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# INCREASE OF NON-LINEAR DISTURBANCES DURING MACHINES OPERATIONS

**Key words:** machine dynamics, nonlinear disturbance, vibroacoustic diagnostics.

**Abstract:** The wearing degradation of the structure is accompanied by the increase of non-linear disturbances of the observed dynamic processes. The first of the mentioned effect can become the basis of the technical diagnostics as well as of the prognosis of the machine state change during its maintenance, if only it is possible to describe and measure such effect. In the further part of this study, it is shown that modern methods of signal analysis allow one to achieve such aims. The example of a gearbox diagnosis is considered.

### Wzrost nieliniowego zaburzenia w trakcie pracy maszyn

**Słowa kluczowe:** dynamika maszyn, zaburzenia nieliniowe, diagnostyka wibroakustyczna.

**Streszczenie:** Zużyciu eksploatacyjnemu maszyny towarzyszy zwykle nieliniowe zaburzenie obserwowanych procesów ynamicznych. To zjawisko może stanowić podstawę diagnostyki technicznej oraz prognozy pozwalającej na przewidywanie stanu maszyny w trakcie eksploatacji, jeżeli tylko potrafimy taki efekt zmierzyć i opisać. W artykule pokazano, że nowoczesne metody analizy sygnałów pozwalają na rozwiązanie tego zadania. Rozpatrzono przykład diagnostyki przekładni zębatej.

## Introduction

Half of the century of considering, in the scientific way, problems related to broadly understood machine maintenance have already passed. This approach, apart from the sudden development of sciences concerning fatigue processes, caused the creation of new fields and even scientific domains. One of them is technical diagnostics, while another one is "oriented maintenance construction." In this last case, it means that, as much as possible, wearing processes should be taken into account at the machine designing stage and, in consequence, the necessary maintenance strategy should be formulated as the optimisation task. It leads to a certain type of task integration. If at the designing stage, all changes observable during the machine service life were predictable and their diagnostic susceptibility and methodology were guaranteed, it would be an ideal situation. Of course, there are some classes of products where knowledge is highly advanced. It mainly concerns devices of large-lot production used for many years, which allows the processing of huge numbers of tests as well as such devices in which the technical progress is based on small modifications introduced every couple of years. However, even in this case certain 'surprises' can be expected concerning the durability changes of some elements or different diagnostic properties after a long operation time. There are several cases and even technical fields where this problem seems to be omitted. For instance, there are several machines for which the condition to be allowed for operations is meeting conditions determining the allowable level of noises and vibrations at the operator stand as well as in external fields. These properties are strictly controlled but only at the acceptance of the new device. What happens next is not controlled by either the producer or by the user, and the allowable standards are exceeded rather quickly. An evolution of the structure dynamic properties is very difficult for a detailed description. In many cases, the reasons for the changes occurring are not known. It should be added that, for many years, rather rough and often false hypotheses were used. Therefore, the problem of the evolution of dynamic properties of a machine during its service life still constitutes a research subject.

To present this problem more accurately one must reach the bases. The postulate that during the machine service life the energy dissipated in parasitic processes, which in the form presented by Natke and Cempel [1] became the base of the 'scientific' technical diagnostics, can be now assumed as a physical law. This law can be formulated more precisely as follows: There is a measure proportional to the dissipated energy, which during the machine operation, runs in accordance with the "life curve" (Fig. 1).

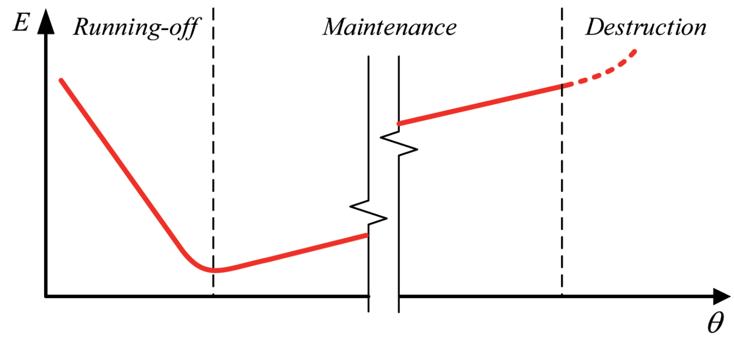


**Fig. 1. Classic 'life curve'**

The fact that physically such a curve exists does not mean that it is easy to find. At the beginning of symptom diagnostics development, especially vibroacoustic diagnostics, searching for such a curve is treated as a duty. The number of studies in which authors were out of touch with reality when looking for at least the central segment (monotonically growing trend) at all cost, reminds one of the Search for the Holy Grail. The lack of effects results from many reasons. Undoubtedly, methods of observation and analysis of signals were not perfect. However, the main reason is based on the fact that the general law of geriatrics concerns the total energy propagated on parasitic processes and not necessarily on the form, which is related to the observed symptom. The energy can be lost through increases of vibrations and noise, but also in thermal, flow, or electric processes, and the fractions of these processes do not need to be proportional (especially linearly proportional) to the total lost energy. However, the form of this propagation can be and usually is changing. A good example can be the rolling bearing defect, which is accompanied by visible changes in vibrations accelerations at the same level of vibrations (Fig. 2) [2].

Observation of this type effects caused the development of the "low-energy diagnostics" [3, 4], which was based on the postulate that, in the early phases of defects, the dissipation energy increase is so small that it is practically immeasurable, while its form changes so much that can become the basis of generating the observable measure. Such actions led to solving several practical tasks; however, the postulate of low-energy changes was justified only in relation to the selected propagation path. In accordance with the second law of thermodynamics, the change of the form of one dynamic process must be accompanied by the change of form of another process (often unknown or unobservable). How the observed forms of dynamic pathways change is seen in Figure 2, which illustrates changes in observed vibrations of roller bearings in a good state and in a defective state. The defect is accompanied by the transformation of the observed system, i.e. the system is or becomes non-linear at the moment the defest occurs. This fact, confirmed in several investigations of the dynamics of various machines and supporting structures [5, 6], leads to the formulation of the thesis preliminarily given in paper [2].

The wear degradation of the structure is accompanied by the increase of non-linear disturbances of the observed dynamic processes. These disturbances can lead to various effects. The stationary dynamic frequency is processed or the chaos effect occurs. This last effect is well observable in the case of oil bearings of various kinds of high-speed systems [7]. The first of the mentioned effects can become the basis of technical diagnostics as well as of the prognosis of the machine state change during its maintenance, if only it is possible to describe and measure this effect. In the further part of this study, we will demonstrate that modern methods of signal analysis allow one to achieve such aims.

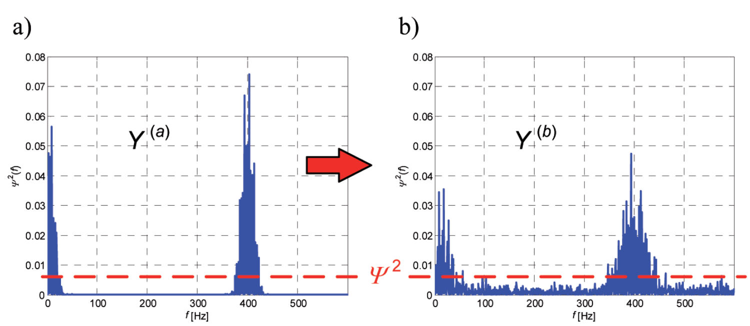


**Fig. 2. The change of the spectral density of vibration accelerations of a defective rolling bearing without increasing the vibration level**

## 1. Short theoretical justification

Let us start the considerations from the relation: *observed signal* ↔ *mathematical model*, defining of which allows one to formulate precisely the task of the dynamic model identification, followed by the diagnostics task (including the prognosis of the state change). Let us assume that, for the new machine, the interesting dynamic process can be described by the system of linear differential ordinary equations and the observed signal is a realisation of a certain random process. With such assumptions, we can write the relation in the time and frequency domains in the following way:

$$\underset{t}{S}\left\{x\left(t,\theta,n,r,\Omega\right)\right\}=\sum p_i\left(t\right)*h_i\left(t-\tau\right)+\varphi+\psi$$

$$\rho\overset{def}{=}\rho\left(.;.\right)$$

$$\underset{\omega}{S}\,\Im\,\underset{t}{S}\left(t,\theta,n,r,\Omega\right)=X\left(\omega,\theta,n,r,\Omega\right)= \\ =\sum P_i\left(\omega\right)\cdot H\left(\omega\right)+\Phi+\Psi \tag{1}$$

$$\rho\overset{def}{=}\rho\left(.;.\right)$$

where

- $\{X\}$ – observed process being the function of the dynamic observation time $t$, evolutional variable $\theta$, which can mean the maintenance time ($\theta >> t$) or the other observed symptom, the location of the measuring sensor $r$, the discriminant of the tested machine $n$ and conditions of the machine operation marked here conventionally as the rotational speed Ω.
- $\underset{t}{S}$ – operator of the selection and averaging in the time domain,
- $\underset{\omega}{S}$ – operator of the selection in the frequency domain (filtration),
- $\Im$ – Fourier transform,
- $p_i$, $P_i$ – excitations and their Fourier transforms,
- $h_i$ – impulse transfer functions being functions of the model parameters ($m_i$, $k_i$, $c_i$),
- $\varphi_i$, $\psi_i$, $\Phi_i$, $\Psi_i$ – errors of observation and modelling and their Fourier transforms, respectively,
- $\rho$ – properly defined metrics.

Generally left and right members of these equations are matrices (the system has "$n$" degrees of freedom). In such assumed notation of the parametric identification of the model, the following dependence is valid:

$$L-R\left(m_i,k_i,c_i,z_i\right)<\delta>\varphi+\psi\quad\Rightarrow\quad z_i \tag{2}$$

where
$L$ – left member of equation (1),
$R$ – right member of equation (1),
$z_i$ – deciding variable selected out of model parameters,
δ – allowable identification error.

The diagnostics task is as follows:

$$\left.\begin{array}{l}L\left(\theta_1,...\right)=R\left(...,z_1\right)\\ L\left(\theta_2,...\right)=R\left(...,z_2\right)\end{array}\right.\quad\Rightarrow\quad\Delta\theta=f\left(\Delta z\right) \tag{3}$$

where $z_i$ is the model state variable. In this last case, the model variable can be omitted when the state variable is known either from the direct observation or from the active diagnostic experiment [2].

In accordance with the thesis stated in the introduction, if the system during its evolution becomes non-linear, the right members of equation (1) become complicated. Non-linear equations do not allow falling into chaos in normal coordinates and their transmittances are mutually dependent. The possibility of solving such task results from the statement given in [2, 8, 9] on the basis of the analytically approximated methods of solving non-linear differential equations, stating that the near accurate solution can be obtained in the form of a certain series (differently organised in dependence on the applied method). This statement can be generalised to the statement of the existence of the operator transforming the solution of linear system into the solution of non-linear system in an additive way, as follows:

$$\bigwedge_{a_n\neq a_0}\sum P_i\,H_i+\Phi\ \rightarrow\ \sum P_i\,H_i+\Phi+\Phi^* \tag{4}$$

where Φ*c is a non-linear "correction" of the solution.

The mentioned statement quite efficiently allows one to model the non-linear disturbance (theoretically with an arbitrary accuracy), but a question of how to measure such a small disturbance still remains unanswered. The authors propose using the generalised notion of the ordinary coherence function. As it is known, the ordinary coherence function is the normalised characteristic of the system of one input and one output defined as the quotient of the system transmittance, calculated in two different ways.

$$H^{(1)}=\frac{G_{xy}}{G_{xx}}\qquad \gamma^2=\frac{H^{(1)}}{H^{(2)}}=\frac{\left|G_{xy}\right|^2}{G_{xx}G_{yy}}\qquad 0<\gamma^2<1\qquad H^{(2)}=\frac{G_{yy}}{G_{yx}} \tag{5}$$

where

- $G_{xy}$ – power cross spectrum density from $x$ to $y$,
- $G_{yx}$ – power cross spectrum density from $y$ to $x$,
- $G_{xx}$, $G_{yy}$ – power spectrum density,
- $\gamma^2$ – ordinary coherence function,
- $H^{(1)}$, $H^{(2)}$ – transmittances calculated in two different ways.

This function for the linear and undisturbed system is equal to unity at its input. A value $\gamma^2$ lower than unity indicates the disturbance of the input signal measurement, the output disturbance, or the system non-linearity. The filtration properties of the coherence function were shown in paper [10]. For the system with the disturbed output (Fig. 3), the following dependences occur:

$$H^{(1)} = H_{xy}$$

$$H^{(2)} = \frac{G_{yy}}{G_{yx}}\left(1+\frac{G_{zz}}{G_{yy}-G_{zz}}\right)H_{xy} \quad \Rightarrow \quad \left(1-\gamma^2\right)G_{yy} = G_{zz} \tag{6}$$

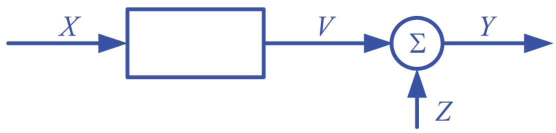


**Fig. 3. System with the output disturbed by the process *z***

These dependences allow one to divide the observed spectrum density into a part originated from the linear transformation of excitation and from the output disturbance. The authors' proposition presented for the first time in papers [11, 12] is based on treating the disturbance as the effect of the system non-linearity. It is worth mentioning that this is one of the proposed ways. As the model of non-linear effect, either the input disturbance or the input and output disturbances in the same time can be used. The choice of the description way should depend on details of the system modelling; therefore, this constitutes a separate problem undertaken in studies [5, 6]. In order to utilise filtrating the properties of the coherence function in the task of the development of dynamic properties of the system during time, we must use the fact that the observed process is a five-dimensional process (1) and define the relation *input* ↔ *output* not according to the spatial variable $r$ but to the evolutional variable. It means that the selected state described by variable $\theta_0$ can be the input, while the arbitrary state described by variable $\theta_i$ can be the output; however, under conditions that the remaining variables $r$, $n$, $\Omega$ will be determined. Especially important is determining the variable $\Omega$ describing machine operation conditions. This requires an accurate synchronisation of rotational speeds, since observations are not performed simultaneously, as in the case of defining input and output according to the spatial variable. Attention should be directed to one more property of the coherence function. When the analysis is reduced to the analysis of modules, the following dependence is valid:

$$G_{xx}\cdot|H|^2 = G_{yy} \tag{7}$$

in which the square of the transmittance function module, i.e. the coefficient of amplification has the scale function meaning. It is then easy to derive the statement that the linear rescaling of the input does not change the coherence function [11]. Thus, generalising further, the discrete spectrum density of the input can be substituted by an arbitrary spectrum of the same frequency distribution. The practical conclusion, resulting from the presented considerations, is that, instead of analysing the evolution of a non-linear disturbance, the evolution of the coherence function versus an arbitrary standard (corresponding the theoretical 'ideal' excitation), can be analysed.

The final confirmation of the considerations presented here can constitute only the example of the technical task.

## 2. Example of measuring the evolution of the non-linear disturbance of the defective vehicle gearbox

As the example, we will consider the task of finding the defect of the toothed wheel of the fifth gear of a significantly worn passenger car, on the basis of measuring vibrations of the motor head during operation on a road. This task seems to be extremely difficult. The effect of the tooth defect very weakly influences the general vibration level of the drive system. The measuring point is situated far from the source and in the vicinity of much stronger excitations and among traffic disturbances. The experiment was performed for four selected states:

– A maintenance state without defects ($x_{(0)}$),
– A maintenance state with a slightly defected tooth ($x_{(1)}$),
– A maintenance state with a strongly defected tooth ($x_{(2)}$), and
– A maintenance state with a new wheel (not run-in) ($x_{(n)}$).

The measurement results were synchronised by means of the signal resampling method (as to have the successive samples corresponding to the same angular displacements of the shaft) [13].

## 3. Analysis of the results

The spectrum densities of vibration accelerations performed with one-hertz resolution for the selected tests are presented in Figure 4.

Of course, in accordance with expectations, average values are identical. Thus, let us introduce the selection operator filtering out the spectrum in the direct vicinity of the tooth mesh frequency. The result is similar. Four spectral lines of a width of 1 Hz do not differ from each other in practice (Fig. 5).

In accordance with the conclusion from the theoretical considerations, we will now calculate the ordinary coherence function between the analysed pathways and the standard (arbitrary input), defined as the pulse train of 0.01 Hz resolution (Fig. 6) and of the amplitude value equal one.

Calculation results are presented in Figure 7. It is clearly seen that, along the increasing defect, the maximum value decreases and the 'broadening' of the

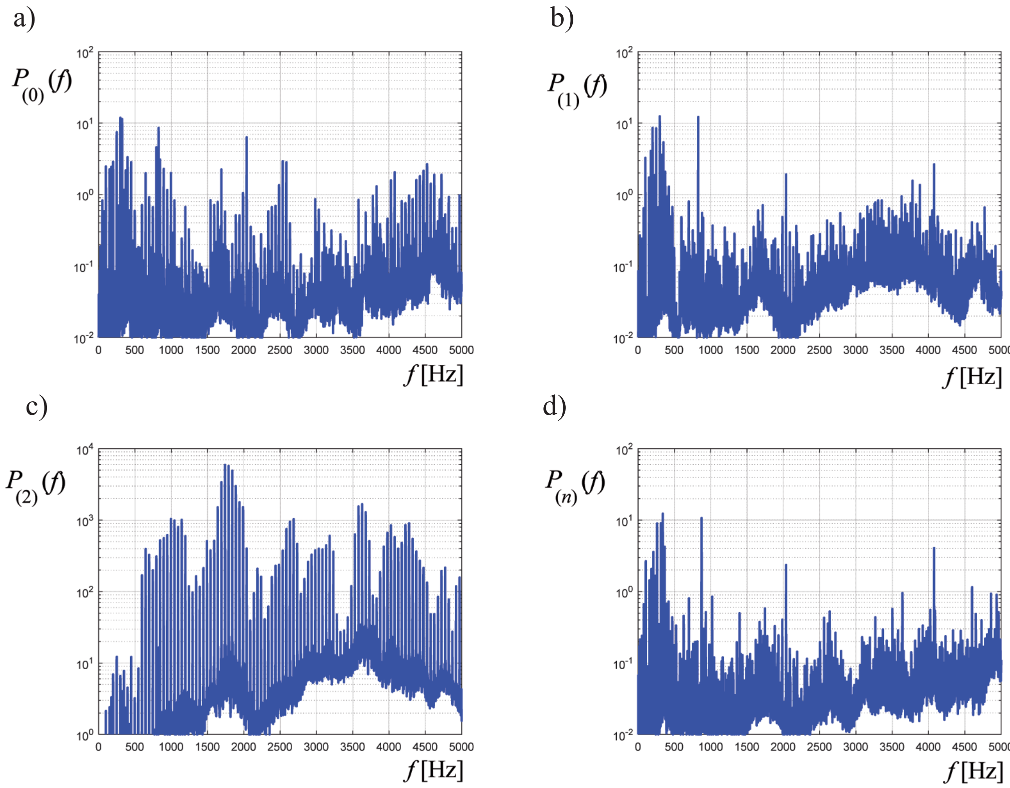


**Fig. 4.** **Power spectrum density for the selected tests: a) maintenance state without defects, b) maintenance state with a slightly defected tooth, c) maintenance state with a strongly defected, d) maintenance state with a new wheel (not run-in)**

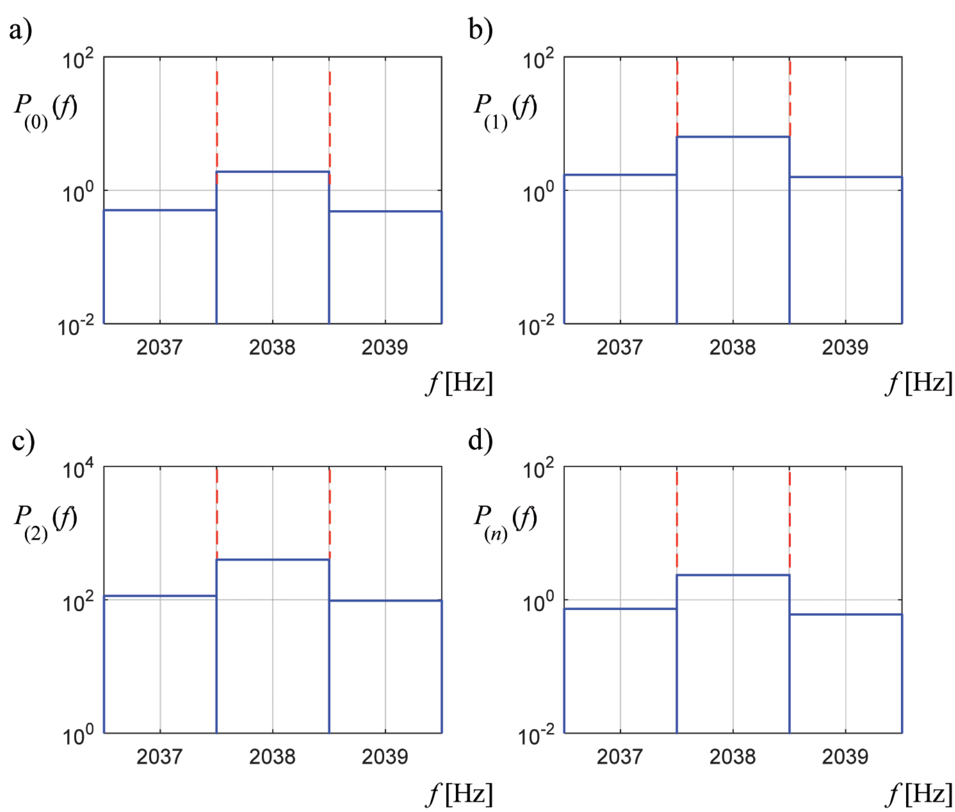


**Fig. 5.** **The filtered bands of power spectrum in the vicinity of the mesh frequency: a) maintenance state without defects, b) maintenance state with a slightly defected tooth, c) maintenance state with a strongly defected, d) maintenance state with a new wheel (not run-in)**

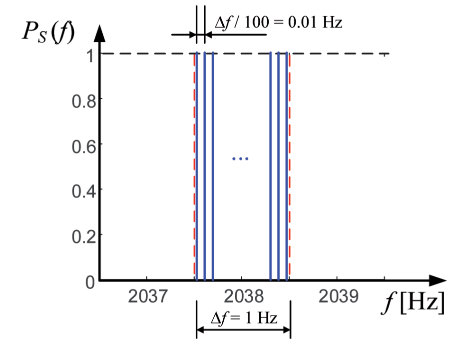


**Fig. 6. Arbitrary input (standard)**

tooth mesh frequency increases. This corresponds to the effect of the non-linear disturbance related to the defect propagation.

Utilising the standard function from the range ⟨0,1⟩ causes that the coherence function corresponds by its shape to the power spectral density function and requires only rescaling. It is also easy to introduce a simple measure, such as the basic harmonic amplitude value or the ratio of its value to the sum of side spectral lines or the ratio of integrals:

$$\frac{\int_{f_1}^{f_2} \gamma^2(f) + \int_{f_3}^{f_4} \gamma^2(f)}{\int_{f_2}^{f_3} \gamma^2(f)} = \overline{X} \qquad 8)$$

The path of this measure is shown in Figure 8.

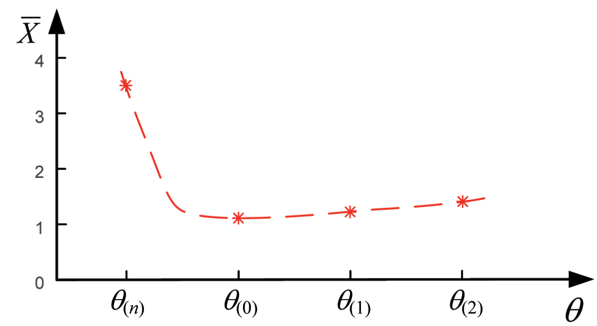


**Fig. 8. The path of the defined measure proportional to the defect propagation**

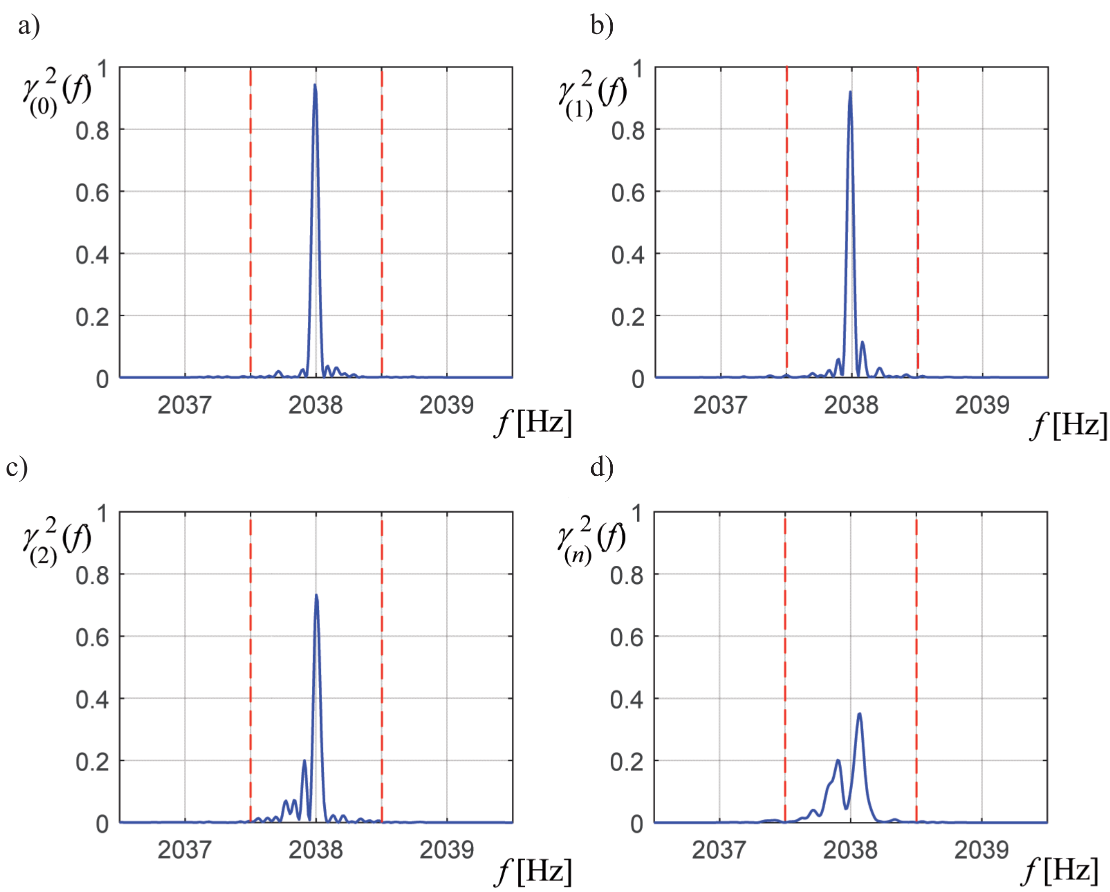


**Fig. 7. Coherence function in the selected band: a) maintenance state without defects, b) maintenance state with a slightly defected tooth, c) maintenance state with a strongly defected, d) maintenance state with a new wheel (not run-in)**

## Conclusions

The characteristic change of the coherence function and, in consequence, the change of the output spectrum density corresponds to the non-linear disturbance increase related to the small defect propagation. In spite of the unfavourable observation conditions mentioned above, the information of the system state was achieved by the proposed method. This fully confirms the efficiency of this tool, the generalised coherence analysis, as well as the thesis (stated at the beginning of this paper) on the increase of the non-linear disturbance within the function of the mechanical defect development. Whether this effect can be described by modelling still remains a question. The Ludwik Müller model [14], generally applied for descriptions of the toothed gear dynamics, represents entering the individual teeth into contact as a stiff body movement on the springs ‘palisade’, i.e. de facto assumes the excitation of vibrations by the pulse train of a constant time spacing. The discussed defect causes a minimal change of this time segment (shape change of the contact line) and – due to the defected tooth elasticity change – the amplitude change. In the model simplification to the pulse train „$\delta$” the defect can be presented visually, as in Figure 9.

This is because the ‘pulse’ model is identifiable in spite of minimal differences between time segments $\Delta t_i$.

The general conclusion, which can be drawn on the bases of the performed considerations, is presented below. Even small mechanical defects cause increases in non-linear disturbances accompanying the dynamic responses of machines. The effects of these disturbances are measurable and possible for modelling, which means that it can be utilised both in the technical diagnostics and in maintenance-oriented design, under condition of choosing the proper signal analysis technique. As an interesting side note, the fact that the properly defined measure is in agreement with the ‘theoretical life curve’, can be added.

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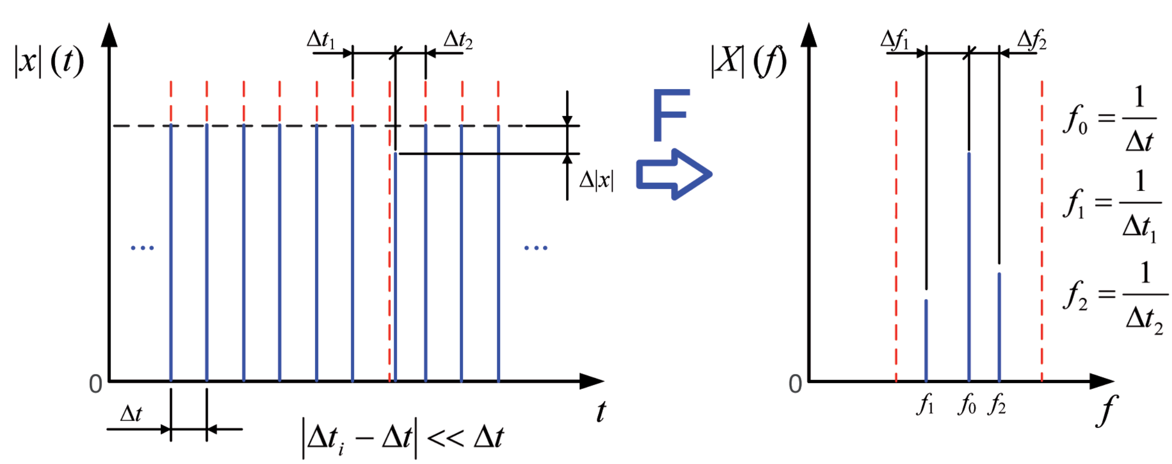


**Fig. 9. The effect of one tooth defect in the time and frequency domains**

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