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Antoni ŚWIĆ[*], Arkadiusz GOLA[**], Mikuláš HAJDUK[***]

MODELLING OF CHARACTERISTICS OF TURNING OF SHAFTS WITH LOW RIGIDITY

Abstract

Numerical studies were conducted with the use of a computer program permitting the determination of the basic dynamic characteristics of the process of machining and the presentation of graphic characteristics of the numerical simulations performed. Analysis was performed of the relation of the output parameters of the dynamic system to the input parameters, relative stiffness coefficient B to feed rate and depth of machining, change of retardation and time constants to rotary speed and depth of machining, and of the frequency and time characteristics of models of the dynamic system of machining of shafts with low rigidity.

1. INTRODUCTION

The numerical studies of the machining process were performed in the program MATMOD. The program allows the determination of the basic dynamic characteristics of the machining process, numerical simulation of the dynamic system of the machining process, and graphic presentation of the characteristics of the numerical simulations performed (Abakumov, Taranenko & Zubrzycki, 2006).

[*] Institute of Technological Systems of Information, Lublin University of Technology, Nadbystrzycka Street 36, 20-618 Lublin, Poland, e-mail: a.swic@pollub.pl
[**] Department of Enterprise Organization, Lublin University of Technology, Nadbystrzycka Street 38, 20-618 Lublin, Poland, e-mail: a.gola@pollub.pl
[***] Technical University of Košice, Department of Production Systems and Robotics, Slovakia, e-mail: mikulas.hajduk@tuke.sk

Simulations were performed for shafts of steel C45, with dimensions of L_1=500 mm, d_1=20 mm and L_2=300 mm, d_2=10 mm. The variable input parameters were the rotation speed, feed, depth of machining, and the tool cutting edge angle. The machined part was fixed in a chuck and supported by the tailstock centre.

The following relations were analysed: of the output parameters of the dynamic system to the input parameters, of the relative stiffness coefficient *B* to feed rate and machining depth, of change of retardation and time constants to rotary speed and machining depth, as well as the frequency characteristics and the time characteristics of the dynamic system of machining of shafts with low rigidity.

The possibility was found of the approximation of the frequency and time characteristics within the range of actual frequencies of the dynamic system, and of the control system of the operator transmittances of typical dynamic elements: integrating element and aperiodic element of the second or first order. The gain factor and the time constants of the approximated models are subject to change, primarily with a change in the rotary speed of the machined part and in the relative dynamic stiffness coefficient *B*, characterising the ratio of stiffness of the elastic system of the machine tool and the gain factors of the machining process (Świć, 2009; Świć, Gola & Wołos, 2014). In the case of use of universal lathes for the machining of the shafts under analysis, with changing conditions and parameters of machining the model parameters can vary within a broad range (Gola, 2014; Świc & Mazurek, 2011).

Comparative analysis of the relative error in the determination of the time constants of the models indicates that the developed mathematic models of turning of shafts are adequate to the actual process.

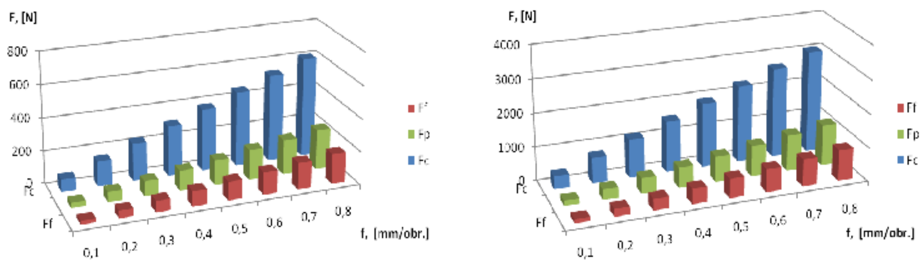
2. NUMERICAL ANALYIS OF PARAMETERS OF MACHINED SHAFT WITH LOW RIDIGITY AND OF PARAMETERS OF ITS MACHINING

The simulations were performed for shafts of steel C45, with dimensions of L_1=500 mm, d_1=20 mm and L_2=300 mm, d_2=10 mm. The variable input parameters were the rotary speed n_p=100–2000 rev/min, feed f=0.1–0.8 mm/rev, machining depth a_p=0.5–3 mm, Δa_p=0.5 mm, and the tool cutting edge angle κ_r =45^o, 90^o. The machined part was fixed in a chuck and supported with the tailstock centre. The results of the simulations for the shafts under analysis and the effect of the input parameters of the dynamic system on the change in the values of the output parameters are presented in Tab. 1.

Tab. 1. Changes of output parameters of dynamic system with change of input parameters (own study)

	Cutting forces [N]	Amplifier gain f [N/mm]	Amplifier gain a_p [N/mm]	Coefficient B	Delay τ [s]	Response to unity pitch f [s]	The amplitude [dB]	The phase [rad/s]
Angle κ_r [º]	**only to f_f**	**only m_x**	**only n_x**	**yes**	**no**	**slight**	**slight**	**slight**
a_p [mm]	**yes**	**yes**	**no**	**yes**	**no**	**yes**	**yes**	**yes**
f [rpm]	**yes**	**no**	**yes**	**yes**	**no**	**yes**	**slight**	**slight**
n_p [rpm]	**no**	**no**	**no**	**no**	**yes**	**yes**	**yes**	**yes**

Changes of machining force components in the case of shaft with L=500 mm, d=20 mm, a_p=0.4 mm and a_p=2.5 mm are presented in Fig. 1 a) and 1 b).



a) b)

Fig. 1. Changes of machining force components in the function of longitudinal feed rate at *L*=500 [mm], *d* = 20 [mm], tool cutting edge angle 45°, depth of machining 0.5 [mm] – a) and 2.5 [mm] – b), n_p=1000 [rev/min] (own study)

Analysing Fig. 1 one can conclude that with a change of machining parameters the machining force components change in a linear manner. Irrespective of the depth of machining and of the feed rate, component F_c assumes higher values, while components F_p and F_f – at tool cutting edge angle of 90° – are equal to each other.

We calculated the relative dynamic stiffness coefficient B. The results of the calculations at $\kappa_r = 45^\circ$ and $\kappa_r = 90^\circ$ are presented in Tab. 2 and 3.

Tab. 2. Change of relative stiffness coefficient *B* under the effect of feed rate and machining depth at *L* = 500 [mm], *d* = 20 [mm], rotary speed of machined part n_p=1000 [rev/min], tool cutting edge angle κ_r= 45° (own study)

		Relative stiffness coefficient B							
Depth of cut a_p [mm]	**0.1**	0.020	0.020	0.019	0.019	0.019	0.019	0.019	0.019
	0.2	0.039	0.039	0.039	0.039	0.038	0.038	0.038	0.038
	0.3	0.059	0.059	0.058	0.058	0.058	0.057	0.057	0.057
	0.4	**0.079**	**0.078**	**0.078**	**0.077**	**0.077**	**0.077**	**0.076**	**0.076**
	0.5	0.099	0.098	0.097	0.097	0.096	0.096	0.095	0.095
	0.6	0.118	0.118	0.117	0.116	0.115	0.115	0.114	0.113
	0.7	0.138	0.137	0.136	0.135	0.135	0.134	0.133	0.132
	0.8	0.158	0.157	0.156	0.155	0.154	0.153	0.152	0.151
	0.9	0.177	0.176	0.175	0.174	0.173	0.172	0.171	0.170
	1.0	0.197	0.196	0.195	0.194	0.192	0.191	0.190	0.189
	1.5	0.296	0.294	0.292	0.290	0.289	0.287	0.285	0.284
	2.0	0.394	0.392	0.389	0.387	0.385	0.383	0.380	0.378
	2.5	0.493	0.490	0.487	0.484	0.481	0.478	0.475	0.473
	3.0	0.591	0.588	0.584	0.581	0.577	0.574	0.570	0.567
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**

Tab. 3. Change of relative stiffness coefficient *B* under the effect of feed rate and machining depth at *L* = 500 [mm], *d* = 20 [mm], rotary speed of machined part n_p=1000 [rev/min], tool cutting edge angle κ_r= 90° (own study)

		Relative stiffness coefficient B							
Depth of cut a_p [mm]	**0.1**	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.017
	0.2	0.036	0.036	0.036	0.036	0.036	0.035	0.035	0.035
	0.3	0.055	0.054	0.054	0.054	0.053	0.053	0.053	0.052
	0.4	**0.073**	**0.072**	**0.072**	**0.071**	**0.071**	**0.071**	**0.070**	**0.070**
	0.5	0.091	0.090	0.090	0.089	0.089	0.088	0.088	0.087
	0.6	0.109	0.108	0.108	0.107	0.107	0.106	0.105	0.105
	0.7	0.127	0.127	0.126	0.125	0.124	0.124	0.123	0.122
	0.8	0.146	0.145	0.144	0.143	0.142	0.141	0.140	0.140
	0.9	0.164	0.163	0.162	0.161	0.160	0.159	0.158	0.157
	1.0	0.182	0.181	0.180	0.179	0.178	0.177	0.175	0.174
	1.5	0.273	0.271	0.270	0.268	0.266	0.265	0.263	0.262
	2.0	0.364	0.362	0.359	0.357	0.355	0.353	0.351	0.349
	2.5	0.455	0.452	0.449	0.447	0.444	0.441	0.439	0.436
	3.0	0.546	0.542	0.539	0.536	0.533	0.530	0.527	0.523
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**

Analysis of the results shows that the value of the relative stiffness coefficient *B* decreases slightly with increase of the tool cutting edge angle. The change in the value of coefficient *B* under the effect of change in the feed and the machining depth a_p (from 0.5 to 3 mm), and κ_r = 45° and 90° are presented in Fig. 2 and Fig. 3.

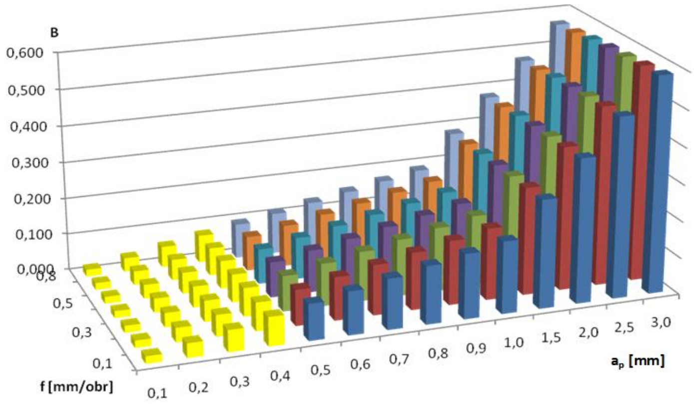


Fig. 2. Change of relative stiffness coefficient *B* under the effect of feed rate and machining depth at *L* = 500 [mm], *d* = 20 [mm], rotary speed of machined part n_p=1000 [rev/min], tool cutting edge angle κ_r= 45° (own study)

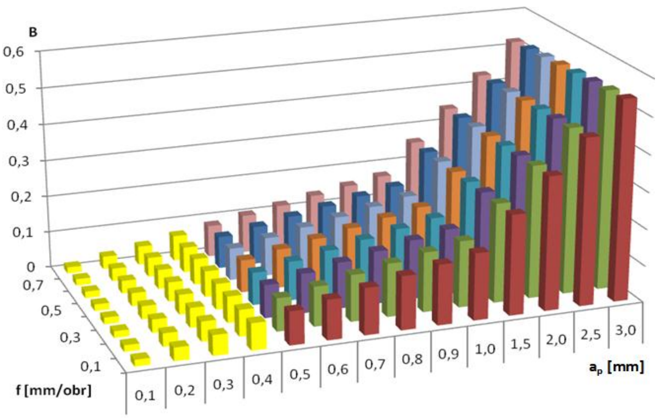


Fig. 3. Change of relative stiffness coefficient *B* under the effect of feed rate and machining depth at *L* = 500 [mm], *d* = 20 [mm], rotary speed of machined part n_p=1000 [rev/min], tool cutting edge angle κ_r= 90° (own study)

Based on the results of modelling (Fig. 2 and 3) we can conclude that the value of coefficient *B* increases in the function of change of machining depth and decreases slightly with increase of feed rate. With the change of tool cutting edge angle from κ_r=90° to κ_r=45° one can observe a slight increase in the value of coefficient *B* (for feed of 0.8 mm/rev and depth of 0.5 mm the difference is 0.008).

Therefore, we need to determine the operator transmittance in control, corresponding to these conditions, as well as the gain factors of the object and the retardation τ. Next, we determine the operator transmittance relative to the interference factor that assumes the form:

$$G_{v_f gi}(s) = \frac{\Delta F_i(s)}{\Delta v_f} = \frac{m_x h_{xx} G_\tau(s)}{s\left[(T_c s+1)(T_{us}^2 s^2 + 2\xi T_{us} s + 1) + B_1 G_\tau(s) + n_y h_{yy}\right]} \quad (1)$$

and the accurate value of coefficient B:

$$B = \frac{m_x h_{xx} + K_{\kappa_r} m_y h_{yy}}{1 + n_y h_{yy}} \quad (2)$$

After calculating the gain factors of the machining process, m_x, m_y and m_z, with the change of feed f and machining depth a_p (these values can be treated as interference), we determine the relative dynamic stiffness coefficient B.

We determine the coefficients of object strengthening K_{ox}, K_{oy}, K_{oz}, retardation τ and coefficients K_{bx}, K_{by}, K_{bz}. The relation to the approximated operator transmittance of the object in control assumes the form (3) (Cardi, Firpi, Bement & Liang, 2008; Świc & Taranenko, 2012; Wu & Liu, 1985):

$$G_o(s) = \frac{\Delta Y_o(s)}{\Delta v_f(s)} = \frac{K_o}{(T_1 s+1)\,(T_2 s+1)} \quad (3)$$

After the determination of the operator transmittance we determine coefficients $K_{a0x}, K_{a0y}, K_{a0z}$ and time constants T_1 and T_2.

Analysis of the simulation results presented in Tab. 2, Tab. 3 and in Fig. 2 and Fig. 3 indicates that in those cases the operator transmittances of the approximated object of control have different forms. In the case of coefficient $B \geq 0.077$ (the limit value of B, at which the form of the operator transmittance changes), with changes of feed f and of machining depth a_p (from 0.4 to 3 mm) the operator transmittance has a form conforming to relation (3). At $B < 0.077$ and changes of machining depth a_p (from 0.1 to 0,4 mm) and feed rate, the operator transmittance of the approximated object has the following form (Świć, Wołos, Zubrzycki, Opielak, Gola & Taranenko, 2014; Ratchev, Liu, Huang, & Becker, 2004)

$$G_o(s) = \frac{\Delta Y_o(s)}{\Delta v_f(s)} = \frac{K_o}{(T_1 s+1)}\,. \quad (4)$$

The results of calculations of retardation and time constants in relation to changes in rotary speed and machining depth, on the basis of which Fig. 4 was elaborated, are presented in Tab. 4.

Tab. 4. Change of retardation and time constants in relation to rotary speed and machining depth at length *L* = 500 [mm], diameter *d* = 20 [mm], tool cutting edge angle κ_r= 90° (own study)

a_p [mm]	0.4				0.6			
n [rpm]	500	1000	1500	2000	500	1000	1500	2000
Delay τ [s]	0.12	0.06	0.04	0.03	0.12	0.06	0.04	0.03
T_1 [s]	0.12	0.06	0.04	0.03	0.048	0.025	0.016	0.012
T_1 [s]	-	-	-	-	0.025	0.012	0.008	0.006
a_p [mm]	1.0				1.5			
n [rpm]	500	1000	1500	2000	500	1000	1500	2000
Delay τ [s]	0.12	0.06	0.04	0.03	0.12	0.06	0.04	0.03
T_1 [s]	0.062	0.031	0.021	0.016	0.076	0.038	0.025	0.019
T_1 [s]	0.019	0.01	0.006	0.005	0.016	0.008	0.005	0.004

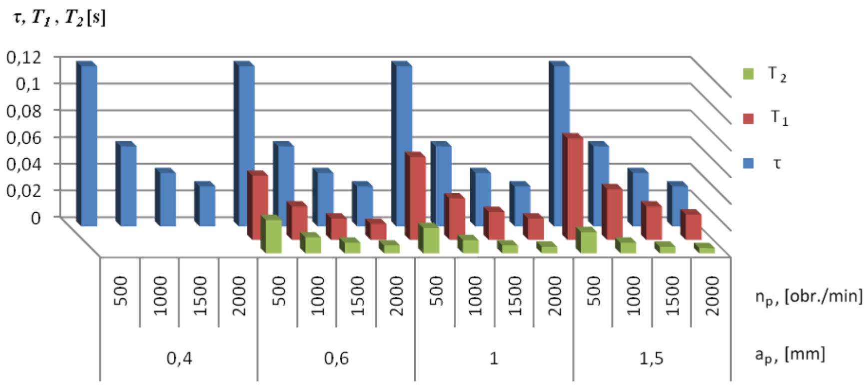


Fig. 4. Change of retardation and time constants in relation to rotary speed and machining depth at length *L* = 500 [mm], diameter *d* = 20 [mm] (own study)

Analysis of the data given in Tab. 4 and in Fig. 4 shows that the operator transmittance at $a_p \leq 0,4$ mm has a form conforming to relation (4).

3. ANALYSIS OF FREQUENCY AND TIME CHARACTERISTICS OF MODELS OF DYNAMIC SYSTEM OF MACHINING OF SHAFTS WITH LOW RIGIDITY

Transition processes in the dynamic system, at step-wise changes in feed rate and various depths of machining, are illustrated in Fig. 5 a. At machining depth of a_p=0.5 [mm] the approximated characteristic conforms to the operator transmittance (4), while at higher values of a_p it assumes the form of (3). This is due to the change of the operator transmittance at the value of coefficient $B \geq 0.077$, dependent on machining depth. The change of the operator transmittance affects both the approximated and the output characteristics. At higher depths of machining the time of response to unit step of feed increases.

This is due to the inclusion of the change of time constants T_1 and T_2. In Figs. 5–9 solid line represents the characteristics of the accurate model (output), and the broken line the characteristics of the approximated model.

The amplitude characteristics at various depths of machining are presented in Fig. 5 b. Irrespective of the depth of machining, at frequency 0 [rad/s] the curves assume the value of 10 dB. At the depth of a_p=0.5 [mm] the approximated characteristic assumes values higher than those of the initial characteristic, and in the case of greater depths of machining the approximated characteristic assumes lower values. The change of the operator transmittance affects only the approximated characteristic.

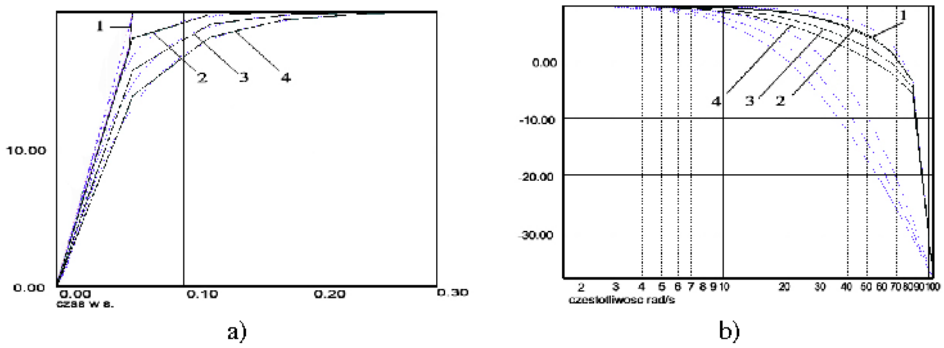


Fig. 5. Responses to unit step of feed rate – a); amplitude characteristic at depth of machining - b); where: 1 – 0.5 [mm], 2 – 0.6 [mm], 3 – 1.5 [mm], 4 – 2.5 [mm], feed 0.4 [mm/rev], speed 1000 [rev/min], angle κ_r= 90° and L = 500 [mm], diameter d = 20 [mm], tool cutting edge angle κ_r= 90° (own study)

The phase-frequency characteristics at various depths of machining are presented in Fig. 6. Irrespective of machining depth the output characteristic attains the value of –180°, at frequency of 100 [rad/s]. The approximated characteristic, in the range from 0° to –90°, assumes higher values at lower depths of machining. At machining depth of 0.6 and higher, after exceeding the value of –45°, the curve of the approximated characteristic diverges increasingly from the curve of the output characteristic.

Within the range from 0^0 to –90°, the approximated characteristic assumes somewhat higher values than the output characteristic in the case of machining depths smaller than 0.5 [mm]. At the angle of –90° the curves cross each other, and therefore the output characteristic assumes higher values with increase of the depth of machining.

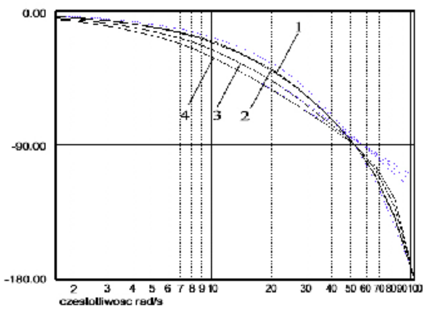


Fig. 6. Phase-frequency characteristics at various values of machining depth, where: 1 – 0.5 [mm], 2 – 0.6 [mm], 3 – 1.5 [mm], 4 – 2.5 [mm], feed 0.4 [mm/rev], speed 1000 [rev/min], angle κ_r = 90° (own study)

The responses to init step of feed are presented in Fig. 6. Irrespective of the feed change, the output characteristic attains the value of –180° at frequency of 100 [rad/s]. Within the range from 0° to –90° the approximated characteristic assumes higher values at smaller machining depths. Feed rate change has no pronounced effect on either the amplitude or the phase characteristics. The feed rate has a slight effect on the relative stiffness coefficient *B*, therefore the change in the shape of the curves of the initial and the approximated characteristics is barely observable. At machining depth of 0.6 mm and higher, and at angles greater than -45^0, the curve of the phase approximating characteristic becomes increasingly divergent from the initial characteristic.

In the case of machining depths smaller than 0.5 mm, in the range from 0° to –90° the approximating phase characteristic assumes values higher than those of the initial phase characteristic. At –90° the curves intersect, and thus the initial characteristic assumes higher values with increasing depth of machining.

Feed rate change has only a slight effect on both the amplitude and the phase-frequency characteristics of the dynamic system of machining of parts with low rigidity.

Feed rate change does not have any significant effect on the relative dynamic stiffness coefficient *B*, and therefore the transition, initial and approximated characteristics of the dynamic system remain practically unchanged.

Analysis of the transition characteristics of the system of machining of shafts with low rigidity at step-wise change of unit feeds and at various speeds of shaft rotation indicates that with increase of the rotation speed of the machined part the duration of the transition process is shortened: at 500 rev/min it equals 0.12 s, at 1000 rev/min – 0.06 s, and at 2000 rev/min – 0.03 s. At the same time, there is a reduction in the difference between the values of the characteristics.

The amplitude-frequency characteristics are presented in Fig. 7. The horizontal axis shows various values of part rotation speed: 500, 1000, 2000 rev/min. Analysis of the curves shows that at various speeds of the component the amplitude=frequency characteristics do not undergo any change in the case of the initial and the approximated models. In the case of the speed of 555 rev/min the curves assume the value of –33 dB, at the speed of 50 rad/s and analogously at 1000 rev/min – 100 rad/s; 2000 obr/min – 200 rad/s.

Irrespective of the change in the speed of rotation, the responses to unit change in feed remain unchanged, whereas there is a change in the response time. With increase of the speed of rotation there is an observable shortening of the response time that at 500 [rev/min] amounts to 0.12 [s], at 1000 [rev/min] – 0.06 [s], at 2000 [rev/min] – 0.03 [s], as well as a decrease of the difference between the values of the characteristics.

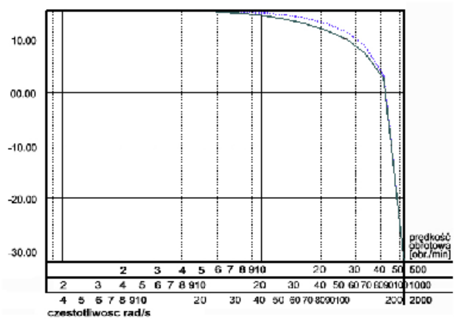


Fig. 7. Amplitude-frequency characteristics, where: a_p = 0.4 [mm], *f*=0.4 [mm/rev], speed: *n*=500, 1000, 2000 [rev/min], angle κ_r= 90° (own study)

The axis of abscissae represents three scales referenced to various speeds of rotation: 500, 1000 and 2000 [rev/min]. Analysis of the graph shows that the shape of the curve, both in the case of the approximated and the output characteristics is the same at each of the speeds of rotation. At the speed of 500 [rev/min] the curves attain the value of –33dB at frequency of 50 [rad/s], and analogously, in the remaining cases: 1000 [rev/min] – 100 [rad/s], 2000 [rev/min] – 200 [rad/s]. Also visible is a small difference in values between the curves of the approximated and the initial characteristics. In the case of the phase-frequency characteristic the speed of rotation has an effect only on the frequency [rad/s], and does not affect the shape of the curve.

The responses to unit step of feed are presented in Fig. 8. At machining depth of 0.6 [mm], the approximated characteristics assume the form of ma curve conforming to the operator transmittance (3). Irrespective of feed change, the time of response remains unchanged. With an increase of longitudinal feed rate one can observe an increasing divergence between the approximated characteristic and the initial characteristic. The initial value increases, e.g. at feed rate of 0.1 [mm/rev] it equals 5 [N], at feed rate of 0.4 [mm/rev] – 20 [N], and at feed of 0.8 [mm/rev] – 40 [N]. After 0.15 [s] the initial characteristic coincides with the approximated characteristic.

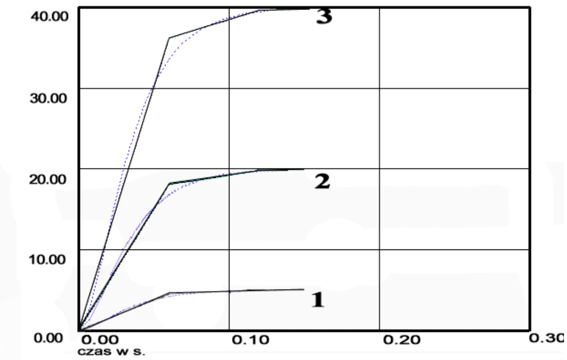
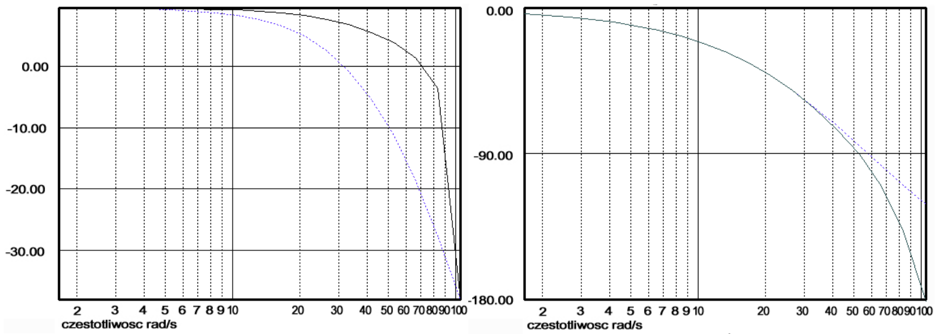


Fig. 8. Response to unit step of feed at machining depth of a_p=0.6 [mm] and feed rate [mm/rev.]: 1 – 0.1; 2 – 0.4; 3 – 0.8; speed 1000 [rev/min], angle κ_r =90° (own study)

The amplitude-frequency characteristic, at machining depth of 0.6 [mm], speed of 1000 [rev/min] and various feed rates, is presented in Fig. 9 a. Based on the Figure it can be concluded that feed change does not affect the shapes of the curves of both the initial and the approximated characteristics. However, there is a visible difference in values between the curves of the initial and the approximated characteristics. Feed rate change has only a slight effect on the relative stiffness coefficient *B*, therefore the change in the shape of the curves is practically non-observable and has no effect on the amplitude characteristic.

The phase-frequency characteristic, at machining depth of 0.6 [mm], speed of rotation of 500 [rev/min] and feed rates of 0.1, 0.4, 0.8 [mm/rev], is presented in Fig. 9 b. The phase-frequency characteristic of the initial model attains the value of −180° at frequency of 100 [rad/s]. After exceeding the value of −45° the curve of the approximated characteristic begins to diverge more and more from the curve of the initial characteristic. Feed rate change has only a slight effect on the relative stiffness coefficient *B*, therefore the change in the shape of the curves is practically non-observable and has no effect on the phase-frequency characteristic.



a) b)

Fig. 9. Amplitude-frequency characteristic at machining depth of 0.6 [mm], feed [mm/rev]: 1 – 0.1; 2 – 0.4; 3 – 0.8; speed 1000 [rev/min] – a); phase characteristic at machining depth of 0.6 [mm], feed [mm/rev]: 1 – 0.1; 2 – 0.4; 3 – 0.8; speed 1000 [rev/min], angle κ_r =90° – b) (own study)

In the determination of the time constants of the models, comparative analysis of the relative error δ (%), that does not exceed 15% (Świć, 2009), indicates that the developed mathematical models of turning of shafts with low rigidity are adequate to the actual process.

4. CONCLUSION

As a result of analysis of the frequency and time models of the dynamic system of machining of shafts with low rigidity, the possibility was found of their approximation within the range of actual frequencies of the dynamic system and of the control system of the operator transmittances of typical dynamic elements: integrating element and aperiodic element of the second or first order. It was determined that the gain factors and the time constants of the approximated models are subject to change, primarily with a change in the speed of rotation of the machined part and in the relative dynamic stiffness coefficient *B*, characterising the ratio of stiffness of the elastic system of the machine tool and the gain factors of the machining process. In the case of use of universal lathes, with relation to the changing conditions and parameters of machining the model parameters can vary within a broad range.

REFERENCES

Abakumov, A., Taranenko, V., & Zubrzycki, J. (2006). Modeling of characteristics of dynamic system of turning process for axial-symmetric shafts. *V-th International Congress "Mechanical Engineering Technologies' 06" (MT''06)* (pp.76-78). Varna, Bulgaria.

Cardi, A. A., Firpi, H. A., Bement, M. T., & Liang, S. Y. (2008). Workpiece dynamic analysis and prediction during chatter of turning process. *Mechanical Systems and Signal Processing*, *22*(6), 1481-1494. doi:10.1016/j.ymssp.2007.11.026

Gola, A. (2014). Economic Aspects of Manufacturing Systems Design. *Actual Problems of Economics*, *156*(6), 205-212.

Ratchev, S., Liu, S., Huang, W., & Becker, A. A. (2004). Milling error prediction and compensation in machining of low-rigidity parts. *International Journal of Machine Tools & Manufacture*, *44*(15), 1629-1641. doi:10.1016/j.ijmachtools.2004.06.001

Świc, A., & Mazurek, L. (2011). Modeling the reliability and efficiency of flexible synchronous production line. *Eksploatacja I Niezawodnosc-Maintenance and Reliability*, *4*, 41-48.

Świc, A., & Taranenko, W. (2012). Adaptive control of machining accuracy of axial-symmetrical low-rigidity parts in elastic-deformable state. *Eksploatacja I Niezawodnosc-Maintenance and Reliability*, *14*(3), 215-221.

Świć, A. (2009). *Technologia obróbki wałów o małej sztywności.* Lublin: Wydawnictwo Politechniki Lubelskiej.

Świć, A., Gola, A., & Wołos, D. (2014). A Method for Increasing the Economic Effectiveness of the Low Rigidity Shafts. *Actual Problems of Economics*, *161*(11), 469-477.

Świć, A., Wołos, D., Zubrzycki, J., Opielak, M., Gola, A., & Taranenko, V. (2014). Accuracy Control in the Machining of Low Rigidity Shafts. *Applied Mechanics and Materials*, *613,* 357-367.

Wu, D. W., & Liu, C. R. (1985). An analytical model of Cutting dynamics. Part 2. Verification. *Journal of Engineering for Industry*, *107*(2), 112. doi:10.1115/1.3185973