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COMPLEX FOURIER SERIES MATHEMATICAL MODEL OF A UNIVERSAL MOTOR SUPPLIED BY A TRIAC

MODEL MATEMATYCZNY SILNIKA UNIWERSALNEGO ZASILANEGO POPRZEZ TRIAK PRZY WYKORZYSTANIU ZESPOLONYCH SZEREGÓW FOURIERA

Abstract: The present contribution shows an analytical method of the calculus of the torque ripple and current waveforms of a universal motor supplied by a triac directly from the network. The triac output voltage waveform is formulated by the complex Fourier series. The armature reaction of the motor is included in the calculus. The motor performance is computed using the circuit parameters determined by measurements. The calculated current waveforms are compared with the measured ones.

1. Introduction

Despite their disadvantages, universal motor belong to the most used electric machines in home appliances as well as workshop hand tools. Thanks to their excellent regulation properties they are employed as drive motors of washing machines, professional mixers or mills. Their versatility is given by the fact that they can be supplied by both direct and alternative voltages. In both cases the motor speed is controlled by the value of the supply voltage.

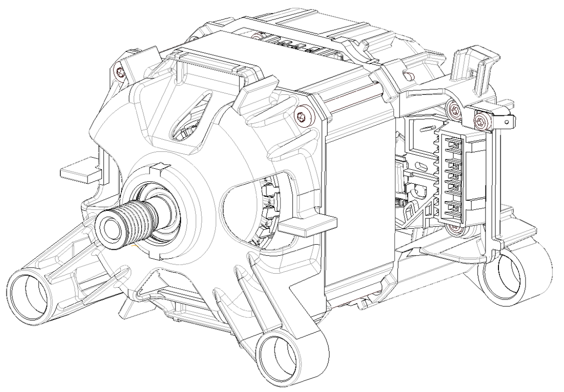

Fig. 1. Construction of universal motor used for washing machine drive

Main advantages of the universal motor include:

- economical and smooth speed regulation in wide range,
- easy start, large acceleration torque and small acceleration current,
- good power factor.

Small universal motors are produced without compensating and commutation winding. The stator contains mostly two salient poles with exciting winding. The number of the commutator brushes is equal to the number of excitation winding poles. Since magnetic field of the machine is alternating, the magnetic circuit is laminated. The stator and the rotor are series connected.

2. Mathematical model of the motor

Mathematical analysis of a universal motor is based on the voltage equations, with certain simplifying assumptions [1]:

- mechanical losses and also losses in iron are neglected,
- commutation influence is neglected. A perfect commutating armature is assumed,
- mutual inductance is supposed to be constant. Saturation effect is neglected.

Fig.2 depicts the equivalent circuit of a two-pole universal motor. The armature has two brushes on the diameter of the commutator and it is shifted by an angle α_a towards the axis of the exciting magnetic flux. The rotor rotates with mechanical angular velocity ω_m .

Suppose that the motor is supplied by a variable voltage u . In such a case the following voltage equation can be written:

$$u = Ri + L\frac{di}{dt} + u_i \qquad (1)$$

For the electromotive force (EMF), the folowing general equation:

$$u_i = M\frac{di}{dt}\sin\alpha_a - M\omega_m\cos\alpha_a \qquad (2)$$

M is a mutual inductance between stator and rotor.

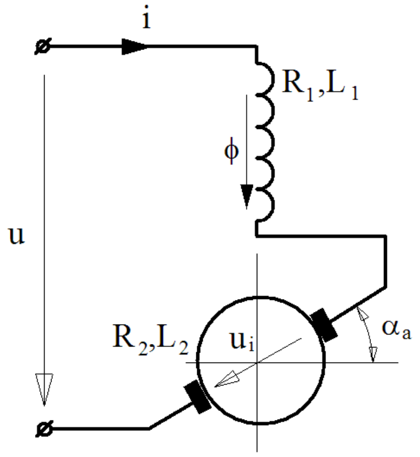

Fig. 2. Equivalent circuit of a universal motor

The instantaneous value of the induced electromagnetic torque is given by the formula:

$$m = Mi^2\cos\alpha_a \qquad (3)$$

3. Mathematical model of a triac convertor

Fig. 3 shows the triac controlled universal motor drive [3], [4]. Suppose that the supply mains voltage is purely sinusoidal.

$$u_1 = U_m\sin\theta = U_m\frac{e^{j\theta}-e^{-j\theta}}{2j} \qquad (4)$$

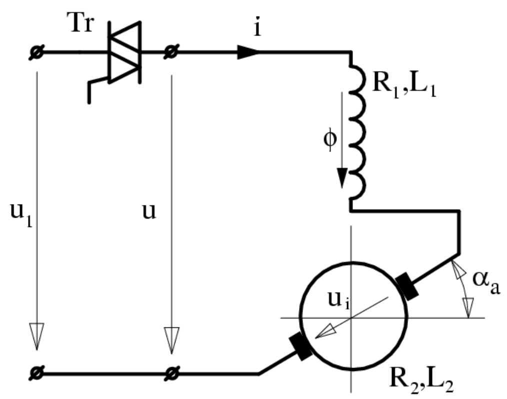

Fig. 3. Triac controlled universal motor drive

Phase angle control of the triac can be used to control motor speed, as shown in Fig.3. The start of each half cycle is delayed by a control angle α. Suppose, motor as a load with ohm and inductive character. For such a load current continues beyond the voltage zero, due to the inductance. Triac turns off after cessation of current. The turn off angle is marked as an angle β.

Triac output voltage can be expressed in a form of the complex Fourier series:

$$u = \sum_{k=-\infty}^{\infty} c_k e^{jk\theta} \qquad (5)$$

Where c_k is a Fourier coefficient, which is for the triac control defined:

$$c_k = \frac{1}{2\pi}\int_{\alpha}^{\beta} u_1 e^{-jk\theta} + \frac{1}{2\pi}\int_{\pi+\alpha}^{\pi+\beta} u_1 e^{-jk\theta} \qquad (6)$$

After calculus we obtain for the Fourier coefficient following formula:

$$c_k = \frac{U_m}{4\pi}\left\{\frac{1}{1-k}\left[e^{j\alpha(1-k)} - e^{j\beta(1-k)}\right] + \frac{1}{1+k}\left[e^{j\alpha(1+k)} - e^{j\beta(1+k)}\right]\right\}$$

For $k \neq \pm 1$

$$c_k = \frac{U_m}{4\pi}\left[j(\alpha-\beta) + \frac{1}{2}\left(e^{-j2\alpha} - e^{-j2\beta}\right)\right]$$

For $k = \pm 1$

In the Fig.4 is shown a typical triac output voltage waveform. The waveform was calculated on the basis a complex Fourier series formula derived above.

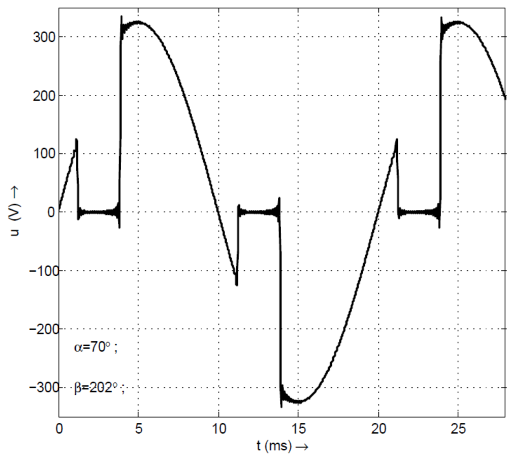

Fig. 4. Triac output voltage waveform

4. Motor current and electromagnetic torque calculation

To determine the motor current waveform, we need to solve equation *(1)*. After substituting *(2)* and *(4)* into *(1)* we obtain.

$$\sum_{k=-\infty}^{\infty} c_k e^{jk\theta} = \sum_{k=-\infty}^{\infty} \Big[\left(R + M\omega_m \sin\alpha_a \right) i + \left(L - M\cos\alpha_a \right) j\omega k i \Big] \qquad (7)$$

The differential equation *(7)* has the following analytical solution:

$$i = \sum_{k=-\infty}^{\infty} \frac{c_k e^{jk\theta}}{\left(R + M\omega_m \sin\alpha_a \right) + j\omega k \left(L - M\cos\alpha_a \right)} \qquad (8)$$

Instantaneous value of intern electromagnetic torque is given by a formula *(3)*.

5. Computation results and their comparison with experimental data

In order to compare the results obtained from the mathematical models of the machine and triac with measured values, the following parameters of the motor were used:

Machine power	$800W / 230V - 50Hz$
Stator resistance	$1{,}61\,\Omega$
Stator inductance	$71{,}4\,mH$
Rotor resistance	$3{,}04\,\Omega$
Rotor inductance	$20{,}7\,mH$
Mutual inductance	$63{,}1\,mH$

Using the equations of the previous sections, the waveforms of the supply voltage, motor current and electromagnetic torque were calculated. The motor worked at constant speed and was loaded by a constant torque. Based on the bibliography [2], was angle of armature reaction was a_a assumed to be 15°.

Fig.5 and Fig.6 show the calculated and measured quantities at steady state at the speed $3000\,rpm$ and control angle $\alpha = 115^\circ$.

Calculated effective current of the is *I=2,976 A* and measured value is *2,98 A*.

Calculated average value of motor moment is M_{AVI}*=0,32 Nm* and measured value is *0,31 Nm*.

To verify the correctness of the above equations were made calculations for other speeds and motor load torque, at different triac switching angles.

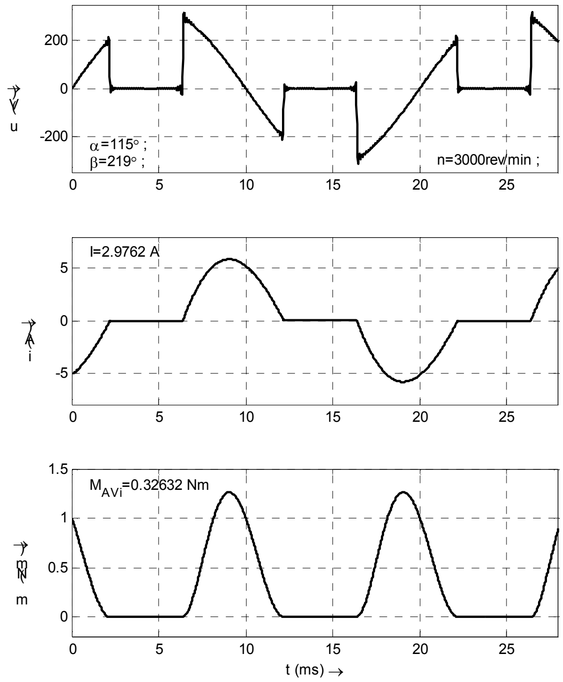

Fig. 5. Calculated motor quantities for 3000 rpm and α=115°

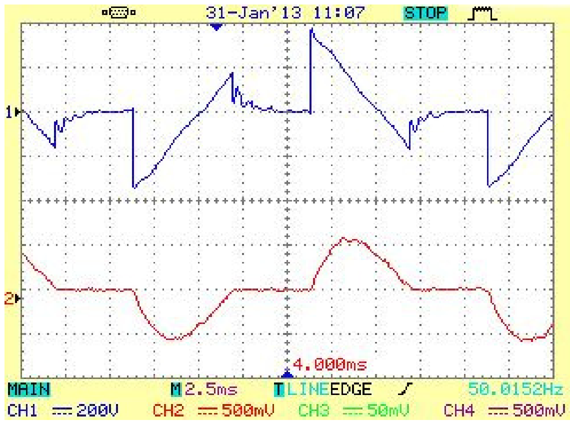

Fig. 6. Measured motor quantities for 3000 rpm and α=115°

So computed behaviours preceded by quantities of motor were by measuring verified. Measurements of the motor confirmed the accuracy of calculated waveforms.

Fig.7 and Fig.8 show the calculated and measured quantities at steady state at the speed $7000\,rpm$ and control angle $\alpha = 103^\circ$.

Calculated effective current of the is I=2,814 A and measured value is 2,80 A.

Calculated average value of motor moment is M_{AVI}*=0, 29 Nm* and measured value is *0,28 Nm*.

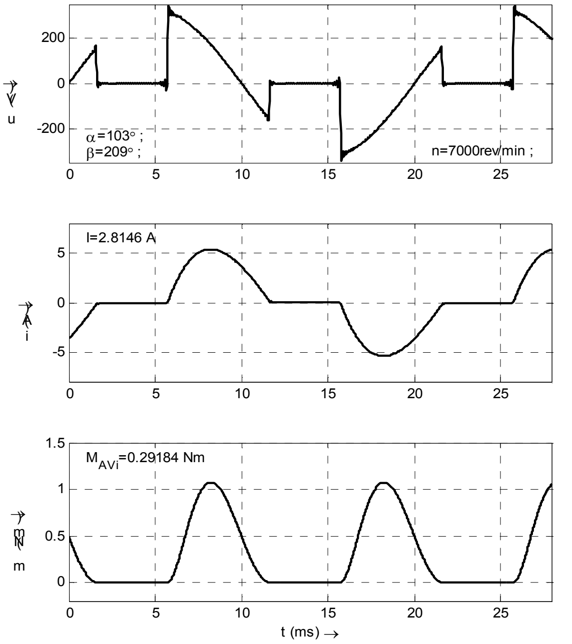

Fig. 7. Calculated motor quantities for 7000 rpm and α =103°

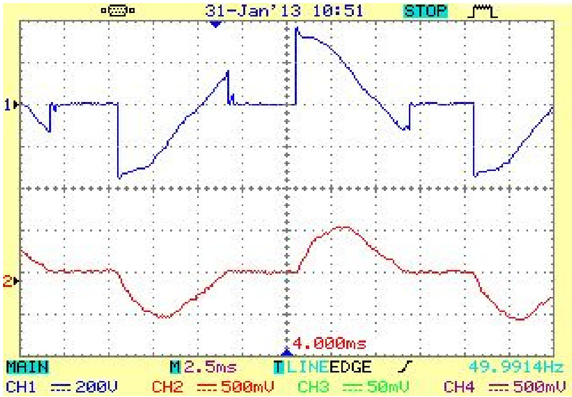

Fig. 8. Measured motor quantities for 7000 rpm and α =103°

6. Conclusion

The paper describes the analytical method for modelling the behaviour of a universal motor supplied by a triac directly from the network. The method is based on the complex Fourier series for the description of the motor supply voltage. Calculation of the supply voltage, current and torque by method shown in the introduction was checked by measurement of the real motor. Calculated waveforms of the supply voltage and the motor current differ very little from the oscilloscope measured waveforms different. The difference in measured and computed motor torque is 0,01 Nm (i.e. 3%), which basically reaffirms the correctness of the analytical calculation.

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