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TENSILE FAULT DISLOCATION IN AN IRREGULAR-LAYERED ELASTIC HALF-SPACE

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In the present paper, an analytical solution for the static deformation of a two dimensional model consisting of an infinite homogeneous isotropic elastic layer of uniform thickness placed over an irregular isotropic elastic half-space due to movement of a long tensile fault has been obtained. The rectangular shaped irregularity is assumed to be present in the lower half-space and assuming that the fault lies in the elastic layer at a finite depth say '*h*' to the upper surface of the layer. For numerical computation, the expressions of displacements and stresses are calculated by using Sneddon's method and the effect of source depth and irregularity on the displacements and stresses has been investigated graphically.

Key words: deformation, tensile fault, layered half-space, rectangular irregularity.

1. Introduction

The main objectives of theoretical seismology are the modelling of the dynamics of an earthquake in the seismically active region of the earth. In this geological process, geologists have to indicate slow aseismic changes of stress and strain in such a region. Earthquakes generate faults created due to various types of movements and are different in geometrical shapes and sizes. A fault may be considered as a dislocation created by the fracture of the rock material separating two rock masses.

To study earth deformation due to fault problems, a two dimensional model is considered. It is an observational fact that the tensile fault model is the generalization of the shear fault model with the assumption that the slip vector can be arbitrarily oriented with respect to the fault and is not constrained to lie within the fault plane. Tensile earthquakes are dipping faults and occur in geothermal and volcanic areas which are rich in fluids. Tensile fault representation has several important geophysical applications, such as modeling of the deformation fields due to dyke injection in the volcanic region, mine collapse and fluid-driven cracks.

The static deformation of a layered or semi-infinite elastic media due to tensile and dip slip faults has been studied by many researchers. Bonafede and Rivalta [2] provide analytical solutions for the elementary tensile dislocation problem in an elastic medium composed of two welded half-spaces. Subsequently, Bonafede and Rivalta [3] derive the solution for the elastic field produced by a vertical tensile crack, opening under the effect of an assigned overpressure within it, in the proximity of the welded boundary between two half-spaces characterized by different elastic parameters. Singh and Garg [4] obtained the Airy stress function for an unbounded elastic medium. Using these results, Singh *et al.* [5] studied a problem of a very long dip slip fault in an isotropic elastic layer overlying a uniform isotropic elastic half-space and the integrals involved were calculated approximately by replacing the integrand by a finite sum of exponential terms (Ben-Menahem and Gillon, [1]). Singh and Singh [6] obtained the coseismic deformation of an elastic layer perfectly connected to a

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half-space that is elastic in dilation and viscoelastic in distortion caused by an infinitesimal thin long tensile fault present in the layer. Kumar *et al.* [7] derived the closed form expressions for the Airy stress function due to a long tensile fault of arbitrary dip source in two welded half-spaces. Bala and Rani [8] studied the deformation of isotropic, homogeneous, perfectly elastic half-spaces in welded contact to an orthotropic elastic half-space caused by a long buried dip-slip fault. Malik *et al.* [9] and Malik *et al.* [10] studied the deformation of isotropic, homogeneous, perfectly elastic half-spaces, respectively, in welded contact and smooth contact to an orthotropic elastic half-space caused by a vertical tensile fault.

Elastic problems with irregular boundaries have gained much importance in geophysics due to closeness to their natural environmental conditions. Their understanding leads to a better predictions for the seismic behaviour at continental margins and mountain roots. Therefore, it is interesting to study the static deformation in elastic models with irregular boundaries. A number of researchers have studied the problem of irregular boundaries (Ray and Singh [11], Selim [12], Madan *et al.* [13] and Madan *et al.* [14]) who studied static and quasi-static deformation on irregular interfacing boundary of two elastic half-space. Savita *et al.* [16] obtained shearing stress components at a point in a monoclinic elastic layer overlying an irregular monoclinic elastic half-space and gave the generalization of the results obtained in Savita *et al.* [15]. Both papers resulted that different sizes of rectangular irregularity produce significant variation in shearing stresses for different types of elastic materials.

In this paper, an attempt has been made to consider a crystal structure having a horizontal isotropic infinite elastic layer connected to an irregular boundary of an isotropic elastic half-space and to determine the plane-strain deformation due to a very long tensile fault of infinitesimal finite thickness that lies in elastic layer in the model. The effect of irregularity and source depth on displacements and stresses has been illustrated graphically.

2. Formulation of the problem

We consider a two phase model consisting of a homogeneous isotropic elastic layer of uniform thickness $'Y'$ placed over an irregular isotropic elastic half-space having a rectangular-shaped irregularity on its boundary surface. The origin of the (cartesian co-ordinate system (x,y,z) is placed at the free surface with the *y*-axis vertically downward. Let λ_i, μ_i; $i=1,2$ be Lame's constants for the elastic layer $0 \le y \le Y$ (Med.I) and elastic half-space $y > Y$ (Med.II) respectively. Let there be a long and infinitesimal thin strip of thickness $'ds'$ (as a line), a dislocation source parallel to the *x*-axis passing through the point $(0,h,0)$ of the elastic layer $(0 \le y \le Y)$ (Fig.1).

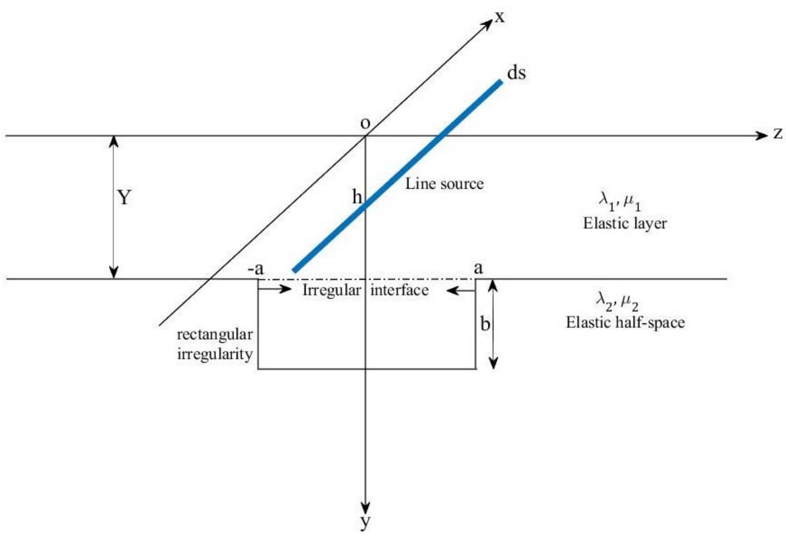

Fig.1. Layer of uniform thickness overlying an irregular elastic half-space with a line dislocation having width $'ds'$ and infinite length parallel to *x*-axis passing through the point $(0,h,0)$.

We have a mathematical representation of the rectangular irregularity:

$$y = \varepsilon f(z) = \begin{cases} b & : |z| \le a \\ 0 & : |z| > a \end{cases} \tag{2.1}$$

where

$$\varepsilon = \frac{b}{2a} << 1,$$

is the perturbation factor.
By applying the Fourier Transform technique to Eq.(2.1), we obtained

$$f(z) = sign(a - z) + sign(a + z). \tag{2.2}$$

3. Airy stress functions

The Airy stress function U_0 for a line source, given by Singh and Garg [4] as

$$U_0 = \int_0^\infty \left[\left(A_0 + B_0 k|y-h| \right) sin(kz) + \left(C_0 + D_0 k|y-h| \right) cos(kz) \right] k^{-1} e^{-k|y-h|} dk \tag{3.1}$$

where the source coefficients A_0, B_0, C_0 and D_0 are independent of k.
For a long dislocation source parallel to the x-axis passing through the point $(0,h,0)$ in the elastic layer $(0 \le y \le Y)$, the Airy stress functions U^I, U^{II} for the elastic layer and half-space, respectively, are of the form

$$U^I = U_0 + \int_0^\infty \left[\left(A_1 + B_1 ky \right) sin(kz) + \left(C_1 + D_1 ky \right) cos(kz) \right] k^{-1} e^{-ky} dk + \\ + \int_0^\infty \left[\left(A_2 + B_2 ky \right) sin(kz) + \left(C_2 + D_2 ky \right) cos(kz) \right] k^{-1} e^{ky} dk , \tag{3.2}$$

$$U^{II} = \int_0^\infty \left[\left(A_3 + B_3 ky \right) sin(kz) + \left(C_3 + D_3 ky \right) cos(kz) \right] k^{-1} e^{-ky} dk . \tag{3.3}$$

The unknowns A_1, B_1, etc. are to be determined by using the boundary conditions.

4. Stresses and displacements in terms of the Airy stress function

The stresses and displacements in the plane strain problem for an isotropic elastic medium can be solved in terms of the Airy stress functions (Sokolnikoff, [17], Section 71)

$$\sigma_{33}^n = \frac{\partial^2 U^n}{\partial y^2}, \quad \sigma_{32}^n = \frac{\partial^2 U^n}{\partial zy}, \quad \sigma_{22}^n = \frac{\partial^2 U^n}{\partial z^2}, \tag{4.1}$$

$$2\mu_n u_2^n = -\frac{\partial U^n}{\partial y} + \frac{1}{2\alpha_n}\int \left(\sigma_{22}^n + \sigma_{33}^n\right) dy\,,$$

(cont.4.1)

$$2\mu_n u_3^n = -\frac{\partial U^n}{\partial z} + \frac{1}{2\alpha_n}\int \left(\sigma_{22}^n + \sigma_{33}^n\right) dz\,,$$

$$\alpha_n = \frac{\lambda_n + \mu_n}{\lambda_n + 2\mu_n} = \frac{1}{2\left(1-\sigma_n\right)}\,. \tag{4.2}$$

In the above Eqs(4.1) and (4.2), $n = I, II$ and indicate the elastic layer and irregular elastic half-space respectively, $'\sigma'$ denotes the Poisson's ratio.

4.1. For the elastic layer

Using Eq.(3.2) in Eq.(4.1) and Eq.(4.2) for n=I, we obtain the stresses and displacements for elastic layer as

$$\begin{gathered}\sigma_{22}^I = -k\int\limits_0^\infty \Big[\left(A_0 + B_0 k|y-h|\right)e^{-k|y-h|} + \left(A_1 + B_1 ky\right)e^{-ky} + \left(A_2 + B_2 ky\right)e^{ky}\Big]\sin\left(kz\right)dk + \\ -k\int\limits_0^\infty \Big[\left(C_0 + D_0 k|y-h|\right)e^{-k|y-h|} + \left(C_1 + D_1 ky\right)e^{-ky} + \left(C_2 D_2 ky\right)e^{ky}\Big]\cos\left(kz\right)dk,\end{gathered} \tag{4.3}$$

$$\begin{gathered}\sigma_{32}^I = k\int\limits_0^\infty \Big\{\pm\left(A_0 + B_0\left(k|y-h|-1\right)\right)e^{-k|y-h|} + \left(A_1 + B_1\left(ky-1\right)\right)e^{-ky} + \\ -\left(A_2 + B_2\left(ky+1\right)\right)e^{ky}\Big\}\cos\left(kz\right)dk + k\int\limits_0^\infty \Big\{\mp\left(C_0 + D_0\left(k|y-h|-1\right)\right)e^{-k|y-h|} + \\ -\left(C_1 + D_1\left(ky-1\right)\right)e^{-ky} + \left(C_2 + D_2\left(ky+1\right)\right)e^{ky}\Big\}\sin\left(kz\right)dk,\end{gathered} \tag{4.4}$$

$$\begin{gathered}\sigma_{33}^I = k\int\limits_0^\infty \Big\{\left(A_0 + B_0\left(k|y-h|-2\right)\right)e^{-k|y-h|} + \left(A_1 + B_1\left(ky-2\right)\right)e^{-ky} + \\ -\left(A_2 + B_2\left(ky+2\right)\right)e^{ky}\Big\}\sin\left(kz\right)dk + k\int\limits_0^\infty \Big\{\left(C_0 + D_0\left(k|y-h|-2\right)\right)e^{-k|y-h|} + \\ +\left(C_1 + D_1\left(ky-2\right)\right)e^{-ky} + \left(C_2 + D_2\left(ky+2\right)\right)e^{ky}\Big\}\cos\left(kz\right)dk,\end{gathered} \tag{4.5}$$

$$\begin{gathered}2\mu_I u_2^I = \int\limits_0^\infty \Bigg[\pm\Bigg\{\Bigg(A_0 + B_0\Bigg(k|y-h| + \frac{1}{\alpha_I} - 1\Bigg)\Bigg) + \sin\left(kz\right) + \Bigg(C_0 + D_0\Bigg(k|y-h| + \frac{1}{\alpha_I} - 1\Bigg)\Bigg)\times \\ \times\cos\left(kz\right)\Bigg\}e^{-k|y-h|} + \Bigg\{\Bigg(A_1 + B_1\Bigg(ky + \frac{1}{\alpha_I} - 1\Bigg)\Bigg)\sin\left(kz\right) + \Bigg(C_1 + D_1\Bigg(ky + \frac{1}{\alpha_I} - 1\Bigg)\Bigg)\Bigg\}e^{-ky} + \\ -\Bigg\{\Bigg(A_2 + B_2\Bigg(ky + \frac{1}{\alpha_I} + 1\Bigg)\Bigg)\sin\left(kz\right) + \Bigg(C_2 + D_2\Bigg(ky + \frac{1}{\alpha_I} + 1\Bigg)\Bigg)\cos\left(kz\right)\Bigg\}e^{ky}\Bigg]dk,\end{gathered} \tag{4.6}$$

$$
\begin{aligned}
2\mu_1 u_3^I = \int_0^\infty \Bigg[& \left\{ \left(-A_0 + B_0 \left(k|y-h| - \frac{1}{\alpha_1} \right) \right) \cos(kz) + \right. \\
& + \left. \left(C_0 + D_0 \left(k|y-h| - \frac{1}{\alpha_1} \right) \right) \sin(kz) \right\} e^{-k|y-h|} + \\
& + \left\{ -\left(A_1 + B_1 \left(ky - \frac{1}{\alpha_1} \right) \right) \cos(kz) + \left(C_1 + D_1 \left(ky - \frac{1}{\alpha_1} \right) \right) \sin(kz) \right\} e^{-ky} + \\
& + \left\{ -\left(A_2 + B_2 \left(ky + \frac{1}{\alpha_1} \right) \right) \cos(kz) + \left(C_2 + D_2 \left(ky + \frac{1}{\alpha_1} \right) \right) \sin(kz) \right\} e^{ky} \Bigg] dk.
\end{aligned} \tag{4.7}
$$

4.2. For the elastic half-space

Using Eq.(3.3) in Eq.(4.1) and Eq.(4.2) for $n=II$, we obtain the stresses and displacements for elastic half-space as

$$\sigma_{22}^{II} = -\int_0^\infty k\left[(A_3 + B_3 ky) e^{-ky} \sin(kz) + (C_3 + D_3 ky) e^{-ky} \cos(kz) \right] dk\,, \tag{4.8}$$

$$\sigma_{32}^{II} = \int_0^\infty k\left[(A_3 + B_3 (ky-1)) e^{-ky} \cos(kz) - (C_3 + D_3 (ky-1)) e^{-ky} \sin(kz) \right] dk\,, \tag{4.9}$$

$$\sigma_{33}^{II} = \int_0^\infty k\left[(A_3 + B_3 (ky-2)) e^{-ky} \sin(kz) + (C_3 + D_3 (ky-2)) e^{-ky} \cos(kz) \right] dk\,, \tag{4.10}$$

$$
\begin{aligned}
2\mu_1 u_2^{II} = \int_0^\infty \Bigg[& \left(A_3 + B_3 \left(ky + \frac{1}{\alpha_1} - 1 \right) \right) e^{-ky} \sin(kz) dk + \\
& + \left(C_3 + D_3 \left(ky + \frac{1}{\alpha_1} - 1 \right) \right) e^{-ky} \cos(kz) \Bigg] dk,
\end{aligned} \tag{4.11}
$$

$$2\mu_1 u_3^{II} = \int_0^\infty \left[-\left(A_3 + B_3 \left(ky - \frac{1}{\alpha_1} \right) \right) e^{-ky} \cos(kz) + \left(C_3 - D_3 \left(ky - \frac{1}{\alpha_1} \right) \right) e^{-ky} \sin(kz) \right] dk. \tag{4.12}$$

5. Boundary conditions

The upper surface of the elastic layer $(i.e.\ y=0)$ is stress free and the mediums (I, II) are in welded contact yields the conditions

$$
\begin{aligned}
&\sigma_{22}^I = \sigma_{32}^I = 0 && \text{at } y=0\,, \\
&\sigma_{22}^I = \sigma_{22}^{II},\ \sigma_{32}^I = \sigma_{32}^{II} && \text{at } y=\varepsilon f(z)\,, \\
&u_2^I = u_2^{II},\ u_3^I = u_3^{II} && \text{at } y=\varepsilon f(z)\,.
\end{aligned} \tag{5.1}
$$

The coefficients A_0, B_0, C_0, D_0 have values A^-, B^-, C^-, D^- for $y<h$ and A^+, B^+, C^+, D^+ for $y>h$ respectively. Using the expressions for stresses and displacements from Eqs (4.3) to (4.12) in Eq.(5.1), we obtained two sets of equations (Appendix 1) and determined the twelve unknowns, namely $A_i, B_i, C_i, D_i; i=1,2,3$.

6. Solution of the problem

Inserting values of the unknowns A_i, B_i, C_i, D_i from (Appendix 1) in Eqs (4.3) to (4.7), we obtained the following expressions for the stresses and displacements in the elastic layer for $y=0$

$$\begin{aligned}
2\mu_1 u_2^I = \int_0^\infty \Bigg[\Bigg\{ \frac{1}{\alpha_1 \Delta_0} \Big[\Big\{ 2\delta^2 T_1 \left(2k\varepsilon f(z)+1\right)\left(A^+ - B^+ kh\right) + \\
+\left(T_4 + \delta^2 T_1 \left(4k^2 \varepsilon f(z)^2 - 1\right)\right) B^+ \Big\} e^{-k(2\varepsilon f(z)-h)} + \left\{2\delta T_3 \left(A^- + B^- kh\right) - \delta T_3 B^-\right\} e^{-kh} + \\
+2\delta T_2 \left(A^+ - B^+ kh\right) e^{-k(4\varepsilon f(z)-h)} - \delta T_2 B^- e^{-k(4\varepsilon f(z)+h)} + \\
+\delta^2 T_1 \left\{2\left(1-2k\varepsilon f(z)\right)\left(A^- + B^- kh\right) - 2B^-\right\} e^{-k(2\varepsilon f(z)+h)} \Big] - \frac{B^-}{\alpha_1} e^{-kh} \Bigg\} \sin(kz) + \\
+\Bigg\{ \frac{1}{\alpha_1 \Delta} \Big[\left\{2\delta T_3 \left(C^- + D^- kh\right) - \delta T_3 D^-\right\} e^{-kh} + 2\delta T_2 \left(C^+ - D^+ kh\right) e^{-k(4\varepsilon f(z)-h)} + \\
-\delta T_2 D^- e^{-k(4\varepsilon f(z)+h)} + \delta^2 T_1 \left\{2\left(1-2k\varepsilon f(z)\right)\left(C^- + D^- kh\right) - 2D^-\right\} e^{-k(2\varepsilon f(z)+h)} + \\
\left\{2\delta T_1 \left(2k\varepsilon f(z)+1\right)\left(C^+ - D^+ kh\right) + \left(T_4 + \delta^2 T_1 \left(4k^2 \varepsilon f(z)^2 - 1\right)\right) D^+\right\} e^{-k(2\varepsilon f(z)-h)} \Big] + \\
-\frac{D^-}{\alpha_1} e^{-kh} \Bigg\} \cos(kz) \Bigg] dk,
\end{aligned} \tag{6.1}$$

$$\begin{aligned}
2\mu_1 u_3^I = \int_0^\infty \Bigg[\Bigg\{ \frac{1}{\alpha_1 \Delta_0} \Big[\left\{2\delta T_3 \left(A^- + B^- kh\right) - \delta T_3 B^-\right\} e^{-kh} - 2\delta T_2 \left\{\left(A^+ - B^+ kh\right) - B^+\right\} \times \\
\times e^{-k(4\varepsilon f(z)-h)} + \delta T_2 B^- e^{-k(4\varepsilon f(z)+h)} + \delta^2 T_1 \Big\{ 2\left(1+2k\varepsilon f(z)\right)\left(C^- + D^- kh\right) + \\
-4k\varepsilon f(z) D^- \Big\} e^{-k(2\varepsilon f(z)+h)} + \Big\{ 2\delta^2 T_1 \left(2k\varepsilon f(z)-1\right)\left(A^+ - B^+ kh\right) + \\
+\left(T_4 + \delta^2 T_1 \left(4k^2 \varepsilon f(z)^2 - 4k\varepsilon f(z) + 1\right)\right) B^+ \Big\} e^{-k(2\varepsilon f(z)-h)} \Big] + \frac{B^-}{\alpha_1} e^{-kh} \Bigg\} \cos(kz) + \\
-\Bigg\{ \frac{1}{\alpha_1 \Delta} \Big[\left\{2\delta T_3 \left(C^- + D^- kh\right) - \delta T_3 D^-\right\} e^{-kh} - 2\delta T_2 \left\{C^+ - D^+ (kh+1)\right\} e^{-k(4\varepsilon f(z)-h)} + \\
+\delta T_2 D^- e^{-k(4\varepsilon f(z)+h)} + +\delta^2 T_1 \left\{2\left(1+2k\varepsilon f(z)\right)\left(C^- + D^- kh\right) - 4k\varepsilon f(z) D^-\right\} e^{-k(2\varepsilon f(z)+h)} + \\
+\Big\{ 2\delta^2 T_1 \left(2k\varepsilon f(z)-1\right)\left(C^+ - D^+ kh\right) + (T_4 + \delta^2 T_1 \left(4k^2 \varepsilon f(z)^2 - 4k\varepsilon f(z) + 1\right)) D^+ \Big\} \times \\
\times e^{-k(2\varepsilon f(z)-h)} \Big] + \frac{D^-}{\alpha_1} e^{-kh} \Bigg\} \sin(kz) \Bigg] dk.
\end{aligned} \tag{6.2}$$

The upper surface ($y=0$) of the elastic layer is stress free, so the stress components σ_{22}^{I} and σ_{32}^{I} vanish and σ_{33}^{I} will be

$$
\begin{aligned}
\sigma_{33}^{I} = -\int_{0}^{\infty}\Bigg[\Bigg\{\frac{2k}{\Delta_0}\Big[\Big\{2\delta T_3\left(A^- + B^- kh\right) - \delta T_3 B^-\Big\}e^{-kh} + \\
-2\delta T_2\Big\{\left(A^+ - B^+ kh\right) - B^+\Big\}e^{-k\left(4\varepsilon f(z)-h\right)} + \delta T_2 B^- e^{-k\left(4\varepsilon f(z)+h\right)} + \\
+\delta^2 T_1\Big\{2\left(1+2k\varepsilon f(z)\right)\left(A^- + B^- kh\right) - 4k\varepsilon f(z) B^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} + \\
+\Big\{2\delta^2 T_1\left(2k\varepsilon f(z)-1\right)\left(A^+ - B^+ kh\right) + \left(T_4 + \delta^2 T_1\left(4k^2\varepsilon f(z)^2 - 4k\varepsilon f(z) + 1\right)\right)B^+\Big\}\times \\
\times e^{-k\left(2\varepsilon f(z)-h\right)}\Big] + 2B^- e^{-kh}\Bigg\}\sin(kz) + \Bigg\{\frac{2k}{\Delta}\Big[\Big\{2\delta T_3\left(C^- + D^- kh\right) - \delta T_3 D^-\Big\}e^{-kh} + \\
-2\delta T_2\Big\{C^+ - D^+\left(kh+1\right)\Big\}e^{-k\left(4\varepsilon f(z)-h\right)} + \delta T_2 D^- e^{-k\left(4\varepsilon f(z)+h\right)} + \\
+\delta^2 T_1\Big\{2\left(1+2k\varepsilon f(z)\right)\left(C^- + D^- kh\right) - 4k\varepsilon f(z) D^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} + \\
+\Big\{2\delta^2 T_1\left(2k\varepsilon f(z)-1\right)\left(C^+ - D^+ kh\right) + (T_4 + \delta^2 T_1\left(4k^2\varepsilon f\left(z^2 - 4k\varepsilon f(z) + 1\right)\right)D^+\Big\}\times \\
\times e^{-k\left(2\varepsilon f(z)-h\right)}\Big] + 2D^- e^{-kh}\Bigg\}\cos(kz)\Bigg]dk.
\end{aligned}
\tag{6.3}
$$

7. Deformation due to a tensile fault

Now, we consider two tensile dislocations; one is a vertical tensile fault and the other is a horizontal tensile fault in the *z*-direction and *y*-direction, respectively.

7.1. Vertical tensile fault

The source coefficients will be

$$
\begin{aligned}
&A^- = A^+ = B^- = B^+ = 0, \\
&C^- = C^+ = D^- = D^+ = \frac{\alpha_1 \mu_1 b_0 ds}{\pi}
\end{aligned}
\tag{7.1}
$$

where 'b_0' is the displacement discontinuity in the direction of the normal to the fault having width 'ds'. By substituting the source coefficients in Eqs (6.1) to (6.3), the expressions for the stresses and displacements for an elastic layer are obtained as:

$$
\begin{aligned}
u_2^{I} = \frac{b_0 ds}{2\pi}\int_{0}^{\infty}\Bigg\{\frac{1}{\Delta_0}\Big(\delta T_3\left(1+2kh\right)e^{-kh} + 2\delta^2 T_1\left(kh - 2k\varepsilon f(z) - 2k^2 h\varepsilon f(z)\right)e^{-k\left(2\varepsilon f(z)+h\right)} + \\
+2\delta T_2\left(1-kh\right)e^{-k\left(4\varepsilon f(z)-h\right)} - \delta T_2 e^{-k\left(4\varepsilon f(z)+h\right)} + \Big[\delta^2 T_1\left(4k^2\varepsilon f(z)^2 + 4k\varepsilon f(z) + \right. \\
\left. -4k^2 h\varepsilon f(z) - 2kh + 1\right) + T_4\Big]e^{-k\left(2\varepsilon f(z)-h\right)}\Big) - e^{-kh}\Bigg\}\cos(kz)\,dk.
\end{aligned}
\tag{7.2}
$$

$$u_3^I = -\frac{b_0 ds}{2\pi}\int_0^\infty \left\{ \frac{1}{\Delta_0}\Big(\delta T_3 (1+2kh) e^{-kh} + 2\delta^2 T_1 \left(1+kh+2k^2 h\varepsilon f(z)\right) e^{-k(2\varepsilon f(z)+h)} + \right.$$
$$+\left[\delta^2 T_1 \left(4k^2 \varepsilon f(z)^2 + 2kh - 4k^2 h\varepsilon f(z) - 1\right) + T_4 \right] e^{-k(2\varepsilon f(z)-h)} + \tag{7.3}$$
$$\left. +\delta T_2 e^{-k(4\varepsilon f(z)+h)} + 2\delta T_2 kh e^{-k(4\varepsilon f(z)-h)} \Big) + e^{-kh} \right\} \sin(kz)\, dk,$$

$$\sigma_{33}^I = -\frac{\mu_1 b_0 ds}{\pi}\int_0^\infty \left\{ \frac{k}{\Delta_0}\Big(\delta T_3 (1+2kh) e^{-kh} + 2\delta^2 T_1 \left(1+kh+2k^2 h\varepsilon f(z)\right) e^{-k(2\varepsilon f(z)+h)} + \right.$$
$$+\left[\delta^2 T_1 \left(4k^2 \varepsilon f(z)^2 + 2kh - 4k^2 h\varepsilon f(z) - 1\right) + T_4 \right] e^{-k(2\varepsilon f(z)-h)} + \tag{7.4}$$
$$\left. +\delta T_2 e^{-k(4\varepsilon f(z)+h)} + 2\delta T_2 kh e^{-k(4\varepsilon f(z)-h)} \Big) + e^{-kh} \right\} \cos(kz)\, dk.$$

7.2. Horizontal tensile fault

The source coefficients will be

$$A^- = A^+ = B^- = B^+ = 0,$$
$$C^- = C^+ = \frac{\alpha_1 \mu_1 b_0 ds}{\pi}, \tag{7.5}$$
$$D^- = D^+ = -\frac{\alpha_1 \mu_1 b_0 ds}{\pi}.$$

On substituting the source coefficients in Eqs (6.1) to (6.3), the expressions for the stresses and displacements for elastic half-space are obtained as

$$u_2^I = \frac{b_0 ds}{2\pi}\int_0^\infty \left\{ \frac{1}{\Delta_0}\Big(\delta T_3 (3-2kh) e^{-kh} + 2\delta T_2 (1+kh) e^{-k(4\varepsilon f(z)-h)} + \right.$$
$$+2\delta^2 T_1 \left(2-kh-2k\varepsilon f(z)+2k^2 h\varepsilon f(z)\right) e^{-k(2\varepsilon f(z)+h)} + \tag{7.6}$$
$$+\left[\delta^2 T_1 \left(4k\varepsilon f(z) + 4k^2 h\varepsilon f(z) + 2kh - 4k^2 \varepsilon f(z)^2 + 3\right) - T_4 \right]$$
$$\left. e^{-k(2\varepsilon f(z)-h)} + \delta T_2 e^{-k(4\varepsilon f(z)+h)} \Big) + e^{-kh} \right\} \cos(kz)\, dk,$$

$$u_3^I = -\frac{b_0 ds}{2\pi}\int_0^\infty \left\{ \frac{1}{\Delta_0}\Big(\delta T_3\left(3-2kh\right)e^{-kh} - 2\delta T_2\left(2+kh\right)e^{-k\left(4\varepsilon f(z)-h\right)} + \right.$$
$$+2\delta^2 T_1\left(1-kh+4k\varepsilon f\left(z\right)-2k^2 h\varepsilon f\left(z\right)\right)e^{-k\left(2\varepsilon f(z)+h\right)} +$$
$$+\left[\delta^2 T_1\left(8k\varepsilon f\left(z\right)+4k^2 h\varepsilon f\left(z\right)-2kh-4k^2\varepsilon f(z)^2-3\right)-T_4\right] \tag{7.7}$$
$$\left. e^{-k\left(2\varepsilon f(z)-h\right)} - \delta T_2 e^{-k\left(4\varepsilon f(z)+h\right)}\Big) - e^{-kh}\right\}\sin\left(kz\right)dk,$$

$$\sigma_{33}^I = -\frac{\mu_1 b_0 ds}{\pi}\int_0^\infty \left\{ \frac{k}{\Delta_0}\Big(\delta T_3\left(3-2kh\right)e^{-kh} - 2\delta T_2\left(2+kh\right)e^{-k\left(4\varepsilon f(z)-h\right)} + \right.$$
$$+2\delta^2 T_1\left(1-kh+4k\varepsilon f\left(z\right)-2k^2 h\varepsilon f\left(z\right)\right)e^{-k\left(2\varepsilon f(z)+h\right)} +$$
$$+\left[\delta^2 T_1\left(8k\varepsilon f\left(z\right)+4k^2 h\varepsilon f\left(z\right)-2kh-4k^2\varepsilon f(z)^2-3\right)-T_4\right] \tag{7.8}$$
$$\left. e^{-k\left(2\varepsilon f(z)-h\right)} - \delta T_2 e^{-k\left(4\varepsilon f(z)+h\right)}\Big) - e^{-kh}\right\}\cos\left(kz\right)dk.$$

8. Special case

By substituting $\alpha_1 = \alpha_2$, $\mu_1 = \mu_2$ in Eqs (7.2) to (7.3), we will obtain the expressions for the displacement components of a uniform half-space due to a vertical tensile fault as follows

$$u_2 = \frac{-bds}{\pi}\int_0^\infty \left(1+kh\right)e^{-kh}\cos\left(ky\right)dk = \frac{-bds}{\pi}\left(\frac{2h^3}{\left(h^2+z^2\right)^2}\right), \tag{8.1}$$

$$u_3 = \frac{bds}{\pi}\int_0^\infty kh\, e^{-kh}\sin\left(ky\right)dk = \frac{bds}{\pi}\left(\frac{2h^3 z}{\left(h^2+z^2\right)^2}\right). \tag{8.2}$$

Similarly, substituting $\alpha_1 = \alpha_2$, $\mu_1 = \mu_2$ in Eqs (7.6) to (7.7), the displacement components due to a horizontal tensile fault for a uniform half-space are

$$u_2 = \frac{-bds}{\pi}\int_0^\infty \left(1-kh\right)e^{-kh}\cos\left(kz\right)dk = \frac{-bds}{\pi}\left(\frac{2hz^2}{\left(h^2+z^2\right)^2}\right), \tag{8.3}$$

$$u_3 = \frac{bds}{\pi}\int_0^\infty \left(2-kh\right)e^{-kh}\sin\left(kz\right)dk = \frac{bds}{\pi}\left(\frac{2z^3}{\left(h^2+z^2\right)^2}\right). \tag{8.4}$$

9. Numerical results and discussion

The expressions in Eqs (7.2-7.4) and Eqs (7.6-7.8) are of the form

$$\int_0^\infty \frac{G}{\Delta_0} e^{-kp} k^q \begin{pmatrix} \cos kz \\ \sin kz \end{pmatrix} dk,$$

$$q = 0, 1, 2; \quad G = -\delta T_3; \quad p = h,\ 2\varepsilon f(z) \pm h,\ 4\varepsilon f(z) \pm h. \tag{9.1}$$

The presence of the factor $\frac{1}{\Delta_0}$ in the integrand makes integration difficult to solve analytically. So, for numerical computation, we are evaluating approximately these integrands by replacing a finite sum of exponential terms with the help of Sneddon's method. Following Singh *et al.* [5], we use the approximation

$$\frac{G}{\Delta} \approx 1 - \left(A + Bk^2 \left(\varepsilon f(z)\right)^2\right) e^{-2k\varepsilon f(z)} + \left(C + \alpha' k^n \left(\varepsilon f(z)\right)^n\right) e^{-\beta' kh} \tag{9.2}$$

where

$$A = \frac{T_4 + \delta^2 T_1}{\delta T_3},\ B = \frac{4\delta T_1}{T_3},\ C = \frac{A^2 + D(A-1)}{1 + A + D},\ D = T_2 / T_3,\ n = 1,2,3...; \tag{9.3}$$

$\alpha',\ \beta'(> 2)$ are chosen in such a way to ensure a best satisfactory fit in the least square sense. The constants α', β' and $'n'$ are to be re-evaluated for each set of values of the parameters $\sigma_1,\ \sigma_2$ and ν. Using the approximation Eq.(9.2), the integral Eq.(9.1) can be expressed as a linear combination of known integrals. Ben-Menahem and Gillon [1] found that for idealistic earth models $n = 2$ yields a satisfactory result and also derived the values of elastic parameters $(\nu, \sigma_1,\ \sigma_2, \alpha', \beta')$ for two different earth crustal models shown in Tab.1.

Table 1. Parameters for two different earth crustal structures.

$\nu = \frac{\mu_2}{\mu_1}$	σ_1	σ_2	α'	β'
1.76 (oceanic)	*0.27*	*0.27*	*0.438716*	*3.31986*
2.22 (continental)	*0.27*	*0.27*	*0.703604*	*3.22888*

We study the effect of irregularity and source depths on the displacements (separately on horizontal and vertical displacements) and stresses with the horizontal distance $'z'$, caused by a tensile (horizontal as well as vertical) faults.

Figures 2 (a-d) and 3 (a-d) shows the variation of the dimensionless horizontal displacement (u_3) and vertical displacement (u_2), respectively, for an elastic layer overlying an irregular elastic half-space with dimensionless horizontal distance $'z'$ due to a vertical tensile fault for rigidity ratio $\nu = 1.76$ (oceanic crustal model) and $\nu = 2.22$ (continental earth crustal model) at three different source depths, i.e. $h = 0.25,\ h = 0.5,\ h = 0.75$. To compare the effect of rectangular irregularity on the displacement components, Figs 2(a, c) and 3(a, c) are plotted by assuming irregularity on the interaction boundary surface connecting elastic layer to elastic half space while Figs 2(b, d) and 3(b, d) are plotted in the absence of irregularity. Irregularity presented in the lower half-space is of length $'2a'$ and depth $'b'$. Figures 2(b, d) and 3(b, d) plotted in the absence of irregularity describe only the influence of source depth on the displacements field while Figs 2(a, c) and 3(a, c) describe the effect of variation in fault depth and rectangular irregularity simultaniously. From the comparision of Figs 2(a, c) respectively with 2(b, d) and 3(a, c) respectively with 3(b, d), it can be observed that the effect of irregularity makes a significant change on the displacement components $'u_2'$ and $'u_3'$ for each different source depth. In Figures 2(b, d) and 3(b, d), displacement components $'u_2'$ and $'u_3'$ for each different source depth $'h'$ are in same pattern and the magnitudes of horizontal and vertical displacements decreases as fault depth increase while displacement components due to the presence of irregularity have two points of discontinuity at $z = -0.5$ and $z = 0.5$ and changes its magnitudes on the horizontal distance $-0.5 < z < 0.5$ (as the length of rectangular irregularity) for each different fault depth. Moreover, effect of irregularity on the displacement components are most influential for higher value of fault depth $h = 0.75$ and also changes the pattren of displacement curves on the irregular horizontal distance $-0.5 < z < 0.5$. In Figures 2(a-d) we observe that, horizontal displacement (u_3) is zero at $z = 0$ for each different source depth. All horizontal and vertical displacements tends to zero as $z \to \infty$.

Figure 4 (a-d) shows the variation of dimensionless stress component (σ_{33}) for an elastic layer overlying the irregular elastic half-space with dimensionless horizontal distance $'z'$ due to a vertical tensile fault with two different rigidity ratios $\nu = 1.76$ and $\nu = 2.22$ at three different source depths, i.e. $h = 0.25$, 0.5 and 0.75. Figures 4(a, c) displays stress component (σ_{33}) due to irregularity on the boundary surface while Figs 4(b, d) in the absence of irregularity. In these figures we obsearve that stress component σ_{33} in the absence of irregularity decreases in magnitude as source depth increase. In Figures 4(a, c), due to the effect of irregularity stress components (σ_{33}) changes its magnitudes in the interval $-0.5 < z < 0.5$ (length of irregularity) for each different fault depth and having two points of discontinuity at $z = -0.5$ and $z = 0.5$ and for $'h = 0.75'$ stress component decrease very rapidly at $z = 0$ on the horizontal axis.

Figures 5 (a-d) and 6 (a-d) show the variation of the dimensionless horizontal displacement (u_3) and vertical displacement (u_2); respectively, for an elastic layer overlying an irregular elastic half-space with dimensionless horizontal distance $'z'$ due to a horizontal tensile fault. These figures are for the rigidity ratio $\nu = 1.76$ and $\nu = 2.22$ for three different source depths, i.e. $h = 0.25$, 0.5 and 0.75. Figures 5(a, c) and 6(a, c) are plotted in the presence of irregularity while Figs 5(b, d) and 6(b, d) are in the absence of irregularity. In these figures we observe that the source depth has a significant effect on the horizontal displacement $'u_3'$ and vertical displacement $'u_2'$. The magnitude of the displacement components decreases as source depth increase. In Figures 5(a-d) horizontal displacement $'u_3'$ for different source depth is zero at $z = 0$. Both horizontal and vertical displacements tends to zero as $z \to \infty$. Also, we observe that, the effect of irregularity in Figs 5(a, c) and 6(a, c) is insignificant or a little influence are experienced in variations of displacements on the horizontal distance $'z'$ within the range $-0.5 < z < 0.5$.

Figures 7 (a-d) shows the variation of the dimensionless stress component (σ_{33}) for an elastic layer overlying an irregular elastic half-space with dimensionless horizontal distance $'z'$ caused by a horizontal tensile fault with rigidity ratio $\nu = 1.76$ and $\nu = 2.22$ at three different source depths. Variations of the stress component σ_{33} decrease as the source depth increases. Also, we observe that the effect of irregularity on stress components due to a horizontal tensile fault, for each different source depths is insignificant. Figures 5-7 show the variations of displacements and stresses due to source dislocation situated in continental earth model having higher frequency compared to dislocations situated in the oceanic earth model.

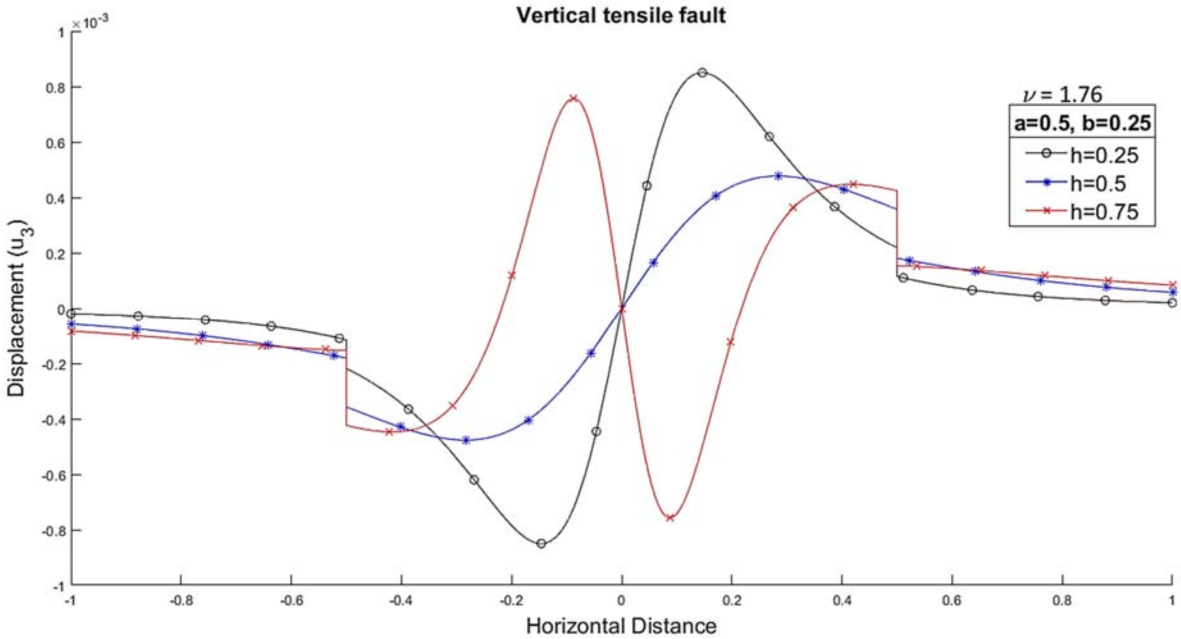

Fig.2a. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the presence a of irregularity for $\nu = 1.76$ due to a vertical tensile fault at three different source depths.

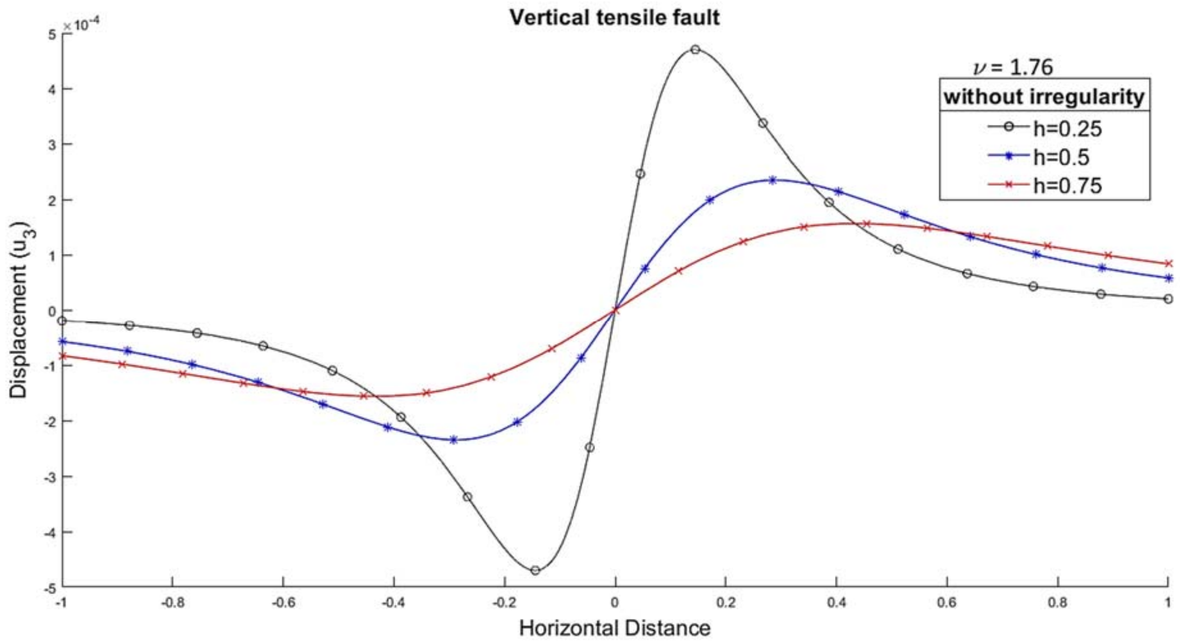

Fig.2b. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the absence of irregularity for $\nu = 1.76$ due to a vertical tensile fault at three different source depths.

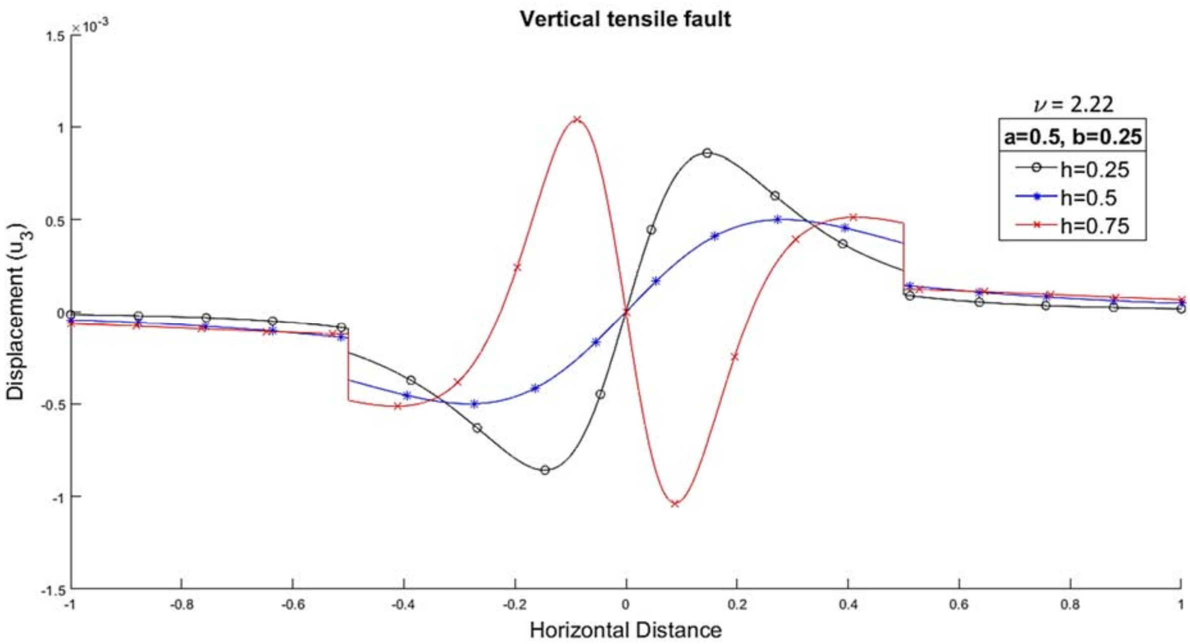

Fig.2c. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the presence of irregularity for $\nu = 2.22$ due to a vertical tensile fault at three different source depths.

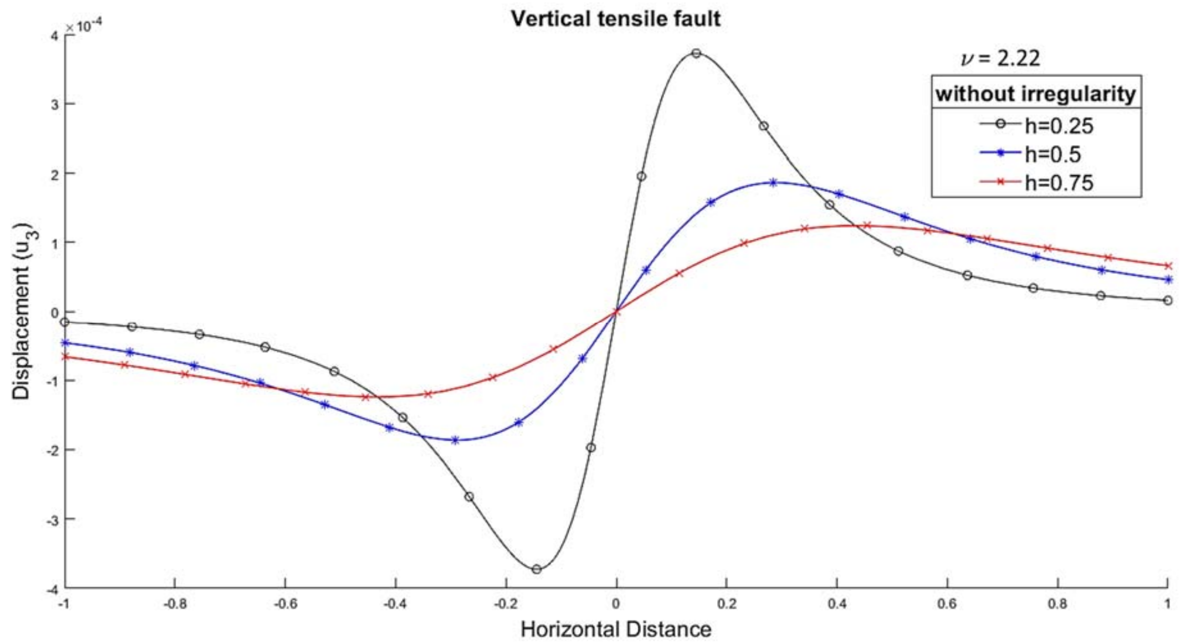

Fig.2d. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the absence of irregularity for $\nu = 2.22$ due to a vertical tensile fault at three different source depths.

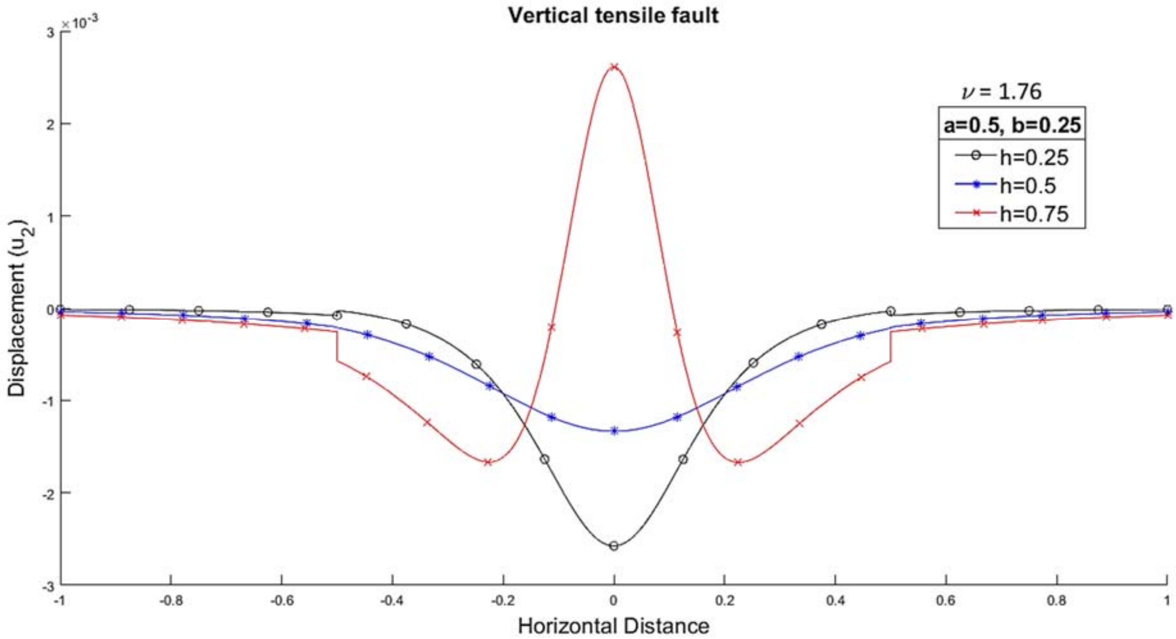

Fig.3a. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the presence of irregularity for $\nu = 1.76$ due to a vertical tensile fault at three different source depths.

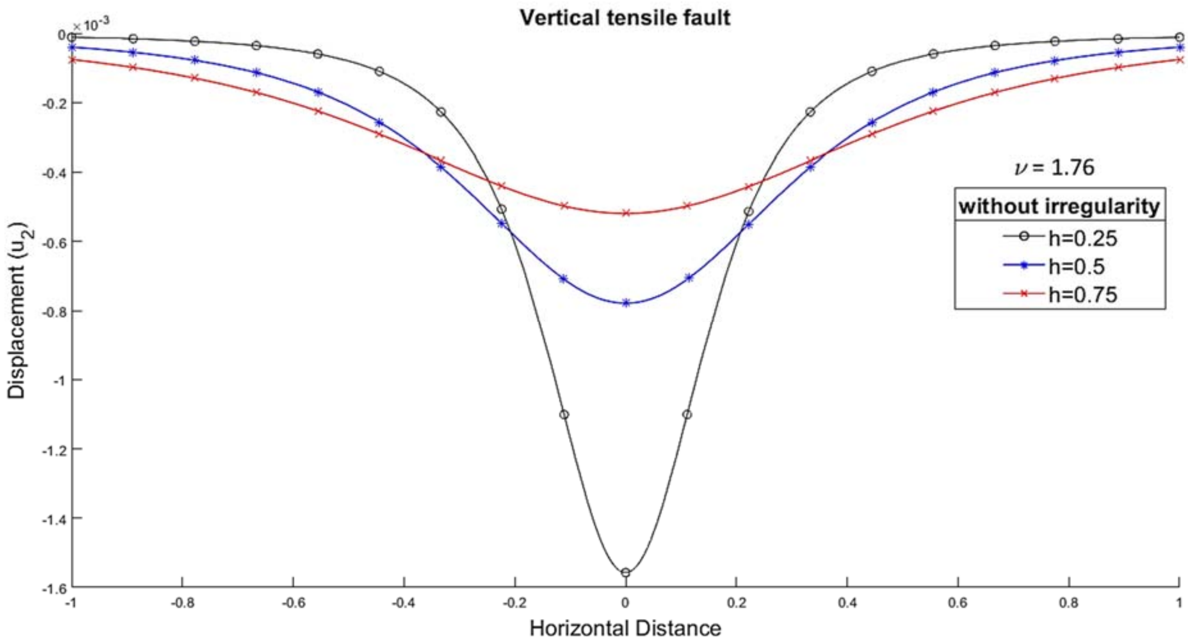

Fig.3b. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the absence of irregularity for $\nu = 1.76$ due to a vertical tensile fault at three different source depths.

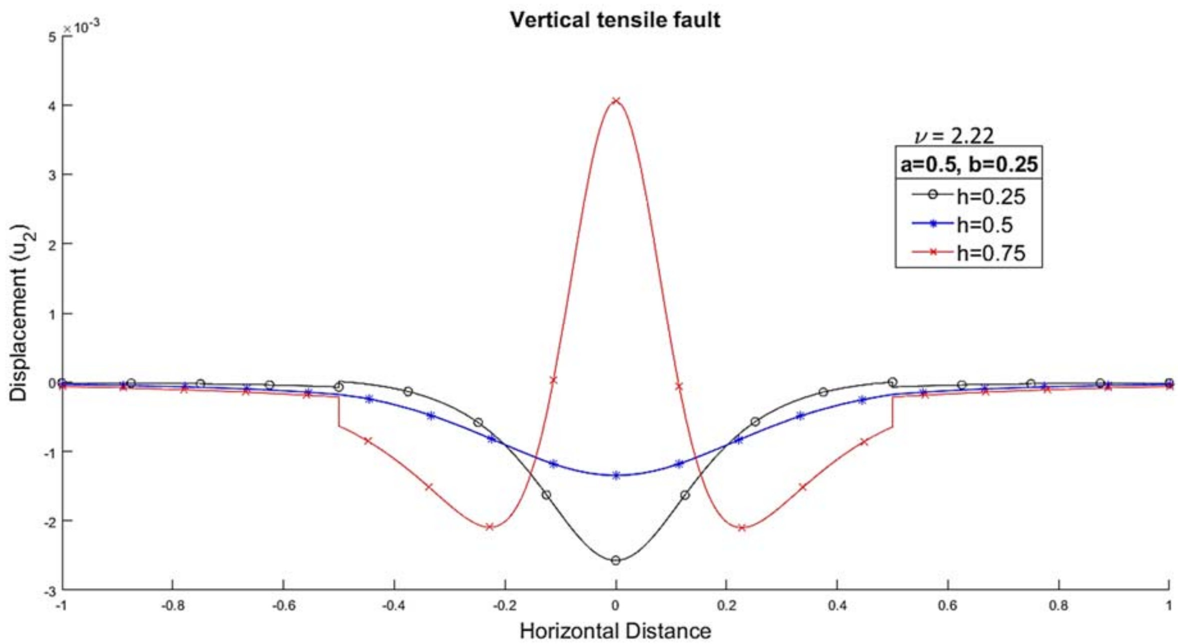

Fig.3c. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the presence of irregularity for $\nu = 2.22$ due to a vertical tensile fault at three different source depths.

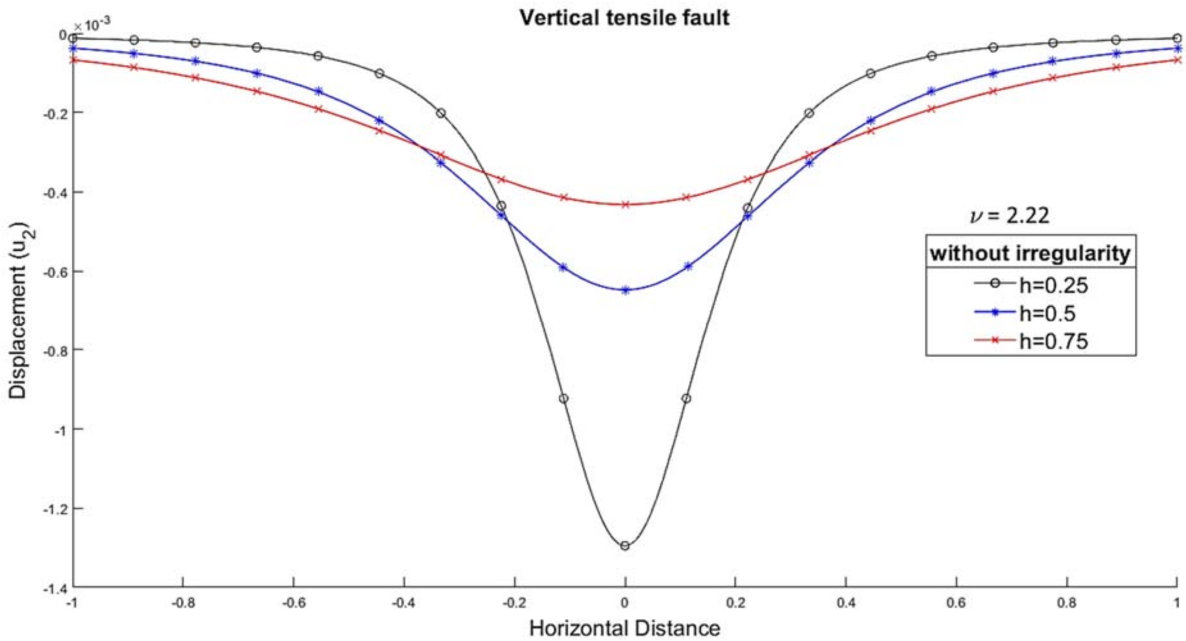

Fig.3d. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the absence of irregularity for $\nu = 2.22$ due to a vertical tensile fault at three different source depths.

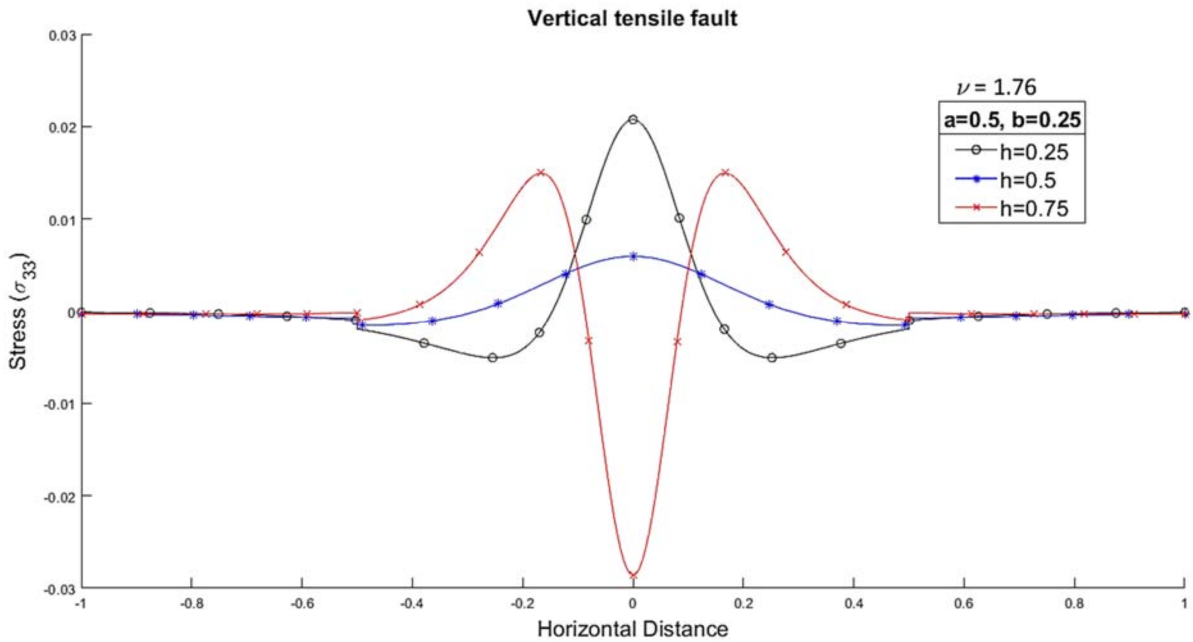

Fig.4a. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the presence of irregularity for $\nu = 1.76$ due to a vertical tensile fault at three different source depths.

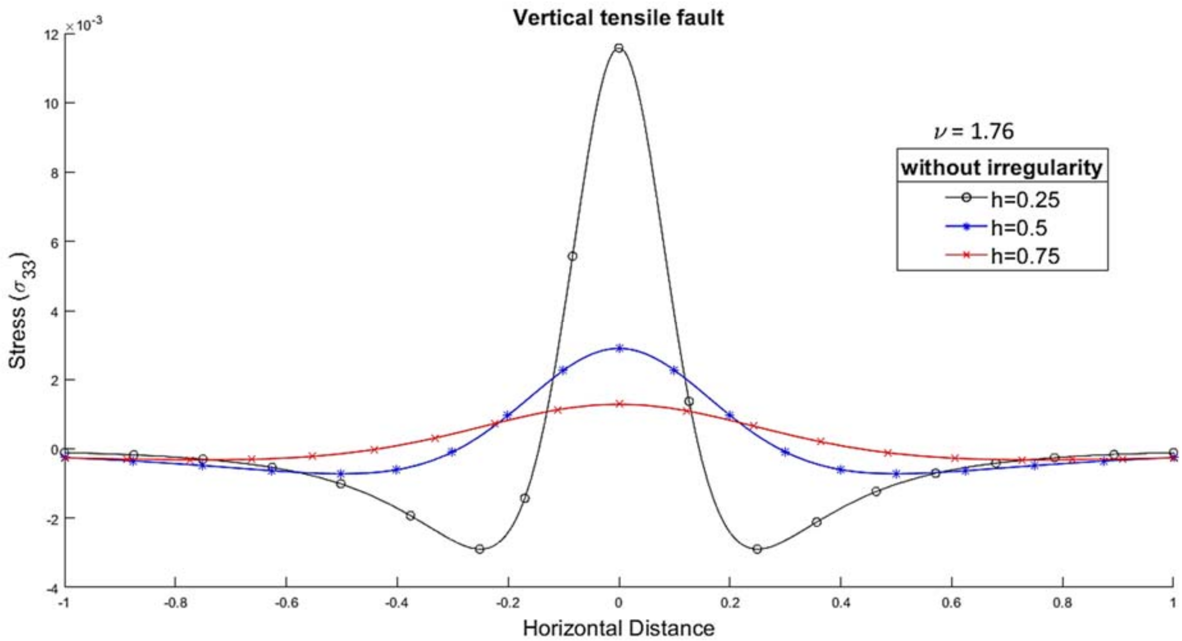

Fig.4b. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the absence of irregularity for $\nu = 1.76$ due to a vertical tensile fault at three different source depths.

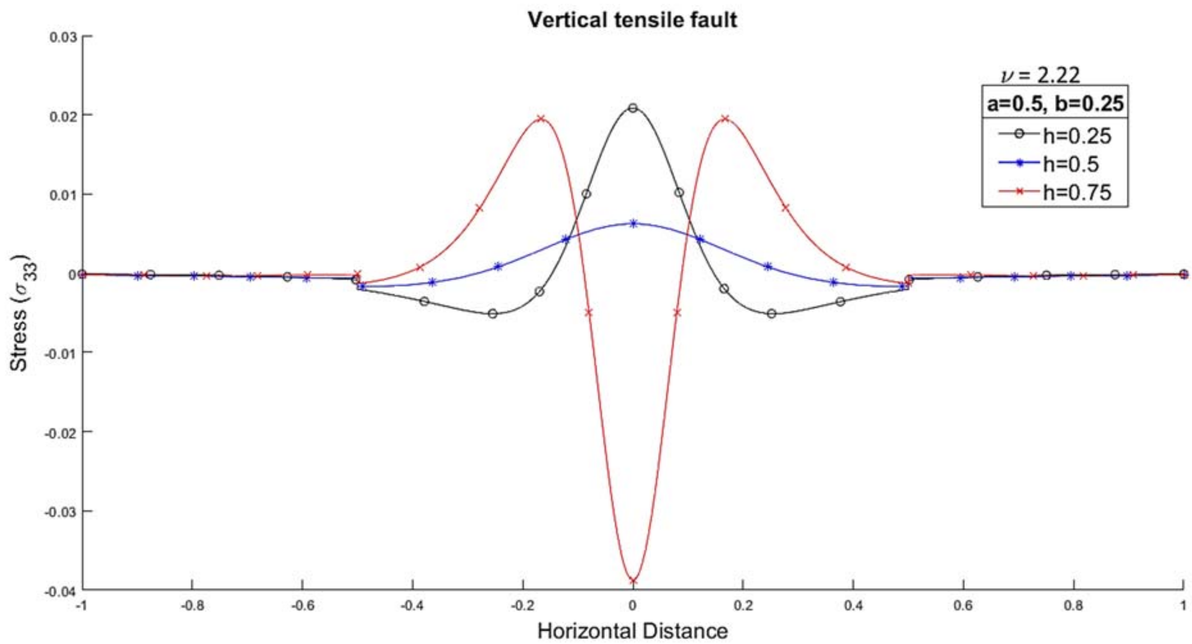

Fig.4c. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the presence of irregularity for $\nu = 2.22$ due to a vertical tensile fault at three different source depths.

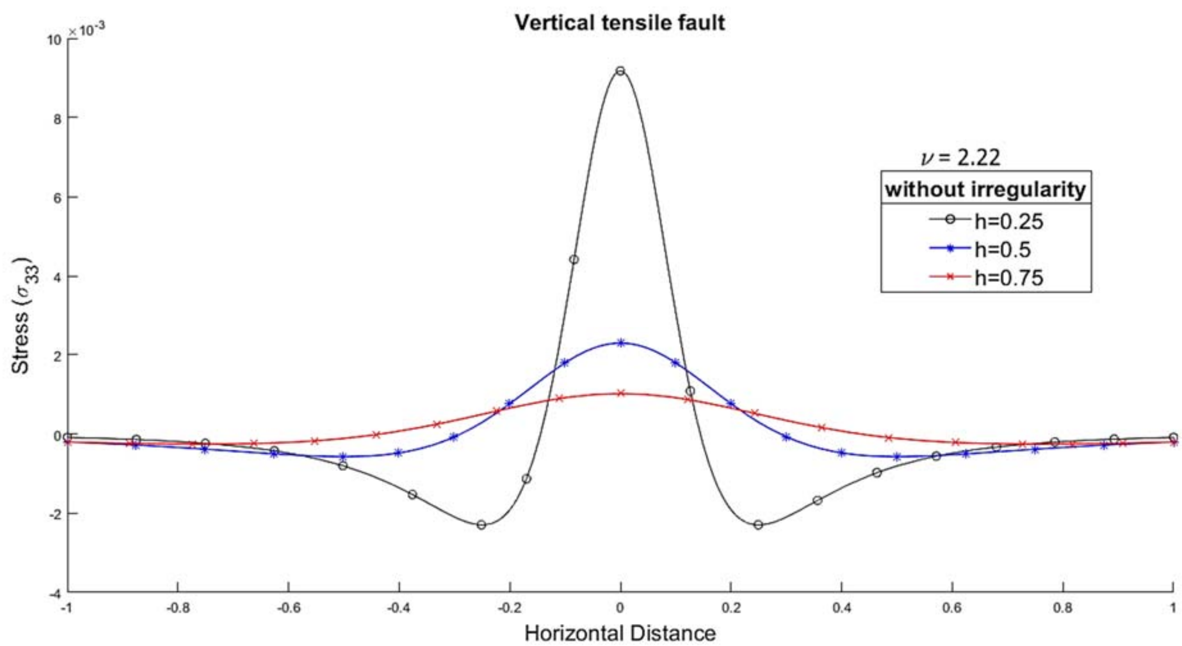

Fig.4d. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the absence of irregularity for $\nu = 2.22$ due to a vertical tensile fault at three different source depths.

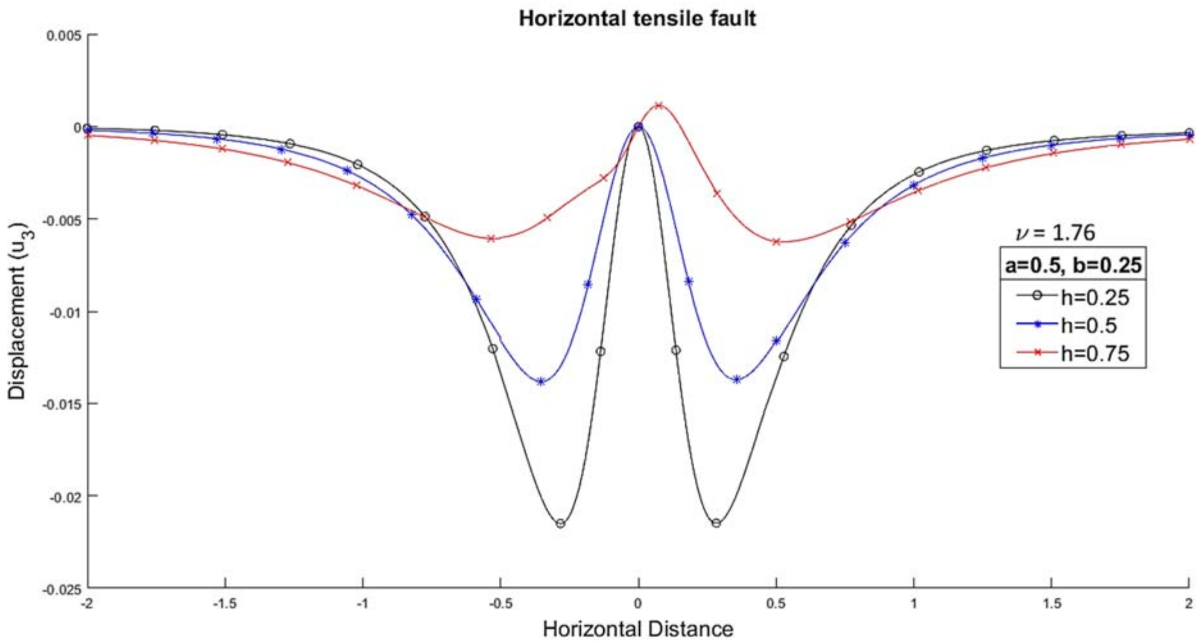

Fig.5a. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the presence of irregularity for $\nu = 1.76$ due to a horizontal tensile fault at three different source depths.

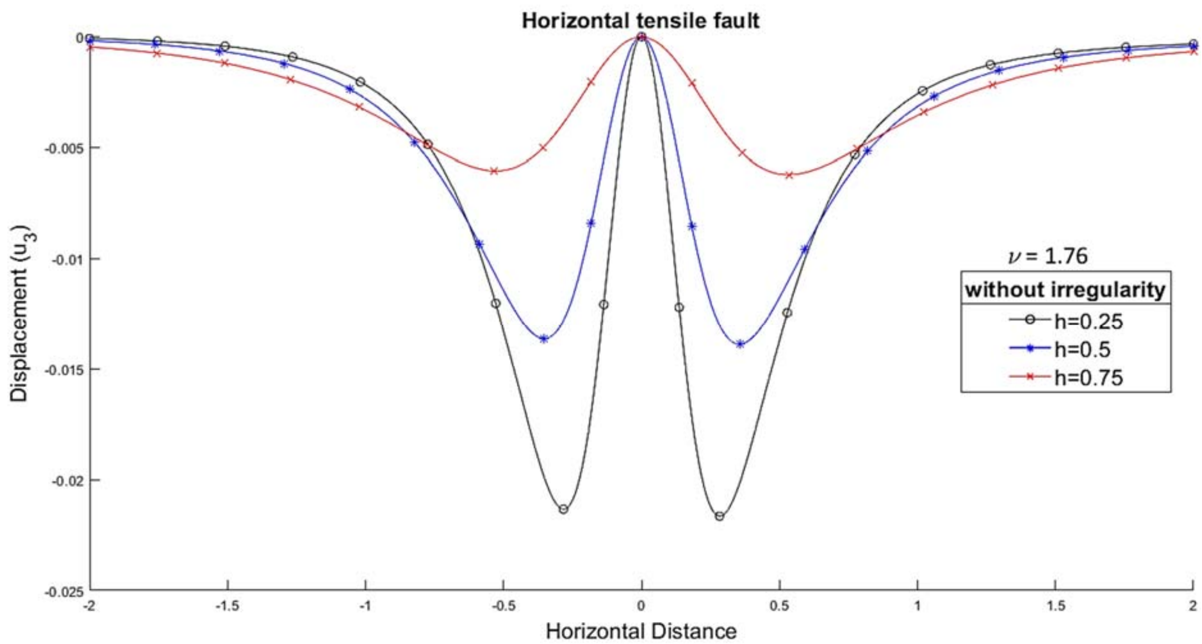

Fig.5b. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the absence of irregularity for $\nu = 1.76$ due to a horizontal tensile fault at three different source depths.

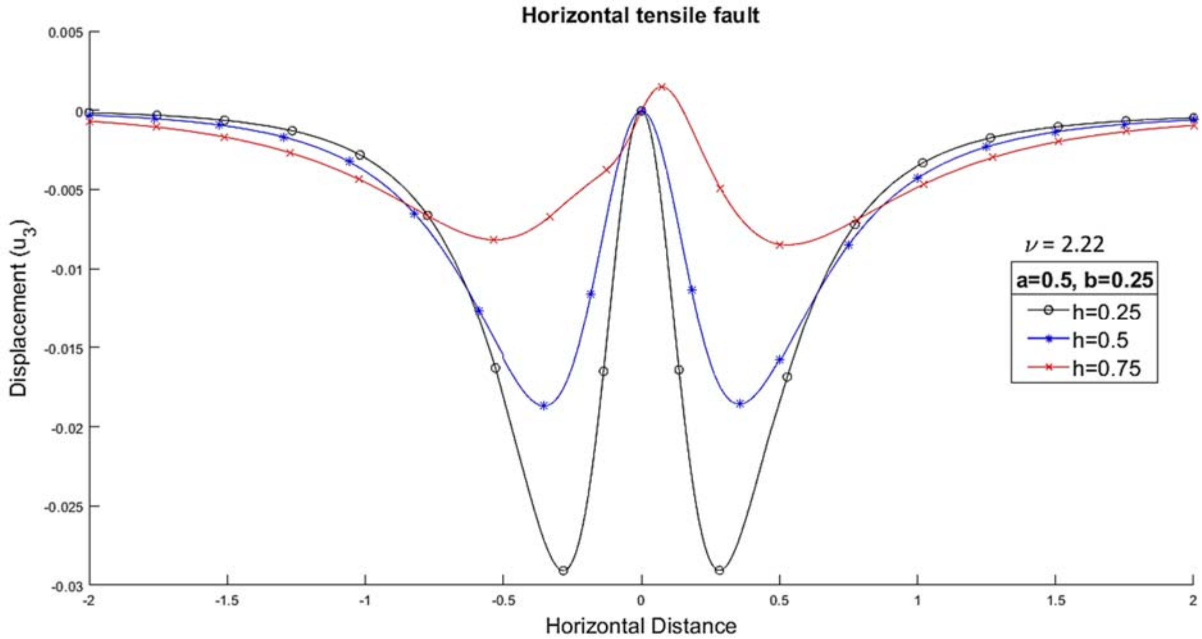

Fig.5c. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the presence of irregularity for $\nu = 2.22$ due to a horizontal tensile fault at three different source depths.

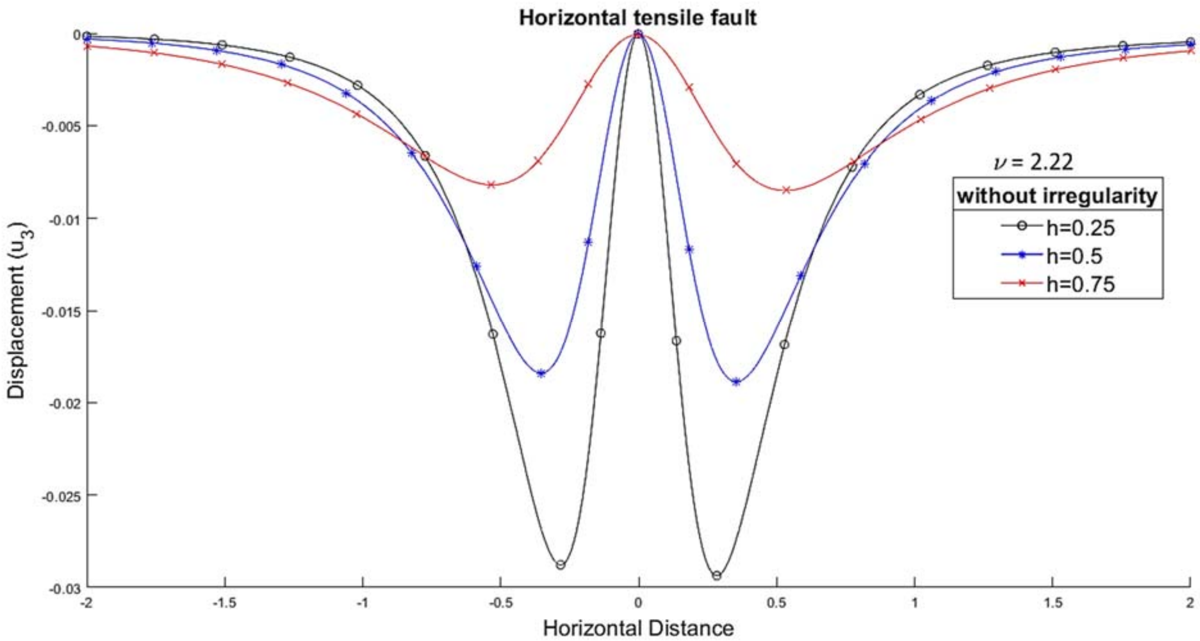

Fig.5d. Variation of the dimensionless horizontal displacement (u_3) for an elastic layer in the absence of irregularity for $\nu = 2.22$ due to a horizontal tensile fault at three different source depths.

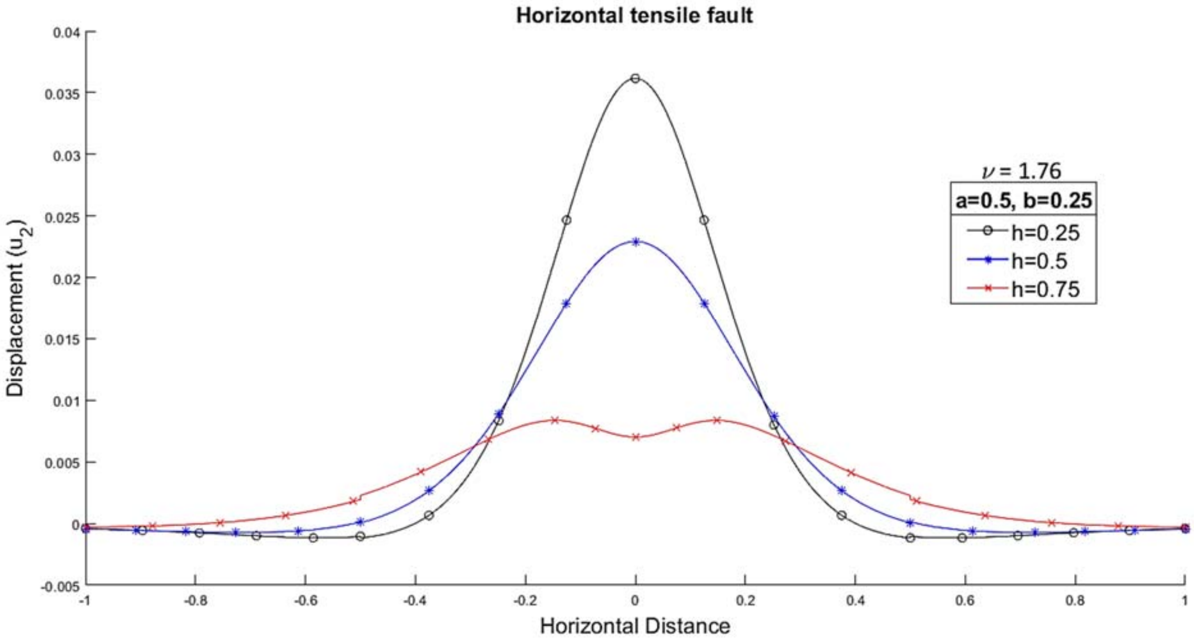

Fig.6a. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the presence of irregularity for $\nu = 1.76$ due to a horizontal tensile fault at three different source depths.

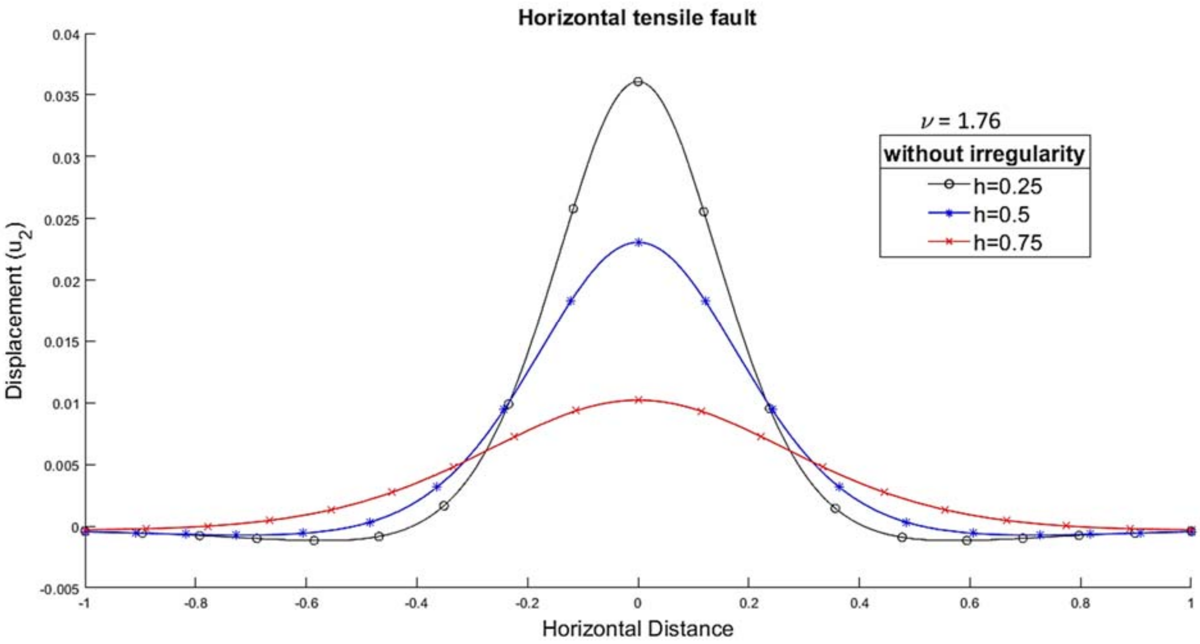

Fig.6b. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the absence of irregularity for $\nu = 1.76$ due to a horizontal tensile fault at three different source depths.

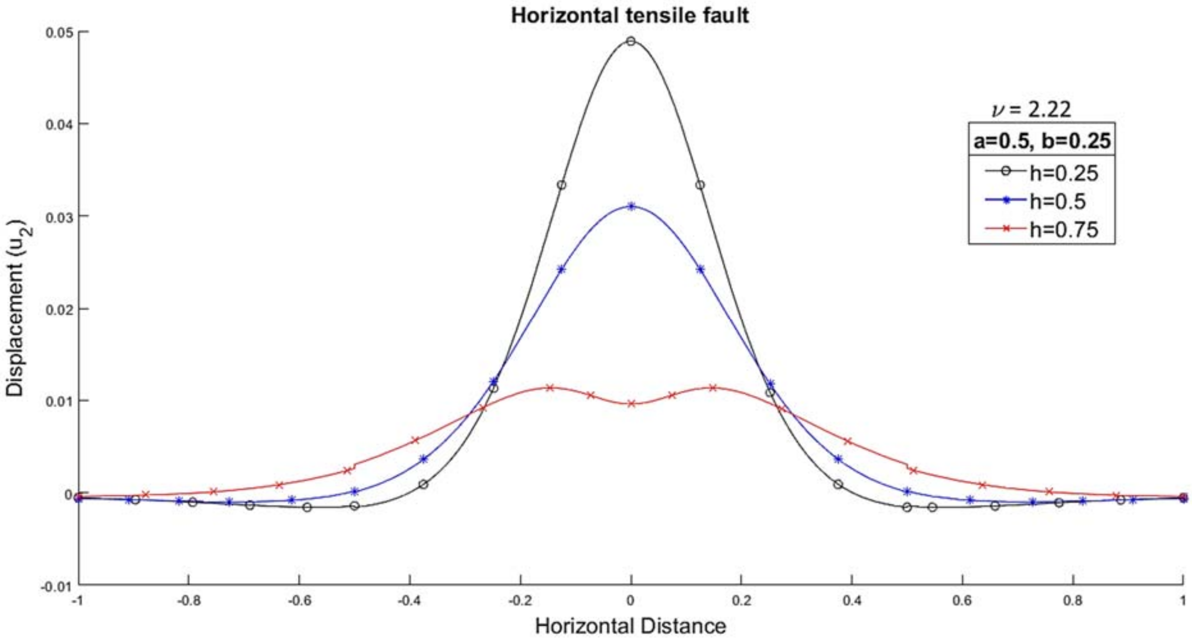

Fig.6c. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the presence of irregularity for $\nu = 2.22$ due to a horizontal tensile fault at three different source depths.

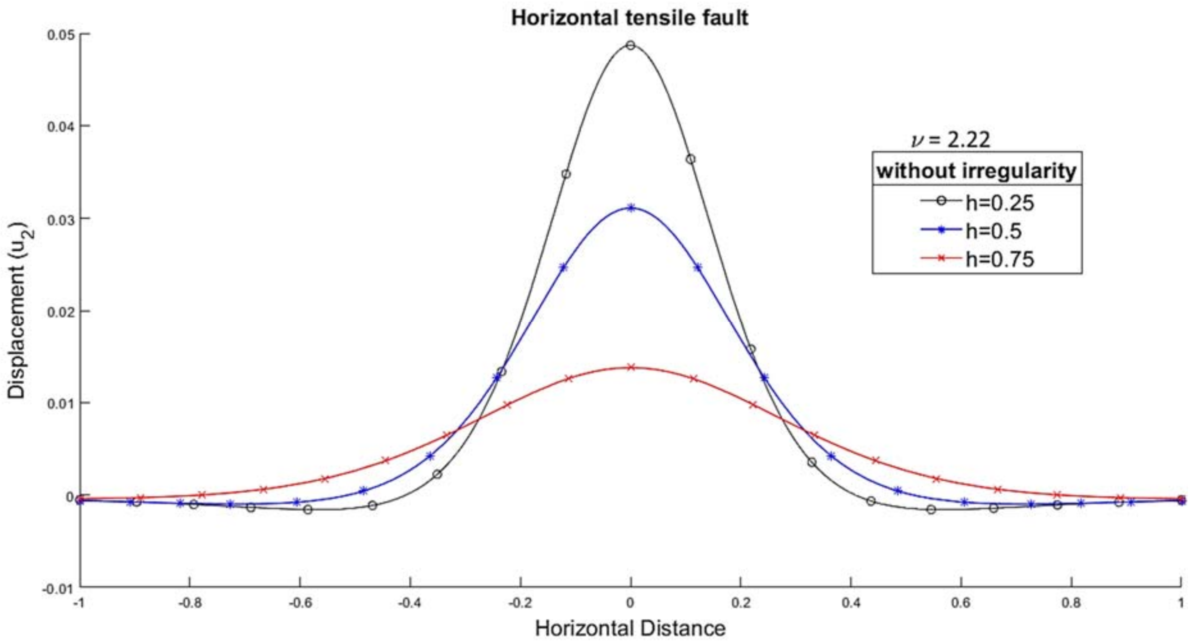

Fig.6d. Variation of the dimensionless vertical displacement (u_2) for an elastic layer in the absence of irregularity for $\nu = 2.22$ due to a horizontal tensile fault at three different source depths.

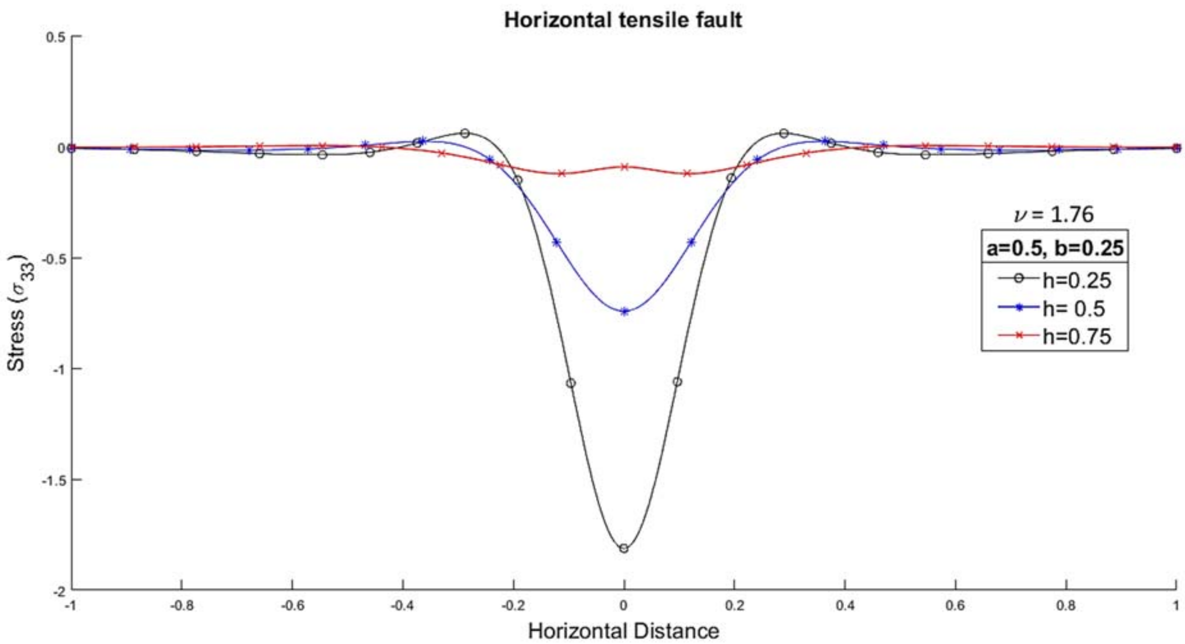

Fig.7a. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the presence of irregularity respectively for $\nu = 1.76$ due to a horizontal tensile fault at three different source depths.

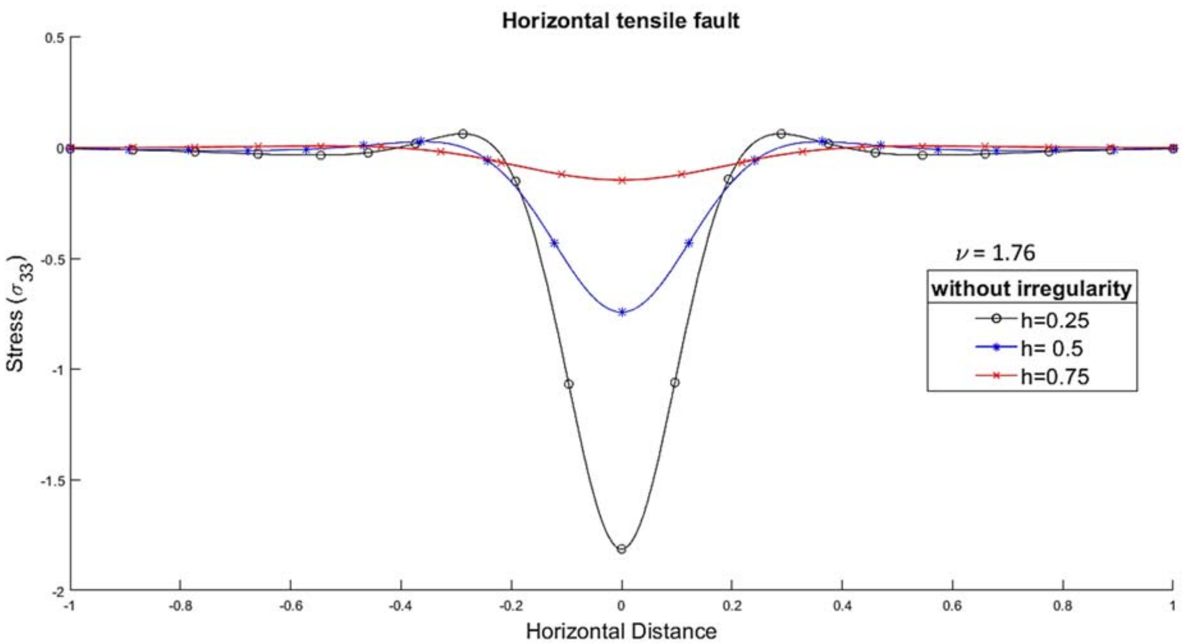

Fig.7b. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the absence of irregularity for $\nu = 1.76$ due to a horizontal tensile fault at three different source depths.

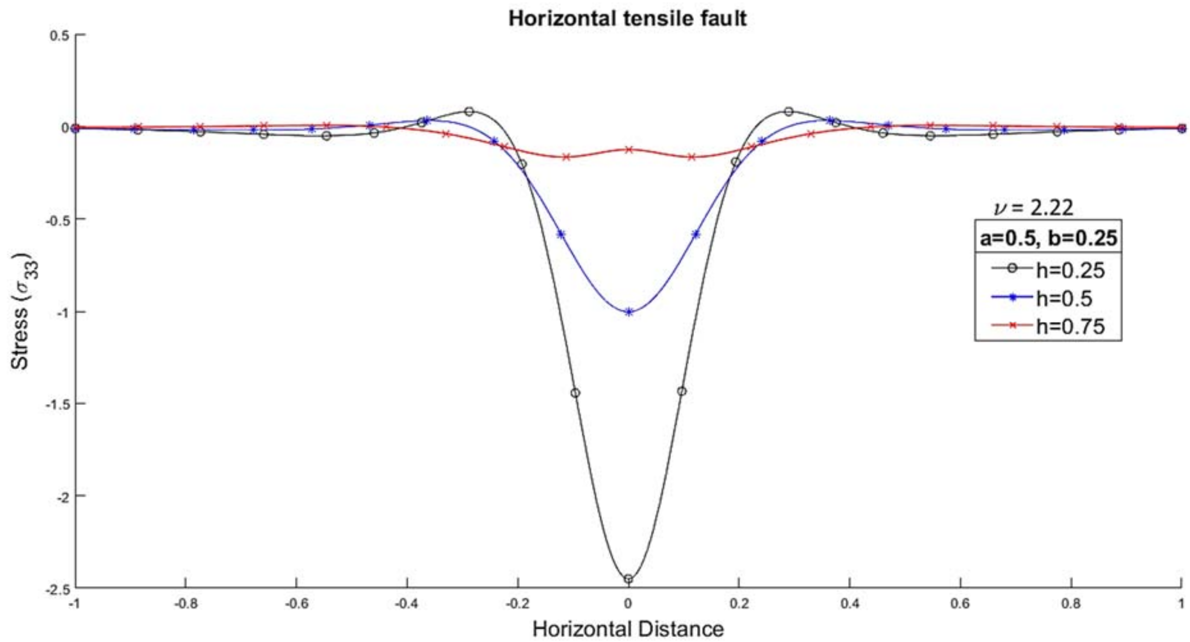

Fig.7c. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the presence of irregularity for $\nu = 2.22$ due to a horizontal tensile fault at three different source depths.

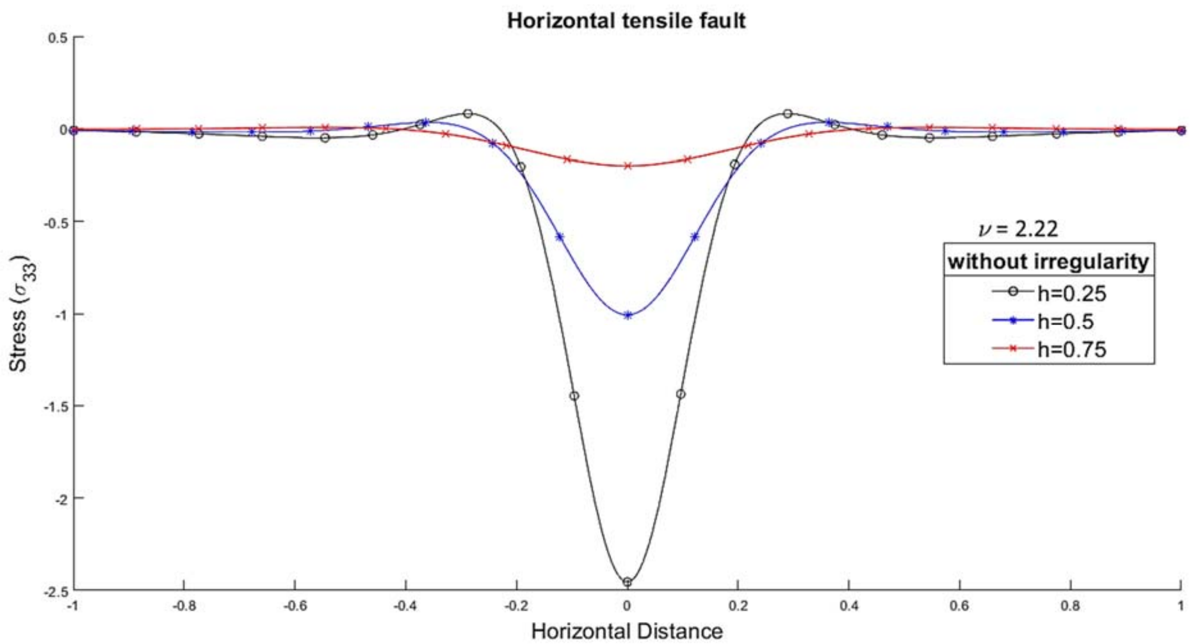

Fig.7d. Variation of the dimensionless stress component (σ_{33}) for an elastic layer in the absence of irregularity for $\nu = 2.22$ due to a horizontal tensile fault at three different source depths.

Acknowledgements

One of the authors (Savita) is thankful to HRDG Council of Scientific & Industrial Research for sanctioning SRF scholarship. The authors are also thankful to the unknown reviewer.

Nomenclature

$2a$	– length of irregularity
b	– depth of irregularity
b_0	– displacement discontinuity
ds	– thickness of discontinuity
h	– depth of discontinuity to free surface along y-axis
(x,y,z)	– co-ordinate axis
Y	– thickness of the elastic layer
ε	– perturbation factor
λ_i	– first Lame's constants
μ_i	– second Lame's constants
ν	– rigidity ratio
σ_i	– Poisson's ratios

Appendix 1

$$A_1 + A_2 = -\left(A^- + B^- kh\right)e^{-kh}, \tag{1.1}$$

$$A_1 - A_2 - B_1 - B_2 = \left(A^- + B^-\left(kh - 1\right)\right)e^{-kh}, \tag{1.2}$$

$$\begin{gathered} e^{-k\varepsilon f(z)}A_1 - e^{k\varepsilon f(z)}A_2 - e^{-k\varepsilon f(z)}A_3 + \left(k\varepsilon f(z) - 1\right)e^{-k\varepsilon f(z)}B_1 - \left(k\varepsilon f(z) + 1\right)e^{k\varepsilon f(z)}B_2 + \\ +\left(1 - k\varepsilon f(z)\right)e^{-k\varepsilon f(z)}B_3 = \left(A^+ + B^+\left(k\left(\varepsilon f(z) - h\right) - 1\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.3}$$

$$\begin{gathered} e^{-k\varepsilon f(z)}A_1 + e^{k\varepsilon f(z)}A_2 - e^{-k\varepsilon f(z)}A_3 + k\varepsilon f(z)e^{-k\varepsilon f(z)}B_1 + k\varepsilon f(z)e^{k\varepsilon f(z)}B_2 + \\ -k\varepsilon f(z)e^{-k\varepsilon f(z)}B_3 = -\left(A^+ + B^+ k\left(\varepsilon f(z) - h\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.4}$$

$$\begin{gathered} -e^{-k\varepsilon f(z)}A_1 - e^{k\varepsilon f(z)}A_2 + \beta e^{-k\varepsilon f(z)}A_3 + \left(\frac{1}{\alpha_1} - k\varepsilon f(z)\right)e^{-k\varepsilon f(z)}B_1 - \left(\frac{1}{\alpha_1} + k\varepsilon f(z)\right)e^{k\varepsilon f(z)}B_2 + \\ + \beta\left(k\varepsilon f(z) - \frac{1}{\alpha_2}\right)e^{-k\varepsilon f(z)}B_3 = \left(A^+ + B^+\left(-\frac{1}{\alpha_1} + k\left(\varepsilon f(z) - h\right)\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.5}$$

$$\begin{gathered} e^{-k\varepsilon f(z)}A_1 - e^{k\varepsilon f(z)}A_2 - \beta e^{-k\varepsilon f(z)}A_3 + \left(\frac{1}{\alpha_1} - 1 + k\varepsilon f(z)\right)e^{-k\varepsilon f(z)}B_1 - \left(1 + k\varepsilon f(z) - \frac{1}{\alpha_1}\right)e^{k\varepsilon f(z)}B_2 + \\ -\beta\left(k\varepsilon f(z) + \frac{1}{\alpha_2} - 1\right)e^{-k\varepsilon f(z)}B_3 = -\left(A^+ + B^+\left(\frac{1}{\alpha_1} + k\left(\varepsilon f(z) - h\right) - 1\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.6}$$

$$C_1 + C_2 = -\left(C^- + D^- kh\right)e^{-kh}, \tag{1.7}$$

$$C_1 - C_2 - D_1 - D_2 = \left(C^- + D^-(kh-1)\right)e^{-kh}, \tag{1.8}$$

$$\begin{gathered} e^{-k\varepsilon f(z)}C_1 - e^{k\varepsilon f(z)}C_2 - e^{-k\varepsilon f(z)}C_3 + \left(k\varepsilon f(z) - 1\right)e^{-k\varepsilon f(z)}D_1 - \left(k\varepsilon f(z)+1\right)e^{k\varepsilon f(z)}D_2 + \\ +\left(1 - k\varepsilon f(z)\right)e^{-k\varepsilon f(z)}D_3 = \left(C^+ + D^+\left(k\left(\varepsilon f(z) - h\right) - 1\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.9}$$

$$\begin{gathered} e^{-k\varepsilon f(z)}C_1 + e^{k\varepsilon f(z)}C_2 - e^{-k\varepsilon f(z)}C_3 + k\varepsilon f(z)e^{-k\varepsilon f(z)}D_1 + k\varepsilon f(z)e^{k\varepsilon f(z)}D_2 + \\ -k\varepsilon f(z)e^{-k\varepsilon f(z)}D_3 = -\left(C^+ + D^+k\left(\varepsilon f(z) - h\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.10}$$

$$\begin{gathered} -e^{-k\varepsilon f(z)}C_1 - e^{k\varepsilon f(z)}C_2 + \beta e^{-k\varepsilon f(z)}C_3 + \left(\frac{1}{\alpha_1} - k\varepsilon f(z)\right)e^{-k\varepsilon f(z)}D_1 - \left(\frac{1}{\alpha_1} + k\varepsilon f(z)\right)e^{k\varepsilon f(z)}D_2 + \\ +\beta\left(k\varepsilon f(z) - \frac{1}{\alpha_2}\right)e^{-k\varepsilon f(z)}D_3 = \left(C^+ + D^+\left(-\frac{1}{\alpha_1} + k\left(\varepsilon f(z) - h\right)\right)\right)e^{-k\left|\varepsilon f(z) - h\right|}, \end{gathered} \tag{1.11}$$

$$\begin{gathered} e^{-k\varepsilon f(z)}C_1 - e^{k\varepsilon f(z)}C_2 - \beta e^{-k\varepsilon f(z)}C_3 + \left(\frac{1}{\alpha_1} - 1 + k\varepsilon f(z)\right)e^{-k\varepsilon f(z)}D_1 - \left(1 + k\varepsilon f(z) - \frac{1}{\alpha_1}\right)e^{k\varepsilon f(z)}D_2 + \\ -\beta\left(k\varepsilon f(z) + \frac{1}{\alpha_2} - 1\right)e^{-k\varepsilon f(z)}D_3 = -\left(C^+ + D^+\left(\frac{1}{\alpha_1} + k\left(\varepsilon f(z) - h\right) - 1\right)\right)e^{-k\left|\varepsilon f(z) - h\right|} \end{gathered} \tag{1.12}$$

where $\beta = \dfrac{\mu_1}{\mu_2}$.

$$\begin{gathered} A_1 = \frac{1}{\Delta_0}\left[\left\{\delta^2 T_1\left(1 + 2k\varepsilon f(z)\right)\left(A^+ - B^+kh\right) + (2\delta^2 T_1 k^2 \varepsilon f\left(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)B^+\right\}\right. \\ e^{-k\left(2\varepsilon f(z) - h\right)} + \left\{\delta^2 T_1\left(1 - 2k\varepsilon f(z)\right)\left(A^- + B^-kh\right) + (2\delta^2 T_1 k^2 \varepsilon f\left(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)B^-\right\} \\ \left. e^{-k\left(2\varepsilon f(z) + h\right)} + \delta T_3\left(A^- + B^-kh\right)e^{-kh} + \delta T_2\left(A^+ - B^+kh\right)e^{-k\left(4\varepsilon f(z) - h\right)}\right], \end{gathered}$$

$$\begin{gathered} B_1 = \frac{1}{\Delta_0}\left[\left\{(T_4 + \delta^2 T_1\left(4k^2\varepsilon f\left(z\right)^2 - 2k\varepsilon f(z)\right)\right)B^+ + 4\delta^2 T_1 k\varepsilon f(z)\left(A^+ - B^+kh\right)\right\} \\ e^{-k\left(2\varepsilon f(z) - h\right)} + \left\{2\delta^2 T_1\left(A^- + B^-kh\right) - \delta^2 T_1\left(1 + 2k\varepsilon f(z)\right)B^-\right\}e^{-k\left(2\varepsilon f(z) + h\right)} + \\ \left. +\delta T_3\left(2A^- + B^-(2kh - 1)\right)e^{-kh} + \delta T_2 B^+ e^{-k\left(4\varepsilon f(z) - h\right)}\right], \end{gathered}$$

$$\begin{gathered} C_1 = \frac{1}{\Delta_0}\left[\left\{\delta^2 T_1\left(1 + 2k\varepsilon f(z)\right)\left(C^+ - D^+kh\right) + (2\delta^2 T_1 k^2 \varepsilon f\left(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)D^+\right\}\right. \\ e^{-k\left(2\varepsilon f(z) - h\right)} + \left\{\delta^2 T_1\left(1 - 2k\varepsilon f(z)\right)\left(C^- + D^-kh\right) + (2\delta^2 T_1 k^2 \varepsilon f\left(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)D^-\right\} \\ e^{-k\left(2\varepsilon f(z) + h\right)} + \delta T_3\left(C^- + D^-kh\right)e^{-kh} + \delta T_2\left(C^+ - D^+kh\right)e^{-k\left(4\varepsilon f(z) - h\right)}, \end{gathered}$$

$$D_1 = \frac{1}{\Delta_0}\Bigg[\Big\{2\delta^2 T_1\left(C^- + D^- kh\right) - \delta^2 T_1\left(1 + 2k\varepsilon f(z)\right)D^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} +$$

$$+\Big\{\Big[T_4 + \delta^2 T_1\left(4k^2\varepsilon f(z)^2 - 2k\varepsilon f(z)\right)\Big]D^+ + 4\delta^2 T_1 k\varepsilon f(z)\left(C^+ - D^+ kh\right)\Big\}e^{-k\left(2\varepsilon f(z)-h\right)} +$$

$$+\delta T_3\left(2C^- + D^-\left(2kh - 1\right)\right)e^{-kh} + \delta T_2 D^+ e^{-k\left(4\varepsilon f(z)-h\right)}\Bigg],$$

$$A_2 = \frac{1}{\Delta_0}\Bigg[\Big\{\Big[\delta^2 T_1\left(1 + 2k\varepsilon f(z)\right) + 4\delta^2 k^2\varepsilon f(z)^2 T_1 - \delta^2 T_1 + T_4\Big]\left(A^- + B^- kh\right) +$$

$$-\left(2\delta^2 T_1 k^2\varepsilon f(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)B^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} + \delta T_2\left(A^- + B^- kh\right)e^{-k\left(4\varepsilon f(z)+h\right)} +$$

$$-\delta T_2\left(A^+ - B^+ kh\right)e^{-k\left(4\varepsilon f(z)-h\right)} - \Big\{\delta^2 T_1\left(1 + 2k\varepsilon f(z)\right)\left(A^+ - B^+ kh\right) +$$

$$+\left(2\delta^2 T_1 k^2\varepsilon f(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)B^+\Big\}e^{-k\left(2\varepsilon f(z)-h\right)}\Bigg],$$

$$B_2 = \frac{1}{\Delta_0}\Bigg[\delta^2 T_1\Big\{-4k\varepsilon f(z)\left(A^- + B^- kh\right) + \left(2k\varepsilon f(z) - 1\right)B^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} +$$

$$+\delta^2 T_1\Big\{2\left(A^+ - B^+ kh\right) + \left(2k\varepsilon f(z) - 1\right)B^+\Big\}e^{-k\left(2\varepsilon f(z)-h\right)} +$$

$$-\delta T_2 B^- e^{-k\left(4\varepsilon f(z)+h\right)} + \delta T_2\Big\{2\left(A^+ - B^+ kh\right) - B^+\Big\}e^{-k\left(4\varepsilon f(z)-h\right)}\Bigg],$$

$$C_2 = \frac{1}{\Delta_0}\Bigg[\Big\{\Big[\delta^2 T_1\left(1 + 2k\varepsilon f(z)\right) + 4\delta_2 k^2\varepsilon f(z)^2 T_1 - \delta_2 T_1 + T_4\Big]\left(C^- + D^- kh\right) +$$

$$-\left(2\delta^2 T_1 k^2\varepsilon f(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)D^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} +$$

$$-\Big\{\delta^2 T_1\left(1 + 2k\varepsilon f(z)\right)\left(C^+ - D^+ kh\right) + \left(2\delta^2 T_1 k^2\varepsilon f(z)^2 - \frac{\delta^2 T_1}{2} + \frac{T_4}{2}\right)D^+\Big\}e^{-k\left(2\varepsilon f(z)-h\right)} +$$

$$\delta T_2\left(C^- + D^- kh\right)e^{-k\left(4\varepsilon f(z)+h\right)} - \delta T_2\left(C^+ - D^+ kh\right)e^{-k\left(4\varepsilon f(z)-h\right)}\Bigg],$$

$$D_2 = \frac{1}{\Delta_0}\Bigg[\delta^2 T_1\Big\{-4k\varepsilon f(z)\left(C^- + D^- kh\right) + \left(2k\varepsilon f(z) - 1\right)D^-\Big\}e^{-k\left(2\varepsilon f(z)+h\right)} +$$

$$-\delta T_2 D^- e^{-k\left(4\varepsilon f(z)+h\right)} + \delta^2 T_1\Big\{2\left(C^+ - D^+ kh\right) + \left(2k\varepsilon f(z) - 1\right)D^+\Big\}e^{-k\left(2\varepsilon f(z)-h\right)} +$$

$$+\delta T_2\Big\{2\left(C^+ - D^+ kh\right) - D^+\Big\}e^{-k\left(4\varepsilon f(z)-h\right)}\Bigg],$$

$$A_3=\frac{1}{\Delta_1}\left[\left\{\left(\frac{2(\delta-\nu\delta_1)}{\alpha_1\nu\delta\delta_1}+\frac{4(\nu-1)(1-2k\varepsilon f(z))k\varepsilon f(z)}{\alpha_1\nu}\right)\left(A^+-B^+kh\right)+\right.\right.$$
$$\left.+\left(\frac{T_4-\delta^2T_1}{8\nu^2\delta^2\delta_1}+\frac{4(\nu-1)k^2\varepsilon f(z)^2}{\alpha_1\nu}\right)B^+\right\}e^{-k(2\varepsilon f(z)-h)}+$$
$$+\left\{\left(\frac{2(\nu+\delta)}{\alpha_1\nu\delta}-\frac{\left(T_4-\delta^2T_1\right)k\varepsilon f(z)}{2\nu^2\delta^2\delta_1}\right)\left(A^-+B^-kh\right)-\frac{1-2k\varepsilon f(z)}{8\nu^2\delta^2\delta_1}\left(T_4-\delta^2T_1\right)B^-\right\}e^{-kh}+$$
$$+\left\{\frac{2k\varepsilon f(z)-1}{8\nu^2\delta^2\delta_1}\left(T_4-\delta^2T_1\right)B^+-\frac{2(\nu\delta_1+1)}{\alpha_1\nu\delta_1}\left(A^+-B^+kh\right)\right\}e^{kh}+$$
$$\left.+\left\{\frac{2(\nu-1)(1-2k\varepsilon f(z))}{\alpha_1\nu}\left(A^-+B^-kh\right)+\left(\frac{T_4-\delta^2T_1}{8\nu^2\delta^2\delta_1}+\frac{4(\nu-1)k^2\varepsilon f(z)^2}{\alpha_1\nu}\right)B^-\right\}e^{-k(2\varepsilon f(z)+h)}\right],$$

$$B_3=\frac{1}{\Delta_1}\left[\left\{-2(\beta-1)\left(A^-+B^-kh\right)+(\beta-1)(1+2k\varepsilon f(z))B^-\right\}e^{-k(2\varepsilon f(z)+h)}-\frac{\nu+\delta}{\nu\delta}B^+e^{kh}+\right.$$
$$\left.+\left\{-4k\varepsilon f(z)\left(A^+-B^+kh\right)+(1+2k\varepsilon f(z))B^+\right\}(\beta-1)e^{-k(2\varepsilon f(z)-h)}+\frac{\nu+\delta}{\nu\delta}\left(2\left(A^-+B^-kh\right)-B^-\right)e^{-kh}\right],$$

$$C_3=\frac{1}{\Delta_1}\left[\left\{\left(\frac{2(\delta-\nu\delta_1)}{\alpha_1\nu\delta\delta_1}+\frac{4(\nu-1)(1-2k\varepsilon f(z))k\varepsilon f(z)}{\alpha_1\nu}\right)\left(C^+-D^+kh\right)+\right.\right.$$
$$\left.+\left(\frac{T_4-\delta^2T_1}{8\nu^2\delta^2\delta_1}+\frac{4(\nu-1)k^2\varepsilon f(z)^2}{\alpha_1\nu}\right)D^+\right\}e^{-k(2\varepsilon f(z)-h)}+$$
$$+\left\{\left(\frac{2(\nu+\delta)}{\alpha_1\nu\delta}-\frac{\left(T_4-\delta^2T_1\right)k\varepsilon f(z)}{2\nu^2\delta^2\delta_1}\right)\left(C^-+D^-kh\right)-\frac{1-2k\varepsilon f(z)}{8\nu^2\delta^2\delta_1}\left(T_4-\delta^2T_1\right)D^-\right\}e^{-kh}+$$
$$+\left\{\frac{2k\varepsilon f(z)-1}{8\nu^2\delta^2\delta_1}\left(T_4-\delta^2T_1\right)D^+-\frac{2(\nu\delta_1+1)}{\alpha_1\nu\delta_1}\left(C^+-D^+kh\right)\right\}e^{kh}+$$
$$\left.+\left\{\frac{2(\nu-1)(1-2k\varepsilon f(z))}{\alpha_1\nu}\left(C^-+D^-kh\right)+\left(\frac{T_4-\delta^2T_1}{8\nu^2\delta^2\delta_1}+\frac{4(\nu-1)k^2\varepsilon f(z)^2}{\alpha_1\nu}\right)D^-\right\}e^{-k(2\varepsilon f(z)+h)}\right],$$

$$D_3=\frac{1}{\Delta_1}\left[\left\{-2(\beta-1)\left(C^-+D^-kh\right)+(\beta-1)(1+2k\varepsilon f(z))C^-\right\}e^{-k(2\varepsilon f(z)+h)}+\right.$$
$$-\frac{\nu+\delta}{\nu\delta}D^+e^{kh}+\left\{-4k\varepsilon f(z)\left(C^+-D^+kh\right)+(1+2k\varepsilon f(z))D^+\right\}(\beta-1)e^{-k(2\varepsilon f(z)-h)}+$$
$$\left.+\frac{\nu+\delta}{\nu\delta}\left(2\left(C^-+D^-kh\right)-D^-\right)e^{-kh}\right]$$

where

$$\Delta=-\left[\delta T_3+\left(T_4+\delta^2T_1+4\delta^2k^2\varepsilon f(z)^2T_1\right)e^{-2k\varepsilon f(z)}+\delta T_2e^{-4k\varepsilon f(z)}\right],$$

$$\frac{1}{\Delta_1}=\frac{\left[4\nu^2\delta^2\delta_1\right]}{\Delta_0},$$

$$\delta=\frac{\alpha_1}{2-\alpha_1}=3-4\sigma_1,\quad \delta_1=\frac{\alpha_2}{2-\alpha_2}=3-4\sigma_2,\quad \nu=\frac{1}{\beta},$$

$$T_1 = 4(\nu - 1)(\nu\delta_1 + 1),\ T_2 = 4(\nu - 1)(\nu\delta_1 - \delta),$$

$$T_3 = 4(\nu + \delta)(\nu\delta_1 + 1),\ T_4 = 4(\nu + \delta)(\nu\delta_1 - \delta),$$

$$f(z) = sign(a - z) + sign(a + z).$$

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