A MONTE CARLO-BASED METHOD FOR ASSESSING THE MEASUREMENT UNCERTAINTY IN THE TRAINING AND USE OF ARTIFICIAL NEURAL NETWORKS

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Abstract

When an artificial neural network is used to determine the value of a physical quantity its result is usually presented without an uncertainty. This is due to the difficulty in determining the uncertainties related to the neural model. However, the result of a measurement can be considered valid only with its respective measurement uncertainty. Therefore, this article proposes a method of obtaining reliable results by measuring systems that use artificial neural networks. For this, it considers the *Monte Carlo Method* (MCM) for propagation of uncertainty distributions during the training and use of the artificial neural networks.

Keywords: artificial neural networks, measurement system, measurement uncertainty, Monte Carlo Method.

1. Introduction

ANN-based tools frequently present themselves as solutions where mathematical models of the physical phenomena are impracticable [1]. They enable effective modelling of complex problems with quite adequate results [2–6]. However, when ANNs are used to present the value of a physical quantity, their results can be contested in the light of metrology: when a measurement result is reported it is mandatory to present it with a kind of quantitative indication of its quality. Without this quality indication it is not possible to compare the result either with other results or with a reference value [7].

The quality of a measured quantity is expressed by its measurement uncertainty, which is defined in the international dictionary of metrology as a parameter associated with the measurement result, which characterizes the dispersion of values that could reasonably be attributed to the measurand [8]. Accordingly, the measurement uncertainty is nothing more than a range of values which indicates, with a certain probability, where is the real value of the measured quantity. Thereby, the measurement uncertainty reflects the lack of knowledge about the exact value of the quantity and it must always be given – because it is impossible to determine the value of any quantity without the existence of doubt. To better illustrate the concept of measurement uncertainty, the measurement of a voltage with a voltmeter can be taken as an example. It is necessary to connect the voltmeter to the electrical circuit and the input impedance of the voltmeter will introduce errors in the measurement related to the real

voltage value. Correction of these errors could be done by knowing the input impedance of the voltmeter. However, this impedance can be known only by measurements which would be uncertain. Thus, it is impossible to execute an error-free measurement. This lack of knowledge about the exact value of the measured quantity must be expressed [7].

In measuring systems where indications of different instruments are combined in a mathematical model for to obtain a final result, the measurement uncertainty must also be propagated in this model. Thus, the final result must also be expressed with its respective measurement uncertainty. As an example, the active power measurement (P) in an electronic circuit as the product of measurements of the direct voltage (V) and direct current (I) can be considered. Thereby, the voltage and current measurement uncertainties are propagated in the mathematical model ($P = V \cdot I$), and so it is possible to estimate the range in which the real value of the power is found.

For all these measurement situations there are very well defined and internationally accepted methods for evaluation of the uncertainty. However, these methods do not make any reference to measuring systems that use *Artificial Neural Networks* (ANNs). In many practical cases ANNs are used to replace the classical mathematical models. Usually, ANNs are obtained with interactive and iterative procedures based on data and heuristics. When this occurs, the training dataset is obtained from measurements of the real world, and as there are errors in these measurements, there will certainly be a difference between the classical mathematical model and the neural model.

Due to the infeasibility to exactly know the errors that measurements cause in the training of an ANN, it is necessary to consider them in evaluation of the measurement uncertainty. Accordingly, the current methods used for determination of *Confidence Intervals* (CI) of ANNs do not satisfy the metrological state-of-art. On the other hand, the current methods of uncertainty evaluation do not consider the use of ANNs and their particularities in measuring processes. Therefore, this article aims to contribute to estimation of *Confidence Intervals* (CI) of measurement results that are generated by measuring processes using multilayer perceptron neural models. The article presents the current methods for estimation of CI in ANNs, the internationally accepted methods for evaluation of the measurement uncertainty, advantages of using the *Monte Carlo Method* (MCM) for propagation of the uncertainty distributions in artificial neural models, and the proposed method illustrated by a practical example.

2. Methods for estimating confidence intervals in ANNs

Different methods were proposed for estimating CI of ANNs. These methods usually depend on the type of architecture of ANN and play an important role when applied in practical situations [9−11]. The authors, like Chryssolouris *et al.* [12], Hwang *et al.* [13] and Veaux *et al.* [14], are still often quoted as the references in many different papers. They established and analysed methods for the CI estimation assuming that the presented errors are normally distributed and independent. The authors used CI estimation techniques applied in non-linear regressions. Moreover, they took into account the information that ANNs were obtained from random noise contaminated data.

Papadopoulos [15] compares three of these main methods with the aim of showing which best suits industrial applications. The tested methods estimate CI by obtaining a variance estimator assigned to each output value during the use of a neural model. It is expected that this estimator can then represent variability of the data used during the training phase. The following methods are considered:

− maximum likelihood training − which established creation of an increased network, according to what is presented in Fig. 1, which objective is obtaining a variance estimator

that represents variability of the data used during the learning of the ANN. The training of both networks is performed simultaneously with a function that contains all the parameters;

− approximate Bayesian training – which also makes use of an increased network as presented in Fig. 1, but accomplishes the training of both networks separately;

− bootstrap method – which starts from creation of a committee of ANNs, which is nothing but a set of k networks with the same configuration. Each ANN is trained with a set of n elements randomly chosen (with replacement) from an N element database ($n < N$). The resulting committee may be a simple average and the variance is the estimator of variability which is used for obtaining the CI.
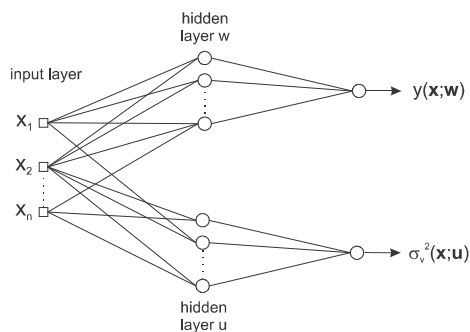

Fig. 1. An increased network [15].

The presented methods show a concern in establishing ways to provide some reliability to the neural models that were trained using contaminated data. However, all methods concern exclusively with random errors, as can be seen also in [16]. Nevertheless, in many practical measurement situations, the systematic errors can have a stronger influence than the random errors. Actually, both random and systematic errors can or cannot be present in a measurement. However, the doubt related to the result will always exist. It is important to stress that the measurement uncertainty is a measure of doubt and it does not correspond to variability of the data. Therefore, in cases when the doubt related to the systematic error is stronger than that of the random error, the presented methods will not provide an adequate CI.

Another metrological situation not addressed by the presented methods is that they do not worry about uncertainties related to measurement instruments during the use of ANNs. They assume that the data will remain with the same variability of the data collected for the training phase. Nonetheless, the measurement instruments used during the training phase may be not the same as those used during the use of ANNs. This reinforces the need of obtaining a metrologically accepted method capable of estimating the uncertainty in the training and use of ANNs, instead of simply considering variability of the data caused by random errors during collection of the training data.

3. Methods evaluating the measurement uncertainty

As previously seen, the result of a measurement deviates from the real value of a quantity by the measurement error, which is generated by the inherent process of measuring and depends on the specific conditions of each measurement. The errors can be classified as systematic and random ones. The former, being a constant and repeated value in all results, and the latter – purely random with an average value equal to zero and assuming different models of probability distribution. Additionally, there are errors caused by wrong execution of the measurement procedures, damage or poor operation of the measuring instruments, and problems in treatment

or transcription of the collected data. These types of errors cause outliers that must be disregarded because they are not relevant for evaluation of the uncertainty.

As mentioned before, by its own nature, the error cannot be known exactly, and this fact led the international metrological community to formulate the concept of measurement uncertainty. In addition, it is now recognized that even if all known error components have been corrected, there still remains an uncertainty of correction itself [7].

There are several sources of errors in measuring instruments that can influence the measurement result. For this reason, the instrument manufacturers indicate in their specifications the *Maximum Permissible Error* (MPE) which informs, with a certain degree of probability, the extreme values that the error of a measurement can reach in a given situation. The MPE is by the manufacturers wrongly referred to as accuracy and encompasses different types of errors related to instruments, for example: linearity, gain, offset and hysteresis. In electronic measuring instruments, the limit of MPE around an indication is usually given as a percentage of this indication plus a fixed value [17].

In a very simplified way, it can be said that when a measuring instrument is individually used, its MPE may be misinterpreted as the uncertainty [17]. However, when different instruments are used for measuring a quantity, a procedure is necessary to combine the uncertainty contributions of each one of them.

The Guide to the expression of uncertainty in measurement establishes general rules to evaluate and express the measurement uncertainty. The Guide proposes two methods: the first one is given by the document JCGM 100 [7], which takes as the basis propagation of uncertainties (standard deviations) in the measurement model; the second one is given by the document JCGM 101 [18], which deals with propagation of the probability distributions in the measurement model using the *Monte Carlo Method* (MCM).

3.1. The propagation method of the measurement uncertainties versus the classical method

The classical method employs a mathematical model for obtaining the uncertainty of output quantity from a combination of the uncertainties of input quantities. The uncertainties of input quantities, also called the standard uncertainties, are defined as the standard deviations of the probability distributions. To obtain the combined standard uncertainty of output quantity the uncertainty propagation law is used, (1). It is worth mentioning that this paper deals just with uncorrelated measurements.

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 \cdot u^2(x_i)} \ , \tag{1}$$

where $u_c(y)$ is the combined standard uncertainty of the output quantity y; $u(x_i)$ is the standard uncertainty of the input quantity x_i; $\frac{\partial f}{\partial x_i}$ is the coefficient of sensitivity of the input quantity x_i.

Using the central limit theorem, the normal probability distribution for the uncertainty of the output quantity is expected. Therefore, the combined standard uncertainty is nothing but the standard deviation of this distribution and represents the probability of 68.2%. However, in practice it is necessary to present an interval with a higher coverage probability. An additional measure that satisfies this requirement is called the expanded uncertainty (*U*), which is defined as $U = k \cdot u_c$, where k is the coverage factor of the *t*-student probability distribution, which is an approximation of the normal distribution [7]. The effective degrees of freedom are calculated from the Welch-Satterthwaite formula, (2), which is used to obtain k from the *t*-student probability distribution.

$$v_{eff} = \frac{u_c^4(y)}{\sum \frac{\left(\partial f/\partial x_i \cdot u(x_i)\right)^4}{v_i}} \ , \tag{2}$$

where v_{eff} are the effective degrees of freedom for the output quantity and v_i are the degrees of freedom of the input quantity x_i.

3.2. The propagation method of the probability distributions versus the Monte Carlo Method

Evaluation of the measurement uncertainty with the MCM is nothing more than the propagation of the distributions of probability in the mathematical model, using numerical simulations of probable input values. So, the method treats the input quantities as random variables with their respective *probability density functions* (pdf). Thereby, random values are generated respecting the pdfs of the input quantities, which are propagated in the mathematical model to form the pdf of the output quantity. The limits for the measurement uncertainty are defined from the output quantity pdf. The MCM assumes that the measurement uncertainty represents the doubt about the measurement result, so each value obtained by random generation is as legitimate as any value indicated by the measuring instrument [19].

It is important to stress that the number of MCM trials has a strong influence on the expected coverage probability of the output quantity pdf. Accordingly, it is important to balance the number of trials with availability of hardware and simulation time [18].

3.3. Propagation of the measurement uncertainty in ANN

When an ANN is used in measuring systems as a substitute for a classical mathematical model of the physical phenomenon, it is also necessary to propagate the uncertainty in this ANN, and both propagation methods can be used. However, the MCM propagation method is easier than the classical one.

3.3.1. The classical method

A trained ANN can be considered as a mathematical model. Thus, the method of propagation of the measurement uncertainties as shown in 3.1 can be applied. If a proper neural network was obtained with a precise representation of the process it could be safely used as a mathematical model of measurement and treated using the classical approach for uncertainty evaluation. Some references, as [20] and [21], follow this way. Gusman [20] presents an example of using this method for a generic ANN with 2 neurons in the input layer, 2 neurons in the hidden layer and 1 single neuron in the output layer, as shown in Fig. 2.

The output function for this ANN is given by (3).

$$y = f(x_1, x_2) = \varphi\{b_3 + w_3.\varphi[b_1 + w_{11}.x_1 + w_{21}.x_2] + w_4.\varphi[b_2 + w_{12}.x_1 + w_{22}.x_2]\}, (3)$$

where: y is the ANN output; x_i are the ANN inputs, ϕ represents the transfer function of each neuron, w_{ij} are the weights assigned to each input of the neurons; and b_i are the biases of each neuron.
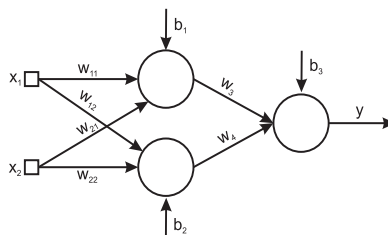

Fig. 2. A generic ANN [20].

If in the mathematical model given by (3) x_i represent the values of measurements of the input quantities and y represents the value of the output quantity, then to obtain the measurement uncertainty of y, it is only necessary to apply the uncertainty propagation law in the function $f(x_1, x_2)$, as demonstrated by (4), (5) and (6):

$$u(y) = \sqrt{\frac{\partial(x_1,x_2)^2}{\partial x_1} \cdot u_{x_1}^2 + \frac{\partial(x_1,x_2)^2}{\partial x_2} \cdot u_{x_2}^2}, \tag{4}$$

where:

$$\frac{\partial(x_1,x_2)}{\partial x_1} = \varphi'(x_1) \cdot [w_3 \cdot \varphi'(x_1) \cdot w_{11} + w_4 \cdot \varphi'(x_1) \cdot w_{12}], \tag{5}$$

$$\frac{\partial(x_1,x_2)}{\partial x_2} = \varphi'(x_2) \cdot [w_3 \cdot \varphi'(x_2) \cdot w_{21} + w_4 \cdot \varphi'(x_2) \cdot w_{22}]. \tag{6}$$

It is possible to verify whether application of the uncertainty propagation law to an ANN requires having by the neurons differentiable transfer functions. It is also possible to notice that the simple propagation of the uncertainty in an ANN during its use does not take into account the uncertainties related to the data of the training process. Another problem in application of the classical method is establishing the partial derivatives related to the number of hidden layers and neurons for larger ANN configurations.

3.3.2. The Monte Carlo Method

Application of the MCM for propagation of the pdf in an ANN is less costly than application of the classical method. In this case, different values in the ANN inputs must be simulated to obtain the pdf of the output quantity, as exemplified by Fig. 3. It can be noticed that there is no need to calculate partial derivatives, the method just uses the already trained ANN for propagation of the distributions. Application of this method does not take into account the errors related to the ANN training process.
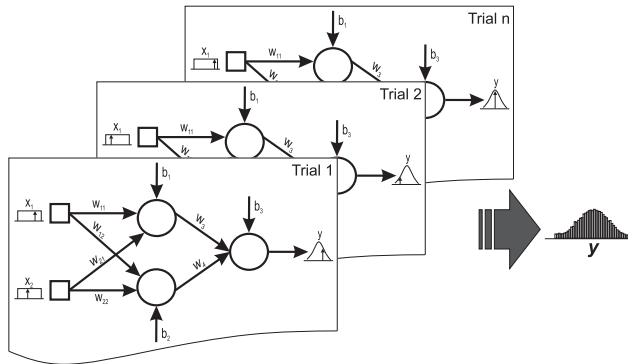

Fig. 3. Application of the MCM in a generic ANN.

Validity of this evaluation method for an ANN is demonstrated by comparison of the results obtained from propagation of the pdfs in a mathematical model and in its neural equivalent. As an example it will be taken determination of the active power (P) using measurements of dc voltage (V) and dc current (I), following the equation P = V·I.

For the neural equivalent model, the following configuration of a feed-forward network was implemented: 2 neurons in the input layer; 4 neurons in the first hidden layer; 4 neurons in the second hidden layer; 1 neuron in the output layer. For the training the back-propagation algorithm was used; Table 1 presents the training set.

Table 1. The training dataset without measurement errors.

Voltage (V)	5.00	6.00	7.00	8.00	9.00	5.00	6.00	7.00	8.00	9.00	5.00	6.00	7.00
Current (A)	5.00	5.00	5.00	5.00	5.00	6.00	6.00	6.00	6.00	6.00	7.00	7.00	7.00
Power (W)	25.00	30.00	35.00	40.00	45.00	30.00	36.00	42.00	48.00	54.00	35.00	42.00	49.00
Voltage (V)	8.00	9.00	5.00	6.00	7.00	8.00	9.00	5.00	6.00	7.00	8.00	9.00	
Current (A)	7.00	7.00	8.00	8.00	8.00	8.00	8.00	9.00	9.00	9.00	9.00	9.00	
Power (W)	56.00	63.00	40.00	48.00	56.00	64.00	72.00	45.00	54.00	63.00	72.00	81.00	

After the training process a test set with different values of voltage and current was presented to the ANN. This enabled verification of the deviation in the neural model related to the mathematical model. The deviations were very close to zero for the test set. As the mathematical model of the physical phenomenon was known and used to generate the training set, there were no errors in the data of the training set.

Now, it is assumed a measuring system comprising the following instruments and related specifications: a 3½ digit voltmeter with 10 mV resolution and the maximum permissible error of ± (0.5% of reading + 4 times the resolution) V; a 3½ digit ammeter with 10 mA resolution and the maximum permissible error of ± (2% of reading + 6 times the resolution) A; in both cases the maximum permissible errors have uniform pdfs.

Thereby, a voltage indication of 7.00 V can be regarded as (7.00 ± 0.08) V, which states that the real value of the voltage must be between 6.92 V and 7.08 V. In the same way, a current indication of 7.00 A can be regarded as (7.00 ± 0.20) A.

Within the ranges defined by (7.00 ± 0.08) V and (7.00 ± 0.20) A 10,000 values of voltage and 10,000 values of current were simulated, respecting uniform pdfs in both cases. Therefore, these values were applied in the mathematical model and in the neural model. The results for the active power are shown in Fig. 4.
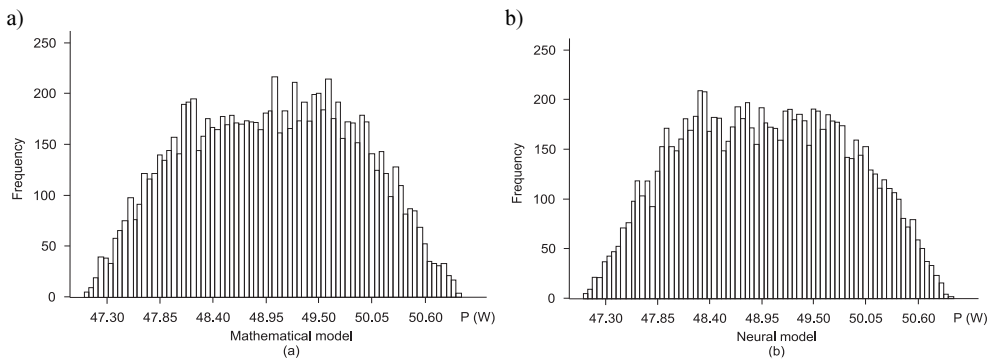

Fig. 4. Application of the MCM: a) in the mathematical model; b) in the neural model.

It is possible to observe that the distributions in both models are very similar in average and spreading. The mathematical model presents an average value of 49.02 W with standard deviation of 0.866 W, whereas the neural model presents an average value of 49.03 W with standard deviation of 0.874 W. The resultant pdfs for both models are also very similar to each other, which agrees with the declaration of feasibility of using the MCM in neural networks. Similar behaviour is observed for the whole test set. It is important to stress that the neural model must be used with the input data from within the range it was trained. Accordingly, the neural model will present proper outputs.

4. Effects of measurement errors on the ANN training

Subsection 3.3.2 showed that the use of the MCM for propagation of the measurement uncertainties of the input quantities in an ANN is possible. However, simple propagation of the pdf in a neural model would not result in proper uncertainties of the output quantity. In practical situations, where application of an ANN is advantageous in comparison with the classical mathematical model, it is impossible to train an ANN without errors imposed by measurements. In practice, a training set is obtained from the measurements and the mathematical modelling is unknown.

To better exemplify this concept the power measurement example will be revisited. Previously, the data were obtained without errors using a known mathematical model. However, the training data could come from measurements of the voltage, current and power. Therefore, the following assumptions are made: the same instruments are used to generate data for the ANN training and use; voltmeter and ammeter specifications are the same as in the previous example; the wattmeter has the following specification: $3^{1}/_{2}$ digit with 1 W resolution and the maximum permissible error of $\pm$ (1.5% of reading + 3 times the resolution) W; all measuring instruments have systematic errors within the maximum permissible error; the training data obtained by measurements are presented in Table 2; the trained ANN has the same configuration as in the previous example.

It can be observed that simple multiplication of the related voltage and current in Table 2 does not result in the respective power. However, the power indications are correct considering the wattmeter specifications. This is due to the fact that the voltmeter, the ammeter and the wattmeter have systematic errors of −0.04 V, −0.06 A and +3 W, respectively, which are in the range of MPE defined for each instrument. Therefore, when the voltmeter indicates 7.00 V and the ammeter indicates 7.00 A, the real value for the quantities are 7.04 V and 7.06 A, which results in the real value of power of 49.7024 W. However, as the wattmeter has the systematic error of +3 W, it will indicate 52 W and not 49 W, as it would be expected. The neural network was trained with data presented in Table 2.

A new test dataset with 20 pairs of measurement results – within the training data range – obtained from the voltmeter and the ammeter was applied to the ANN considering 10,000 MCM trials for a 95% coverage probability.

Figure 5 presents the ANN output for different levels of current. Accordingly, it would be expected that at least 95% of the true power values (dashed lines) were within the limits defined by the uncertainty U (dotted lines). However, one can observe that most of the true values are out of the uncertainty limits. This demonstrates that propagation of pdf inputs is not enough to establish reliable limits for the ANN output.

Table 2. The training data with the systematic error in the measurement.

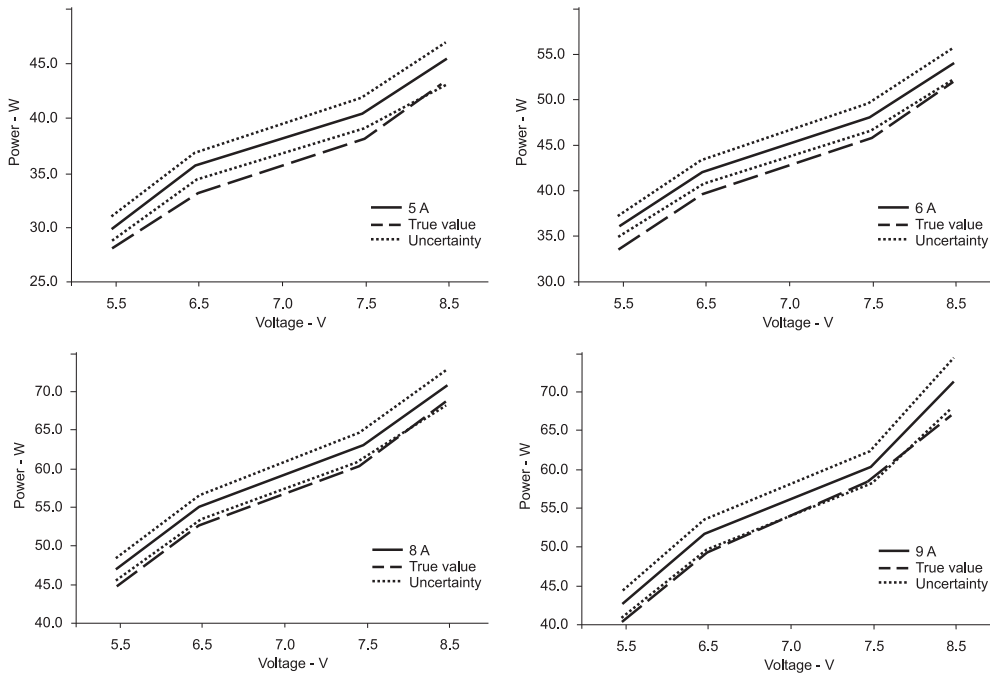| Voltage (V)	5.00	6.00	7.00	8.00	9.00	5.00	6.00	7.00	8.00	9.00	5.00	6.00	7.00
Current (A)	5.00	5.00	5.00	5.00	5.00	6.00	6.00	6.00	6.00	6.00	7.00	7.00	7.00
Power (W)	28.00	33.00	38.00	43.00	48.00	33.00	39.00	45.00	51.00	57.00	38.00	45.00	52.00
Voltage (V)	8.00	9.00	5.00	6.00	7.00	8.00	9.00	5.00	6.00	7.00	8.00	9.00	
Current (A)	7.00	7.00	8.00	8.00	8.00	8.00	8.00	9.00	9.00	9.00	9.00	9.00	
Power (W)	59.00	66.00	43.00	51.00	59.00	67.00	75.00	48.00	57.00	66.00	75.00	84.00	

Fig. 5. Output considering only the MCM use in an ANN.

5. Evaluation of the measurement uncertainty in the training of an ANN

As demonstrated in Section 4, the measurement errors in the training data directly affect the learning of an ANN and its final performance. In the light of metrology, the measurement errors that are not corrected must be considered in evaluation of the uncertainty. This article proposes assessing the measurement uncertainty – related to the training data set – using a Monte Carlo-based method during the learning phase of a neural model.

As in the bootstrap method described in Section 2, the proposal starts from creating an ANN committee. Each one of k ANNs is trained with a set of n examples, which are randomly chosen from a set of N examples. As the data were randomly selected with replacement there will be some repeated data and some other will not appear in any of the k different training sets. Then, it is expected that the ANN committee has included the data variability contained in the data set of N examples. It is possible to notice that the bootstrap method is not appropriate for determination of a confidence interval in the previous example, given the fact that it would not include the information related to the measurement uncertainty in the committee. The doubt about each data point used in the training still exists and the systematic errors are in it.

Defining each example for the training as $g_x(\xi)$ or $g_y(\xi)$, $\xi \in]-\infty, +\infty[$ (where g_x and g_y are the pdfs related to the measurement uncertainty of input and output data, respectively, and ξ is the variable that describes all the possible values), one could use this fact to create different training sets from the same knowledge basis, without random selection with replacement.

In this proposal, in each new learning process, the data are randomly modified using the MCM respecting the measurement uncertainty pdfs, as illustrated in Fig. 6. In the same way as in the propagation method of the distributions, presented in Subsection 3.2, each value generated by simulation is as legitimate as the data originally obtained for the training set. It is expected that when an input is applied to the neural committee, each ANN presents a distinct answer and when combined they present a pdf. This pdf will be a combination of all pdfs related

to the training data set (Fig. 7). In this proposal all the data are used in the learning process; such a situation does not occur in the bootstrap method. However, for every value obtained in the measurement process, it is necessary to know the measurement uncertainties and their respective pdfs. An additional advantage of the proposal focused on the use of a committee is the fact that different networks trained with consistent training sets can present different results, so that a simple average process is favourable for the final result [22−25]. Accordingly, the individual network errors partially compensate themselves when the outputs are combined, reaching better performance when compared with a single ANN [26, 27].
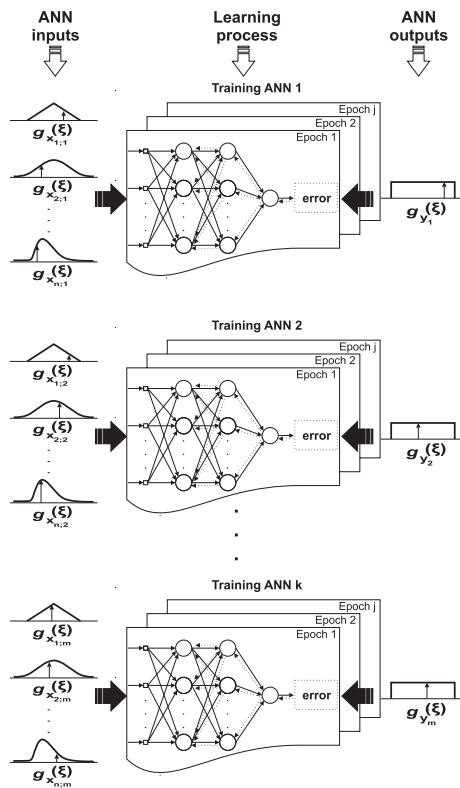
Fig. 6. The training process with data selected from the MCM.

It is now possible to apply the MCM for propagation of the uncertainties of the input quantities when using the neural model. Thus, it is expected an influence of all the measurement errors on the final result: those related to the input quantities during the use and those related to the training set.

Returning to the example of power measurement. Like in Section 4, the same considerations were made for this example, including the training set with the same measurement errors. However, 1,000 ANNs were trained considering MCM randomly generated values based on the uncertainty pdfs of each input-output example of the training data. Using a new committee for the inputs 7.00 V and 7.00 A, the answer is (52.00 ± 3.44) W, with a coverage probability of 95%, as presented in Fig. 8. The average value of 52 W perfectly represents the training data obtained with measurement errors, as can be observed in Table 2. However, there is now an interval around this value, as represented by a histogram of the ANN outputs shown in Fig. 8.
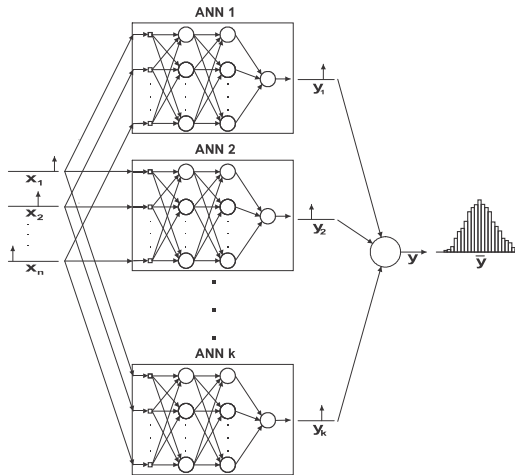
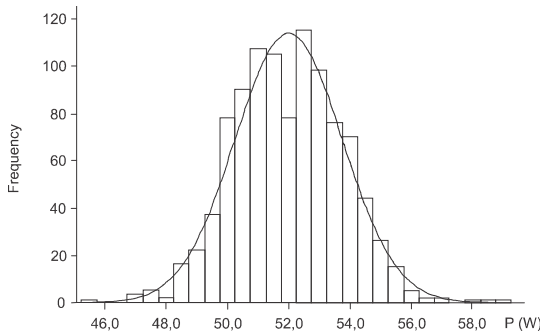Fig. 7. Simulations of an ANN committee trained from the MCM.

Fig. 8. The uncertainty related to the neural model.

Now, suppose a situation where one wants to use an ANN committee with data generated by the measurement instruments – voltmeter and ammeter – that are different from those used to generate the training data. In such a situation the measurement uncertainties would be different from those considered in training the ANNs. The MCM can be used for propagation of the uncertainty distributions propagation through the ANN committee.

So, for the indications of 7.00 V and 7.00 A there are the following measurement results: (7.00 ± 0.08) V and (7.00 ± 0.20) A, with a uniform pdf for both cases. Thus, when these results are presented to the ANN committee using the MCM, the obtained power measurement result is (51.99 ± 4.00) W, with a coverage probability of 95% and a normal pdf, as shown in Fig. 9. For evaluation of this uncertainty 1,000 values of voltage and current were generated, which – when combined and propagated by the ANN committee – generated 1,000,000 different power values.

In the same way as in Section 4, a dataset with 20 pairs of measurement results was presented to the neural committee. The 20 power results were obtained using the MCM and are presented in Fig. 10. The solid lines represent the committee output and are located between the dotted lines that represent the 95% limit comprising the committee output values. However, in this example the dashed lines – which represent the real value of power – are within the 95% limit. Accordingly, this indicates that the measurement results consider all the measurement errors, those present in the training and those present in the use of the neural committee.
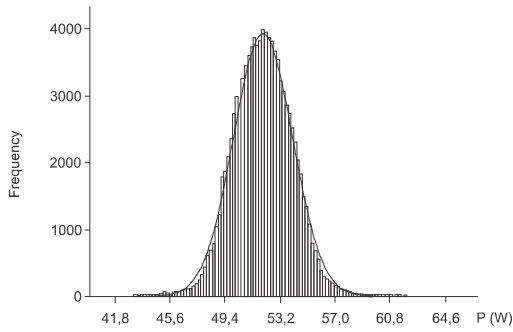
Fig. 9. The committee output uncertainty.

6. Conclusions

The article presents methods of creating an artificial neural model composed by ANNs trained from the same training set. The proposed method makes use of the MCM that enables obtaining different ANNs, which can be combined and express a proper value of the uncertainty. From the obtained measurement uncertainty, the neural model can be a part of measuring systems. The presented method enables using different instruments to obtain the training data and to use the model. So, it is possible to dispose of more advanced instruments, consequently more expensive and more sensitive, for obtaining the training data, with the purpose of minimizing the measurement uncertainty of the neural model, as well as instruments with a greater measurement uncertainty, consequently cheaper or more robust, during the use of the ANNs. This is the situation that can be really desirable in numberless cases of industrial measurements.
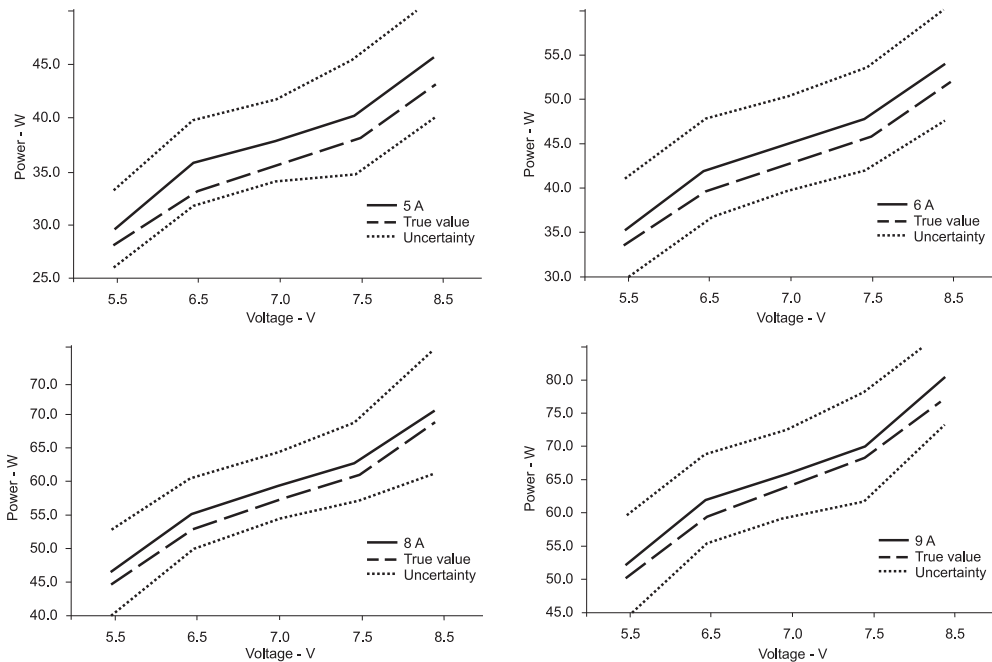

Fig. 10. The ANN committee output considering the uncertainties in training and use.

Acknowledgements

References

[1] Haykin, S. (1999). *Neural Networks: A Comprehensive Foundation*. 2nd ed., Prentice Hall.

[2] Singaram, L. (2011). ANN prediction models for mechanical properties of AZ61 MG alloy fabricated by equal channel angular pressing. *Int. J. of Res. and Reviews in Appl. Sciences*, (8), 337–345.

[3] Ghobadian, B., Rahimi, H., Nikbakht, A.M., Najafi, G., Yusaf, T.F. (2009). Diesel engine performance and exhaust emission analysis using waste cooking biodiesel fuel with an artificial neural network. *Renew. Energy*, (34), 976–982.

[4] Ertunc, H.M., Hosoz, M. (2006). Artificial neural network analysis of a refrigeration system with an evaporative condenser. *Appl. Therm. Eng.*, (26), 627–635.

[5] Arcaklioğlu, E., Çavuşoğlu, A., Erişen, A. (2004). Thermodynamic analyses of refrigerant mixtures using artificial neural networks. *Appl. Energy*, (78), 219–230.

[6] Russel, S., Norvig, P. (2003). *Artificial Intelligence: A Modern Approach*. 2nd ed., Prentice Hall.

[7] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, OIML, (2008). JCGM 100: Evaluation of measurement data – Guide to the expression of uncertainty in measurement, 134.

[8] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, OIML, (2008). JCGM 200: International vocabulary of metrology – Basic and general concepts and associated terms, 104.

[9] Neto, A.C.R., Neves, C.A.M., Roisenberg, M. (2013). Comparative study on local and global strategies for confidence estimation in neural networks and extensions to improve their predictive power. *Neural Comput. & Appl.*, (22), 1519–1530.

[10] Zapranis, A., Livanis, E. (2005). Prediction intervals for neural network models. *Proc. 9th WSEAS International Conference on Computers*, Athens, Greece, 7.

[11] Methaprayoon, K., Yingvivatanapong, C., Wei-Jen, L., Liao, J.R. (2007). An Integration of ANN Wind Power Estimation Into Unit Commitment Considering the Forecasting Uncertainty. *IEEE Trans. Ind. Appl.*, (43), 1441–1448.

[12] Chryssolouris, G., Lee, M., Ramsey, A. (1996). Confidence interval prediction for neural network models. *IEEE Trans. on Neural Netw.*, (7), 229–232.

[13] Hwang, J.T.G., Ding, A.A. (1997). Prediction intervals for artificial neural networks. *J. of the American Statistical Association*, (92), 748–757.

[14] De Veaux, R.D., Schweinsberg, J., Schumi, J., Ungar, L.H. (1998). Prediction intervals for neural networks via nonlinear regression. *Technometrics*, (40), 273–282.

[15] Papadopoulos, G., Edwards, P.J., Murray, A.F. (2001). Confidence estimation methods for neural networks: a practical comparison. *IEEE Trans. on Neural Netw.*, (12), 1278–1287.

[16] Bakhary, N., Hao, H., Deeks, A.J. (2007). Damage detection using artificial neural network with consideration of uncertainties. *Eng. Structures*, (29), 2806–2815.

[17] Fluke Corporation. (1994). *Calibration: Philosophy in Practice*. 2nd ed.

[18] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, OIML, (2008). JCGM 101: Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method. 90.

[19] Moscati, G., Mezzalira, L.G., Santos, F.D. (2004). Measurement uncertainty using the Monte Carlos method in the context of GUM-Supplement 1. *Proc. ENQUALAB National Meeting for Laboratories Quality*, São Paulo, Brazil, 4.

[20] Gusman, C.S.A. (2011). *Measurement uncertainty evaluation for neural networks applied on predictive maintenance of power transformers. Master Degree Dissertation on Metrology.* Pontifícia Universidade Católica, Rio de Janeiro.

[21] Zhu, J., Wang, Z., Xia, X., Lei, J. (2005). One parameterized model of indirect measurement based on neural network and its sensitivity coefficient computing. *Proc. 7th Int. Symp. on Measurement Technol. and Intell. Instrum., J. of Phys.: Conf. Series 13.*

[22] Fortuna, L., Giannone, P., Graziani, S., Xibilia, M.G. (2007). Virtual instruments based on stacked neural networks to improve product quality monitoring in a refinery. *IEEE Trans. on Instrum. and Measurement*, (56), 95–101.

[23] Hu, Y.H., Hwang, J. (2002). *Handbook of neural network signal processing.* CRC Press LLC, USA, 384.

[24] Edwards, P.J., Peacock, A.M., Renshaw, D., Hannah, J.M., Murray, A.F. (2002). Minimizing risk using prediction uncertainty in neural network estimation fusion and its application to papermaking. *IEEE Trans. on Neural Netw.*, (13), 726–731.

[25] Ahmad, Z., Zhang, J. (2002). A comparison of different methods for combining multiple neural networks models. *Proc. Int. Joint Conf. on Neural Netw.*, (1), 828–833.

[26] Efron, B., Tibshirani, R. (1993). *An introduction to the bootstrap.* Chapman & Hall, NY, 450.

[27] Tibshirani, R. (1994). *A comparison of some error estimates for neural network models.* Department of Preventive Medicine and Biostatistics, University of Toronto.