Control and Cybernetics

vol. **42** (2013) No. 4

Control in loan transactions: adjusting the payments to the real inflation rates* †

by

Salvador Cruz Rambaud and José González Sánchez

Departamento de Economía y Empresa
University of Almería
Cañada de San Urbano, 04120 Almería, Spain
scruz@ual.es, jgonza@ual.es

Abstract: This paper explores the effect of inflation on general financial transactions and particularly on loans. For this purpose, inflation is specified in the mathematical expression of a capitalization or discounting function. Thus, we write these functions as the product of an inflation-free capitalization or discounting function and the law of variation of the money purchasing power. As an immediate consequence, we show that the instantaneous rate of a capitalization or discounting function is the sum of the instantaneous rates of each component. Later, this decomposition is applied to simple and compound financial transactions, leading to the equivalence among deflated amounts by means of an inflation-free capitalization or discounting function. This analysis is applied to a loan transaction where we introduce six methods to adjust the payments to the real inflation, because, in a beginning, these payments were calculated with the predicted inflation rates.

Keywords: adjustment, loan, payment, inflation, interest rate

1. Introduction

It is well known that the *time value of money* is the value of money figuring in a given amount of interest earned over a given amount of time (Brigham and Daves, 2007). This is equivalent to saying that investments generate cash-flows to compensate for the time value of money. This principle must get reflected in the mathematical expression of the capitalization or discounting function used to value an investment (Cruz and Ventre, 2008). This trade-off between time and money can be due to different causes and is measured by the rate of return

*Submitted: November 2011; Accepted: November 2013

†We are very grateful for the comments and suggestions of an anonymous referee. This paper has been partially supported by the project "Valoración de proyectos gubernamentales a largo plazo: obtención de la tasa social de descuento", reference: P09-SEJ-05404, Proyectos de Excelencia de la Junta de Andalucía and Fondos FEDER.

(interest or discount rate). The main factors used by investors to determine the rate of return, at which they are willing to invest money, include (see Bodie and Merton, 2000):

- Estimates of future *inflation rates*.
- Estimates regarding the *risk of the investment*.
- Availability of money with the investor for other uses (*liquidity*).

If $F(t,p)$ denotes the capitalization (respectively discounting) function describing the equivalent value at instant p of EUR 1 available at moment $t \leq p$ (respectively $t \geq p$), this underestimate or minor value of future money amounts, can be represented by stating that $F(t,p)$ may be strictly decreasing with respect to t and strictly increasing with respect to p (Gil, 1992; Cruz and Valls, 2008). In other words, the monotonicity of $F(t,p)$ with respect to t and p reflects, in an implicit way, among others, the renounce to liquidity, the risk of the financial transaction and the loss of value of the future money amount due to inflation.

In this paper, we will focus on the monetary inflation, which is defined as the depreciation motivated by the increase in the level of prices. In effect, except for the rare periods of significant deflation, where the opposite may be true, one euro in cash is worth less today than it was yesterday, and worth more today than it will be worth tomorrow (Bodie et al., 2004). At this point, we would like to say that the aim of this paper is not to analyze the factors that, in a monetary economy, generate inflation. The study of these factors, as well as of the destabilizing effects of inflation and the suitable anti-inflationary measures, corresponds to the so-called Economic Policy.

The aim of this paper is to decompose a capitalization (discounting) function with implicit inflation into two components: on the one hand, the mathematical expression of the capitalization (discounting) function including all factors except inflation; and, on the other hand, the mathematical expression of the monetary inflation. But, why is this decomposition interesting? This is because, on some occasions, we can find financial transactions (such as loans) in which the lender or the borrower can pay some amounts of money in advance or can delay the payment of some periods, and in this case it is clear that his/her decision will depend on the forecast of inflation rates (Brealey et al., 2006).

The process leading to specifying the inflation in the mathematical expression of a capitalization (discounting) function can be summative or multiplicative. Nevertheless, we will opt for the multiplicative solution because, as shown in Section 2, the instantaneous rate of a function can be decomposed into the sum of the instantaneous rate corresponding to the function without inflation and the instantaneous rate of the inflation. Later, in Section 3, we will describe the effect of a capitalization (discounting) function without inflation on general financial transactions, this study being particularized to the case of a loan. In Section 4, we will study the financial transactions, in which inflation is uncertain, and again the obtained results are particularized to loans. In this section, we will introduce two methodologies for adjusting the payments to the real inflation rates: the first one does not verify the equation of financial equivalence at the transaction origin, and the other one does satisfy this equation. The section finishes with two theorems which relate, in terms of inequality, the payments obtained in both methodologies. Finally, Section 5 summarizes and concludes.

2. Capitalization (discounting) functions with specified inflation

Let us suppose that the mathematical expression of a capitalization (discounting) function, $F(t,p)$, includes n factors (and among them, the monetary inflation). Therefore, initially, F will be a function of n functions denoted by $F_1, F_2, \ldots, F_n$, that is to say:

$$F = \Psi(F_1, F_2, \ldots, F_n).$$

However, the treatment of inflation in this general framework would be very difficult, so that we are going to just assume that the capitalization (discounting) function can be multiplicatively decomposed into the ordinary product of inflation and the rest of the $n-1$ factors, both quantifying the future underestimate of money.

Therefore, this assumption allows us to decompose the capitalization (discounting) function $F(t,p)$ into the product of two functions:

$$F(t,p) = \hat{F}(t,p) \cdot H(t,p),$$

where:

- $\hat{F}(t,p)$ represents the capitalization (discounting) function in constant monetary units corresponding to instant t, called the *deflated* capitalization (discounting) function, and
- $H(t,p)$ represents the *law of variation of the money purchasing power*, that is, the amount of money at p with the same purchasing power as the monetary unit (EUR 1) at t.

Our entire reasoning is going to be done for both capitalization and discounting functions. Thus, once the existence of inflation has been accepted, there should be:

$H(t,p) > 1$, if $t < p$,

or

$H(t,p) < 1$, if $t > p$.

Nevertheless, there can exist some periods of time, during which inflation behaves on the contrary, though we have to say that these periods are not significant for our study. As an example, we can mention in Spain the period from March to October 2009, whose annual variations in the consumer prices index were as given in Table 1.

Summarizing, the consumer price index is usually a positive variable and it takes negative values solely in very exceptional circumstances, like those exemplified in Table 1. In this way, and in order to reach other economic objectives, the monetary authorities of all countries try to keep this index bounded.

Table 1. An example of a period of negative consumer prices index (Spain)

Year	**Month**	**Variation**
2009	March	-0.1%
	April	-0.2%
	May	-0.9%
	June	-1.0%
	July	-1.4%
	August	-0.8%
	September	-1.0%
	October	-0.7%

Finally, in this section, we will deduce the expression of the instantaneous capitalization or discount rate at instant t:

$$\begin{aligned}\tau(t,p) &:= \frac{-\frac{\partial F(t,p)}{\partial t}}{F(t,p)} = \frac{-\frac{\partial[\hat{F}(t,p)\cdot H(t,p)]}{\partial t}}{\hat{F}(t,p)\cdot H(t,p)} = \\ &= \frac{-\frac{\partial \hat{F}(t,p)}{\partial t}\cdot H(t,p) - \frac{\partial H(t,p)}{\partial t}\cdot \hat{F}(t,p)}{\hat{F}(t,p)\cdot H(t,p)} = \\ &= \frac{-\frac{\partial \hat{F}(t,p)}{\partial t}}{\hat{F}(t,p)} + \frac{-\frac{\partial H(t,p)}{\partial t}}{H(t,p)} =: \hat{\tau}(t,p) + \omega(t,p).\end{aligned}$$

That is, the instantaneous rate in current euro or euro with inflation can be decomposed into the sum of the deflated instantaneous rate or in constant euro:

$$\hat{\tau}(t,p) = -\frac{\partial \hat{F}(t,p)/\partial t}{\hat{F}(t,p)}$$

and the instantaneous rate of inflation:

$$\omega(t,p) = -\frac{\partial H(t,p)/\partial t}{H(t,p)}.$$

3. Financial transactions with specified inflation

In this section, we are going to study the consequences that the use of a capitalization or discounting function with specified inflation has on certain financial transactions. This analysis is of special interest when, in a financial transaction, an advance or a delay in the payments of some of its amounts of money is agreed upon. Moreover, this section may turn out very interesting when one tries to adjust the payments in a financial transaction (calculated with the predicted inflation rates) to the real inflation rates.

Logically, these consequences might be studied in all financial transactions, but in this section we will focus only on both simple and compound amortization transactions because, currently, they have wider practical applications.

3.1. Simple financial transactions

A financial transaction is said to be *simple* if it has a unique initial contribution and a unique final refund. Thus, let us consider a simple financial transaction with the initial contribution (C_0, t_0) and the final refund (C_1, t_1), with i being the interest rate to be applied during the time period $[t_0, t_1]$ including inflation, $\hat{i}$ being the interest rate in constant euro or without inflation, and g being the average rate of inflation corresponding to the same period.

Taking into account the relationship between the former interest rates (Suárez, 1991), i.e.:

$$(1+i) = (1+\hat{i}) \cdot (1+g),$$

the financial equivalence of the transaction at t_1 can be written as follows:

$$C_1 = C_0(1+i) = C_0(1+\hat{i}) \cdot (1+g) = C_0 + C_0\hat{i} + C_0 g + C_0\hat{i}g,$$

which indicates the decomposition of the refunded money into four parts (Gil Luezas and Gil Peláez, 1987):

- C_0 : contributed money,
- $C_0\hat{i}$: interests without inflation corresponding to C_0,
- $C_0 g$: money addressed to compensate the depreciation of C_0, and
- $C_0\hat{i}g$: money addressed to compensate the depreciation of interests $C_0\hat{i}$.

Denote by $\hat{C}_1$ the amount at t_1, equivalent to (C_0, t_0) in a non-inflation situation, that is, $\hat{C}_1 = C_0(1+\hat{i})$, or, equivalently, $\hat{C}_1 = C_1(1+g)^{-1}$. The difference:

$$C_1 - \hat{C}_1 = C_0(1+\hat{i}) \cdot g = C_0 g + C_0\hat{i}g$$

represents the increment to be paid at t_1, as a consequence of inflation. This is shown graphically in Fig. 1.

Next, we are going to study a multiperiod simple transaction, in a general framework of variable interest rates. To do this, we are going to consider a simple transaction with initial contribution (C_0, t_0) and final refund (C_n, t_n), whose duration $[t_0, t_n]$ is divided into n time periods with partial interest rates $i_1, i_2, \ldots, i_n$.

The financial equivalence of the transaction at t_n allows us to write: $C_n = C_0 \prod_{h=1}^{n} (1+i_h)$. In this expression, the inflation is implicit. Nevertheless, in order to specify its influence, we have to write the factors $(1+i_h)$ with inflation or in current euro, as the product of the factors in constant euro $(1+\hat{i}_h)$ and the inflationary factors $(1+g_h)$, which results in:

$$C_n = C_0 \prod_{h=1}^{n} (1+\hat{i}_h) \cdot (1+g_h) = C_0 \prod_{h=1}^{n} (1+\hat{i}_h) \cdot \prod_{h=1}^{n} (1+g_h).$$

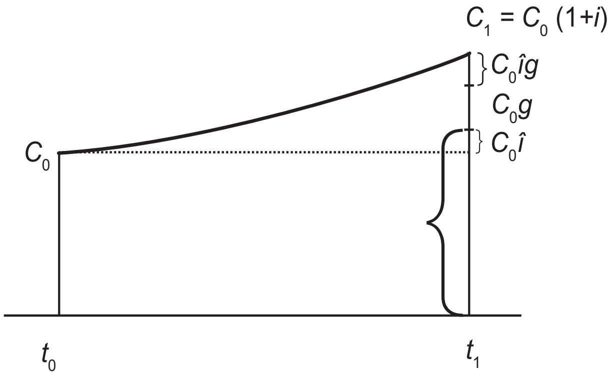

Figure 1. Decomposition of interest in a simple financial transaction

On the other hand, this equation, written as:

$$C_n \prod_{h=1}^{n} (1+g_h)^{-1} = C_0 \prod_{h=1}^{n} (1+\hat{i}_h),$$

can be considered as the financial equivalence of the transaction in constant monetary units at t_0, because it indicates the amount C_n deflated as the result of capitalizing C_0 during the interval $[t_0, t_n]$ according to the assumption of no inflation or in constant monetary units interest rates $\hat{i}_h$.

Representing by $\hat{C}_n$ the amount equivalent at t_n to the amount (C_0, t_0) in case of no inflation, that is, under the hypothesis $g_h = 0$, for every h:

$$\hat{C}_n = C_0 \prod_{h=1}^{n} (1+\hat{i}_h),$$

the difference $C_n - \hat{C}_n$ will represent the increment to be paid at t_n as a consequence of inflation. Thus,

$$C_n - \hat{C}_n = C_0 \prod_{h=1}^{n} (1+\hat{i}_h) \left[\prod_{h=1}^{n} (1+g_h) - 1\right] = \hat{C}_n \left[\prod_{h=1}^{n} (1+g_h) - 1\right].$$

3.2. Compound financial transactions

In general, given a financial transaction whose contributions are:

$$\mathrm{P} = \{(C_1, t_1), (C_2, t_2), \ldots, (C_m, t_m)\}$$

and whose refunds are:

$$\mathrm{C} = \{(C'_1, t'_1), (C'_2, t'_2), \ldots, (C'_n, t'_n)\},$$

the equation of financial equivalence at instant p, according to a capitalization or discounting function $F(t,p)$ with implicit inflation, is the following expression:

$$\sum_{i=1}^{m} C_i \cdot F(t_i,p) = \sum_{j=1}^{n} C'_j \cdot F(t'_j,p).$$

By specifying the inflation in the former expression, we obtain:

$$\sum_{i=1}^{m} C_i \cdot \hat{F}(t_i,p) \cdot H(t_i,p) = \sum_{j=1}^{n} C'_j \cdot \hat{F}(t'_j,p) \cdot H(t'_j,p).$$

By dividing both sides of the last equation by $H(\tau,p)$, with τ being the benchmark to calculate the inflation, we obtain the following expression:

$$\sum_{i=1}^{m} C_i \cdot \hat{F}(t_i,p) \cdot \frac{H(t_i,p)}{H(\tau,p)} = \sum_{j=1}^{n} C'_j \cdot \hat{F}(t'_j,p) \cdot \frac{H(t'_j,p)}{H(\tau,p)},$$

which can be written as:

$$\sum_{i=1}^{m} \hat{C}_i^{\tau} \cdot \hat{F}(t_i,p) = \sum_{j=1}^{n} \hat{C}_j'^{\tau} \cdot \hat{F}(t'_j,p),$$

where $\hat{C}_i^{\tau} = C_i \frac{H(t_i,p)}{H(\tau,p)}$ and $\hat{C}_j'^{\tau} = C'_j \frac{H(t'_j,p)}{H(\tau,p)}$ are the deflated amounts, at instant τ, corresponding to C_i and C'_j, respectively. Usually, the inflationary factors verify the property of transitivity:

$$H(t_i,p) = H(t_i,\tau) \cdot H(\tau,p) \quad \text{and} \quad H(t'_j,p) = H(t'_j,\tau) \cdot H(\tau,p),$$

whereby we can simply write that $\hat{C}_i^{\tau} = C_i \cdot H(t_i,\tau)$ and $\hat{C}_j'^{\tau} = C'_j \cdot H(t'_j,\tau)$. Finally, it is very usual that $\tau = t_1$, the origin of the transaction.

3.3. Amortization transactions

Let us consider a financial transaction with the initial unique contribution (C_0,t_0) and the final refunds (a_1,t_1), $(a_2,t_2),\ldots,$ (a_n,t_n). It is well known that, if i_1, i_2, ..., i_n are the interest rates with inflation, corresponding to time periods $[t_0,t_1], [t_1,t_2], \ldots, [t_{n-1},t_n]$, respectively, $\hat{i}_1,\ \hat{i}_2,\ \ldots,\ \hat{i}_n$, the interest rates without inflation, and $g_1,\ g_2,\ \ldots,\ g_n$, the inflation rates of such periods, we can write down:

- Equation of financial equivalence at origin:

$$C_0 = \sum_{r=1}^{n} a_r \prod_{h=1}^{r} (1+i_h)^{-1} = \sum_{r=1}^{n} a_r \prod_{h=1}^{r} (1+\hat{i}_h)^{-1} \cdot (1+g_h)^{-1} =$$
$$= \sum_{r=1}^{n} a_r \prod_{h=1}^{r} (1+\hat{i}_h)^{-1} \prod_{h=1}^{r} (1+g_h)^{-1}.$$

- Outstanding principal at the end of period s:

$$C_s = \sum_{r=s+1}^{n} a_r \prod_{h=s+1}^{r} (1+i_h)^{-1} = \sum_{r=s+1}^{n} a_r \prod_{h=s+1}^{r} (1+\hat{i}_h)^{-1} \prod_{h=s+1}^{r} (1+g_h)^{-1}.$$

Denoting by $\hat{a}_r$ the deflated payment equivalent to a_r, in constant monetary units of t_0, it is verified that:

$$\hat{a}_r = a_r \prod_{h=1}^{r} (1+g_h)^{-1}.$$

Therefore, the initial equation of financial equivalence can be written down as:

$$C_0 = \sum_{r=1}^{n} \hat{a}_r \prod_{h=1}^{r} (1+\hat{i}_h)^{-1},$$

which indicates the financial equivalence of the transaction in real terms or without inflation. Put in other words, the payments $(\hat{a}_r, t_r)$ amortize the amount $\hat{a}_r$ according to the capitalization or discounting function in constant monetary units or without inflation, whose interest rates are $\hat{i}_1, \hat{i}_2, \ldots, \hat{i}_n$.

Observe that the conditions imposed on the current payments, a_r, are not translated in general into the deflated payments, $\hat{a}_r$, and vice versa; and thus, for example, if the payments a_r are constant, the resulting payments $\hat{a}_r$, would, in general, be decreasing, etc.

The expression for the outstanding principal, by recurrence

$$\begin{aligned} C_s = C_{s-1}(1+i_s) - a_s = C_{s-1}(1+\hat{i}_s)(1+g_s) - a_s = \\ = C_{s-1}(1+\hat{i}_s + g_s + \hat{i}_s g_s) - a_s, \end{aligned}$$

leads to the following structure of the payment:

$$a_s = C_{s-1}(\hat{i}_s + g_s + \hat{i}_s g_s) + (C_{s-1} - C_s) = C_{s-1}(\hat{i}_s + g_s + \hat{i}_s g_s) + A_s,$$

where we denoted $A_s = C_{s-1} - C_s$.

As observed, the payment a_s can be decomposed into four summands:

- Interest without inflation: $C_{s-1}\hat{i}_s$.
- Inflation of the outstanding principal: $C_{s-1}g_s$.
- Inflation of interests: $C_{s-1}\hat{i}_s g_s$.
- Principal repaid: $A_s = C_{s-1} - C_s$.

Also the summand $C_{s-1}g_s$ can be incorporated to the principal repaid, obtaining:

$$A'_s = A_s + C_{s-1}g_s = C_{s-1}(1+g_s) - C_s$$

and so:

$$a_s = C_{s-1}\hat{i}_s(1+g_s) + A'_s.$$

Thus, the interest due is lesser, since it only includes the interests with inflation, and the principal repaid has increased to compensate the inflation of the outstanding principal during the period. Obviously, this does not affect the amortization-related dynamics, that is to say, the financial equivalence and the outstanding principal remain invariable.

This situation is depicted in the following Fig. 2, where we have considered a transaction with three periods.

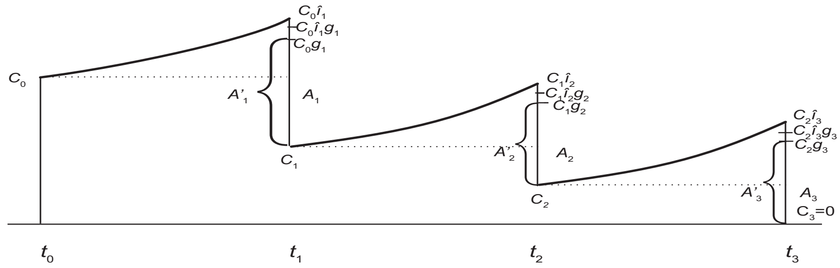

Figure 2. Decomposition of interest in a compound financial transaction

Observe that (considering $C_n = 0$):

$$C_0 = \sum_{s=1}^{n} A_s.$$

Nevertheless, the sum of all A'_ss is greater than C_0. In effect:

$$\sum_{s=1}^{n} A'_s = \sum_{s=1}^{n} A_s + \sum_{s=1}^{n} C_{s-1}g_s = C_0 + \sum_{s=1}^{n} C_{s-1}g_s > C_0.$$

4. Financial transactions with uncertain inflation

In this section, we are going to explore those financial transactions, in which the contributed and refunded amounts, and the deflated capitalization or discounting function, are known, except for the law of variation of the money purchasing power which is uncertain. For the sake of clarity of the presentation, we are going to divide this section into two subsections: the first one, dealing with simple transactions, and the second one, in which we will study the compound transactions.

4.1. Simple transactions with uncertain inflation

For the development of this subsection, we are going to generalize the approach presented in subsection 3.1. In effect, let (C_0, t_0) and (C_n, t_n) be two equivalent amounts according to the capitalization (discounting) function $F(t, p)$:

$$C_0 \cdot F(t_0, p) = C_n \cdot F(t_n, p).$$

Let us suppose that $F(t, p)$ can be decomposed into the product of the deflated capitalization (discounting) function, $\hat{F}(t, p)$, which is well known, and the law of variation of the money purchasing power, $H(t, p)$, which is known provided that the expirations of all contributed and refunded amounts, and the benchmark p are previous to the moment at which the transaction has been agreed. In this case,

$$C_0 \cdot \hat{F}(t_0, p) \cdot H(t_0, p) = C_n \cdot \hat{F}(t_n, p) \cdot H(t_n, p).$$

In the opposite case, $H(t, p)$ is known in terms of risk or uncertainty for some of the expirations t_0, t_n or p (in the case in which some of them is later than the moment at which the transaction has been agreed). In any case, we can use the average law of variation of the money purchasing power. Therefore, by the transitive property of this law

$$H(t_0, p) = H(t_0, t_n) \cdot H(t_n, p),$$

by substituting $H(t_0, p)$ in the left-hand side, the former equation can be written as:

$$C_0 \cdot \hat{F}(t_0, p) = \frac{C_n}{H(t_0, t_n)} \cdot \hat{F}(t_n, p),$$

or, equivalently,

$$C_0 \cdot \hat{F}(t_0, p) = \hat{C}_n \cdot \hat{F}(t_n, p),$$

where $\hat{C}_n = C_n \cdot H(t_0, t_n)^{-1}$ is the final amount of the transaction in constant euro of the instant t_0. Representing by τ an instant later than t_0, the true amount to be paid at t_n with the information available at τ is:

$$C_n^\tau = \hat{C}_n \cdot H_\tau(t_0, t_n),$$

or, equivalently:

$$C_n^\tau = C_n \cdot \frac{H_\tau(t_0, t_n)}{H(t_0, t_n)},$$

where $H_\tau(t_0, t_n)$ is the factor to be applied with the information available at τ. Observe that the two equations, in spite of being equivalent, have different readings. In the first one, the final amount of the transaction was inflation-free and has been multiplied by the adjusting inflation factor. In the second one, the inflation was implicit in the final amount of the transaction, but it has been adjusted by the real inflationary factor, known at this instant. In most cases, the point τ considered in order to revise the inflation is the final maturity t_n.

It is precisely this second option that should be used when the payment of a rent or the perception of a salary are adjusted by the real inflation. Nevertheless, alternatively one can use the factor $1 + H_\tau(t_0, t_n) - H(t_0, t_n)$ which, obviously, in a real inflationary context, is greater:

$$\frac{H_\tau(t_0, t_n)}{H(t_0, t_n)} = \frac{H(t_0, t_n) + H_\tau(t_0, t_n) - H(t_0, t_n)}{H(t_0, t_n)} < 1 + H_\tau(t_0, t_n) - H(t_0, t_n). \quad (1)$$

4.2. Compound transactions with uncertain inflation

Assume a financial transaction which has been agreed upon at an instant τ (occurring before t_1) with initial contributions:

$$\mathrm{P} = \{(C_1, t_1), (C_2, t_2), \ldots, (C_m, t_m)\}$$

and refunds:

$$\mathrm{C} = \{(C'_1, t'_1), (C'_2, t'_2), \ldots, (C'_n, t'_n)\}.$$

Analogously as in the preceding section, 4.1, let us suppose that $F(t, p)$ can be decomposed into the product of the deflated capitalization (discounting) function $\hat{F}(t, p)$, which is well known, and the law of variation of the money purchasing power, $H(t, p)$, which is known provided that the expirations of all contributed and refunded amounts, and the benchmark p, are all antecedent with respect to the moment at which the transaction has been agreed upon. However, if the transaction has been agreed upon at an instant τ preceding t_1:

$$\sum_{i=1}^{m} \hat{C}_i^\tau \cdot \hat{F}(t_i, p) = \sum_{j=1}^{n} \hat{C}_j^{\prime\tau} \cdot \hat{F}(t'_j, p),$$

where $\hat{C}_i^\tau = C_i \frac{H_\tau(t_i, p)}{H_\tau(\tau, p)} = C_i \cdot H_\tau^{-1}(\tau, t_i)$ and $\hat{C}_j^{\prime\tau} = C'_j \frac{H(t'_j, p)}{H(\tau, p)} = C'_j \cdot H_\tau^{-1}(\tau, t'_j)$ are the deflated amounts, at instant τ, corresponding to C_i and C'_j, respectively. So, once the time moment t_1 has come, the money purchasing power is $H_1(t, p)$ and, in general, it is not true that:

$$\sum_{i=1}^{m} \hat{C}_i^1 \cdot \hat{F}(t_i, p) = \sum_{j=1}^{n} \hat{C}_j^{\prime 1} \cdot \hat{F}(t'_j, p),$$

where $\hat{C}_i^1 = C_i \frac{H_1(t_i, p)}{H_1(t_1, p)} = C_i \cdot H_1^{-1}(t_1, t_i)$ and $\hat{C}_j^{\prime 1} = C'_j \frac{H_1(t'_j, p)}{H_1(t_1, p)} = C'_j \cdot H_1^{-1}(t_1, t'_j)$ are the deflated amounts, at instant t_1, corresponding to C_i and C'_j, respectively. Therefore, it should be necessary to adjust the transaction by introducing the payment to be really contributed at such instant, $\tilde{C}_1$. To obtain this effect, we write:

$$\tilde{C}_1 \cdot \hat{F}(t_1, p) + \sum_{i=2}^{m} \hat{C}_i^1 \cdot \hat{F}(t_i, p) = \sum_{j=1}^{n} \hat{C}_j^{\prime 1} \cdot \hat{F}(t'_j, p),$$

from which:

$$\tilde{C}_1 = \frac{1}{\hat{F}(t_1,p)}\left[\sum_{j=1}^{n}\hat{C}'^{1}_{j}\cdot\hat{F}(t'_j,p) - \sum_{i=2}^{m}\hat{C}^{1}_{i}\cdot\hat{F}(t_i,p)\right].$$

Nevertheless, once the instant t_2 has come (if it follows the expiration time in the financial transaction; in the contrary case, we would reason with $t'_1 > g$), the predicted inflation for the entire transaction is $H_2(t,p)$. Therefore, the amount $\tilde{C}_2$ must be adjusted in the following way:

$$\tilde{C}_1\cdot\hat{F}(t_1,p)\cdot H_2(t_1,t_2) + \tilde{C}_2\cdot\hat{F}(t_2,p) + \sum_{i=3}^{m}\hat{C}^{2}_{i}\cdot\hat{F}(t_i,p) = \sum_{j=1}^{n}\hat{C}'^{2}_{j}\cdot\hat{F}(t'_j,p),$$

where $\hat{C}^2_i = C_i\frac{H_2(t_i,p)}{H_2(t_2,p)} = C_i\cdot H_2^{-1}(t_1,t_i)$
and
$\hat{C}'^2_j = C'_j\frac{H_2(t'_j,p)}{H_2(t_2,p)} = C'_j\cdot H_2^{-1}(t_2,t'_j)$
are the deflated amounts, at instant t_2, corresponding to C_i and C'_j, respectively, from where:

$$\tilde{C}_2 = \frac{1}{\hat{F}(t_2,p)}\left[\sum_{j=1}^{n}\hat{C}'^{2}_{j}\cdot\hat{F}(t'_j,p) - \sum_{i=3}^{m}\hat{C}^{2}_{i}\cdot\hat{F}(t_i,p) - \tilde{C}_1\cdot\hat{F}(t_1,p)\cdot H_2(t_1,t_2)\right],$$

and so on.

4.3. Adjusting the payments of a loan with the real inflation rates

Let us consider a general loan whose principal is C_0 and where the interest rate (previously established) is variable $(i_1, i_2, \ldots, i_n)$. It is well known that the payments, a_s, must verify the equation of financial equivalence at the origin of the transaction:

$$C_0 = \sum_{s=1}^{n} a_s \prod_{h=1}^{s}(1+i_h)^{-1}.$$

For the sake of clarity of the presentation, we are going to assume that the average inflation rate predicted during the entire transaction is g, whereby we can decompose the discounting factor in the following way:

$$C_0 = \sum_{s=1}^{n} \underbrace{a_s(1+g)^{-s}}_{\hat{a}_s} \prod_{h=1}^{s}(1+\hat{i}_h)^{-1},$$

or, equivalently:

$$C_0 = \sum_{s=1}^{n} \hat{a}_s \prod_{h=1}^{s}(1+\hat{i}_h)^{-1}.$$

For the development of this section, we are going to assume an upward trend of future inflation rates, that is, $g_1, g_2, \dots g_n > g$. Moreover, we are going to distinguish two cases: the adjusting methods, in which the resulting payments do not verify the equation of financial equivalence of the loan, and the adjusting methods, in which the payments continue verifying the equation of financial equivalence at the origin of the transaction.

4.3.1. Methods not preserving the equation of financial equivalence

In this subsection, we will distinguish the following methods:

Calculation of the adjusted payments by means of the following expression (this method will be referred to as M1):

$$a_s^{(1)} = \hat{a}_s(1+g_1)(1+g_2)\cdots(1+g_s).$$

Calculation of the adjusted payments by means of the following expression (this method will be referred to as M2):

$$a_s^{(2)} = a_s\frac{(1+g_1)(1+g_2)\cdots(1+g_s)}{(1+g)^s}.$$

Calculation of the adjusted payments by means of the following expression (this method will be referred to as M3):

$$a_s^{(3)} = a_s\left[1+(1+g_1)(1+g_2)\cdots(1+g_s)-(1+g)^s\right].$$

Calculation of the adjusted payments by means of the following expression (this method will be referred to as M4):

$$a_s^{(4)} = a_s(1+g_1-g)(1+g_2-g)\cdots(1+g_s-g).$$

In these conditions, we can enunciate the following

Theorem 1. *In general, regardless of the evolution of inflation rates, there holds $a_s^{(2)} < a_s^{(4)} < a_s^{(3)}$, for every s. However, in the case of increase of inflation rates with respect to the initially predicted ones, the following is true*:

$$\sum_{s=1}^{n} a_s < \sum_{s=1}^{n} a_s^{(1)},$$

where a_s are the payments calculated with the initial inflation rates.

Proof. Firstly, the inequality $a_s^{(2)} < a_s^{(4)}$, for every s, is a direct consequence of the fact that, in general, $\frac{1+g_h}{1+g} = 1+\frac{g_h-g}{1+g} < 1+g_h-g$. On the other hand, if we develop the expression for $a_s^{(4)}$, without considering a_s, the following summands will appear:

- $(1+g_1)(1+g_2)\cdots(1+g_s)$.
- $-g$ multiplied by the sum of all products of order $s-1$ of factors of the type $(1+g_h)$.
- $+g^2$ multiplied by the sum of all products of order $s-2$ of factors of the type $(1+g_h)$.
- and so on.

Leaving aside the first summand, the second one is less than

$$-g\binom{s}{s-1}(1+g)^{s-1} = -g\binom{s}{1}(1+g)^{s-1}.$$

On the other hand, the third summand is greater than

$$+g^2\binom{s}{s-2}(1+g)^{s-2} = +g^2\binom{s}{2}(1+g)^{s-2}.$$

And so on. By substituting each summand and taking into account that the increase of each of them is greater than the decrease of the following one, we have:

$$\begin{aligned} a_s^{(4)} &< a_s\left[(1+g_1)(1+g_2)...(1+g_s) - g\binom{s}{l}(1+g)^{s-1} + g^2\binom{s}{2}(1+g)^{s-2} - ...\pm g^s\right] \\ &= a_s\left[(1+g_1)(1+g_2)...(1+g_s) + (1+g-g)^s - (1+g)^s\right] \\ &= a_s\left[(1+g_1)(1+g_2)...(1+g_s) - (1+g)^s + 1\right] \\ &= a_s^{(3)}. \end{aligned}$$

Next, for the rest of the demonstration, first we are going to assume that we have used the French amortization system (Ayres, 1963). In this case, it is verified that:

$$a = \frac{C_0}{a_{\overline{n|}\, i}}$$

and

$$\hat{a} = \frac{C_0}{a_{\overline{n|}\, \hat{i}}}.$$

The following sequence of equalities and inequalities:

$$\begin{aligned} (1+g)^n \cdot \hat{a} = (1+g)^n \frac{C_0}{a_{\overline{n|}\, \hat{i}}} = (1+g)^n \frac{C_0(1+\hat{i})^n}{s_{\overline{n|}\, \hat{i}}} = \\ \frac{C_0(1+i)^n}{s_{\overline{n|}\, \hat{i}}} > \frac{C_0(1+i)^n}{s_{\overline{n|}\, i}} = \frac{C_0}{a_{\overline{n|}\, i}} = a \end{aligned}$$

holds true, because $s_{\overline{n|}\, i}$ is an increasing function of i, and $i > \hat{i}$. Therefore,

$$a_n^{(1)} = (1+g_1)(1+g_2)\cdots(1+g_n)\cdot\hat{a} > (1+g)^n\cdot\hat{a} > a.$$

On the other hand, it is also true that $\hat{a} < a$, because $a_{\overline{n|}\, i}$ is a decreasing function of i, and $i > \hat{i}$. Thus, despite the fact that it is possible that, theoretically, $a_1^{(1)} = (1 + g_1) \cdot \hat{a}$ can be greater than a, in practice there must be $a_1^{(1)} < a$. Second, we are going to assume that we have used the amortization system of the constant principal repaid. In this case, $\hat{a}_n = A(1 + \hat{i})$. Therefore,

$$(1 + g)^n \hat{a}_n = (1 + g)^n A(1 + \hat{i}) = (1 + g)^{n-1} A(1 + i) = (1 + g)^{n-1} a_n > a_n.$$

Consequently, $a_n^{(1)} = (1 + g_1)(1 + g_2) \cdots (1 + g_n)\hat{a}_n > a_n$. On the other hand, $\hat{a}_1 = A(1 + n \cdot \hat{i})$, whereby $(1 + g)\hat{a}_1 = (1 + g)A(1 + n \cdot \hat{i}) < A(1 + n \cdot i) = a_1$. Thus, $a_1^{(1)} = (1 + g_1)\hat{a}_1 > a_1$. Finally, let us assume that we have used the American amortization system. In this case, $\hat{a}_n = C_0(1 + \hat{i})$. Therefore,

$$(1 + g)^n \hat{a}_n = (1 + g)^n C_0(1 + \hat{i}) = (1 + g)^{n-1} C_0(1 + i) = (1 + g)^{n-1} a_n > a_n.$$

Consequently, $a_n^{(1)} = (1 + g_1)(1 + g_2) \cdots (1 + g_n)\hat{a}_n > a_n$. On the other hand, $\hat{a}_1 = C_0 \cdot \hat{i}$, whereby $(1 + g)\hat{a}_1 = (1 + g) \cdot C_0 \cdot \hat{i} < C_0 \cdot i = a_1$. Thus, $a_1^{(1)} = (1 + g_1)\hat{a}_1 > a_1$.

Therefore, in the three aforementioned cases, there must be an s such that:

$$a_1^{(1)}, a_2^{(1)}, \ldots, a_s^{(1)} < a$$

and

$$a_{s+1}^{(1)}, a_{s+2}^{(1)}, \ldots, a_s^{(1)} > a,$$

because of the increasing character of the sequence $\left\{a_s^{(1)}\right\}$.

Nevertheless, for all amortization systems, the inequality $\sum_{s=1}^{n} a_s^{(1)} > \sum_{s=1}^{n} a_s$ holds. In effect,

$$\sum_{s=1}^{n} a_s^{(1)} \prod_{h=1}^{s} (1 + i_h)^{-1} = \sum_{s=1}^{n} \hat{a}_s \prod_{h=1}^{s} (1 + g_h) \prod_{h=1}^{s} (1 + i_h)^{-1} =$$

$$= \sum_{s=1}^{n} a_s (1 + g)^{-s} \prod_{h=1}^{s} (1 + g_h) \prod_{h=1}^{s} (1 + i_h)^{-1} > \sum_{s=1}^{n} a_s \prod_{h=1}^{s} (1 + i_h)^{-1} = C_0.$$

In particular, taking $i_h = 0$, for every h, the inequality $\sum_{s=1}^{n} a_s^{(1)} > \sum_{s=1}^{n} a_s$ holds.

As a comment to this theorem, we would like to point out that, in general, the inequality $\sum_{s=1}^{n} a_s^{(3)} < \sum_{s=1}^{n} a_s^{(1)}$ does not hold. It can be stated, though, that this inequality is true for certain values of $g_1, g_2, \ldots, g_n$, slightly greater than g, and that, for values of $g_1, g_2, \ldots, g_n$ sufficiently larger than g, the former inequality changes to $\sum_{s=1}^{n} a_s^{(3)} > \sum_{s=1}^{n} a_s^{(1)}$. To demonstrate this, it suffices to take into account that (we continue with the French amortization system):

$$\sum_{s=1}^{n} a_s^{(3)} = a\left[n + \sum_{s=1}^{n}(1+g_1)(1+g_2)\cdots(1+g_s) - \ddot{s}_{\overline{n|}\, g}\right]$$

and that:

$$\sum_{s=1}^{n} a_s^{(1)} = \hat{a}\sum_{s=1}^{n}(1+g_1)(1+g_2)\cdots(1+g_s).$$

Therefore, transposing some summands, the comparison between both sums is reduced to the comparison between the expressions:

$$(a-\hat{a})\sum_{s=1}^{n}(1+g_1)(1+g_2)\cdots(1+g_s)$$

and

$$\sum_{s=1}^{n} a_s^{(3)} = a\left(\ddot{s}_{\overline{n|}\, g} - n\right),$$

which demonstrates our statement.

4.3.2. Methods preserving the equation of financial equivalence

In this subsection, we will distinguish the following methods:

Table 2. Amortization schedule according to the M5 method

Period (s)	**Interest due** (I_s)	**Principal repaid** (A_s)	**Payment** (a_s)	**Outstanding principal** (C_s)
0	-	-	-	$C_0 = n \cdot A$
1	$I_1 = C_0\ \boxed{(1+\hat{i})(1+g_1)-1}$	A	a_1	$C_1 = (n-1)\cdot A$
2	$I_2 = C_1\ \boxed{(1+\hat{i})(1+g_2)-1}$	A	a_2	$C_2 = (n-2)\cdot A$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$I_n = C_{n-1}\ \boxed{(1+\hat{i})(1+g_n)-1}$	A	a_n	0

- Calculation of the payments by the method of constant principal repaid, and then determining the interest due with the deflated interest rate plus the inflation rate corresponding to the accrual period (this method will be referred to as M5). This method is similar to the one proposed by Ferruz (1994) or by López and Sebastián (2008) for the calculation of the payments corresponding to a loan in case of variable interest rates, as shown in Table 2.
- Calculation of the payments adjusted in the following way (this method will be referred to as M6):
  - The payments a_s are calculated with the inflation rate corresponding to the time instant 0.
  - Once the expiration instant 1 has come, the payment $\tilde{a}_1$ is calculated with the inflation rate corresponding to time instant 1 and leaving the payments $a_2, \ldots, a_n$ invariable.
  - Once the expiration instant 2 has come, the payment $\tilde{a}_2$ is calculated with the inflation rate corresponding to time instant 1 and leaving the payments $\tilde{a}_1, a_3, \ldots, a_n$ invariable.
  - and so on.

Under these conditions, we can enunciate the following

Theorem 2. *In the case of a generalized raise of inflation rates with respect to the initially predicted ones, it is true that $\tilde{a}_s > a_s$, for every s, with a_s being the payments calculated with the initial inflation rates.*

Proof. The payments $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$ are successively calculated in an iterative way, as follows:

$$C_0(1+\hat{i}_1)(1+g_1) = \tilde{a}_1 + \sum_{s=2}^{n} a_s \prod_{h=1}^{s} (1+i_h)^{-1},$$

$$C_0(1+\hat{i}_1)(1+g_1)(1+\hat{i}_2)(1+g_2) = \tilde{a}_1(1+\hat{i}_2)(1+g_2)+\tilde{a}_2+\sum_{s=3}^{n} a_s \prod_{h=1}^{s} (1+i_h)^{-1},$$

$$\vdots$$

We can observe that, by applying the prospective method:

$$\tilde{C}_1 = C_1,$$

$$\tilde{C}_2 = C_2,$$

$$\vdots$$

On the other hand, by applying the retrospective method to the former equalities:

$$C_0(1+\hat{i}_1)(1+g_1) - \tilde{a}_1 = C_0(1+\hat{i}_1)(1+g) - a_1,$$

$$\underbrace{C_0(1+\hat{i}_1)(g_1-g)}_{>0} = \tilde{a}_1 - a_1,$$

from where:

$$\tilde{a}_1 > a_1.$$

By the recurrent method:

$$\tilde{C}_1(1+\hat{i}_2)(1+g_2) - \tilde{a}_2 = C_1(1+\hat{i}_2)(1+g) - a_2.$$

Taking into account that $\tilde{C}_1 = C_1$,

$$\underbrace{C_1(1+\hat{i}_2)(g_2-g)}_{>0} = \tilde{a}_2 - a_2,$$

from which:

$$\tilde{a}_2 > a_2,$$

and so on.

A noteworthy particular case arises when the payment, the interest rate and the initial inflation rate are all constant. In this case, the equation of financial equivalence at the transaction origin:

$$C_0 = a \cdot a_{\overline{n|}\, i},$$

when considering the deflated interest rate:

$$C_0 = \sum_{s=1}^{n} a(1+\alpha)^{-s}(1+\hat{i})^{-s},$$

turns into the present value of an annuity variable in geometric progression:

$$C_0 = A_{(a,1+\alpha)\overline{n|}\, \hat{i}}.$$

Thus, when considering the inflation rates really performed, in order to adjust the payments, we have two options:

- **Option I**: To successively write:

$$C_0(1+\alpha_1) = \tilde{a}_1^1 + A_{(a,1+\alpha_1)\overline{n-1|}\, \hat{i}},$$
$$C_0(1+\alpha_1)(1+\alpha_2) = \tilde{a}_1^1(1+\alpha_2) + \tilde{a}_2^1 + A_{(a,1+\alpha_2)\overline{n-2|}\, \hat{i}},$$
$$\vdots$$

- **Option II**: To successively propose:

$$C_0 = A_{(\tilde{a}_1^2,1+\alpha_1)\overline{n|}\, \hat{i}},$$
$$C_0(1+\alpha_1) = \tilde{a}_1^2 + A_{(\tilde{a}_2^2,1+\alpha_2)\overline{n-2|}\, \hat{i}},$$
$$\vdots$$

Nevertheless, in this paper we will only consider option I.

5. Practical application

Let us consider a general loan whose payments have been calculated according to the inflation rates expected at the transaction agreement moment. If the lender and the borrower do not want to bear any losses or earnings due to the effect of inflation, they can adopt, as previously seen, several positions to adapt the payments to the inflation effect.

For the development of this practical application, we are going to start with a loan of the principal EUR 100,000, with a nominal interest rate of 6%, an inflation rate of 3% and an amortization period of 5 years.

Firstly, we are going to determine the inflation-free interest rate corresponding to the starting situation, since this will be the pure interest rate to be assigned to the financial transaction and from which we will calculate the amount corresponding to the inflation-free interests situation.

To determine the inflation-free interest rate, we will use the following expression:

$$\hat{i} = \frac{1+i}{1+g} - 1 = \frac{1+0.06}{1+0.03} - 1 = 0.0291262.$$

Next, we are going to construct the amortization tables corresponding to the former loan, by considering the following classic amortization systems: French, constant principal repaid, and American (see, respectively, Tables 3, 4, and 5).

Table 3. Schedule of the French amortization system, in EUR

Period	**Interest due**	**Principal repaid**	**Payment**	**Outstanding principal**
0	-	-	-	100,000.00
1	6,000.00	17,739.64	23,739.64	82,260.36
2	4,935.62	18,804.02	23,739.64	63,456.34
3	3,807.38	19,932.26	23,739.64	43,524.08
4	2,611.44	21,128.20	23,739.64	22,395.89
5	1,343.75	22,395.89	23,739.64	0.00

5.1. Methods not preserving the equation of financial equivalence

Once the amortization schedule tables constructed, we are going to elaborate the amortization tables determining the payments by means of three different procedures: one of them (M1) by using the inflation-free interest rate and adjusting the obtained payments with the really performed inflation rates; another one (M2) by deflating the amounts of the payments obtained in the former paragraph with the expected inflation, and adjusting them with the real inflation rates of the amortization periods; the third, (M3), by adjusting the values with

Table 4. Schedule of the constant principal repaid amortization system, in EUR

Period	**Interest due**	**Principal repaid**	**Payment**	**Outstanding principal**
0	-	-	-	100,000.00
1	6,000.00	20,000.00	26,000.00	80,000.00
2	4,800.00	20,000.00	24,800.00	60,000.00
3	3,600.00	20,000.00	23,600.00	40,000.00
4	2,400.00	20,000.00	22,400.00	20,000.00
5	1,200.00	20,000.00	21,200.00	0.00

Table 5. Schedule of the American amortization system, in EUR

Period	**Interest due**	**Principal repaid**	**Payment**	**Outstanding principal**
0	-	-	-	100,000.00
1	6,000.00	0.00	6,000.00	100,000.00
2	6,000.00	0.00	6,000.00	100,000.00
3	6,000.00	0.00	6,000.00	100,000.00
4	6,000.00	0.00	6,000.00	100,000.00
5	6,000.00	100,000.00	106,000.00	0.00

the differences between the rates of inflation performed in the periods of reference; and the fourth, (M4), which is a mix of the former methods. The results are illustrated in Tables 6 through 11.

Table 6. Schedule of the French amortization system (inflation-free interest rate), in EUR

Period	**Interest due**	**Principal repaid**	**Payment**	**Outstanding principal**
0	-	-	-	100,000.00
1	2,912.62	18,868.39	21,781.01	81,131.61
2	2,363.06	19,417.95	21,781.01	61,713.66
3	1,797.49	19,983.52	21,781.01	41,730.13
4	1,215.44	20,565.57	21,781.01	21,164.57
5	616.44	21,164.57	21,781.01	0.00

Table 7. Summary of payments according to M1, M2, M3 and M4 methods, in EUR

Real inflation	**Payment (M1)**	**Payment (M2)**	**Payment (M3)**	**Payment (M4)**
3.00%	-	-	-	-
3.50%	22,543.34	23,854.88	23,858.34	23,858.34
4.00%	23,445.08	24,086.48	24,107.60	24,096.92
4.25%	24,441.49	24,378.79	24,438.06	24,398.13
3.75%	25,358.05	24,556.31	24,658.81	24,581.12
3.50%	26,245.58	24,675.51	24,824.57	24,704.02

Observe that the arithmetic sum of the payments adjusted by inflation is greater in the first of the used methods (M1) than in the third one (M3), this being shown, in particular, in Table 12, containing the arithmetic sums of the payments obtained by every amortization method and by every method of calculation (M1, M2, M3 and M4). Moreover, by applying inequality (1), we obtain that the payments obtained by means of M3 are greater than their counterparts calculated with M4, and these ones are greater than the calculated ones with M2.

5.2. Methods preserving the equation of financial equivalence

Next, we are going to calculate the payments adapted to the inflation effect, using two procedures: the first one (M5), consistent in applying the amortization system of constant principal repaid, calculating the interest due with

Table 8. Schedule of the constant principal repaid amortization system (inflation-free interest rate), in EUR

Period	**Interest due**	**Principal repaid**	**Payment**	**Outstanding principal**
0	-	-	-	100,000.00
1	2,912.62	20,000.00	22,912.62	80,000.00
2	2,330.10	20,000.00	22,330.10	60,000.00
3	1,747.57	20,000.00	21,747.57	40,000.00
4	1,165.05	20,000.00	21,165.05	20,000.00
5	582.52	20,000.00	20,582.52	0.00

Table 9. Summary of payments according to M1, M2, M3 and M4 methods, in EUR

Real inflation	**Payment (M1)**	**Payment (M2)**	**Payment (M3)**	**Payment (M4)**
3.00%	-	-	-	-
3.50%	23,714.56	26,126.21	26,130.00	26,130.00
4.00%	24,036.12	25,162.33	25,184.40	25,173.24
4.25%	24,403.97	24,235.39	24,294.31	24,254.62
3.75%	24,640.93	23,170.58	23,267.30	23,193.99
3.50%	24,801.44	22,035.76	22,168.87	22,061.22

Table 10. Schedule of the American amortization system (inflation-free interest rate), in EUR

Period	**Interest due**	**Principal repaid**	**Payment**	**Outstanding principal**
0	-	-	-	100,000.00
1	2,912.62	0.00	2,912.62	100,000.00
2	2,912.62	0.00	2,912.62	100,000.00
3	2,912.62	0.00	2,912.62	100,000.00
4	2,912.62	0.00	2,912.62	100,000.00
5	2,912.62	100,000.00	102,912.62	0.00

Table 11. Summary of payments according to M1, M2, M3 and M4 methods, in EUR

Real inflation	**Payment (M1)**	**Payment (M2)**	**Payment (M3)**	**Payment (M4)**
3.00%	-	-	-	-
3.50%	3,014.56	6,029.13	6,030.00	6,030.00
4.00%	3,135.15	6,087.66	6,093.00	6,090.30
4.25%	3,268.39	6,161.54	6,176.52	6,166.43
3.75%	3,390.95	6,206.41	6,232.31	6,212.68
3.50%	124,007.18	110,178.78	110,844.35	110,306.08

Table 12. Arithmetic sums of payments according to M1, M2, M3 and M4 methods, in EUR

Amortization system	**Method M1**	**Method M2**	**Method M3**	**Method M4**
French	122,033.55	121,551.97	121,887.38	121,638.53
Constant principal repaid	121,597.02	120,730.27	121,044.88	120,813.07
American	136,816.24	134,663.52	135,376.18	134,805.49

the inflation-free interest rate and the inflation really affecting the outstanding principal of the previous period; and the second one (M6), using the described method of invariance of the outstanding principal with respect to the original transaction (French system (1), system of constant principal repaid (2) and American amortization system (3)). The respective results are summarized in Table 13.

Table 13. Summary of payments according to M5 and M6 methods, in EUR

Real inflation	**Payment (M5)**	**Payment (M6) (1)**	**Payment (M6) (2)**	**Payment (M6) (3)**
3.00%	-	-	-	-
3.50%	26,514.56	24,254.20	26,514.56	6,514.56
4.00%	25,623.30	24,586.20	25,623.30	7,029.13
4.25%	24,371.84	24,555.95	24,371.84	7,286.41
3.75%	22,708.74	24,075.58	22,708.74	6,771.84
3.50%	21,302.91	23,854.88	21,302.91	106,514.56

6. Conclusions

The aim of this paper has been to develop the ways to control the payments in a loan transaction in view of the inflation effect. Indeed, both lender and borrower must not necessarily bear the losses or obtain earnings due to the future evolution of interest rates. Thus, in this paper, we have presented a general model for the treatment of inflation in financial transactions, based on Gil Luezas and Gil Peláez (1987) and De Pablo (1998 and 2000). First, we assume a multiplicative behaviour of the law of variation of prices (inflation) inside the general expression of a capitalization (discounting) function.

Once the effect of inflation is isolated from the rest of the different reasons that justify the mathematical expression of a capitalization (discounting) function, we show that the instantaneous rate is the sum of the instantaneous rate of the capitalization (discounting) function without inflation and the instantaneous rate of the law of variation of the money purchasing power. Later, we apply this decomposition for the treatment of simple transactions, and then of the compound ones. A noteworthy particular case is the analysis of amortization transactions whose treatment with inflation-free interest rates is going to be of crucial importance for the rest of the paper.

Indeed, one of the main contributions of this paper is the treatment of the financial transactions with uncertain inflation, six methods being described to adjust the payments deduced from the inflation-free transaction with the real inflation rates occurring in the different periods involved in the transaction. We introduce two models for adjusting the payments: a group of payments which do not verify the equation of financial equivalence at origin, and another group that do verify it. For these two settings, we have shown two theorems that relate, in terms of inequality, the payments obtained in each group.

References

AYRES, F. (1963) *Mathematics of Finance.* McGraw-Hill, New York.

BODIE, Z., KANE, A. and MARCUS, A.J. (2004) *Essentials of Investments.* 5th Edition. McGraw-Hill/Irwin, New York.

BREALEY, R.A., MYERS, S.C. and ALLEN, F. (2006) *Principles of Corporate Finance*, 8th Edition. McGraw-Hill/Irwin, New York.

BRIGHAM, E.F. and DAVES, P.R. (2007) *Intermediate Financial Management*, 9^{th} Edition. Thomson/South-Western, Mason, OH.

CRUZ RAMBAUD, S. and VALLS MARTÍNEZ, M.C. (2008) *Introducción a las Matemáticas Financieras.* Editorial Pirámide, Madrid.

CRUZ RAMBAUD, S. and VENTRE, A.G.S. (2008) Joint investment strategies with a superadditive capitalization function. *Control and Cybernetics* **37** (3), 649-667.

DE PABLO LÓPEZ, A. (1998) Some Factors for the Correction of the Inflation Effect in Financial Transactions. *Proceedings of the First Spanish-Italian Meeting on Financial Mathematics.* Servicio de Publicaciones de la Universidad de Almería, 29-40.

DE PABLO LÓPEZ, A. (2000) *Matemática de las Operaciones Financieras, Tomos I y II.* Universidad Nacional de Educación a Distancia (UNED), Madrid.

FERRUZ AGUDO, L. (1994) *Operaciones Financieras. Descripción, análisis y valoración.* Ed. Ariel S.A., Barcelona.

GIL LUEZAS, M.A. and GIL PELÁEZ, L. (1987) *Matemáticas de las Operaciones Financieras.* Universidad Nacional de Educación a Distancia, Madrid.

GIL PELÁEZ, L. (1992) *Matemática de las Operaciones Financieras.* Editorial AC, Madrid.

LÓPEZ PASCUAL, J. and SEBASTIÁN GONZÁLEZ, A. (2008) *Gestión Bancaria: Factores Claves en un Entorno Competitivo.* Ed. McGraw-Hill, Madrid.

SUÁREZ SUÁREZ, A.S. (1991) *Decisiones Óptimas de Inversión y Financiación en la Empresa.* Ediciones Pirámide, S.A., Madrid.