Computer Dynamic Modelling of Communal Sewage Networks

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Abstract:
In the paper two algorithms of dynamic modeling of communal sewage networks are presented. It is assumed that the hydraulic parameters of segments, namely canal shape, canal dimension and roughness are constant. The goal of the algorithms is to calculate the main sewage network parameters using only the continuity equation and the Manning formula. In the algorithms, fundamental algebraic formulas describing the filling heights in the canals and the sewage flow velocities are also used. The network model based only on the Manning formula and continuity equation in difference form and not on the liquid equations as it is used commonly by modelling the sewage networks is simpler and faster in calculations. While modeling the networks fixed network structure and slowly changing sewage inflows into the canals are assumed. The forecasted inflow values are stated and the investigation presented concerns the sanitary and mixed gravitational sewage networks.

Keywords: *command sewage network, flow computer modelling, Manning formula*

1. Simplified Flow Models of Sewage

The commonly known flow models of wastewater networks are based on two Saint-Venant equations, i.e. on the continuity and the dynamic equations ([15], [1], [7], [12], [14]). The models presented below concern the housekeeping, i.e. sanitary or combined, i.e. sanitary connected with rain sewage networks consisted of segments and nodes. The nodes are the points in which few segments join together or into/from which the wastewater inflows/outflows. The equations of flow continuity hold in the nodes and the conditions of concordane of sewage surface levels hold in the canals, which are combining each other. It is assumed that the main hydraulic parameters of the network such as shape, canal dimension, canal slope and roughness are constant at any one time, the sewage inflows are slow-changing in time and the nets investigated are of gravitational type.

The formulations of the wastewater nets models proposed in the paper are based on the Saint-Venant continuity equation and on the Manning formula, which have the following forms ([7]):

a) the continuity equation

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}-\zeta=0, \tag{1}$$

b) the Manning formula

$$Q=\frac{1}{n}R^{2/3}\cdot J^{1/2}\cdot A, \tag{2}$$

where: A – cross sectional area of the network canal, [m^2], Q – flow rate, [m^3/s], ζ – sewage inflow calculated for the length unit of the canal,

$v=\frac{Q}{A}$ – mean velocity of the sewage flow, [m/s],

J – canal slope, [%], R – hydraulic radius of the canal, [m], n – roughness coefficient, [s/m$^{1/3}$].

In the following there will be assumed that network investigated is divided by the nodes into N segments (canals) and each j-th canal is divided into Mj subsegments with the relative lengths $\Delta x_1, \Delta x_2, \Delta x_3, \dots, \Delta x_{Mj}$ as is shown in Fig. 1. In the following relations j means the canal index and i means the subsegment index of j-th canal.

The flow changes in respective subsegments of a j-th canal can be written in form of equations (for $j = 1, \dots, N$):

$$\Delta Q_{1j}=Q_{1j}-W_j-\zeta_{1j},$$

$$\Delta Q_{ij}=Q_{ij}-Q_{i-1j}-\zeta_{ij} \quad i=1,\dots,M_j, \tag{3a}$$

$$\Delta Q_{M_j j}=Q_{M_j j}-Q_{M_j-1j}-\zeta_{ij},$$

$$\zeta_{ij}(t)=\zeta_j(t)\cdot\Delta x_i,$$

where: N – number of segments(canals), M_j – number of subsegments in the j-th canal, Q_{ij} – flow in i-th subsegment of the j-th canal given by the Manning formula, ζ_{ij} – sewage inflow to the j-th canal calculated for the length unit of the i-th subsegment, W_j – sewage inflow to the j-th canal being the sum of the outputs from other canals combined with canal j.

$$W_j=\sum_{k\neq j}^{N}P_{kj}\cdot Q_{M_j j}+\gamma_j \quad j=1,\dots,\mathrm{N}, \tag{3b}$$

where: P_{kj} – matrix of elements 0 and 1 describing the connections between the network segments, $Q_{M_j j}$ – outflow from the *j*-th segment of the network, γ_j – sewage inflow to the *j*-th network segment.

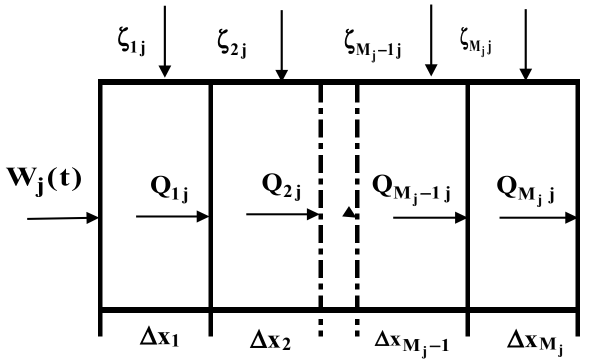

Fig. 1. Division of the j-th canal segment into subsegments

Under assumption that roughness coefficient *n* and the canal slope *J* are constant along the whole length of the segment considered we can write the formula describing the flow rate:

$$Q_{ij}(t) = \frac{1}{n} R_{ij}(t)^{2/3} \cdot J_j^{1/2} \cdot A_{ij}(t). \quad (3c)$$

After rearranging equation (1) to the form:

$$\frac{\Delta Q}{\Delta x} + \frac{\Delta A}{\Delta t} = \zeta \quad (4)$$

and after taking into account equations (3a), we receive equations set determining the change of cross sectional area ΔA in time Δt:

$$\frac{\Delta A_{ij}(t)}{\Delta t} = \frac{Q_{i-1j}(t) - Q_{ij}(t)}{\Delta x_i} + \zeta_j(t). \quad (5)$$

The calculated changes are used to determine A_{ij} for the next time step. From (5) we obtain for each *j*-th network segment for *j*=1,…,*N*:

$$A_{1j}(t+\Delta t) = A_{1j}(t) + \frac{\Delta t}{\Delta x_i} \cdot \left(\sum_{k \neq j}^{M_j} P_{kj} \cdot Q_{M_j j}(t) + \gamma_j(t) - Q_{1j}(t) \right) + \zeta_j(t), \quad (6a)$$

$$A_{ij}(t+\Delta t) = A_{ij}(t) + \frac{\Delta t}{\Delta x_i} \cdot \left(Q_{i-1j}(t) - Q_{ij}(t)\right) + \zeta_j(t), \quad i = 1, \dots, M_j. \quad (6b)$$

where: Δt – time step, Δx_i – lenght of the *i*-th canal subsegment.

Solving equations (6a)–(6b), beginning from the moment *t*=0 up to time *T* with given sewage inflow $\zeta_i(t)$, we will receive the set of values A_{ij} (for *j*=1,…, *N*, *i* = 1,…, M_j and t = 0,…, T) for each Δx_i and each time step Δt, where *T* is the total simulation time.

For equations (6a)–(6b) the initial conditions for *t*=0, for cross sectional area *A* and the flow rate *Q* have to be given.

The flow model is described by relations (3b), (3c) and (6a)–(6b). For each moment *t* and for each *j*-th network segment (*j*= 1,…, *N*) the flows $Q_{ij}(t)$ can be then calculated.

Knowing the flow values for the time *t*, we calculate the cross sectional area *A* for the next time period $t+\Delta t$ by means of (6a)–(6b).

According to the Manning formula, the sewage flow depends on the hydraulic radius *R* and on the cross sectional area *A* whereas *R* and *A* depend on the canal filling height *H*.

To simplify the description of relations (7a)–(8c), the indexes *i* and *j* in them are omitted but they concern each section of the investigated network segment.

From the Manning formula and taking into account canal geometry, one can formulate the following relations [2]:

for $H \le 0.5\,d$:

$$A = \frac{d^2}{8} \cdot (\varphi - sin\varphi), \quad (7a)$$

$$\varphi = 2 \cdot arccos\left(1 - 2 \cdot \frac{H}{d}\right), \quad (7b)$$

$$R = \frac{1}{4} d \left(1 - \frac{sin\varphi}{\varphi}\right), \quad (7c)$$

for $H > 0.5\,d$:

$$A = \frac{\pi d^2}{4} - \frac{d^2}{8} \cdot (\varphi - sin\varphi), \quad (8a)$$

$$\varphi = 2 \cdot arccos\left(2 \cdot \frac{H}{d} - 1\right), \quad (8b)$$

$$R = \frac{d}{4} + \frac{d}{8} \cdot \frac{sin\varphi}{\pi - 0.5\varphi}, \quad (8c)$$

where: *H* – canal filling height, φ – canal central angle, *d* – canal inside diameter.

From the above relations, one can see that for circular canals the cross sectional area *A* and the hydraulic radius *R* depend on the canal filling height *H* and these relations can be described as $A = F_1(H)$ and $R = F_2(H)$.

In this way, while knowing cross sectional area *A*, one can determine filling height *H* and hydraulic radius *R*. In Fig. 2 the relations between *A* and canal filling degree *H/d* for different diameter values *d* are shown [2].

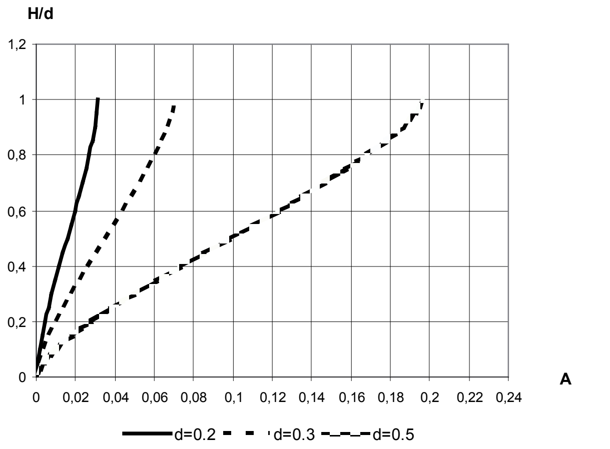

Fig. 2. Relation between canal filling degree H/d and cross sectional area A for different values d

The algorithm for calculating the sewage flow model consists for given time t of the following steps:

- determining cross sectional area $A_{ij}(t)$ by solving equation (6a) – (6b) at the moment t for each j-th network segment and the i-th canal subsegment $i=1,\dots,M_j$;
- calculating filling height $H_{ij}(t)$ by solving equation $F(H_{ij}) - A_{ij} = 0$ for two options: (for the j-th network segment, $j=1,\dots,N$, and for all canal subsegments $i=1,\dots,M_j$:

  for $H_{ij} \le 0.5\, d_j$

$$F(H_{ij})=\frac{d_j^2}{8}\cdot\left(2arccos\left(1-2\frac{H_{ij}}{d_j}\right)-sin\left(2arccos\left(1-2\frac{H_{ij}}{d_j}\right)\right)\right), \tag{9a}$$

  for $H_{ij} > 0.5\, d_j$

$$F(H_{ij})=\frac{\pi d_j^2}{4}-\frac{d_j^2}{8}\cdot\left(2arccos\left(2\frac{H_{ij}}{d_j}-1\right)-sin\left(2arccos\left(2\frac{H_{ij}}{d_j}-1\right)\right)\right), \tag{9b}$$

- calculating hydraulic radius $R_{ij}(t)$ from relations (7c) or (8c);
- determining flow $Q_{ij}(t)$ from Manning formula (3c).

Function $F(H)$ is continuous and for $H = 0.5d$ it has the value $\frac{\pi d^2}{8}$. Equation $F\left(H_{ij}\right) - A_{ij} = 0$ for calculating the canal filling degree is nonlinear and the standard numerical methods for solving nonlinear algebraic equations can be applied.

The algorithm is rather complicated and to calculate it the solution of the additional equation $F(H)$-A=0 must be solved. Function $F(\cdot)$ has the same form as functions (9a)–(9b). The calculations are done in sequence for each segment of network, beginning from the furthest segment and completing for the segment closest to wastewater treatment. The scheme of the algorithm is shown in Fig. 3.

The second version of the network model takes into account the calculation of canal filling height H. For canals with the circular cross-sections the relations between active cross sectional area A and filling height H can be used. As function $F(\cdot)$ iscontinues and differentiable, then we can write down:

$$\frac{\partial A}{\partial t}=\frac{\partial F}{\partial H}\cdot\frac{\partial H}{\partial t}, \tag{10a}$$

where $F(\cdot)$ is given by (9a) or (9b).

After a transformation of (10a), we obtain:

$$\frac{\partial F}{\partial H}=\frac{d}{4}\cdot\frac{1-cos\varphi}{\sqrt{\frac{H}{d}-\left(\frac{H}{d}\right)^2}}, \tag{10b}$$

where φ has the form (7b) or (8b).

From equation (4) and from relation (10b) transformed to the difference form, we receive:

$$\frac{\Delta Q_{ij}(t)}{\Delta x_i}+\frac{d_j}{4}\cdot\frac{1-cos(\varphi)}{\sqrt{\frac{H_{ij}(t)}{d_j}-\left(\frac{H_{ij}(t)}{d_j}\right)^2}}\cdot\frac{\Delta H_{ij}}{\Delta t}=0, \tag{11a}$$

$$\frac{\Delta H_{ij}}{\Delta t}=\frac{H_{ij}(t+\Delta t)-H_{ij}(t)}{\Delta t}. \tag{11b}$$

To transform the above relation, let us determine the change of the filling height H_{ij} during time Δt:

$$H_{1j}(t+\Delta t)=H_{1j}(t)+4\frac{\sqrt{\frac{H_{1j}(t)}{d_j}-\left(\frac{H_{1j}(t)}{d_j}\right)^2}}{d_j\cdot\left(1-cos(\varphi_{1j}(t))\right)}\cdot\left(\sum_{k\neq j}^{M_j}P_{kj}\cdot Q_{M_j j}(t)+\gamma_j(t)-Q_{1j}(t)+\zeta_{ij}(t)\right)\cdot\frac{\Delta t}{\Delta x_i}, \tag{12a}$$

$$H_{ij}(t+\Delta t)=H_{ij}(t)+4\frac{\sqrt{\frac{H_{ij}(t)}{d_j}-\left(\frac{H_{ij}(t)}{d_j}\right)^2}}{d_j\cdot\left(1-cos(\varphi_{ij}(t))\right)}\cdot\left(Q_{i-1j}(t)-Q_{ij}(t)+\zeta_{ij}(t)\right)\cdot\frac{\Delta t}{\Delta x_i}. \tag{12b}$$

where: $Q_{ij}(t)$ – flow rate in i-th canal subsegment calculated from Manning formula (3b) for the *j*-th network segment, d_j – internal diameter of *j*-th network segment, φ_i – canal central angle given by formula (7b) or (8b), Δx_i – length of *i*-th canal segment, Δt – time step.

The second version of the network model is described by relations (3b), (7a)–(8c) and (12a)–(12b). In this model, flow $Q_{ij}(t)$ is calculated for moment *t* and for each *j*-th network segment (from $j=1$ to $j=N-1$) in all canal segments $i=1,\ldots,M_j$. Then knowing the flows and filling heights H_{ij} for time period *t*, we can calculate the filling heights for next period $t+\Delta t$ according to (12a)–(12b).

2. Calculation of Sewage Inflows into Combined Wastewater Networks

The housekeeping as well as industrial or rain sewages are flowing into the combined wastewater network. Depending on the kind of sewage its inflow rate is calculated in different way. For the housekeeping and industrial sewage its inflow towards a given canal is considered as maximal hourly flow γ_{dj} defined by the relation:

$$\gamma_{dj} = \frac{N_{hmax} M \cdot q_{mv}}{24}, \qquad (13)$$

where: M – number of inhabitants assigned to the given network canal, q_{mv} – mean value of the sewage outflow from a house unit, N_{hmax} – coefficient of daily unevenness regarding the housekeeping sewage production.

An exact description of methods calculating the inflows of rain sewage into combined wastewater networks can be found in numerous literature ([8], [14], [9], [5], [6]).

The inflow of rain sewage into the network canals can be calculated by determining the functions describing the rainfalls and the drainage basin on which the wastewater network is located. For calculating the rainfall sewage the following formula can be used:

$$\gamma_d(t) = q_d \cdot \psi \cdot F \cdot \tau, \qquad (14)$$

where: γ_d– drift of the rain sewage from the terrain on which the wastewater network is located, [dm^{3}l/s], F –surface of drainage basin from which the sewage is drifting towards the given canal section, [ha], Ψ – coefficient of surface drift being the quotient between the rain sewage amount reaching the net canal and the total rainfall amount that dropped at the regarded soil part, τ– delay coefficient, q_d– rainfall intensity in [dm^3/s ha], being the rainfall amount in dm^3 that dropped at the soil surface of 1 ha in the time of 1 s.

Not all amounts of rain water runs off of the drainage basin towards the net canals and the process occurs gradually with regard

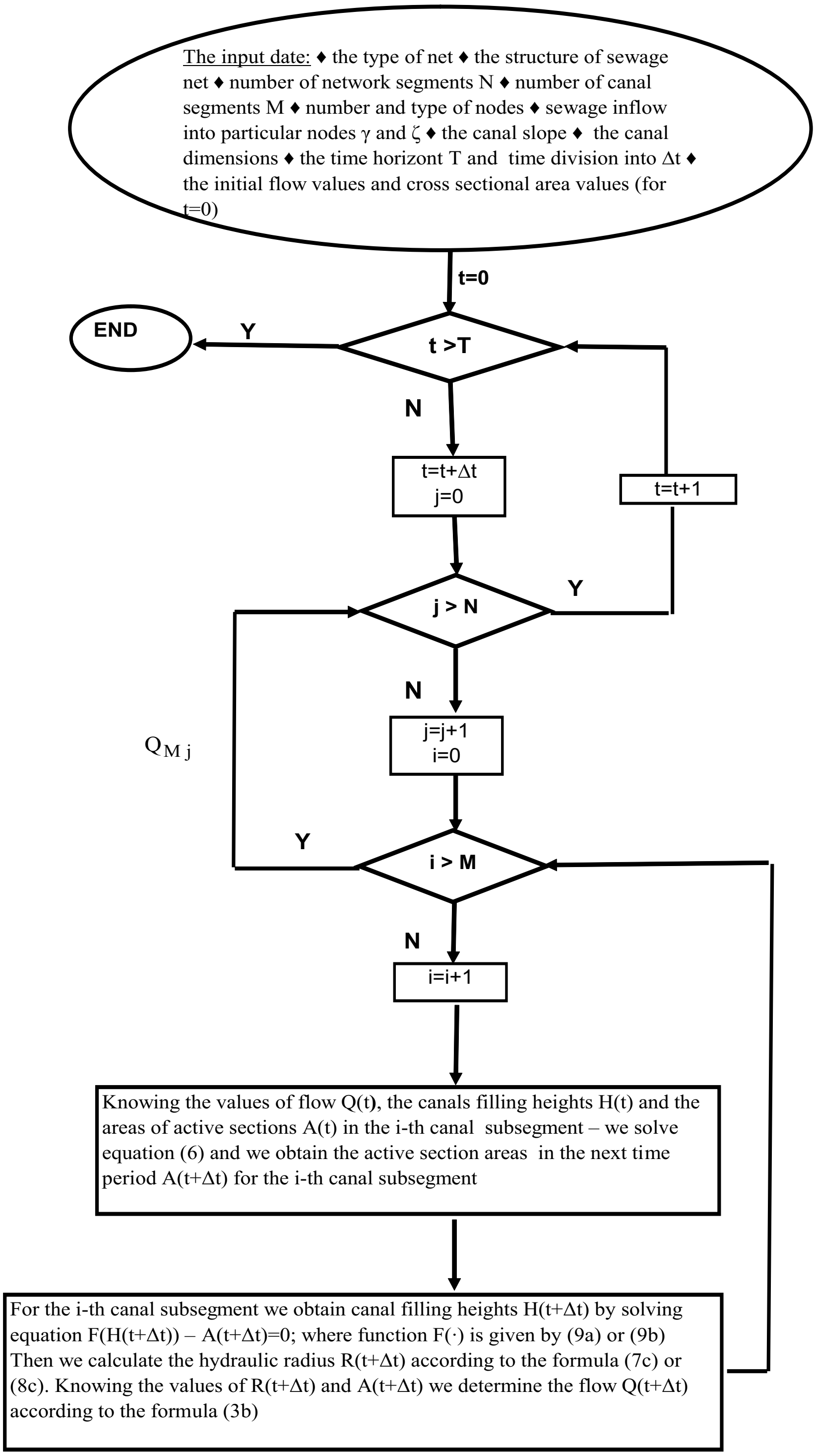

Fig. 3. Schema of the computing algorithm for the first version of the network model

to the phenomenon of local retention. It depends on the form of drainage basin, on canals situation, on field slope etc. The amount of water that does not reach the canals but will seep into the ground or will steam away can be estimated by means of the runoff coefficient ψ calculated from the Reinhold formula:

$$\psi = M \cdot q^{0.567} \cdot t_d^{\ 0.228}, \tag{15}$$

where: q – rainfall intensity [dm³/h], t_d – rainfall duration [min], M – factor characterizing the drainage basin and climatic conditions.

For the given network structure, the surface of the drainage basin must be determined regarding the shape and configuration of the ground. In this way, the real directions of water runoff towards the canals can be defined. Then the values of ψ for individual field parts can be calculated. These values depend on the density of buildings situated at the land and on the kind of land covering.

The intensity of significant rainfall is calculated on the base of long term meteorological observations. This rainfall is described by the following parameters:

- duration t, [min],
- rainfall height h_o, [mm],
- intensity $I = h_o/t$, [mm/min],
- range F, [ha],
- probability of appearance p, [%] or
- incidence $c = 100/p$, [years].

There are several relations combining the rainfall intensity, rainfall duration and probability of rainfall appearance. One of the most used relation for calculating the runoff of significant rainfall is the following Błaszczyk formula:

$$q = \frac{6{,}63\sqrt[3]{h_o^{\ 2} c}}{t_d^{\ 0{,}67}}, \tag{16}$$

where: h_o – mean value of yearly rainfall, [mm], q – rainfall intensity [dm³/h], c – rainfall incidence [years], t_d – rainfall duration [min], p – appearance probability (%), p=100/c.

Duration of significant rainfall t_d can be calculated from the formula:

$$t_d = 1{,}2 \cdot \sum t_p + t_k = \frac{1}{50}\sum_{i=1}^{N} \frac{L_i}{v_i} + t_k. \tag{17a}$$

There is also an another formula for calculating the rainfall duration t_w in which the network and drainage basin retentions are taken into consideration:

$$t_w = \frac{\alpha}{60}\sum_{i=1}^{N} \frac{A_i L_i + V_i F_i}{q_{pi}}, \tag{17b}$$

where: N – number of network segment, t_p – time of sewage flow through individual canal segments beginning from the upper network node to the point in which the calculation is currently doing, [min], L_i – length of i-th canal segment, [m], v_i – mean value of flow velocity in i-th canal segment, t_k – time of soil concentration, A_i – surface of lateral canal section, F_i – surface of drainage basin part belonging to the i-th canal segment, V_i – factor of canal volume and of land retention regarding the i-th canal segment, q_{pi} – assumed flow of the rain sewage in i-th segment of the network, α – factor of used capacity of the network retention.

The total sewage flow in a canal is calculated as the sum of housekeeping wastewater, industrial sewage and rainfall water. From this united sewage flow the wastewater outlet taking place in overflow points, which are situated above the canal segment investigated has to be subtracted.

Another way of determining the intensity of significant rainfall is the method of constant intensities in which the rainfall duration t_d = 10 min and the rainfall incidence c = 2 are defined.

Delay coefficient τ depends on the surface of drainage basin, on its shape and slope and it can be calculated from the following Burklieg-Ziegler formula:

$$\tau = \frac{1}{\sqrt[m]{F}}. \tag{17c}$$

Coefficient τ can take values from 2 up to 8 and these values are bigger for larger drainage basins with bigger slopes.

The appearance of soil retention can be also considered while doing the calculation by using the function $f(t)$ depending on time (Fig. 4). In this picture the following function parameters are specified:

- t_r – duration of soil retention,
- t_d – rainfall duration,
- t_k – total time of the runoff of rainfall water towards the canal investigated.

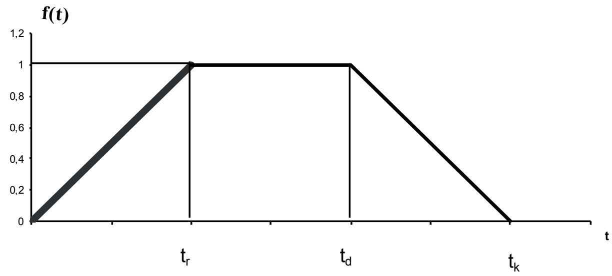

Fig. 4. Function f(t) of wastewater inflow to the sewage network canals

For estimated function $f(t)$, the wastewater inflow to the sewage net canal $\gamma_d(t)$ can be calculated with the following formula:

$$\gamma_d(t) = q_d \cdot \psi \cdot F \cdot f(t) \tag{18}$$

with F – surface of the drainage basin.

Inflow γ_d calculated by means of (18) can be considered by the modeling of a sewage network as the point-wise inlet introduced into the network nodes but the better approach from the computational point of view, is to regard it as the sectional inlet assigned to the canal unit.

3. Algorithm for the Wastewater Network Simulation

Taking into account the forecasted values of sewage inflow, the velocities v_j and the canal filings H_j for each interval Δt and for each j-th network segment can be calculated. Using the calculated canal outflow $Q_{Mj}j$ as the additional inflow to the next canal segment we can simulate with this method any part of the net investigated.

The algorithm shown in Fig. 5 is based on the second version of the network model given by equations (12a)–(12b) and (3b) describing the change of filling height H during time Δt and on the Manning formula for determining the flow Q. In this model the heights of segment fillings are determined for individual time periods. To build the model we have to define the following network parameters:

- type of the net – housekeeping or combined sewage network;
- structure of the net – number of network segments N, number of canal segments M, types of canals, number and type of nodes;
- network segment parameters, i.e. canal dimensions, slopes, lengths and roughness coefficients;
- initial data for computing, i.e. initial flows and initial filling height;
- date describing the simulation process, i.e. simulation time, time steps, network division into segments;
- sewage inflows into particular nodes.

The inflows of rainfall water to the canals can be given directly according to the functions $\zeta_i(t)$ determined as a result of soil investigations or indirectly using some approximating functions [4].

The task of the algorithm is to determine the values of the following parameters for each net segment and for fixed time period:

- the filling height;
- the flow velocity;
- the flow rates.

It is assumed that the hydraulic parameters of segments, namely canal shape, canal dimension and roughness are constant. The sewage inflows occur in the network nodes.

In the following the main elements of the algorithm will be desribed:

Step 1. The net structure defined by: number of nodes NW, number of segments N, M_j – number of subsegments in the j-th canal, the set of nodes $W = \{k\}$, the set of segments $\{j\}$, j=1,…,N, the set of subsegments in the j-th canal $\{i\}$, i=1,…,M_j, the set of diameters $\{d_j\}$, slopes for the segments J_j, j=1,…,N, roughness n_i for each segment, the initial flow values Q_{ij} and canals filling height values H_{ij} (for t=0) for each segment j=1,…,N and each canal subsegnent i=1,…,M_j, the time horizont T and time division into Δt, ζ_{ij} – sewage inflow to the j-th canal calculated

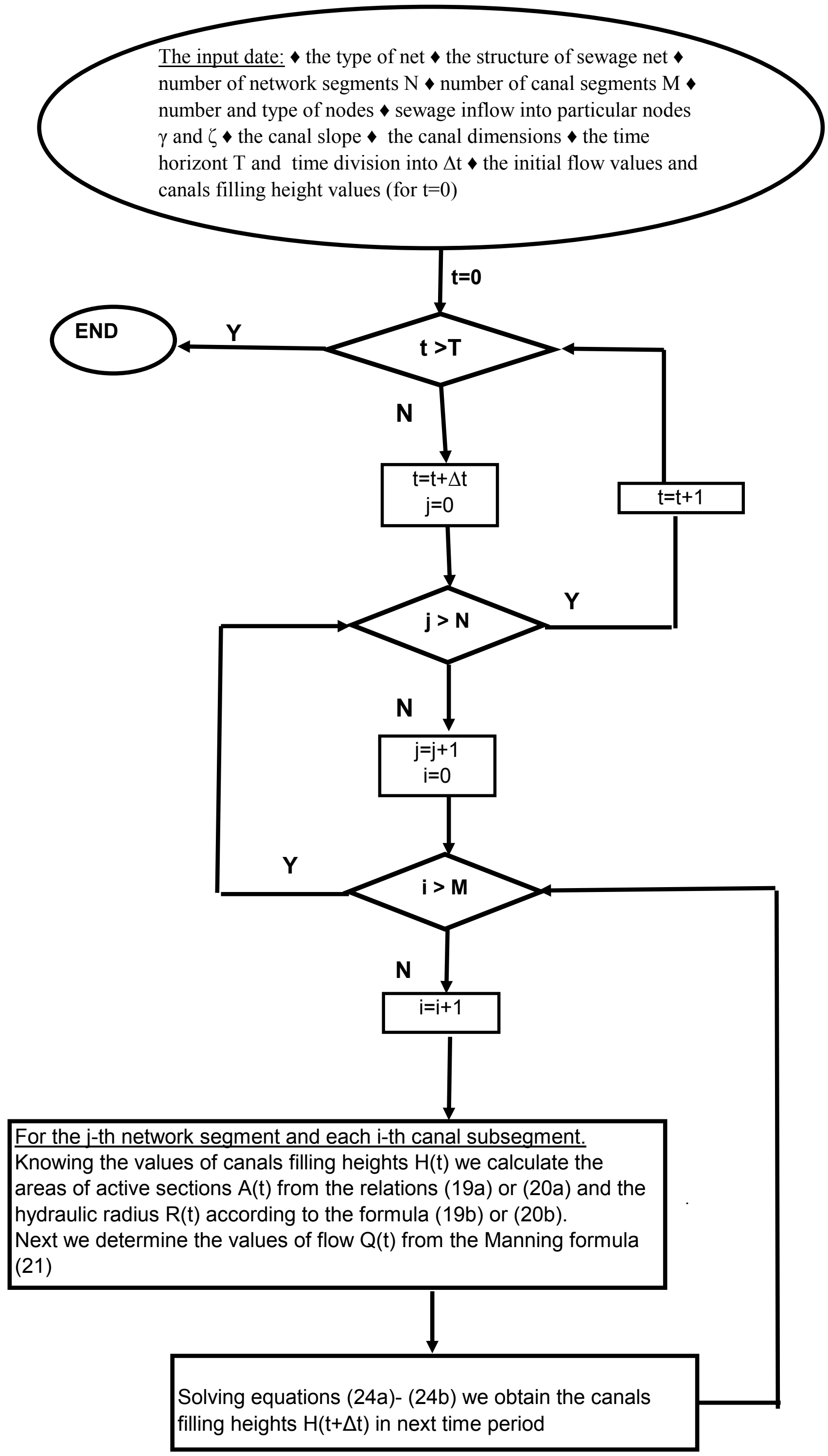

Fig. 5. Schema of the computing algorithm for the second version of the network model

for the length unit of the i-th subsegment, γ_j – sewage inflow to the j-th network segment should be entered into the algorithm.

Step 2. The sewage inflow $\zeta_{ij}(t)$ the j-th canal calculated for the length unit of the i-th subsegment, γ_j – sewage inflow for individual net nodes are calculated for the given time period $t_k = t_{k-1} + \Delta t$.

Depending on the kind of the network (housekeeping or combined sewage net) the rate of inflow for each segment is calculated using the relations given in point 2.

Step 3. For cosidered time t_k, the cross sectional areas A_{ij} and hydraulic radiuses R_{ij} for the known filling heights $H_{ij}(t_k)$ (calculated according to (12a)–(12b)) are determined (for the particular j-th net segments j=1,…,N and for each i-th canal subsegment i=1,…,M_j) as follows:

for $H_{ij} \le 0.5\, d_j : j = 1,\dots,N \quad i = 1,\dots,M_j$

$$A_{ij}(t_k) = \frac{d_j^{\,2}}{8} \cdot \left(\varphi_{ij}(t_k) - sin\left(\varphi_{ij}(t_k)\right)\right), \quad (19a)$$

$$R_{ij}(t_k) = \frac{d_j \cdot \left(\varphi_{ij}(t_k) - sin(\varphi_{ij}(t_k))\right)}{4 \cdot \varphi_{ij}(t_k)}, \quad (19b)$$

$$\varphi_{ij}(t_k) = 2 \cdot arccos\left(1 - 2 \cdot \frac{H_{ij}(t_k)}{d_j}\right), \quad (19c)$$

for $H_{ij} > 0.5\, d_j : \; j = 1,\dots,N \quad i = 1,\dots,M_j$,

$$A_{ij}(t_k) = \frac{\pi d_j^2}{4} - \frac{d_j^{\,2}}{8} \cdot \left(\varphi_{ij}(t_k) - sin\left(\varphi_{ij}(t_k)\right)\right), \quad (20a)$$

$$R_{ij}(t_k) = \frac{d_j}{4} + \frac{d_j}{8} \cdot \frac{sin\left(\varphi_{ij}(t_k)\right)}{\pi - 0.5 \cdot \varphi_{ij}(t_k)}, \quad (20b)$$

$$\varphi_{ij}(t_k) = 2 \cdot arccos\left(2 \cdot \frac{H_{ij}(t_k)}{d_j} - 1\right), \quad (20c)$$

where: d_j –diameter of j-th network segment.

Step 4. Knowing hydraulic radiuses $R_{ij}(t_k)$ and active areas of canal segments $A_{ij}(t_k)$, we can calculate (for individual segments j=1,…,N and for the i-th canal subsegment, where i=1,…,M_j):

a) flow rates Q_i:

$$Q_{ij}(t_k) = \tfrac{1}{nj}\left(R_{ij}(t_k)\right)^{2/3} \cdot J_j^{\,1/2} \cdot A_{ij}(t_k), \quad (21)$$

b) flow velocities v_i:

$$v_{ij}(t_k) = \tfrac{1}{nj}(R_{ij}(t_k))^{2/3} \cdot J_j^{\frac{1}{2}}, \quad (22)$$

where: n_j – roughness coefficient of j-th segment, J_j – canal slope of j-th segment of the network.

The calculations are done sequentially for each segment of the network, beginning from the furthest segment and completing the computing for the segment, which is closest to the wastewater treatment plant. Each segment is divided subsequently into M_j subsegments for which the calculations shown are repeated.

Step 5. In each followed node the relation is calculated:

$$W_j(t_k) = \sum_{k \ne j}^{N} P_{kj} \cdot Q_{M_j j}(t_k) + \gamma_j(t_k), \quad (23)$$

where: P_{kj} – matrix consisting of 0 and 1 elements describing the connections between the network segmenst, $Q_{Mj}\,j$ – outflow from the j-th segment of the network, γ_j – sewage inflow to the j-th network segment, W_j – sewage inflow to the j-th canal being the sum of the outflowws form other canals connected with the j-th canal.

Step 6. Knowing the values of flow rates $Q_{ij}(t_k)$ in all segments of the net we can determine the canal filing heights for the next time period $t_{k+1} = t_k + \Delta t$ as:

$$H_{1j}(t_k + \Delta t) = H_{1j}(t_k) + 4\frac{\sqrt{\frac{H_{1j}(t_k)}{d_j} - \left(\frac{H_{1j}(t_k)}{d_j}\right)^2}}{d_j \cdot \left(1 - cos(\varphi_{1j}(t_k)\right)} \cdot$$

$$\cdot\left(W_j(t_k) - Q_{1j}(t_k) + \zeta_{ij}(t_k)\right) \cdot \frac{\Delta t}{\Delta x_i}, \quad (24a)$$

$$H_{ij}(t_k + \Delta t) = H_{ij}(t_k) + 4\frac{\sqrt{\frac{H_{ij}(t_k)}{d_j} - \left(\frac{H_{ij}(t_k)}{d_j}\right)^2}}{d_j \cdot \left(1 - cos(\varphi_{ij}(t_k)\right)} \cdot$$

$$\cdot\left(Q_{i-1\,j}(t_k) - Q_{ij}(t_k) + \zeta_{ij}(t_k)\right) \cdot \frac{\Delta t}{\Delta x_i}. \quad (24b)$$

After calculating the canal filling heights in all segments of the net for the time period $t_{k+1} = t_k + \Delta t$, we calculate active areas of segments A, hydraulic radiuses R, flow rates Q and velocities v_i for the whole network and for the whole time od simulation.

Using the simplified flow models, the sewage networks of any kind can be calculated for dynamical case.

4. Conclusions

In the paper two algorithms for dynamic modelling and planning of communal sewage networks are proposed. In the first algorithm the network investigated is described with the relations (3b) and (6) from which the lateral area A for each canal segment j and for the simulation time t can be determined. Subsequently an additional equation in form $F(H)$-A=0 is formulated with function $F(H)$ defined by (9a) or (9b). From this equation canal filling height H and canal hydraulic radius *R* can be calculated and using them the sewage flow *Q* can be determined. This approach seems to be simple but the necessity to formulate and solve the equation $F(H)$-A=0 complicates the process of the modeling.

In the second algorithm the canal filing height *H* for given simulation times *t* are determined directly by solving the difference equation (24a)–(24b). Afterwards, the sewage flows *Q* can be calculated from (21). The network modelled can be calculated step by step for all network segments taking subsequently the outflows from some canal segments as the inflows to other ones.

For both algorithms, the essential problem is to determine the models of sewage inflows to the individual network segments. In the case of communal and industrial sewage these inflows are relatively simple to define knowing the data of the water consumption regarding the end users of the water network connected with the sewage one. These inflows can be modelled as the curves with constant values for subsequent time sections. A problem arises by modeling the rain fall water flowing into the sewage canals. The rain fall sewage inflows can be defined directly by means of some wastewater functions resulted from special field investigation or indirectly by means of the functions describing the rainfall and the referred drainage basin. In the second case several parameters describing the soil like surface and shape of the terrain, field decrease, buildings density on the drainage basin, soil covering etc. must be defined what complicates essentially the problem of inflow modeling.

The algorithms for modeling and planning the sewage networks presented in the paper are in our opinion an indirect approach between the standard method using the nomogramms and the more sophisticated method using the hydraulic models of the networks like SWMM developed by EPA (*US Environment Protection Agency*). The modeling with the nomogramms is very simple but not very exact and its application is pure mechanical without any need to understand the process of modeling. On the other side, the modelling with hydraulic models is very exact and also very difficult because of the need to determine many network and terrain parameters. In the case of our algorithms, the exactness is better than by the nomogramms and the complications are lower than by the hydraulic models.

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