Implementation of Ffowcs Williams and Hawkings Aeroacoustic Analogy in OpenFOAM

Konrad JAROSZ
AGH University of Science and Technology
Faculty of Mechanical Engineering and Robotics
Department of Power Systems and Environmental Protection Facilities
Al. Mickiewicza 30, 30-059 Kraków, jaroszk@agh.edu.pl

Ireneusz CZAJKA
AGH University of Science and Technology
Faculty of Mechanical Engineering and Robotics
Department of Power Systems and Environmental Protection Facilities
Al. Mickiewicza 30, 30-059 Kraków, iczajka@agh.edu.pl

Andrzej GOŁAŚ
AGH University of Science and Technology
Faculty of Mechanical Engineering and Robotics
Department of Power Systems and Environmental Protection Facilities
Al. Mickiewicza 30, 30-059 Kraków, ghgolas@cyf-kr.edu.pl

Abstract

This paper presents the development of post-processing aeroacoustics utility for OpenFOAM, based on Ffowcs Williams-Hawkings aeroacoustic analogy. Although the FH-W analogy is well known for almost 50 years, there is a lack of open-source software which is using it, hence decision to perform this implementation. This is the veryfirst version of utility, so only one formulation of FH-W were implemented. Presented application allows to compute far-field acoustic pressure from near field CFD solution. Validation is based on NASA Tandem Cylinder Case. Comparison of the results from simulation show fairly good agreement with experimental data.

Keywords: *aeroacoustics, CFD, FH-W analogy, CAA, OpenFOAM, FVM*

1. Introduction

Engineering problems like far-field noise prediction of aircraft landing gear or helicopter rotor it is still a challenge, despite there is a constant progress in computational aeroacoustics(CAA). Complexity of these cases and large distances to far-field, causes that accurate solution of acoustic fluctuations propagation inside computational domain would beineffective.

There is a way to bypass this difficulty by introducing some integral methods. Those methods are using data obtained from time-dependent CFD(computational fluid dynamics) solution or PIV measurements. That data should be accurate enough to capture all potential noise sources. The next step is to use anaeroacoustic analogy to propagate near-field results(sources)to the far-field observers. This paper is a try to extend the research conducted in [10].

It is worth mentioning that apart from the technical aspects of modeling of the sound distribution, more and more research uses the modeling of wave phenomena to the sound synthesis [11].

2. Lighthill equation

The Lighthill analogy[1][2] is applicable to unbounded, incompressible, low Mach number flow. These equations are derived from Navier-Stokes[7] equations, which are reorganised into inhomogeneous wave equation, in form presented below:

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j} T_{ij} \tag{1}$$

$$T_{ij} = \rho u_i u_j + p_{ij} - a_0^2 \rho \delta_{ij} \tag{2}$$

The source term from equation (1) and described in equation (2) has been named a Lighthill stress tensor. It contains acoustical sources, which are represented as a flow parameters. Mathematically speaking, equation (1) is a hyperbolic differential equation, which describes acoustic wave propagation with speed of sound a_0. Because of assumptions that were made, these analogy have some obvious limitations:

- propagation of sound is through unbounded domain,
- level of sound pressure is relatively small,
- acoustic wave have no influence on the flow.

So it is clear, that Lighthill analogy is applicable only on subsonic flows.

3. Ffowcs-Williams Hawkings analogy

FH-W analogy[3] is an extended version of Lighthill analogy. It introduces the so called source surfaces, which are taken into account when computing the sound pressure level at the observer. Those surfaces can be set as surfaces of solid body(impermeable) or as a any free surface located in domain(permeable). In contradiction to Lighthill analogy, FH-W analogy allows the motion of the bodies inside fluid domain, that fact extends its applicability to predict noise generated by rotors. Analogy is govern by equations below:

$$\begin{aligned} &\frac{1}{a_0}\frac{\partial^2(\rho-\rho_0)}{\partial t^2} - \nabla^2(\rho-\rho_0) = \frac{\partial}{\partial t}[Q_n \delta(f)] - \frac{\partial}{\partial x_i}[L_i \delta(f)] + \\ &+ \frac{\partial^2}{\partial x_i \partial x_j}[T_{ij} H(f)] \end{aligned} \tag{3}$$

Where Q_n and L_i are defined as:

$$Q_n = Q_i \hat{n}_i = [\rho_0 v_i + \rho(u_i - v_i)]\hat{n}_i \tag{4}$$

$$L_i = L_{ij} \hat{n}_i = [P_{ij} + \rho u_i (u_j - v_j)]\hat{n}_i \tag{5}$$

The source surface mentioned before(also called integration surface) is described as $f(x,t)=0$ and $\hat{n}_i = \nabla f$ is a unit normal vector pointed out from surface f. In equations (4) and (5) $v_i$denotes the velocity of surface *f*, while u_i is the velocity of the fluid at the integration surface. If the source surface is equal to the solid body surface then $u_i=v_i$.

In equation (5) there is a compressible stress tensor:

$$P_{ij} = (p - p_0)\delta_{ij} - \tau_{ij} \tag{6}$$

Because of the contribution of the last term of equation (6) to total acoustic power is relatively small, it can be neglected. Also we can assume that the disturbances of density outside the source surface are also small, so the term $(\rho\text{-}\rho_0)$ can be replaced by p', which is considered to be acoustic pressure.

4. Formulation 1A

For a complex geometry it is hard to find the direct solution of equation (3). Therefore some numerical formulations of FH-W analogy were introduced.One of them is formulation 1A proposed by Farassant[5][6]. It is suitable for moving solid bodies in fluid at rest. That formulation was developed to improve prediction of noise generated by helicopter rotor.

The acoustic pressure p'that is generated by solid body with subsonic velocity, measured by observer in position x and time t is given by:

$$p'(x,t) = p'_T(x,t) + p'_L(x,t) \tag{7}$$

$$\begin{aligned} 4\pi p'_T(x,t) &= \int_{f=0} \left[\frac{\dot{Q}_n + Q_{\dot{n}}}{r(1-M_r)^2} \right]_{ret} dS + \\ &+ \int_{f=0} \left[\frac{Q_n(r\dot{M}_r + a_0(M_r - M^2))}{r^2(1-M_r)^3} \right]_{ret} dS \end{aligned} \tag{8}$$

$$\begin{aligned} 4\pi p'_L(x,t) &= \int_{f=0} \left[\frac{\dot{L}_r}{r(1-M_r)^2} \right]_{ret} dS + \\ &+ \int_{f=0} \left[\frac{L_r - L_M}{r^2(1-M_r)^3} \right]_{ret} dS + \\ &+ \int_{f=0} \left[\frac{L_r(r\dot{M}_r + a_0(M_r - M^2))}{r^2(1-M_r)^3} \right]_{ret} dS \end{aligned} \tag{9}$$

Where M denotes Mach number of a source, with components $M_i=v_i/a_0$, the dot "·" means time derivative with respect to emission time τ. Other components of equations (8) and (9) are following:

$$M_r = M_i r_i \quad \dot{M}_r = \frac{\partial M_i}{\partial \tau} r_i$$
$$Q_n = Q_i \hat{n}_i \quad \dot{Q}_n = \frac{\partial Q_i}{\partial \tau} \hat{n}_i \quad Q_{\dot{n}} = Q_i \frac{\partial \hat{n}_i}{\partial \tau} \tag{10}$$
$$L_i = L_{ij} \hat{n}_i \quad \dot{L}_r = \frac{\partial L_i}{\partial r} \hat{r}_i \quad L_r = L_i \hat{r}_i \quad L_M = L_i M_i$$

Subscript *ret* means that the integral is evaluated at the emission time. The retarded time equation has a form presented below:

$$g = \tau_{ret} - t + \frac{r}{a_0} = 0 \tag{11}$$

Where $r = |x - y(ret)|$, and is a distance between observer and the source at the emission time.

5. Formulation GT

In case, that could be defined as flow inside wind tunnel, there is situation when both observer and source remain motionless. Only fluid has a velocity. Also there is a need to assume that mean flow velocity has a direction $+x_1$, which leads to $U_0 = (U_{01}, 0, 0)$. These situation is equivalent to situation when source and observer are in motion with velocity $-U_0$ but the fluid is at rest. With those assumptions there is an ability to use special case of formulation A1.

For source in subsonic, rectilinear and uniform motion equation (11) given by Garrick [4] simplifies to form:

$$\tau_{ret} = t = \frac{R}{a_0} \tag{12}$$

And distance between source and observer is given by:

$$R = \frac{-M_0(x_1 - y_1) + R^*}{\beta^2} \tag{13}$$

Where:

$$R^* = \sqrt{(x_1 - y_1)^2 + \beta((x_2 - y_2)^2 + (x_3 - y_3)^2)} \tag{14}$$

$$\beta = \sqrt{1 - M_0^2} \tag{15}$$

In this particular case R is an effective acoustic distance, rather than geometric. Components of unit distance vector are defined as:

$$\hat{R}_1 = \frac{-M_0 R^* + (x_1 - y_1)}{\beta^2 R} \quad \hat{R}_2 = \frac{(x_2 - y_2)}{R} \quad \hat{R}_3 = \frac{(x_3 - y_3)}{R} \quad M_R = M_i \hat{R}_i \tag{16}$$

Variables Q_n and L_i are identical as in equations (4) and (5), but in this formulation velocity of integration surface v_i is replaced by $-U_{0i}$, because all of the velocity components has to expressed in stationary reference frame. So equations (4) and (5) could be rewrite as:

$$Q_n = [\rho_0 U_{0i} + \rho(u_i + U_{0i})]\hat{n}_i \tag{17}$$

$$L_i = [P_{ij} + \rho u_i (u_j + U_{0j})]\hat{n}_i \tag{18}$$

In contradiction to formulation 1A, distance between source and observer is not a function of time, so R=*const* also as other variables which depend on time.Those variables which are not function of time could be computed at initial step.Also derivatives of those variables could be neglected.That leads to simplification of computation. Simplified equations (8) and (9) could be written as:

$$\begin{aligned} 4\pi p'_T(x,t) &= \int_{f=0} \left[\frac{\dot{Q}_n}{R(1-M_R)^2} \right]_{ret} dS + \\ &+ \int_{f=0} \left[\frac{Q_n + a_0(M_R - M^2)}{R^2(1-M_R)^3} \right]_{ret} dS \end{aligned} \tag{19}$$

$$\begin{aligned} 4\pi p'_L(x,t) &= \int_{f=0} \left[\frac{\dot{L}_R}{R(1-M_R)^2} \right]_{ret} dS + \\ &+ \int_{f=0} \left[\frac{L_R - L_M}{R^2(1-M_R)^3} \right]_{ret} dS + \\ &+ \int_{f=0} \left[\frac{L_R(M_R - M^2)}{R^2(1-M_R)^3} \right]_{ret} dS \end{aligned} \tag{20}$$

6. Numerical implementation

In the first version of the utility presented in this paper, the GT formulation were implemented. The decision were made to make this application working as a post-processing utility of OpenFOAM(open source FVM software). Due to fact that acoustic pressure measured at the observer is a function of time, the CFD simulation,which will provide input data, needs to be time dependent. Each cell of finite volume mesh will be treated as a separate acoustic source region.

The retarded time equation (12) could be resolved in 2 ways. In the first option, commonly called retarded time algorithm, receive time t is set, and then the emission time τ_{ret} has to be found, and finally the integrals are evaluated. Considering a numerical calculations this could be confusing, because of the possibility of not having input data at computed emission time.

The second approach is to set constant emission times, which in fact will be equal to CFD time steps, then appropriate receive times needs to be calculated. That algorithm was described in [6].

For purpose of these implementation, the second approach was chosen. In constant emission time algorithm, there is a need to interpolate calculated data of each source region at the same receive times. That is necessary to correctly sum noise deriving from all sources.

The last simplification in this version of utility, is to allow use only solid body surfaces(impermeable) as a source surfaces. It will reduce the computational complexity of utility. Equation (7) will be simplified to form:

$$p'(x,t) = p'_L(x,t) \tag{21}$$

Computations made with usage of theaeroacuostical analogies have that advantage, that observer could be located outside of computational domain. That allows to reduce size of computational mesh, and simulate only area of interest. But there is also a disadvantage, those analogies does not take into account the reflections of the acoustic wave. They are also "blind" to solid reflecting surfaces. The final decision, if use or not to use, always depend on user.

7. Validation of implementation

To check if implemented analogy works properly, some validation was performed.It was a CFD simulation of tandem cylinder case, which is well described in [8], also a experimental data are available[8]. Geometry and flow parameters of the test case are presented below. All microphones are located at the center plane of the span.

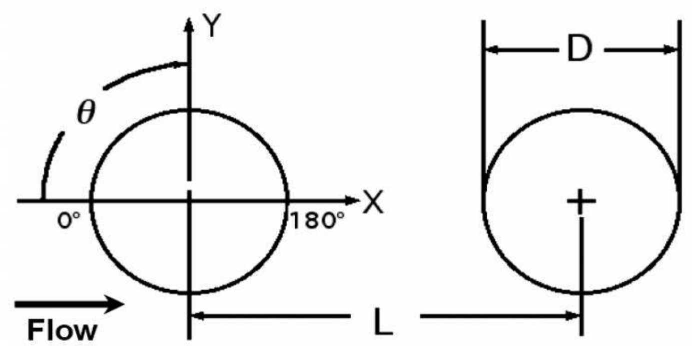

Figure 1. Configuration of test case

- D = 0.05715 [m] L=3.7 D Re=166000 M=0.128 (44 m/s)
- Span = 12 D
- Mic. A(-8.33D, 27.815D) Mic.B(9.11D, 32.49D) Mic.C(26.55D, 27.815D)

To obtain results, transient simulation with Spalart-Allmaras[7][9] turbulence model was performed. Due to fact that used solver demands Courant number lower than 1 and very fine quality of computational mesh (around 5 million of finite volume elements), time step value was $\Delta t=10^{-5}s$. Because $y+$ parameter value were lower than 1, no wall functions were used. Simulation results served as input to implemented utility. Computation took almost 2 weeks on Zeus HPC cluster (24 cores).

Results from microphone B are shown on Figure 2. There is slight difference between simulation and experimental data, but the dominant frequency is almost the same(around 170 Hz), with similar levels. Better results could be obtained with more accurate computational grid. But that needs more hardware resources to use, and also drastically extends simulation time.

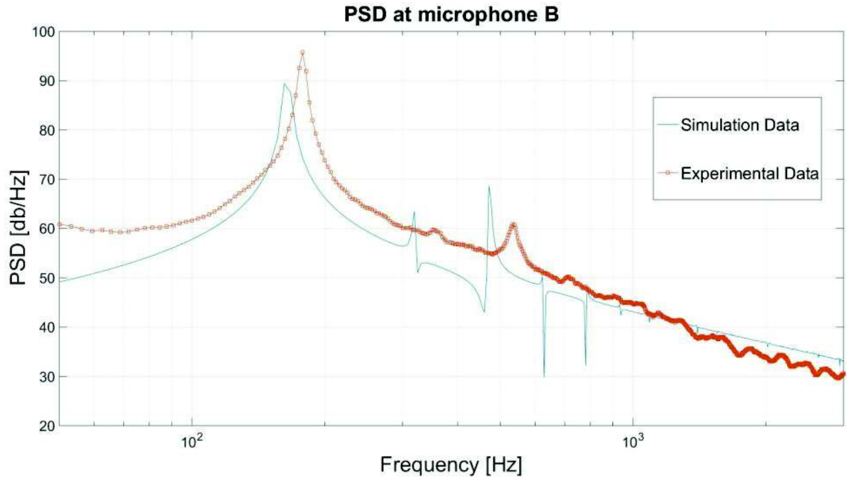

Figure 2. PSD at microphone B

8. Conclusions

Benchmark test that was conducted, shows that presented implementation of FH-W analogy works more or less properly. It is a desirable tool for predicting acoustic pressure at far-field observer. Acoustic analogies allows to compute acoustic pressure outside of numerical domain, what causes in significant reduce in computation time. The next will be an implementation of 1A formulation, which extend utility potential to permeable surfaces and more general cases, for example helicopter rotor noise.

Acknowledgments

This paper was supported in part by Cyfronet AGH infrastructure.

References

1. M. J. Lighthill, *On Sound Generated Aerodynamically I. General Theory*, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, **211**(1107) (1952) 564 – 587.
2. M. J. Lighthill, *On Sound Generated Aerodynamically II. Turbulence as a Source of Sound*, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, **222**(1148) (1954).
3. J. E. Ffowcs Williams, D. L. Hawkings, *Sound generation by turbulence and surfaces in arbitrary motion*, Proc. R. Soc. Lond. A, **264** (1969) 321 – 342.

4. I. E Garrick, E. W. Watkins, *A Theoretical Study of the Effect of Forward speed on the Free Space Sound Pressure Field Around Helicopters*, Tech. Rep. TR-1198, NACA, 1954.
5. F. Farassant, *Derivation of Formulation 1 and 1A of Farassant*, Tech. Rep. TM-2007-214853, NASA, 2007.
6. K S. Brentner, F.Farassant, *Modeling aerodynamically generated sound of helicopter rotors*, Prog. Aerospace Sci., **39** (2003) 83 – 120.
7. D. C. Wilocox, T*urbulence Modeling for CFD*, 2006.
8. D. P. Lockard, M. M Choudhari, M. R Khorrami, D. H Neuhart, F. V. Hutcheson, T. F Brooks, D. J. Stead, *Aeroacoustic Simulations of Tandem Cylinders with Subcritical Spacing*, AIAA-2008-2862, 2008.
9. P. R. Spalart, S. R. Allmaras, *A One-Equation Turbulence Model for Aerodynamic Flows*, AIAA Paper 92-0439, 1992.
10. I. Czajka, K. Jarosz, K. Suder-Dębska, *Modelling of an aerodynamic noise of a horizontal axis wind turbine using ANSYS/Fluent and OpenFOAM packages*, 7th forum acusticum 2014, Kraków 7-12.09.2014.
11. M. Pluta, B. Borkowski, I. Czajka, K. Suder-Dębska, *Sound synthesis using physical modeling on heterogeneous computing platforms*, Acta Physica Polonica. A, **128** (2015).