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Discriminant Analysis and Optimization Applied to Vibration Signals for the Quality Control of Rotary Compressors in the Production Line

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In this paper, the applications of the multivariate data analysis and optimization on vibration signals from compressors have been tested on the assembly line to identify nonconforming products. The multivariate analysis has wide applicability in the optimization of weather forecasting, agricultural experiments, or, as in this case study, in quality control. The techniques of discriminant analysis and linear program were used to solve the problem. The acceleration and velocity signals used in this work were measured in twenty-five rotating compressors, of which eleven were classified as good baseline compressors and fourteen with manufacturing defects by the specialists in the final acoustic test of the production line. The results obtained with the discriminant analysis separated the conforming and nonconforming groups with a significance level of 0.01, which validated the proposed methodology.

Keywords: discriminant analysis; noise; optimization; quality control.

1. Introduction

Due to the high competitiveness of the market in terms of technological change internet development of communications and transportation, globalization among other factors, the quality of products has become an important competitive advantage in the industrial setting (Ganapavarapu, Prathigadapa, 2015). The quality can be defined as the ability of the product to meet the implicit and explicit needs for which it was designed. In this way, the production process must comprise small variations within the projected safety margin. It is considered as a more adequate quality assessment, the quality check after the production process final product usually carried out in the automobile and electronics industries (Nahmias, Olsen, 2015). A study by Duarte *et al.* (2015) showed that the human ear presents itself as a good tool in the noise quality control of compressors in production lines. However, it has the vulnerability of being affected by emotional and environmental problems such as background noise. In addition, assessment by the human ear does not return measurable values for decision making by the company. Wang *et al.* (2017) is concerned about the consequences of exposure to noise, such as the effect on human health and quality of life. These effects can result, among others, in annoyance and cardiovascular diseases. Sánchez *et al.* (2018) points out that one of the main problems of today's society is the high exposure to noise. The high level of exposure to noise can cause damage to people's quality of life. The author reports that WHO (World Health Organization) considers noise pollution as the most important problem in the world after air and water pollution. The use of vibration signals has been successfully performed in the detection of failures in mechanical systems (Lamim Filho *et al.*, 2014; Amarnath, 2016; Rai, Upadhyay, 2016; Vishwakarma *et al.*, 2017). A natural extension of the techniques based on vibration measurement was its use as quality control tools for production line equipment (Carnero *et al.*, 2010; Duarte, 2013). According to D'Elia *et al.* (2014) 'Vibration signals can be successfully captured and analysed for quality control at the end of the production line'. D'Elia *et al.* (2014) in their conclusion demonstrate the importance of selecting proper signal processing tools in order to extract the most reliable information from the signals. The same conclusion was drawn by Duarte (2013), who in his work used a set of 185 vibratory parameters in conjunction with the differential evolution procedure to list the best parameters to be used in the classification of good baseline compressors or manufacturing defects compressors.

Sartorio (2008) applies techniques of multivariate data analysis to agricultural experiments and performs the comparison with univariate analysis techniques, confirming the superiority of the efficiency of multivariate techniques in his dissertation. To apply the technique of multivariate analysis of the data, the author used the statistical analysis software R. Matter and Stutzer (2015) propose a tool developed in software R to make accessible the data of the political sphere of the United States to the wide scientific community, revolutionizing the current reality due to the great complexity of the data. This tool enables communication between statistical techniques and the large database.

Thus, this technique has wide applicability in the optimization of weather forecasting, agricultural experiments or, as in this case study, in quality control compressors noise levels. For the simulation, we used the statistical analysis software R. This report describes the statistical theoretical development and simulation. In agreement with the initial proposal of the work, we used the calculation of the main vibroacoustic symptoms diagnosed by Duarte (2013): kurtosis skewness crest factor K4 energy level of the filtered envelope and difference between the maximum and minimum of the filtered envelope. Classic metrics are listed by Duarte (2013) from the analysis of 185 symptoms. In this study, we decided to choose the symptoms that presented the best results in Duarte's work (2013). The symptoms were calculated and used as data entry in the multivariate model with the objective of determining Fisher's Linear Discriminant Function (FDLF). One of the results is from the data of a chosen compressor, to determine if it is adequate or not, through application of the FDLF found. Another result is the application of a program in the Gurobi software to classify the compressors.

Carletti (2013) presents in his article the results of a study on the "customization" of a sound quality methodology for the noise control of construction machines. Construction machines as well as rotary compressors generate high noise levels which can lead to health and performance problems at work. A rotary compressor can be defined as an industrial equipment to increase the pressure of a gas in the gaseous state (Fagundes Neto, Duarte, 2015). Expanding the definition, it is a device that uses the rotating action of an internal cylinder to a chamber of the same format, whose function is the compression of the refrigerant gas (Tecumseh, 2016). Rotary compressors have fewer components compared to other compression technologies and are commonly used in air-conditioning and refrigerators.

The noise spectrum of the rotary compressors is considered complex due to the kinematics of the compression process and to the large compressor frame area (Fagundes Neto, Duarte, 2015). Thus, the noise spectrum consists of low, medium and high frequencies. The low frequencies are controlled by the mechanism kinematics, electric motor and vane. The medium frequencies originate from the gas flow, valve, shaft and roller. High frequencies are generated by friction.

Barbosa *et al.* (2002) points out that the traditional analysis of vibratory systems in machines were studied in the first modes, i.e. at low frequencies. However, the analysis of vibration in high frequency have already aroused interest at some point. Barbosa *et al.* (2003) points out that the region of high frequency is responsible for most of the general level of sound power, that is, any method of noise control should be to minimize the noise radiated in this region of frequency.

For *ASHRAE Handbook* (2008), the main sources of noise of rotary compressors are the internal turbulences, impacts of the valves, friction and electric motor. Gerges (2000) defines the dominant sources of noise in compressors as flow turbulence as a function of the non-smooth flow of the fluid, separation of the flow by the interaction of flow – rotating parts and flow – fixed parts (stators) or by means of the flow with other structural parts and the non-stationary, that is, irregular flow in the blades of the rotors, which generates noise for the rotation frequency and its harmonics.

Figure 1 shows a fixed vane-type rotary compressor in which are detailed some parts of the compressor such as the vane, discharge port, suction port, rotor, compressor shaft, compressor chamber and cylinder.

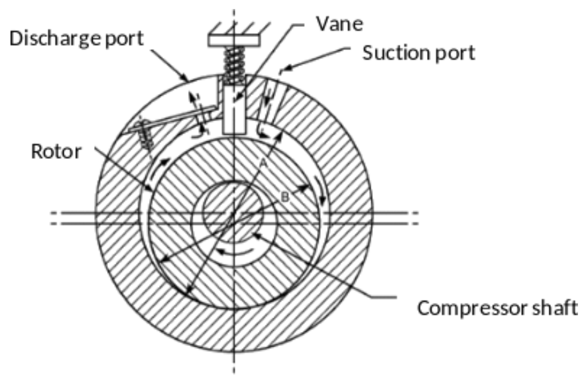

Fig. 1. Rotary compressor.

Fisher addressed the problem of multivariate analysis AD discriminant analysis in the year 1936 with the achievement of a linear combination of measured characteristics that presented the highest potential discrimination among the groups studied according to Sartorio (2008).

Santos *et al.* (2003) believes that the solution to research problems when there are two or more groups of units for which a number of characteristics has been calculated and we want to classify new units based on the same characteristics, we find in the multivariate analysis discriminant analysis (AD). Zuge and Chaves Neto (1999) affirm that AD is a multivariate technique to verify a classification made *a priori*.

Reis (1997) defines AD as the construction of a classification rule, that is, the objective of AD is to find a linear combination of independent variables that makes it possible to minimize the probability of erroneous classification of units/individuals. The first step is to identify the discriminant variables in the model. The discriminant variables are the response variables with the greatest discrimination power between the groups analysed in the multivariate model. From the discriminant variables, discriminant functions are estimated, whose objective is the classification of new units/individuals.

The discriminant function is used to identify the discriminant score of the data of the model studied. The discriminant score is the value found after the use of the discriminant function. The cutoff point is the determinant to perform a new classification of a new individual/unit analysed. The cutoff point is calculated by means of the means of the discriminant scores of each group analysed in the multivariate model.

The three hypotheses of the multivariate discriminant analysis method are presented below.

- H1.1: The discriminant variables present normal multivariate distribution.
- H1.2: The covariance matrices of the clusters are the same.
- H1.3: Clusters differ in their means.

For Khattree and Naik (2000), discriminant analysis is a type of multivariate statistics whose focus is the separation of units from a population into two or more classes according to the characteristics of the proposed mathematical model.

Regazzi (2000) reports that the record of the first approach to the problem of discrimination between two or more groups for later classification dates back to 1936, by Fisher. The procedure tries to classify an individual Z or experimental unit Z in one of the populations or groups studied. Thus, measures of a number of characteristics are taken to minimize the probability of misclassification, i.e. to minimize the probability of classifying an experimental unit in the population i, when in fact it is the population j.

2. Material and methods

The work methodology was divided into four parts. The first one refers to the technique of bibliographical revision of the vibroacoustic symptoms used (kurtosis, asymmetry, mean square value, crest factor, K4, filtered envelope energy level, and difference between maximum and minimum of the filtered envelope), rotary compressors, and statistical techniques for multivariate data analysis.

The second part of the methodology corresponds to the structuring of the work. In this part, the data acquired by Duarte (2013) for the application of the statistical technique of multivariate analysis denominated discriminant analysis in the software of statistical analysis were used. The vibration acceleration signals were measured at two distinct points of the carcasses, with a sampling frequency of 33 333 Hz according to Fig. 2.

Fig. 2. Used compressor.

The following signs were acquired:

- Acceleration on the compressor cover.
- Speed on the compressor cover.
- Acceleration at the point near the soldering point of the compressor kit.
- Speed at the point near the soldering point of the compressor kit.

The acceleration signals used in this work were those acquired near the soldering point of the compressor kit. The multivariate model was developed from the calculated symptoms with the objective of determining Fisher's Linear Discriminant Function (FDLF). One of the results of the work is, from the data of a chosen compressor, to determine if it is conforming or nonconforming, through application of the FDLF found.

Another result of the work is the development of a program in Gurobi software, with an academic license, object oriented in C++ with dynamic allocation of memory to identify nonconforming products.

For the computer simulation, a computer with the following technical specifications was used: Ubuntu 16.04 LTS operating system, 3.8 GB memory, Intel® Core i5 processor, 64-bit system type and 582.8 GB disk.

In the fourth part of the work, a comparison was made of the results obtained with the multivariate analysis technique implemented in the R software and the results obtained with the linear programming in the Gurobi software with the results obtained by Duarte (2013).

2.1. Fisher's linear discriminant function

The methodology used in this work to obtain Fisher's linear discriminant function (FDLF) is based on the following steps:

(i) Input of data in software R.
(ii) Matrices construction.
(iii) Estimation of the groups arithmetic means.
(iv) Calculations of groups variance and covariates matrices.
(v) Estimation of the common variance.
(vi) Calculation of the inverse matrix of the common variance.
(vii) Calculation of the discriminant vector estimator.
(viii) Obtaining FDLF.
(ix) Reliability evaluation of the FDLF function.

The numbered steps from (i) to (vii) allow the calculations of the 5 parameters required to construct the FDLF function, as explained below.

- *Parameter 1:* Matrices correspond to the values of the symptoms.
- *Parameter 2:* Arithmetic means of the groups, obtained by the formula of Eq. (1), where is the input vector of the data of each group:

$$\sum_{i=1}^{n} x_i. \tag{1}$$

- *Parameter 3:* Matrices of variances and covariance of the groups given by Eq. (2), where $\mathbf{Y}$ is the data matrix for each population group. In Eq. (2) we consider v the number of variables observed in each experiment. The variances and covariances are defined according to the matrix of Eq. (2). In Eq. (2) I corresponds to the identity matrix of dimension v, $x_{jj} = \mathrm{var}(Y_j)$ the variance of the j-th variable, $x_{jj'} = \mathrm{cov}(Y_j, Y_{j'})$ sample covariance between the variables j and j', $j, j' = 1, 2, ..., v$ and $j \neq j'$. For all variables j and j', we have $x_{jj'} = x_{j'j}$. The cov matrix is composed of v variances and $\frac{1}{2}v(v-1)$ potentially different covariances

$$\mathrm{cov} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1v} \\ x_{21} & x_{22} & \dots & x_{2v} \\ \vdots & \vdots & & \vdots \\ x_{v1} & x_{v2} & \dots & x_{vv} \end{bmatrix} = \frac{1}{n-1} Y' \left(I - \frac{1}{n} 11' \right) Y. \tag{2}$$

- *Parameter 4:* Common variance, Sc according to Eq. (3), where A represents the studied population A, SA is the common variance of population A, and SB represents the common variance of population B

$$Sc = \frac{(nA-1) \cdot SA + (nB-1) \cdot SB}{nA + nB - 2}. \tag{3}$$

- *Parameter 5:* Corresponds to the inverse of the common variance according to Eq. (4) and Eq. (5) for square matrices of dimension n, where ISc represents the inverse of the combined common variance of groups A and B and I is the identity matrix of order n

$$ISc \cdot Sc = I_n, \tag{4}$$

$$Sc \cdot ISc = I_n. \tag{5}$$

After the calculation of these 5 parameters, the estimator L is calculated, which is obtained by the product of the arithmetic mean of the groups by the inverse of the common variance according to Eq. (6)

$$L = XAB \cdot ISc. \tag{6}$$

Thus, we obtain the equation for the classification of new compressors according to Eq. (7), that is, the FDLF function. X represents the input

$$D(x) = L \cdot x[XA - AB]' \cdot ISc \cdot x. \tag{7}$$

After calculating the FDLF function, the mean point m of the groups is determined. Based on the calculated m-point m, Fisher's classification rule can be obtained.

$$m = \frac{(\text{groupmean A} + \text{groupmean B})}{2}, \tag{8}$$

$$ra_0 \text{ should be allocated to group A if } D(x) \geq m, \tag{9}$$

$$ra_0 \text{ should be allocated to group B if } D(x) < m. \tag{10}$$

2.2. Simplex method

From the vibroacoustic symptoms defined according to the procedure detailed in Sec. 1 and Subsec. 2.1, the mathematical restrictions for the model were prepared. Thus, there is the need to divide the problem into two parts to perform the modelling in the Gurobi software, because it allows the segregation of the conforming products and the nonconforming products. The model with only one function without dividing into two parts resulted in the non-viable response by the software.

Gurobi is an academic-licensed optimization software. We used dynamic memory allocation, C++ programming language, object-oriented programming, non-linear. The mathematical function of the lower model can be observed in Eqs (11) and (12)

$$\begin{gathered}\min \sum_{i=1}^{25} x_i,\\ LimLower_{ij} \quad Valor_{ij} x_i \quad \forall i = 1, ..., 25 \quad \forall j = 1, ..., 6,\\ x_i \in \mathbb{Z} \quad \forall i = 1, ..., 25.\end{gathered} \tag{11}$$

The mathematical function of the upper model can be observed in Eq. (12)

$$\begin{gathered}\max \sum_{i=1}^{25} x_i,\\ Value_{ij} x_i \quad LimUpper_{ij} \quad \forall i = 1, ..., 25 \quad \forall j = 1, ..., 6,\\ x_i \in \{0, 1\} \quad \forall i = 1, ..., 25.\end{gathered} \tag{12}$$

The idea of the model created is to cross the two parts, bottom and top, and from there the software can issue the list of conforming and nonconforming products.

The lower limit was observed by 24 of the 25 analysed compressors. The upper limit was not respected by 12 of the 25 compressors analysed. In this way, it is possible to observe that, in mathematical terms, the lower limit is easier to be attended compared to the upper limit.

Gurobi software took seconds to solve this problem. In this way, you can increase the amount of symptoms and the number of units/compressors analysed easily in the initial lines of the source code.

The method used in this work is the Simplex optimization method. According to Taha (2008), two assumptions must be considered for modelling, as presented below.

- The constraints are represented by equations, so that the right side of the equations is non-negative.
- Non-negative variables.

The observation of these two assumptions is important because it will standardize and increase the efficiency of the Simplex Method. In inequalities, the right side represents the limit imposed on the availability of a resource and the left side refers to the use of this resource limited by the model variables. Thus, the difference between the right side and the left side of the constraint represents the amount of unused resource, also called the gap.

3. Results and discussion

Duarte (2013) used NWS sound power levels measured in 1/3 octave-centered bands between 100 and 10 000 Hz. The NWS values in 1/3 octave bands, the global values in dBL, the global values in dBA and the mean values of electric current, resulted in 24 symptoms to be compared.

From the experiments obtained on the three initial phases of the research, the fourth phase of the tests was carried out. We adopted two groups (group A and group B) and six vibroacoustic symptoms, which are the variables of the multivariate model created. The following symptoms were adopted as variables. Symptoms calculated on the basis of regions of the spectrum related to frequencies of compressor components for the studied signals, parameters X1, X2, X3, X4, X5 e X6.

- $X1$DE 10 000 Hz (difference between the maximum and minimum of the filtered envelope with high pass filter with cutoff frequency of 10 000 Hz).
- $X2$NEE 10 000 Hz (energy level of the filtered envelope with high pass filter with cutoff frequency of 10000 Hz).
- $X3$DE 6000 Hz (difference between the maximum and minimum of the filtered envelope with high pass filter with cutoff frequency of 6000 Hz).
- $X4$NEE 6000 Hz (energy level of the filtered envelope with high pass filter with cutoff frequency of 6000 Hz).
- $X5$DE 8000 Hz (difference between the maximum and minimum of the filtered envelope with high pass filter with cutoff frequency of 8000 Hz).
- $X6$NEE 8000 Hz (energy level of the filtered envelope with high pass filter with cutoff frequency of 8000 Hz).

For the group of conforming products, there are the vibroacoustic symptoms mentioned as exemplified in Table 1.

Table 1. Group A of the conforming products.

Compressor	NEE (6000 Hz)	DE (6000 Hz)	NEE (10 000 Hz)	DE (10 000 Hz)	NEE (8000 Hz)	DE (8000 Hz)
ra_1	10.30	71.4	8.46	63.0	9.46	67.1
ra_2	7.28	47.7	5.30	37.8	6.02	41.6
ra_3	7.73	49.6	4.73	38.9	6.05	45.0
ra_4	8.01	71.6	5.28	46.1	6.39	53.7

Table 2 shows the model for discriminant analysis for nonconforming products. Group A refers to 1 of conforming compressors and group B to 2 nonconforming compressors.

Table 2. Fourth modelling of discriminant analysis.

$X1_A$	$X2_A$	$X3_A$	$X4_A$	$X5_A$	$X6_A$
46.7	5.56	70.0	8.77	61.7	7.19
55.6	6.67	69.4	8.98	64.0	8.06
38.1	4.82	54.4	7.12	48.0	6.23
55.0	6.32	62.3	7.32	59.0	6.81
$X1_B$	$X2_B$	$X3_B$	$X4_B$	$X5_B$	$X6_B$
89.0	10.90	96.1	11.60	91.7	11.20
114.0	11.40	123.0	12.80	120.0	12.20
81.4	7.66	83.9	8.57	81.8	8.01
58.8	8.66	67.6	9.29	63.4	9.03

From the Fisher Linear Discriminant Function (FDLF), the equation for the classification of new compressors was estimated, resulting in Eq. (13)

$$D(x) = [1.2568 - 18.2164 - 0.0108 - 0.7007 - 1.3783] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}. \tag{13}$$

As the midpoint of populations A and B is −3.432, the classification rule based on Fisher's Linear Discriminant Function (FDLF) will be given by Eqs (14) and (15):

$$ra_0 \text{ should be allocated to group A if } D(ra_0) \geq -3.432, \tag{14}$$

$$ra_0 \text{ should be allocated to group B if } D(ra_0) < -3.432. \tag{15}$$

A new compressor x_0 whose variables x_1 x_2 x_3 x_4 x_5 x_6 are used to test the new classification rule created in this work based on Fisher's Linear Discriminant Function (FDLF) 88.1; 10.2; 122.1; 4.5; 118.8 and 12.6.

As the function $D(ra_0)$ is −6.968916 value less than −3.432 the compressor ra_0 will be allocated in group B, nonconforming products, that is, it was allocated in the correct group according to the classification made by Duarte (2013).

3.1. Significance test

Significance analysis resulted in F_0 of 7.869 being greater than the value of tabulated F_{tab} = 4.388. Since H_0 is rejected, it can be concluded that the separation between groups is significant.

The very promising results open a great perspective for the use of the classification rule based on Fisher's Linear Discriminant Function (FDLF) in the construction of pass-through filters on production lines using vibroacoustic signals. The construction of the filter consists of the following steps.

1) Construct a database with the vibroacoustic signals of a conforming group and a database of a group that does not conform.
2) Evaluate all vibroacoustic symptoms applicable to the studied problem.
3) Using an optimization procedure, construct a Discriminant Function that results in maximizing the alternative hypothesis of the Significance Test.

3.2. Optimization

Another result of the work is the development of a program in Gurobi software, with an academic license, object oriented in C++ with dynamic allocation of memory to identify conforming and nonconforming products. For this, the steps presented in section 3 of this article were performed.

First, the problem to be studied was defined: the distinction between conforming and nonconforming products based on calculated vibroacoustic symptoms. The data were acquired by Duarte (2013) and from these, the symptoms were calculated.

The variables used in the model are: a_i energy level of the filtered envelope with high pass filter with cutoff frequency of 6000 Hz, b_i difference between maximum and minimum of the filtered envelope with high pass filter with cutoff frequency of 6000 Hz, with level of filtered envelope energy with high pass filter with cutoff frequency of 8000 Hz, difference between maximum and minimum of the filtered envelope with high pass filter with cutoff frequency of 8000 Hz, with energy level of the envelope filtered with high pass filter with frequency 10 000 Hz cut, and difference between maximum and minimum of the filtered envelope with high pass filter with cutoff frequency of 10 000 Hz according to Table 3, exemplified for some compressors.

Table 3. Optimization: model variables.

Compressor	ELFE (6000 Hz)	DFE (6000 Hz)	ELFE (8000 Hz)	DFE (8000 Hz)	ELFE (10 000 Hz)	DFE (10 000 Hz)
ra_1	10.30	71.4	9.46	67.1	8.46	63.0
ra_2	9.85	79.0	8.31	66.6	6.97	58.2
ra_3	7.28	47.7	6.02	41.6	5.30	37.8
ra_4	7.73	49.6	6.05	45.0	4.73	38.9

From the study of the vibroacoustic symptoms the restrictions for the construction of the mathematical model were defined. The mathematical model was divided into two parts: part 1 of the model, lower limit and part 2 of the model, upper limit. The Gurobi software from the implemented code crosses the information of the two parts of the model, in order to verify what meets the two parts of the model. The code implemented in the C++ language is available in the dissertation work of Reis (2017).

The mathematical function of the lower model can be observed in Eqs (16) and (17)

$$\begin{gathered}\min \sum_{i=1}^{25} x_i,\\ LiInf_{ij} \quad Valor_{ij} x_i \quad \forall i = 1, ..., 25 \quad \forall j = 1, ..., 6,\\ x_i \in \mathbb{Z} \quad \forall i = 1, ..., 25.\end{gathered} \tag{16}$$

The mathematical function of the upper model can be seen in Eq. (17)

$$\begin{gathered}\max \sum_{i=1}^{25} x_i,\\ Valor_{ij} x_i \quad LiSup_{ij} \quad \forall i = 1, ..., 25 \quad \forall j = 1, ..., 6,\\ x_i \in \{0, 1\} \quad \forall i = 1, ..., 25.\end{gathered} \tag{17}$$

Figure 3 shows the results obtained with Gurobi software. It is possible to observe that the output of the program presents the 11 suitable compressors and the 14 inadequate compressors in accordance with the results obtained by Duarte (2013).

Compressor 1 is good.	Compressor 14 is good.
Compressor 2 is bad.	Compressor 15 is good.
Compressor 3 is good.	Compressor 16 is bad.
Compressor 4 is good.	Compressor 17 is bad.
Compressor 5 is good.	Compressor 18 is good.
Compressor 6 is bad.	Compressor 19 is bad.
Compressor 7 is good.	Compressor 20 is bad.
Compressor 8 is bad.	Compressor 21 is bad.
Compressor 9 is bad.	Compressor 22 is bad.
Compressor 10 is good.	Compressor 23 is bad.
Compressor 11 is bad.	Compressor 24 is bad.
Compressor 12 is good.	Compressor 25 is bad.
Compressor 13 is good.	
Detailed solution:	
The total of good compressors is: 11.	

Fig. 3. Output of Gurobi.

In Fig. 4 it can be observed that the lower limit was respected by 24 compressors, only one compressor was not respected.

Detailed solution:	
The total of good compressors is: 11.	
F0 lower: 26	
X[1]: 1.	X[14]: 1.
X[2]: 1.	X[15]: 1.
X[3]: 1.	X[16]: 1.
X[4]: 1.	X[17]: 1.
X[5]: 1.	X[18]: 1.
X[6]: 2.	X[19]: 1.
X[7]: 1.	X[20]: 1.
X[8]: 1.	X[21]: 1.
X[9]: 1.	X[22]: 1.
X[10]: 1.	X[23]: 1.
X[11]: 1.	X[24]: 1.
X[12]: 1.	X[25]: 1.
X[13]: 1.	

Fig. 4. Detailed solution of Gurobi: lower limit.

In Fig. 5 it can be observed that the upper limit was respected by 12 compressors, and 13 compressors did not respect the limit.

F0 lower: 12	
X[1]: 1.	X[14]: 1.
X[2]: 0.	X[15]: 1.
X[3]: 1.	X[16]: 0.
X[4]: 1.	X[17]: 0.
X[5]: 1.	X[18]: 1.
X[6]: 1.	X[19]: 0.
X[7]: 1.	X[20]: 0.
X[8]: 0.	X[21]: 0.
X[9]: 0.	X[22]: 0.
X[10]: 1.	X[23]: 0.
X[11]: 0.	X[24]: 0.
X[12]: 1.	X[25]: 0.
X[13]: 1.	

Fig. 5. Detailed solution of Gurobi: upper limit.

It is worth mentioning that the code implemented according to (Reis, 2017) can be adapted to work with as many compressors as it seems appropriate. Suffice it to change the number of compressors and the input file with the vibroacoustic symptoms calculated for the compressors to be evaluated. It is also possible to implement more vibroacoustic symptoms.

4. Conclusions

The objective of this work was to apply the technique of multivariate analysis and optimization to control the noise quality of compressors using R software and Gurobi software. Through the literature review, it was possible to conclude that the discriminant analysis technique can be applied to the noise quality control of compressors. Thus, the methodology of discriminant analysis, multivariate technique, was applied to the case study for the noise quality control with the signals acquired by Duarte (2013) for 25 compressors.

In phase 4 of the tests were used six vibroacoustic symptoms for which discriminant analysis was performed and satisfactory results were obtained with a level of significance of 0.01 and it was possible to conclude that the separation between groups A and B is statistically significant by the tests implemented in software R.

Thus, the results obtained with the discriminant analysis were satisfactory with a level of significance of 0.01 and it was possible to conclude that the separation between groups A and B is statistically significant by the statistical tests implemented in software R for phases 1, 2 and 4 modelling as details available in (Reis, 2017). While the results of phase 3 show that the mathematical model of this phase did not allow the significant separation between groups A and B, according to statistical tests performed, which indicates the redundancy of the symptoms used.

The results obtained using the methodology proposed in this work are superior to the classical methods in relation to the cost attribute, since the statistical analysis software R used in the computational tests is free software, i.e. it does not have license costs and allows the processing of the order of millions. One of the results of the work is, from the data of a chosen compressor, to determine if it is adequate or inadequate, through application of the FDLF found.

Another result is the development of a Gurobi software program, with an academic license, object oriented in C++ with dynamic memory allocation to identify conforming and nonconforming products. Like software R, Gurobi software allows the processing of data in the order of millions, which justifies the choice of these two software for this work. All the objectives of this work presented in the summary and introduction were satisfactorily fulfilled.

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