_g$, (b) a "Rawlsian EO" maximizes $\inf\{u_1, u_2, ...\}$ ($u = (u_g) \in \mathbf{\Omega}$). Problem (a) will be called program *U*, and problem (b) – program *S* (for: utility and sustainability, respectively). The general philosophy to be taken is derived from the expected utility framework of von Neumann–Morgenstern. The significant *novum* in the model is the randomness of the lifetime of the world. The authors assume that the length of the world's duration *D* has a geometrical distribution (discrete time):

$$P(D = g) = p(1 - p)^{g-1}, \quad p \in (0, 1); \quad g = 1, 2, ... \tag{24}$$

So we deal with realizations of random vectors of the form $(G; u_1, u_2, ..., u_G)$, whose cardinal utilities can be written as $W(G, u_1, ..., u_G)$ and, finally, the expected utility of the above "lottery", say *L*, is equal to

$$E(L) = p\sum_{g=1}^{\infty}(1 - p)^{g-1} W\left(g; u_1, u_2, ..., u_g\right) \tag{25}$$

Hence, for a given G, the utility of the program U is defined as $W^U\left(G; u_1, ..., u_G\right)$ $= \sum_{g=1}^{T} u_g$, and its expectation

$$E^U(L) = p\sum_{g=1}^{\infty}(1-p)^{g-1}\left(u_1 + ... + u_g\right) \tag{26}$$

which, after simple calculations, can be expressed as

$$E^U(L) = \sum_{g=1}^{\infty}(1-p)^{g-1}u_g \tag{27}$$

Relation (27) states that the above „stochastic-geometrical-utilitarian" (SGU) program is equivalent to the „simple" (non-stochastic!) „discounted utilitarian" (DU) program, with discount weights $\Phi_g = (1 - p)^{g-1}$: one has to maximize the expression

$$W_\phi^U = \sum_{g=1}^{\infty}\phi^{g-1}u_g, \qquad u \in \mathbf{\Omega} \tag{28}$$

Analogously, for the Rawlsian („sustainable") evaluation one works with $W^R\left(G; u_1, ..., u_6\right) = \min\left\{u_1, ..., u_G\right\}$. Consequently, the expected utility of such a lottery for the "stochastic-geometrical-Rawlsian" (SGR) program is given by the formula

$$E^R(L) = p\sum_{g=1}^{\infty}(1-p)^{g-1}\min\left\{u_1, ..., u_g\right\}, \tag{29}$$

which has to be maximized (over $\mathbf{\Omega}$). The genuinely surprising result of paper [33] is the establishment of the equivalence between the SGU and SGR programs in the case of the cake-eating, single good economy. The authors proved that under a simplified model of an economy without production, when the lifetime of this process has a geometric distribution, then a Rawlsian and utilitarian consumer (or EO) will choose the same consumption streams.

It should be stressed that the randomness of the duration of the system is caused by outside factors which enter the model exogenously, independently of the course of events (contrary to typical stopping times, depending on the history of processes – determined endogenously).

Let us suppose now that the lifetime of a multi-period undertaking is governed by the Poisson distribution $P(\lambda)$. It is worth noting that the Poisson distribution is related to the model of "scattering" points on the positive half-line (say – the time axis).

$$P(D=g)=\frac{\lambda^{g-1}}{(g-1)!}\mathrm{e}^{-\lambda}, \quad \lambda>0, \quad g=1,2,\ldots \tag{30}$$

Thus the expected utility of the utilitarian evaluation functional is given by the formula

$$E^{U}_{P(\lambda)}(L)=\mathrm{e}^{-\lambda}\sum_{g=1}^{\infty}\frac{\lambda^{g-1}}{(g-1)!}\left(u_1+\ldots+u_g\right)=\mathrm{e}^{-\lambda}\sum_{g=0}^{\infty}\frac{\lambda^{g}}{g!}\left(u_0+\ldots+u_g\right), \quad u_0=0$$

This equation can be written as

$$E^{U}_{P(\lambda)}(L)=\mathrm{e}^{-\lambda}\sum_{g=0}^{\infty}R_g(\mathrm{e},\lambda)\,u_g \tag{31}$$

where $R_g(e,\lambda)$ denotes the g-th residual of the series representation of the quantity $\exp(\lambda)$.

So the „stochastic-Poisson-utilitarian" (SPU) program is, in fact, equivalent to the non-stochastic DU sum program with discount factors $f(g)=\mathrm{e}^{-\lambda}R_g(e,\lambda)$:

$$\max\sum_{g=0}^{\infty}f(g)u_g, \quad u\in\Omega \tag{32}$$

Similarly to the previously discussed case, the randomness is "lost", but other reflection may be more interesting. The condition for the convergence of the series (31) is that the sequence of utilities tend to zero at "the appropriate speed" (rank). When $\sum_{g=0}^{\infty}u_g=\mathrm{e}^{\lambda}$, the condition is trivially fulfilled, thus the question arises at to what extent the "tail" of the sequence (u_g) might be thickened? The natural class of candidates is spread across the area between exponential and hyperbolic sequences of utilities. It seems to be interesting to check the "usefulness" of sub-exponential functions (the discrete version), successfully applied to the theory of reliability and risk [41].

6. Conclusions

In the paper, the basic notions, questions (answers) and some history of the development of ideas and topics from research into the fields of intergenerational equity, concepts of sustainability and dilemmas regarding discounting are outlined. Special attention was focused on aspects of conflicts of interest between generations and "just" evaluation (through the "proper" discounting) of infinite streams of economic quantities. The goal is to influence the most important economic parameters of the future world or to "manage" the future course of events (of course: indirectly and partially). There is no universal, unambiguous solution to these questions, but efforts to achieve "golden rules" should be continued, especially those carried out in the last 30 years (these may be regarded as searching for "green golden rules"). The aim is (among others) the exclusion of methodological dictatorships of the future, as well as the present, which may be seen as a development and rigorous exposition of the ideas contained in the famous Brundtland Report. Partial successes were achieved by Chichilnisky, Asheim, Mitra (and many others). On the other hand, investigations which link psychological observations with economic behavior are being carried out. The crucial point here is dynamic inconsistency (concerning sequential decision problems) and incoherency of choice (changing tastes). Non-classical discount formulas have been defined, which "cope" with these problems (in particular: hyperbolic discounting and its mutations). The pioneering challenge of Strotz was elaborated by E. Phelps, R. Pollak, G. Leowenstein, R. Thaler, D. Leibson, C. Harvey, M. Weitzman, C. Gollier, C. Price and T. Cowen (among others). Interesting effects were observed when the life of the world was assumed to have (some kind of) random duration.

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