Control and Cybernetics

vol. **44** (2015) No. 2

Optimal credit period and ordering quantity for credit dependent trended demand and deteriorating items with maximum lifetime*

by

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Abstract: In this paper, an inventory system is analyzed regarding a trended demand. The seller offers its buyer a credit period for settling the account, which attracts more buyers and enhances market demand. However, the offer of credit period leads to default risk for the supplier. The units in inventory are deteriorating continuously and also have definite maximum lifetime. The aim is to maximize the profit of the retailer. Numerical examples are given to validate the developed mathematical model. Sensitivity analysis is performed to obtain the critical inventory parameters and deduce the managerial strategies.

Keywords: inventory, trended demand, trade credit, default risk, deterioration, maximum lifetime

1. Introduction

In the market, if a retailer has an option to pay for goods with a delay without interest charges, then he gets attracted to buy. This may be demonstrated through a real practice regarding the suppliers, as a tool for surviving in global competition. During this allowable period, the retailer can earn interest on sold items. However, on the other hand, the offer of a delay period leads to default risk for the seller, related to the potential situation when the buyer declares his inability to pay off. Obviously, the lengthy delay period increases the default risk. Goyal (1985) discussed an inventory model when delay in payments is

*Submitted: May 2014; Accepted: July 2015

permissible. Teng (2008) made a review on inventory lot-size models with trade credits. Shah et al. (2010) reviewed inventory models with trade credits. Lou and Wang (2013) discussed the seller's decision about setting the delay payment period. They considered deterministic constant demand. Almost all of the researchers ended up with the conclusion that the length of the trade credit increases the demand rate. One can refer to the research articles by Chern et al. (2013), and Teng and Lou (2012).

Deterioration is referred to as the decay, spoilage, evaporation and loss of utility of a product from the original one. Fruits and vegetables, cosmetics and medicines, radioactive chemicals, beverages, electronic items, blood components, agriculture products are some of the examples of deteriorating commodities. Ghare and Schrader (1963) derived the first inventory model with items which decay exponentially. Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001), and Bakker et al. (2012) studied the results on inventory modeling with deterioration. In almost all the above cited articles, it is assumed that the items in inventory have infinite lifetime. In fact, each product has its maximum lifetime (see Sarkar, 2012, 2013; Sarkar and Sarkar, 2013a, b; Sarkar et al., 2013; Chung and Cardenas-Barron, 2013; Chung et al., 2014; Ouyang et al., 2013; Chen et al., 2014; Wu et al., 2014; Shah and Cardenas-Barron, 2015; Sarkar et al., 2015, and many more).

In this paper, we develop an economic order quantity model for the seller to incorporate two prevailing facts: (1) trade-credit dependent trended demand and default risk due to the offer of trade credit, and (2) deterioration of items with a maximum lifetime. The focus is to maximize total profit per unit time for the retailer. At last, we employ sensitivity analysis to study the consequences concerning the optimal solution with respect to one inventory parameter at a time. For the retailer, managerial decisions are furnished.

2. Notations and assumptions

We shall use the following notations and assumptions to form the mathematical model of the problem.

2.1. Notations

A	Ordering cost in \$ per order
C	Purchase cost in \$ per unit
P	Unit sale price in \$; where $P > C$
h	Inventory holding cost in \$ (excluding interest charges) /unit / unit time
M	Credit period (in years) offered by the seller to his buyers (a decision variable)

$R(M,t)$	Time and credit period dependent annual demand rate
$\theta(t)$	Time varying deterioration rate, where $0 \leqslant \theta(t) \leqslant 1$
m	Maximum lifetime of the deteriorating item (in years)
$I(t)$	Inventory level at any instant of time t, $0 \leqslant t \leqslant T$
T	Cycle time in years (a decision variable)
Q	Procurement quantity per cycle
$\pi(M,T)$	Seller's profit in $ per unit time

2.2. Assumptions

1. The inventory system is studied for a single product.
2. The replenishment rate of inventory is infinite.
3. Lead-time is zero or negligible. Shortages are not allowed.
4. Sellers keep selling price invariant to bind the respective retailers globally.
5. Teng and Lou (2012) argued that trade credit is similar to price discount. We consider demand rate increasing with time, which is observed for branded product and credit period as

$$R(M,t) = a(1+bt)M^{\beta} \tag{1}$$

where $a > 0$ is demand scale coefficient, $0 \leqslant b < 1$ is rate of change of demand with respect to time, and $\beta > 0$ is a constant.

6. When longer credit period is offered to the buyer, the default risk of the seller increases. Here, the rate of default risk while offering the credit period M to the buyer is assumed to be

$$F(M) = 1 - M^{-\gamma} \tag{2}$$

where $\gamma > 0$ is a constant.

7. The units in inventory are deteriorating with respect to time and also have an expiry rate. The deterioration rate tends to 1 when time reaches the maximum lifetime m. As argued in Sarkar (2012b), Chen and Teng (2014), and Wang et al. (2014), the functional form for deterioration rate is

$$\theta(t) = \frac{1}{1+m-t} \;;\; 0 \leqslant t \leqslant T < m.$$

Items having once fully deteriorated can neither be repaired nor replaced during the cycle time.

3. Mathematical model

The seller's inventory is depleting due to increasing demand, offer of credit period, and deterioration. The rate of change in the inventory is governed by the differential equation

$$\frac{dI(t)}{dt} = -R(M,t) - \theta(t)I(t), \quad 0 \leqslant t \leqslant T \tag{3}$$

with $I(T)=0$. The solution of the differential equation (3) is

$$I(t)=aM^{\beta}(1+m-t)\left((1+b(1+m))\ln\left(\frac{1+m-t}{1+m-T}\right)+b(t-T)\right). \quad (4)$$

At the beginning, the seller has Q units in the inventory system i.e.

$$Q=I(0)=aM^{\beta}(1+m)\left((1+b(1+m))\ln\left(\frac{1+m}{1+m-T}\right)-bT\right). \quad (5)$$

The costs associated per cycle for the seller are

- net revenue after default risk:

$$SR=P\int_0^T R(M,t)\,dt\,(1-F(M))$$

- purchase cost: $PC=CQ$
- ordering cost: $OC=A$
- holding cost: $HC=h\int_0^T I(t)\,dt$.

Consequently, the seller's annual yield per unit time is

$$\pi(M,T)=\frac{1}{T}[SR-PC-OC-HC]. \quad (6)$$

The necessary condition to have maximum annual profit per unit time with respect to credit period and cycle time is

$$\frac{\partial\pi(M,T)}{\partial M}=0 \quad \text{and} \quad \frac{\partial\pi(M,T)}{\partial T}=0. \quad (7)$$

Here, equations (6) and (7) are nonlinear and hence closed form of solution cannot be obtained. So, we adopt the following algorithm for the determination of the solution:

Step 1: Input the values to the inventory parameters.

Step 2: Solve equations (7) simultaneously by some mathematical software. We have used Maple XIV.

Step 3: Check second order (sufficiency) conditions analytically or graphically.

Step 4: Determine profit $\pi(M,T)$ per unit time from equation (6) and ordering quantity Q from equation (5).

The aim is to ensure maximum total profit per unit time with respect to cycle time and credit period when units in inventory are deteriorating continuously and have maximum lifetime. We analyze the model with numerical values for the inventory parameters in the next section.

4. Numerical examples and sensitivity analysis

Example 1: Take A= \$ 300/order, C= \$ 8/unit, h= \$ 0.1 per unit per year, a= 1,000 units, $b = 115\%$, $\gamma = 1.15$, P= \$ 12 per unit, m= 2 years, and $\beta = 3$. Then, the optimum credit period is 0.7768 years, the optimum cycle time is 0.9496 years, and the consequent profit is \$ 4,140.80. The retailer's optimum purchase is 846 units. Fig. 1 shows the concavity of the profit. The behavior of the demand with time and the delay period are shown in Fig. 2.

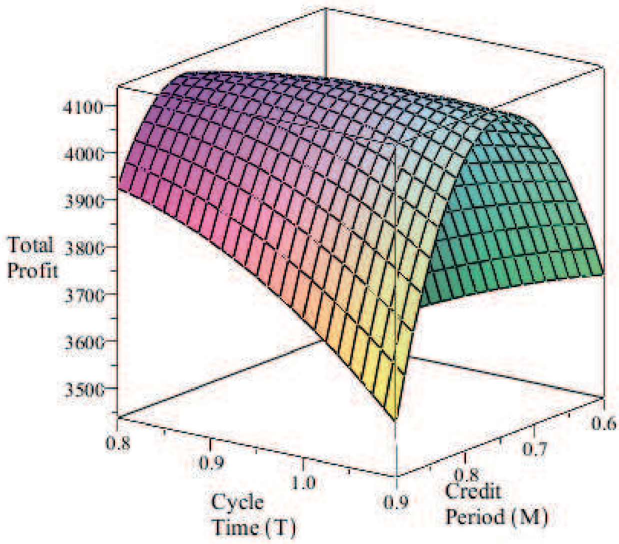

Figure 1. Concavity of total profit with respect to cycle time (T) and credit period (M)

Example 2: Take A= \$ 250/order, C= \$ 15/unit, h= \$ 0.15 per unit per year, a= 5,000 units, b= 50%, $\gamma = 1.15$, P=\$ 25/ unit, m= 2 years, and β= 3. Then, the resulting optimum credit period is 1.89 years, optimum cycle time is 0.06 years, and the resulting profit is \$ 99,566. The retailer's purchase is 2,128 units.

Results from these two examples suggest that if retailer's demand is smaller, then the delay period should be smaller than the cycle time in order to maximize the profit. Profit can be improved for higher demand if settlement is made after the cycle time.

Next, we examine the variations in optimal solutions and in the profit function by altering the inventory parameters, given in Example 1, by -20%,-10%,+10%,+20%; one at a time. The resulting variations in credit period, cycle time, profit of the seller and ordering quantity are demonstrated in Figs.

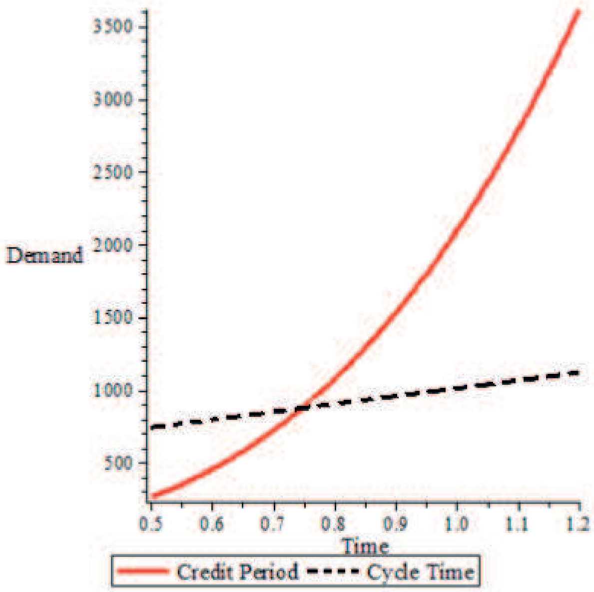

Figure 2. Demand versus time

3 through 6, respectively. It is observed from Fig. 3 that credit period gets a positive impact regarding retail price, scale of demand and mark-up for trade credit, and it decreases in unit purchase cost and rate of change of demand b. This justifies the long repeated argument that delay period boosts demand. Fig. 4 gives just the reverse pattern for the optimal cycle time. Fig. 5 shows that the seller's optimal profit increases with increase in selling price, demand scale coefficient and rate of change of demand b, and decreases with increase in unit purchase cost and trade credit elasticity. Now, it can be seen from Fig. 6 that ordering quantity increases with increase in retail price, rate of change of demand, demand scale coefficient and maximum life time, and decreases with purchase cost and mark up for trade credit.

5. Conclusions

The decision about setting a tolerable credit period is not easy for the seller. The default risk gets enhanced when the longer credit period is given. The problem becomes tougher when demand is linearly increasing with time. This paper studies the above concerns for the deteriorating products with fixed lifetime. The non-linear profit function is obtained. The necessary conditions are expressed to get the optimal solution based on the respective model. It is observed that higher values of demand scale and of retail price should motivate the seller to offer longer credit period, what will bring larger orders. The results

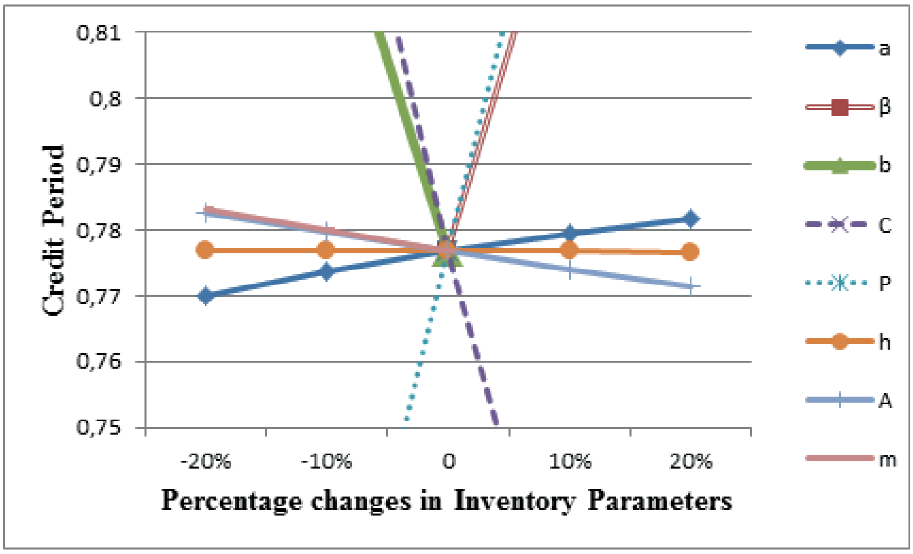

Figure 3. Sensitivity analysis for credit period (M)

from this study should help appreciably the seller in fixing the optimal credit period.

For future research, this concept can be extended for stochastic demand and fuzzy demand. One can think of allowing shortages, as well. Additionally, one can consider preservation technology to control the deterioration.

Acknowledgement

The authors would like to thank the reviewers for their constructive comments. The first author thanks DST-FIST file # MSI-097 for technical support.

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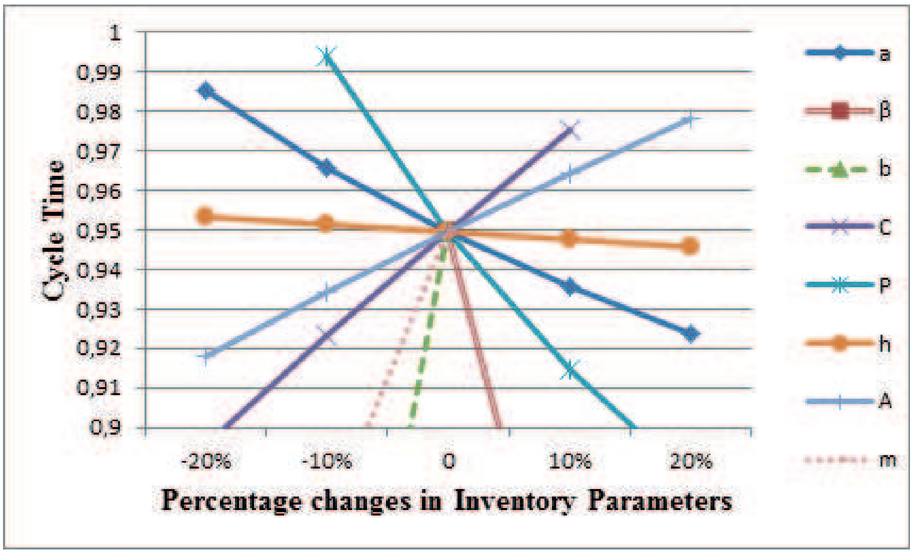

Figure 4. Sensitivity analysis for cycle time (T)

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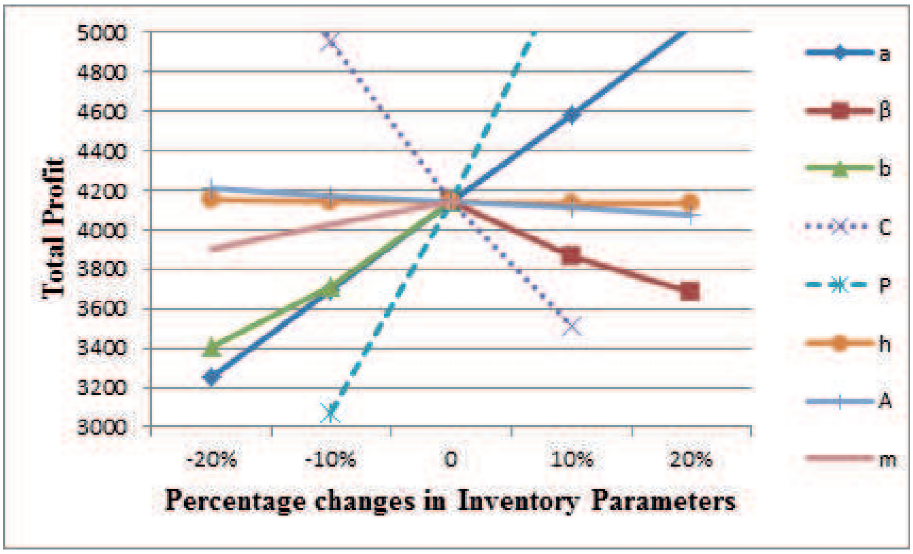

Figure 5. Sensitivity analysis for total profit

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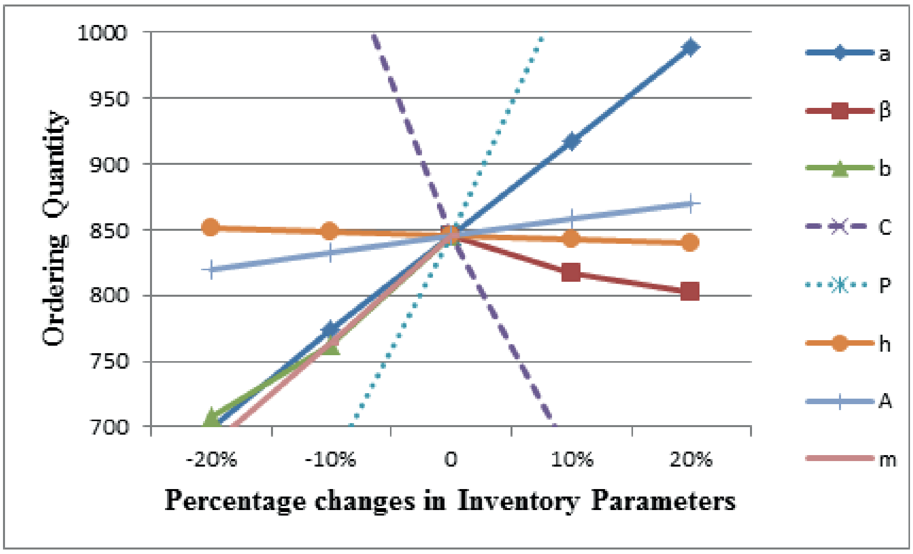

Figure 6. Sensitivity analysis for ordering quantity

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