**Kudzys Antanas**
*KTU Institute of Architecture and Construction, Kaunas, Lithuania*

# Transformed conditional probabilities in the analysis of stochastic sequences

**Keywords**
safety margin, stochastic sequence, conditional probability, system safety

**Abstract**

The need to use unsophisticated probability-based approaches and models in the structural safety analysis of the structures subjected to annual extreme service, snow and wind actions is discussed. Statistical parameters of single and coincident two extreme variable actions and their effects are analysed. Monotone and decreasing random sequences of safety margins of not deteriorating and deteriorating members are treated, respectively, as ordinary and generalized geometric distributions representing highly-correlated series systems. An analytical analysis of the failure or survival probabilities of members and their systems is based on the concepts of transformed conditional probabilities of safety margin sequences whose statistically dependent cuts coincide with extreme loading situations of structures. The probability-based design of members exposed to coincident extreme actions is illustrated by a numerical example.

## 1. Introduction

The stochastic systems and their subsystems consist of some particular members representing the only possible failure mode. To particular members belong cross and oblique sections of tension, compression, flexural and torsional structures. The structural members (beams, slabs, columns, walls) of buildings consist of two or three design particular members and may be treated as auto systems representing multicriteria failure modes. An overloading of members during severe service and climate actions may provoke a failure of structures. Therefore, the requirements of design codes should be satisfied at all sections along structural members.

Structural failures and collapses in buildings and construction works can be caused not only by irresponsibility and gross human errors of designers, builders or erectors but also by some conditionalities of recommendations and directions presented in design codes and standards. A possibility to ensure objectively the safety degree of structures subjected to extreme service loads, wind gust and snow pressures or wave surfs is hardly translated into reality using the traditional deterministic design methods of partial safety factors in Europe or load and resistance factors in the USA.

It is understandable that probabilistic design approaches are inevitable for the calibration of partial factors. However, it should be more expedient to analyse the structural safety of particular members and their systems by probability-based methods. Regardless of efforts to improve and modify deterministic design approaches, it is inconceivable to fix a real reliability index of structures a failure domain of which changes with time. The time-dependent safety assessment and prediction of deteriorating members and systems using unsophisticated methods is a significant concern of researchers.

Despite of fairly developed up-to-date concepts of reliability, hazard and risk theories, including the general principles on reliability for structures [6], [7], [15], it is difficult to apply probability-based approaches in structural safety analysis. These approaches may be acceptable to designers and building engineers only under the indispensable condition that the safety performance of members and their systems may be considered in a simple and easy perceptible manner. In other words, probabilistic methods may be implanted into structural design practice only using unsophisticated mathematical models helping us to assess all uncertainties due to the features of resistances and action effects of structures.

This paper deals with probability-based safety analysis of deteriorating and not deteriorating members and their systems under extreme gravity and lateral (horizontal) actions using unsophisticated but fairly exact design models.

## 2. Time dependent safety margin

According to probability-based approaches (design level III), the time-dependent safety margin as the performance of deteriorating particular members may be presented as follows:

$$Z(t) = g[\boldsymbol{\theta}, \mathbf{X}(t)] = \theta_R R(t) - \theta_g S_g - \theta_{q_1} S_{q_1}(t) -$$

$$- \theta_{q_2} S_{q_2}(t) - \theta_w S_w(t) \;, \qquad (1)$$

where $\boldsymbol{\theta}$ is the vector of additional variables characterizing uncertainties of models which give the values of resistance $R$, permanent $S_g$, sustained $S_{q_1}$ and extraordinary $S_{q_2}$ service and extreme wind $S_w$ action effects of members (Figure 1, a). This vector may represent also the uncertainties of probability distributions of basic variables.

According to Rosowsky and Ellingwood [11], the annual extreme sum of sustained and extraordinary occupancy live action effects $S_q(t) = S_{q_1}(t) + S_{q_2}(t)$ can be modelled as an intermittent process and described by a Type 1 (Gumbel) distribution with the coefficient of variation $\boldsymbol{\delta} S_q = 0.58$, characteristic $S_{qk}$ and mean $S_{qm} = 0.47 S_{qk}$ values. Latter on Ellingwood and Tekie [4] recommended modelling extreme values of this sum during a 50 years period by a Type 1 distribution with the coefficient of variation $\boldsymbol{\delta} S_q = 0.25$ and mean value $S_{qm} = S_{qk}$.

It is proposed to model the annual extreme climate (wind and snow) action effects by Gumbel distribution law with the mean values equal to $S_{wm} = S_{wk} / (1 + k_{0.98} \boldsymbol{\delta} S_w)$ and $S_{sm} = S_{sk} / (1 + k_{0.98} \boldsymbol{\delta} S_s)$ [3, 6, 7, 13, 15]. According to meteorological data, the strong wind conditions are characterized by a small wind extreme velocity variation, i.e. $\boldsymbol{\delta} v \approx 0.1$. On the contrary, a large variation is characteristic of strong snow loading. Therefore, the coefficients of variation of wind and snow loads depending on the feature of a geographical area are equal to $\boldsymbol{\delta} w = 0.2 - 0.4$ and $\boldsymbol{\delta} s = 0.3 - 0.7$.

Probability distributions of material properties are close to a Gaussian distribution [3], [6], [9], [12]. Therefore, a normal distribution or a log-normal distribution may be convenient in resistance analysis models [5], [6], [7]. The permanent action effect $S_g$ can be described by a normal distribution law [4], [5], [6], [10], [12]. Thus, for the sake of design simplifications, it is expedient to present the expression (1) in the form:

$$Z(t) = R_c(t) - S(t), \qquad (2)$$

where the component process

$$R_c(t) = \theta_R R(t) - \theta_g S_g \,, \qquad (3)$$

may be considered as the conventional resistance of members which may be modelled by a normal distribution;

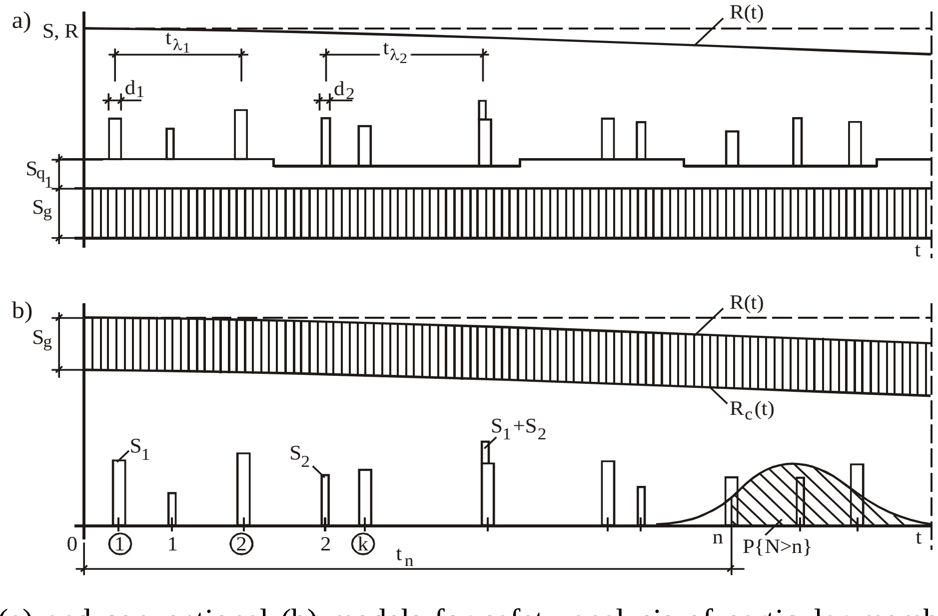


*Figure 1.* Real (a) and conventional (b) models for safety analysis of particular members (sections) of deteriorating structures

$$S(t) = \theta_q S_q(t) + [\theta_w S_w(t)], \quad (4)$$

$$S(t) = \theta_s S_s(t) + [\theta_w S_w(t)], \quad (5)$$

are the joint processes of two annual extreme action effects when floor and roof structures, respectively, are under consideration. The components in square brackets belonging to the wind action effect are used in design analysis of wind-resistant members and systems. The action effect $S_s(t)$ in Equation (5) is caused by extreme snow loads.

## 3. Safety margin sequences with independent cuts

The data presented in Section 2 allow us to model extreme service and climate action effects as intermittent rectangular pulse renewal processes. These time-variant intermittent action effects belong to persistent design situations in spite of the short period of extreme events being much shorter than the design working life of structures. When variable action effects may be treated as rectangular pulse processes, the time-dependent safety margin (2) may be expressed as the finite rank random sequence and written as:

$$Z_k = R_{ck} - S_k, \; k = 1, \; 2, \; 3, \; \ldots, \; n-1, \; n. \quad (6)$$

There

$$R_{ck} = \theta_R R_k - \theta_g S_g, \quad (7)$$

$$S_k = \theta_q S_{qk} + \theta_w S_{wk} \text{ or } S_k = \theta_s S_{sk} + \theta_w S_{wk}, \quad (8)$$

are the components of this non-stationary sequence; $n = \lambda t_n$ is the number of sequence cuts as critical events (situations) during design working life $t_n$ of members (*Figure 1*, *b*), where $\lambda = 1/t_\lambda$ is a mean renewal rate of these events per unit time when their return period is $t_\lambda$.

Usually the components $R_{ck}$ and $S_k$ are stochastically independent. The instantaneous survival probability of a member at *k*-th extreme situation (assuming that it was safe at the situations $1, 2, \ldots, k-1$) is:

$$\boldsymbol{P}_{sk} = \boldsymbol{P}\{R_{ck} > S_k\} = \int_0^\infty f_{R_{ck}}(x) F_{S_k}(x) dx, \quad (9)$$

where $f_{R_{ck}}(x)$ and $F_{S_k}(x)$ are the density and distribution functions of a conventional resistance $R_{ck}$ by (7) and an extreme action effect $S_k$ by (8). In this case, the instantaneous failure probability of members may be presented as:

$$\boldsymbol{P}_{fk} = (1 - \boldsymbol{P}_{sk}) \prod_{i=1}^{k-1} \boldsymbol{P}_{si}. \quad (10)$$

Thus, the random sequence of safety margins may be treated as a geometric distribution with ranked instantaneous survival probabilities of members $\boldsymbol{P}_{f1} < \boldsymbol{P}_{f2} < \ldots < \boldsymbol{P}_{fk} < \ldots \boldsymbol{P}_{f,n-1} < \boldsymbol{P}_{fn}$ calculated by Equation (10).

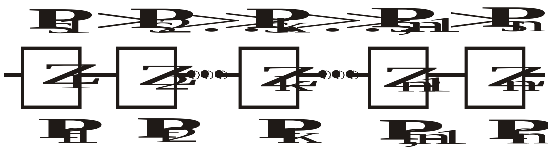

*Figure 2*. The scheme of series systems

Failure probabilities of structures should always be defined for some reference period $t_n$ or as a number of extreme events $n$ during this period. The scheme of series systems representing the safety margin sequences is given in *Figure 2*. When the cuts of rank random sequences are statistically independent, the cumulative distribution function and similarly a failure probability of members during their service life $[0, t_n]$ with $n$ extreme situations may be presented as follows:

$$\boldsymbol{P}_f = F_N(n) = \boldsymbol{P}\{N \le n\} = \sum_{k=1}^{n} \left[ (1 - \boldsymbol{P}_{sk}) \prod_{i=1}^{k-1} \boldsymbol{P}_{si} \right]$$

$$= 1 - \boldsymbol{P}_{s1} + (1 - \boldsymbol{P}_{s2}) \boldsymbol{P}_{s1} + \ldots + (1 - \boldsymbol{P}_{sk}) \prod_{i=1}^{k-1} \boldsymbol{P}_{si}$$

$$\ldots + (1 - \boldsymbol{P}_{sn}) \prod_{i=1}^{n-1} \boldsymbol{P}_{si} = 1 - \prod_{k=1}^{n} \boldsymbol{P}_{sk}. \quad (11)$$

When the resistance $R(t)$ is a time-invariant function and treated as a stationary process, the instantaneous survival probability $\boldsymbol{P}_{sk}$ by (9) is characterized by the same value for all cuts of the monotone sequence. In this case, Equation (11) becomes a cumulative distribution function of an ordinary geometric distribution as follows:

$$\boldsymbol{P}_f = F_N(n) = \boldsymbol{P}\{N \le n\} = 1 - (1 - \boldsymbol{P}_{fk})^{\,n}. \quad (12)$$

The failure probability of members may be approximated by Equations (11) and (12) only for situations in which a variance of the action effect $\boldsymbol{\sigma}^2 S$ is much larger than the value $\boldsymbol{\sigma}^2 R_c$ for their conventional resistance by (7).

## 4. Safety margin sequences with dependent cuts

In design practice, only recurrent extreme action effects caused by extraordinary service and climate loads may be treated as stochastically independent variables. Usually, random sequence cuts of the safety margin (6) are dependent. The value of a coefficient of autocorrelation $\rho_{kl}$ of sequence cuts depends on uncertainties of material properties and dimensions of members. This coefficient may be defined as:

$$\rho_{kl} = \rho(Z_k, Z_l) = Cov(Z_k, Z_l)/(\boldsymbol{\sigma} Z_k \times \boldsymbol{\sigma} Z_l), \qquad (13)$$

where $Cov(Z_k, Z_l)$ and $\boldsymbol{\sigma} Z_k, \boldsymbol{\sigma} Z_l$ are an autocovariance and standard deviations of the random safety margins $Z_k$ and $Z_l$.

The finite random sequence of member safety margins may be treated as a series stochastic system. The survival probability of highly correlated series systems consisting of two dependent elements can be expressed as follows:

$$\boldsymbol{P}\{Z_1 > 0 \cap Z_2 > 0\} = \boldsymbol{P}_{s1} \times \boldsymbol{P}\{Z_2 > 0 | Z_1 > 0\}$$

$$= \boldsymbol{P}_{s1} \times \boldsymbol{P}_{s2} + \rho^a (\boldsymbol{P}_{s2} - \boldsymbol{P}_1 \times \boldsymbol{P}_2), \qquad (14)$$

where $a \approx 4.5/(1 - 0.98\rho_{12})$ is the bond index of survival probabilities of second-order series systems. The data calculated by (14) and computed by the complex numerical integration method presented by Ahammed and Melchers [1] are very close. Thus, a conditional probability $\boldsymbol{P}\{Z_2 > 0 | Z_1 > 0\}$ may be transformed to a probability $\boldsymbol{P}_{s2}\left[1 + \rho^a \left(\frac{1}{\boldsymbol{P}_{s1}} - 1\right)\right]$.

Therefore, Equation (14) may be presented in the form:

$$\boldsymbol{P}\{Z_1 > 0 \cap Z_2 > 0\} \approx \boldsymbol{P}_{s1} \times \boldsymbol{P}_{s2} \times \left[1 + \rho_{12}^a \left(\frac{1}{\boldsymbol{P}_{s1}} - 1\right)\right] \qquad (15)$$

For not deteriorating structures, a member resistance is a time-invariant fixed random function the numerical values of which are random only at the beginning of a process. Therefore, the coefficient of correlation (13) of monotone sequence cuts may be expressed as:

$$\rho_{kl} = 1/\left(1 + \boldsymbol{\sigma}^2 S_k / \boldsymbol{\sigma}^2 R_c\right). \qquad (16)$$

When the monotone rank sequence of safety margins consists of $n$ dependent elements, a failure probability of members is:

$$\boldsymbol{P}_f = \boldsymbol{P}\{N \le n\} = \boldsymbol{P}\left\{\bigcup_{k=1}^{n} Z_k \le 0\right\} = 1 - \boldsymbol{P}\left\{\bigcap_{k=1}^{n} Z_k > 0\right\}$$

$$\approx 1 - \boldsymbol{P}_{sk}^n \left[1 + \rho_{kl}^a \left(\frac{1}{\boldsymbol{P}_{sk}} - 1\right)\right]^{n-1}. \qquad (17)$$

When a ratio of variances $\boldsymbol{\sigma}^2 S_k / \boldsymbol{\sigma}^2 R_c > 1$, the coefficient $\rho_{kl}^a \approx 0$ and the failure probability (17) becomes $\boldsymbol{P}_f = 1 - (1 - \boldsymbol{P}_{fk})^n$ as it is expressed by Equation (12).

A long-term survival probability of not deteriorating members is:

$$\boldsymbol{P}_s = 1 - \boldsymbol{P}_f = \boldsymbol{P}_{sk}^n \left[1 + \rho_{kl}^a \left(\frac{1}{\boldsymbol{P}_{sk}} - 1\right)\right]^{n-1}. \qquad (18)$$

The decreasing rank sequence of safety margins of deteriorating members may be treated as a generalized geometric distribution. Similar to Equation (17), the failure probability of these members as series systems may be calculated by the formula:

$$\boldsymbol{P}_f = \boldsymbol{P}\{N \le n\} \approx 1 - \prod_{k=1}^{n} \boldsymbol{P}_{sk} \left[1 + \rho_{n\ldots1}^a \left(\frac{1}{\boldsymbol{P}_{s,n-1}} - 1\right)\right]$$

$$\ldots \times \left[1 + \rho_{k\ldots1}^a \left(\frac{1}{\boldsymbol{P}_{s,k-1}} - 1\right)\right]$$

$$\ldots \times \left[1 + \rho_{21}^a \left(\frac{1}{\boldsymbol{P}_{s1}} - 1\right)\right], \qquad (19)$$

where the transformed rank coefficient of correlation is

$$\rho_{k\ldots1} = (\rho_{k,k-1} + \rho_{k,k-2} + \ldots + \rho_{k2} + \rho_{k1})/(k-1) \quad (20)$$

The long-term survival probability of deteriorating members $\boldsymbol{P}_s = 1 - \boldsymbol{P}_f$, where the probability $\boldsymbol{P}_f$ is given in (19).

The presented method of transformed conditional probabilities may also be successfully used in the reliability analysis of random systems consisting of individual components and characterizing different failure modes of structures. In this case, it is expedient to base the structural safety analysis of systems on the ranked survival probabilities of their members as: $\boldsymbol{P}_{s1} > \boldsymbol{P}_{s2} > \ldots > \boldsymbol{P}_{sk} > \ldots > \boldsymbol{P}_{s,n-1} > \boldsymbol{P}_{sn}$ (*Figure 2*). A rank correlation matrix of systems is constructed taking into account this analysis rule.

## 4. The system of safety margin sequences

Due to the complexity of mathematical models, it is rather difficult to assess and predict a failure probability of structures subjected to two and more coincident recurrent and different by nature extraordinary actions. The methods based on the Markov-chain model and Turkstra's rule [14] may be quite unacceptable in a probabilistic analysis of not only deteriorating but also not deteriorating members and their systems. The Markov-chain model may be quite inaccurate for reliability analysis of members exposed to multiple combination of action-effect processes [12]. The Turkstra's rule may be assumed only in the case when the principal extreme load is strongly dominant [10].

Failure probabilities of members may be computed by modified numerical integration methods. It is suggested to use the theoretical expression of the cumulative distribution function of the maximum intensity of two load processes [10], the load overlap method [12] and the improved upper bounding techniques [13]. It leads to sufficiently accurate values but it is hard to realize these recommendations in engineering practice.

The need to simplify a reliability analysis of deteriorating structures is especially urgent. In any analysis case, it must be taken into account that a member failure caused by two statistically independent extreme action effects may occur not only in the case of their coincidence but also when the value of one out of two effects is extreme. Therefore, three finite random sequences of safety margins should be considered:

$$M_{1k} = R_{ck} - S_{1k},\ k = 1,\ 2,\ \ldots,\ n_1, \tag{21}$$

$$M_{2k} = R_{ck} - S_{2k},\ k = 1,\ 2,\ \ldots,\ n_2, \tag{22}$$

$$M_{3k} = R_{ck} - S_{3k},\ k = 1,\ 2,\ \ldots,\ n_3. \tag{23}$$

There $S_{3k} = S_{1k} + S_{2k}$ is the joint action effect, the recurrence number of which during the period of time $[0, t_n]$ may be calculated by the equation:

$$n_3 = t_n(d_1 + d_2)\ \lambda_1\lambda_2, \tag{24}$$

where $d_1$, $d_2$ and $\lambda_1$, $\lambda_2$ are durations and renewal rates of extreme actions [8].

Mostly, the duration $d_q$ of annual extreme gravity service loads is from 1 to 3 days. The durations of annual extreme snow and wind loads, respectively, are: $d_s = 14$-28 days and $d_w = 8$-12 hours. The renewal rates of these actions are: $\lambda_q = \lambda_s = \lambda_w = 1$/year. Therefore, for 50 years reference period, the recurrence numbers of extreme actions are: $n_{qw} = 0.2$-0.5 and $n_{sw} = 2$-4.

When probability distributions of random variables $X$ and $Y$ obey a Gumbel distribution law, the bivariate density function of the random variable $Z = X + Y$ may be presented in the form:

$$f_z(z) = \int_{-\infty}^{\infty} f_x\left(z - y, X_m - 0.45\boldsymbol{\sigma}X\right) \times f_y\left(y, Y_m - 0.45\boldsymbol{\sigma}Y\right)dy, \tag{25}$$

where $X_m$, $Y_m$ and $\boldsymbol{\sigma}X$, $\boldsymbol{\sigma}Y$ are means and standard deviations of these variables.

Taking into account that $\boldsymbol{\sigma}^2 Z = \boldsymbol{\sigma}^2 X + \boldsymbol{\sigma}^2 Y$ is the variance of bivariate probability distribution, the joint density function may be expressed as:

$$f_Z(z) \approx f_Z(z, a_z), \tag{26}$$

$$a_z = X_m + Y_m - 0.19(\boldsymbol{\sigma}X + \boldsymbol{\sigma}Y) - 0.45(\boldsymbol{\sigma}^2 X + \boldsymbol{\sigma}^2 Y)^{1/2}.$$

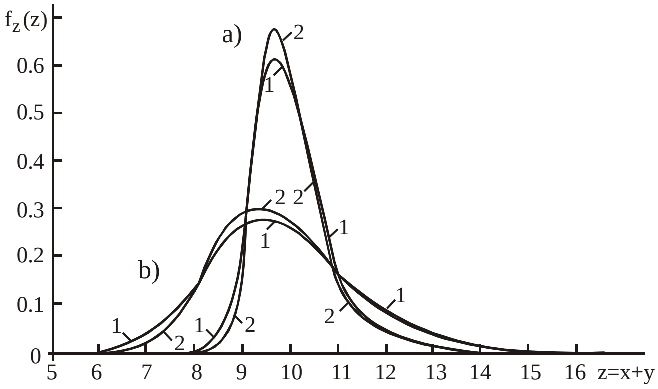


*Figure 3.* Bivariate density functions calculated by Equations (25) – 1 and (26) – 2: the coefficients of correlation $\boldsymbol{\delta}X = \boldsymbol{\delta}Y = 0.10$ (a) and 0.224 (b)

The probability density curves of joint extreme variable $Z = X + Y$ are given in Figure 3. It is not difficult to ascertain that the difference between the values computed by Equations (25) and (26) is fairly small. Besides, the upper tails of both density curves coincides. Therefore, in design practice it is expedient to use the conventional bivariate distribution function of two independent extreme action effects with the mean $S_{3k,m} = S_{1k,m} + S_{2k,m}$ and the variance $\boldsymbol{\sigma}^2 S_{3k} = \boldsymbol{\sigma}^2 S_{1k} + \boldsymbol{\sigma}^2 S_{2k}$.

## 6. Numerical example

The knee-joints of not deteriorating concrete frames of reliability class RC2 are under exposure of shear forces during 50 years period (*Figure 4*). The shear resistance of knee-joints is expressed as: $R = 0.068bhf_c$. The characteristic, design and mean values of the concrete compressive strength and shear resistance of knee-joints are:

$$f_{ck} = 30 \text{ MPa}, \; f_{cd} = 20 \text{ MPa}, \; f_{cm} = 38 \text{ MPa};$$

$$R_k = 306 \text{ kN}, \; R_d = 204 \text{ kN}, \; R_m = 387.6 \text{ kN}.$$

The variance of shear resistance of knee-joints is:

$$\boldsymbol{\sigma}^2 R = (0.128 \times 387.6)^2 = 2461.4 \text{ (kN)}^2.$$

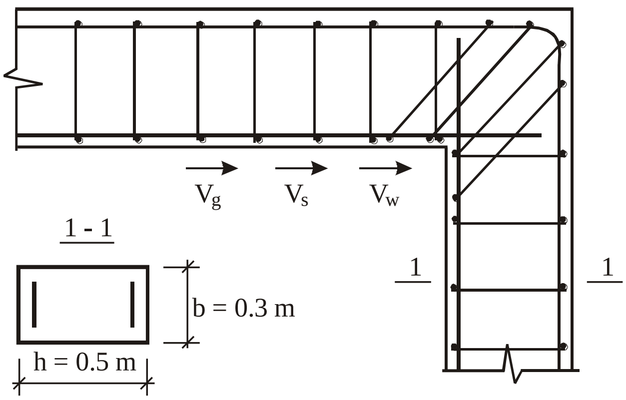


*Figure 4*. The knee-joint of concrete frames

The characteristic and design values of shear forces caused by permanent, snow and wind loads are:

$$V_{gk} = 77.72 \text{ kN},$$

$$V_{sk} = V_{wk} = 38.86 \text{ kN};$$

$$V_{gd} = 77.72 \times 1.35 = 104.92 \text{ kN},$$

$$V_{sd} = 38.86 \times 1.5 = 58.29 \text{ kN},$$

$$V_{wd} = 38.86 \times 0.7 \times 1.5 = 40.8 \text{ kN}.$$

Thus, the joint design shear force

$$V_d = V_{gd} + V_{sd} + V_{wd} = 204 \text{ kN} = R_d.$$

Therefore, according to deterministic calculation data, the frame knee-joints are reliable.

The coefficients of variation, means and variances of these extreme shear forces are:

$$\boldsymbol{\delta} V_g = 0.1,$$

$$V_{gm} = V_{gk} = 77.72 \text{ kN},$$

$$\boldsymbol{\sigma}^2 V_g = 60.4 \text{ (kN)}^2;$$

$$\boldsymbol{\delta} V_s = 0.6,$$

$$V_{sm} = V_{sk} / (1 + k_{0.98} \boldsymbol{\delta} V_s) = 15.21 \text{ kN},$$

$$\boldsymbol{\sigma}^2 V_s = 83.25 \text{ (kN)}^2;$$

$$\boldsymbol{\delta} V_w = 0.3,$$

$$V_{wm} = V_{wk} / (1 + k_{0.98} \boldsymbol{\delta} V_w) = 21.86 \text{ kN};$$

$$\boldsymbol{\sigma}^2 V_w = 43.0 \text{ (kN)}^2.$$

The parameters of additional variables are:

$$\theta_{Rm} = 1.0,$$

$$\boldsymbol{\delta}\theta_R = 0.1;$$

$$\theta_{Vm} = 1.0,$$

$$\boldsymbol{\delta}\theta_g = \boldsymbol{\delta}\theta_s = \boldsymbol{\delta}\theta_w = 0.1,$$

$$\boldsymbol{\delta}\theta_{sw} = 0.15.$$

Thus, the variances of revised shear forces are:

$$\boldsymbol{\sigma}^2(\theta_g V_g) = 120.8 \text{ (kN)}^2,$$

$$\boldsymbol{\sigma}^2(\theta_s V_s) = 85.56 \text{ (kN)}^2,$$

$$\boldsymbol{\sigma}^2(\theta_w V_w) = 47.8 \text{ (kN)}^2,$$

$$\boldsymbol{\sigma}^2(\theta_{sw} V_{sw}) = 157.17 \text{ (kN)}^2.$$

The parameters of conventional shear resistance (3) are:

$$R_{cm} = 387.6 - 77.72 = 309.9 \text{ kN},$$

$$\sigma^2 R_c = 1.0 \times 2461.4 + 387.6^2 \times 0.01$$
$$+ 120.8 = 4084.6 \text{ (kN)}^2.$$

According to (16), the coefficients of autocorrelation of the safety margins $Z_w = R_c - V_w$, $Z_s = R_c - V_s$ and $Z_{sw} = R_c - V_s - V_w$ of considered knee-joints are:

$$\rho_{w,kl} = 0.9884,$$

$$\rho_{s,kl} = 0.9795,$$

$$\rho_{sw,kl} = 0.9629.$$

The recurrence number of joint action effect $V_s + V_w$ calculated by Equation (24) is:

$$n_3 = 50 \; [21/365 + 12/(24 \times 3.65)] \; 1 \times 1 = 2.945.$$

According to (9), the instantaneous survival probabilities of members are:

$$\boldsymbol{P}_{sk,w} = 0.99999617,$$

$$\boldsymbol{P}_{sk,s} = 0.99999728,$$

$$\boldsymbol{P}_{sk,sw} = 0.9999837.$$

Therefore, according to (18), the partial long-term survival probabilities of analysed knee-joints are:

$$\boldsymbol{P}_{sw} = 0.9999717,$$

$$\boldsymbol{P}_{ss} = 0.9999710,$$

$$\boldsymbol{P}_{s,sw} = 0.9999747.$$

According to (13), the coefficients of cross-correlation of safety margins are:

$$\rho_{sw} = 0.9839,$$

$$\rho_{w,sw} = 0.9871,$$

$$\rho_{s,sw} = 0.9914.$$

From Equation (19), the total survival probability of knee-joints is:

$$\boldsymbol{P} = 0.9999747 \times 0.9999717 \times 0.9999710$$

$$\times \left[1 + 0.98767^{11.84}\left(\frac{1}{0.9999717} - 1\right)\right]$$

$$\times \left[1 + 0.9871^{11.73}\left(\frac{1}{0.9999747} - 1\right)\right] = 0.9999635.$$

It corresponds to the reliability index $\beta = 3.97 > \beta_{\min} (= 3.8)$ [5].

Despite high-correlated cuts of the safety margin sequences $Z_w$, $Z_s$ and $Z_{sw}$ of knee-joints, considerable differences among their instantaneous and long-term survival probabilities are corroborated.

The reliability verification of knee-joints of concrete frames by the deterministic partial factor method and probability-based approaches practically gave the same results.

## 7. Conclusion

When the system may be subjected to annual extreme service and climate actions, it is expedient to express its member performance processes by finite random sequences of safety margins, the dependent cuts of which coincide with the extreme loading situations of structures. Therefore, the generalized geometric distribution as the decreasing stochastic sequence may be successfully used in failure or survival probability analysis of highly correlated series systems. It leads to considerable perfections of probability-based analysis of deteriorating structures subjected to recurrent single and coincident actions as intermittent rectangular pulse renewal processes. A Gumbel distribution law may be used not only for joint sustained and extraordinary variable service loads but also for the sum of annual extreme action effects.

For the sake of simplifications of probabilistic time-dependent safety analysis of members, it is recommended to use design models with their conventional resistances and correlated sequence cuts of safety margins representing a variety of load combinations. The presented unsophisticated probability-based approaches and models may stimulate engineers having minimum appropriate skills to use full probabilistic methods in their engineering practice more courageously and effectively. It should be one more remedy in the struggle against deterministic approaches in the structural design.

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