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TEMPERATURE DISTRIBUTION IN A BRAKE DISC WITH VARIABLE CONTACT PRESSURE

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Abstract: The aim of this study was to investigate the influence of the time of pressure increase during single braking on the temperature in a brake disc. The case of linear pressure increase from zero to nominal value in the initial stage of braking and maintaining this value to standstill was considered. The time distribution of the sliding velocity of frictional elements was determined from the differential equation of motion with the initial condition. Based on the time distributions of pressure and sliding velocity, the intensity of the frictional heat flux, which affects on the disc surface, was determined. Spatio-temporal distribution of the temperature in a brake disc was found from analytical solution of the heat conduction boundary–value problem for semi–space heated on the outer surface heat flux with known *a priori* intensity. The numerical analysis conducted allowed to determine engineering equation, which describes relation between maximum temperature and the time of pressure increase.

Keywords: braking, frictional heating, temperature, brake disc.

Nomenclature: a – effective depth of the heat penetration [m]; A_a – nominal area of the contact surface [m^2]; erf(x) – Gauss error function; erfc(x) = 1 – erf(x) – complementary error function; $\mathrm{ierfc}(x) = \pi^{-1/2}\exp(-x^2) - x\,\mathrm{erfc}(x)$ – integral of the complementary error function; f – friction coefficient; F – friction force [N]; $H(x)$ – Heaviside step function; K – thermal conductivity [W K^{-1} m^{-1}]; k – thermal diffusivity [m^2 s^{-1}]; p – contact pressure [Pa]; p_0– nominal contact pressure [Pa]; q – intensity of the frictional heat flux [W m^{-2}]; t – time [s]; t_m – time of pressure increase [s]; t_s – braking time [s]; T – temperature, [K]; T^* – dimensionless temperature; T_0 – temperature scaling factor [K]; T_a – ambient temperature [K]; W_0 – initial kinetic energy [J]; V – velocity sliding [m s^{-1}]; V_0 – initial velocity [m s^{-1}]; τ – dimensionless time, τ_m – dimensionless time of pressure increase; τ_s – dimensionless braking time; ζ – dimensionless spatial coordinate.

Introduction

Temperature field in frictional elements of braking systems is the subject of long–term research and analysis. Knowledge of its distribution is a priority during design of the brake mechanism. Due to high costs and difficulty in performing experimental research, the temperature is estimated from the analytical solutions of the thermal problem of friction. Experimental research have demonstrated, that during a single, rapid braking with high initial velocity about 95% of the heat pervade to the frictional elements in perpendicular direction to the friction surface [12], therefore considered thermal problem of friction is often one-dimensional [4, 5]. It was demonstrated that the temperature values obtained analytically are sufficiently compatible with experimental results [6].
Analysis of the temperature fields in a brake disc replaced by semi–space during braking with constant deceleration and with constant or linearly increasing contact pressure were conducted in articles, respectively [10, 11]. In this study the basis of the analytical calculations of temperature distribution are the differential equation of motion with initial condition and one–dimensional heat conduction boundary–value problem. Evolution of the contact pressure was determined based on the approximation of the general equation [3]. At the beginning of the braking process pressure increases linearly from zero to nominal value in the time moment t_m, next it maintains this value to standstill. Conducted numerical analysis investigated the influence of the t_m on temperature distribution in a brake disc.

Statement to the problem

Distribution of the pressure on the contact surface between disc and pad depends on specified external load and type of braking system. The general equation of the contact pressure p in time t has the following form [3]:

$$p(t) = p_0 p^*(t),\ p^*(t) = 1 - \exp(-t/t_m),\ 0 \le t \le t_s. \qquad (1)$$

Using the power series expansion of the exponential function and reducing it to the first two elements, from formula (1) we found:

$$p^*(t)=\frac{t}{t_m}H(t_m-t)+H(t-t_m), 0\le t\le t_s. \tag{2}$$

Relative sliding velocity of the disc was determined from the differential equation of motion with initial condition [6]:

$$\frac{2W_0}{V_0^2}\frac{dV(t)}{dt}=-2F(t),\ 0\le t\le t_s. \tag{3}$$

$$V(0)=V_0, \tag{4}$$

where

$$F(t)=F_0p^*(t), F_0=fp_0A_a, \tag{5}$$

and braking time t_s was found using the stop condition:

$$V(t_s)=0\cdot \tag{6}$$

Solution to the ordinary differential equation of first order (3), which fulfills the initial condition (4), has the following form:

$$V(t)=V_0V^*(t),\ V^*(t)=1-\frac{1}{t_s^0}\int_0^t p^*(s)ds, 0\le t\le t_s, \tag{7}$$

where

$$t_s^0=\frac{W_0}{F_0V_0{}^2}, \tag{8}$$

is a braking time with immediate $(t_m\to 0)$ pressure increase to nominal value $p^*(t)=1$ and with linear velocity reduction $V^*(t)=1-t/t_s^0$, $0\le t\le t_s^0$. Taking into account in the solution (7) the time distribution of the pressure (2), we obtained:

$$V^*(t)=1-\frac{1}{t_s^0}\{V_m(t)H(t_m-t)+[V_m(t_m)+V_s(t)]H(t-t_m)\}, 0\le t\le t_s, \tag{9}$$

where

$$V_m(t)=\frac{t^2}{2t_m},\ V_s(t)=t-\frac{t_m}{2}. \tag{10}$$

Substituting the solution (9), (2.10) to the stop condition (2.6), the braking time t_s was determined:

$$t_s=t_s^0+0{,}5t_m\cdot \tag{11}$$

Specific power of friction during braking is equal to [7]:

$$q(t)=fp(t)V(t),\ 0\le t\le t_s. \tag{12}$$

Substituting to formula (12) function $p(t)$ (1), (2) and $V(t)$ (7)–(10), we obtained:

$$q(t)=q_0q^*(t), 0\le t\le t_s, \tag{13}$$

where

$$q_0=fp_0V_0,\ q^*(t)=q_m(t)H(t_m-t)+q_s(t)H(t-t_m), \tag{14}$$

$$q_m(t)=\frac{t}{t_m}\left(1-\frac{t^2}{2t_s^0t_m}\right),\ q_s(t)=1-\frac{1}{t_s^0}\left(t-\frac{1}{2}t_m\right). \tag{15}$$

Graphs of the functions $p^*(t)$ (2), $V^*(t)$ (9), (10) and $q^*(t)$ (14), (15) are presented in Fig. 1.

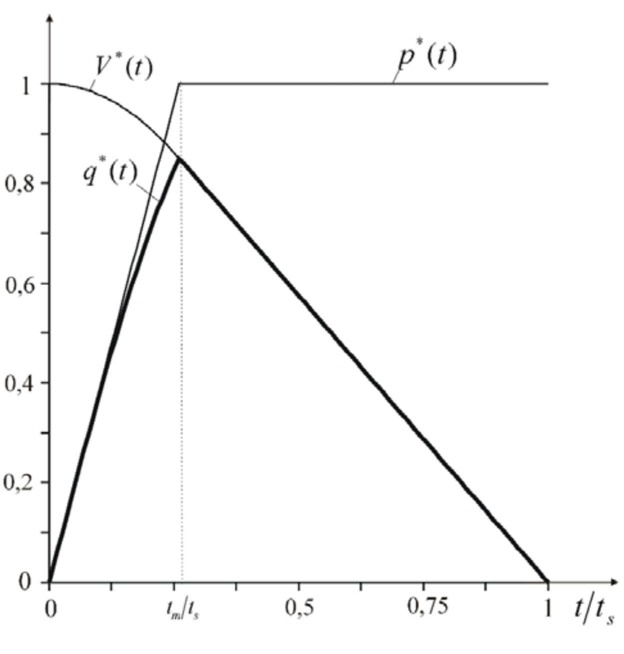

Fig. 1 Time distributions of: specific power of friction q^* (bolded line), contact pressure p^* and velocity V^* with $t_m=0{,}26t_s$, $t_s^0=0{,}87t_s$.

It was assumed that, the material of the disc is homogeneous, convective cooling has negligible influence on the temperature, and gradients of temperature in radial and circumferential directions were neglected. Taking into account abovementioned assumptions, the heating process of the brake disc was replaced by a simplified one-dimensional model – semi-space $z\ge 0$ heated on the outer surface $z=0$ by the heat flux with intensity $q(t)$ (13)–(15). Temperature distribution in semi–space was found from the solution to the following boundary–value heat conduction problem:

$$\frac{\partial^2 T(z,t)}{\partial z^2} = \frac{1}{k}\frac{\partial T(z,t)}{\partial t},\ z>0,\ 0\le t\le t_s, \tag{16}$$

$$K\left.\frac{\partial T(z,t)}{\partial z}\right|_{z=0} = -q(t),\ 0\le t\le t_s, \tag{17}$$

$$T(z,t)\to 0,\ z\to\infty,\ 0\le t\le t_s, \tag{18}$$

$$T(z,0)=T_a,\ z\ge 0. \tag{19}$$

Applying the following dimensionless variables and parameters:

$$\zeta=\frac{z}{a},\ \tau=\frac{kt}{a^2},\ \tau_m=\frac{kt_m}{a^2}\ \tau_s=\frac{kt_s}{a^2},\ \tau_s^0=\frac{kt_s^0}{a^2}\ T_0=\frac{q_0 a}{K},\ T^*=\frac{T-T_a}{T_0}, \tag{20}$$

considered boundary–value problem (16)–(19) was written in the following dimensionless form:

$$\frac{\partial^2 T^*(\zeta,\tau)}{\partial \zeta^2}=\frac{1}{k^2}\frac{\partial T^*(\zeta,\tau)}{\partial \tau},\ \zeta>,0\le\tau\le\tau_s, \tag{21}$$

$$\left.\frac{\partial T^*(\zeta,\tau)}{\partial\zeta}\right|_{\zeta=0}=-q^*(\tau),\ 0\le\tau\le\tau_s, \tag{22}$$

$$T^*(\zeta,\tau)\to 0,\ \zeta\to\infty,\ 0\le\tau\le\tau_s, \tag{23}$$

$$T^*(\zeta,0)=0,\ \zeta\ge 0, \tag{24}$$

where, having regard to relations (14), (15), we have:

$$q^*(\tau)=q_m(\tau)H(\tau_m-\tau)+q_s(\tau)H(\tau-\tau_m), \tag{25}$$

$$q_m(\tau)=\frac{\tau}{\tau_m}\left(1-\frac{\tau^2}{2\tau_s^0\tau_m}\right),\ q_s(\tau)=1-\frac{1}{\tau_s^0}\left(\tau-\frac{1}{2}\tau_m\right). \tag{26}$$

Solution to the problem

Solution to the boundary-value problem (21)–(26) was found from the following Duhamel's theorem [8]:

$$T^*(\zeta,\tau)=\int_0^\tau q^*(s)\frac{\partial}{\partial\tau}T^{*(0)}(\zeta,\tau-s)ds,\ \zeta\ge 0,\ 0\le\tau\le\tau_s, \tag{27}$$

where

$$T^{*(0)}(\zeta,\tau)=2\sqrt{\tau}\,\mathrm{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right),\ \zeta\ge 0,\ 0\le\tau\le\tau_s, \tag{28}$$

is the solution of this problem with $q^*(\tau)=1$ [2]. Substituting to the formula (27) the following partial derivative [1]:

$$\frac{\partial}{\partial\tau}\left[T^{*(0)}(\zeta,\tau-s)\right]=\frac{e^{-\left(\frac{\zeta}{2\sqrt{\tau-s}}\right)^2}}{\sqrt{\pi(\tau-s)}}, \tag{29}$$

we obtained:

$$T^*(\zeta,\tau)=T_m(\zeta,\tau)H(\tau_m-\tau)+[T_m(\zeta,\tau_m)+T_s(\zeta,\tau)]H(\tau-\tau_m),\zeta\ge 0,\ 0\le\tau\le\tau_s, \tag{30}$$

where

$$T_m(\zeta,\tau)=\frac{1}{\sqrt{\pi}\,\tau_m}\int_0^\tau\left(s-\frac{s^3}{2\tau_s^0\tau_m}\right)\frac{e^{-\left(\frac{\zeta}{2\sqrt{\tau-s}}\right)^2}}{\sqrt{\tau-s}}ds,\quad T_s(\zeta,\tau)=\frac{1}{\sqrt{\pi}}\int_{\tau_m}^\tau\left(1+\frac{\tau_m}{2\tau_s^0}-\frac{s}{\tau_s^0}\right)\frac{e^{-\left(\frac{\zeta}{2\sqrt{\tau-s}}\right)^2}}{\sqrt{\tau-s}}ds. \tag{31}$$

The function $T_m(\zeta,\tau)$ (31) was written in the form:

$$T_m(\zeta,\tau)=\frac{1}{\sqrt{\pi}\,\tau_m}\left[\tau\int_0^\tau\frac{e^{-\left(\frac{\zeta}{2\sqrt{\tau-s}}\right)^2}}{\sqrt{\tau-s}}ds-\int_0^\tau\sqrt{\tau-s}\,e^{-\left(\frac{\zeta}{2\sqrt{\tau-s}}\right)^2}ds-\frac{1}{2\tau_s^0\tau_m}\int_0^\tau\frac{s^3}{\sqrt{\tau-s}}e^{-\left(\frac{\zeta}{2\sqrt{\tau-s}}\right)^2}ds\right],\zeta\ge 0,0\le\tau\le\tau_m, \tag{32}$$

and using substitution $x=1/\sqrt{\tau-s}$, we obtained:

$$T_m(\zeta,\tau)=\frac{1}{\sqrt{\pi}\tau_m}\left[\left(2\tau-\frac{\tau^3}{\tau_s^0\tau_m}\right)L_2(\zeta,\tau)+\left(\frac{3\tau^2}{\tau_s^0\tau_m}-2\right)L_4(\zeta,\tau)-\frac{3\tau}{\tau_s^0\tau_m}L_6(\zeta,\tau)+\frac{1}{\tau_s^0\tau_m}L_8(\zeta,\tau)\right],\zeta\ge 0,\ 0\le\tau\le\tau_m, \tag{33}$$

where

$$L_k(\zeta,\tau)=\int_{\frac{1}{\sqrt{\tau}}}^{\infty}e^{-\left(\frac{\zeta}{2}\right)^2x^2}\frac{dx}{x^k},k=2,4,6,8. \tag{34}$$

Exploiting the following recurrence relation [9]:

$$\int\frac{e^{-(ax)^2}}{x^n}dx=-\frac{e^{-(ax)^2}}{(n-1)x^{n-1}}-\frac{2a^2}{n-1}\int\frac{e^{-(ax)^2}}{x^{n-2}}dx,$$
$$a>0,\ n=2,3,..., \tag{35}$$

were counted:

$$L_2(\zeta,\tau)=\sqrt{\pi\tau}\,\mathrm{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right), \tag{36}$$

$$L_4(\zeta,\tau)=\frac{\tau\sqrt{\pi\tau}}{3}\left\{\left[1-2\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\right]\mathrm{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)+\frac{\zeta}{2\sqrt{\tau}}\mathrm{erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)\right\}, \tag{37}$$

$$L_6(\zeta,\tau)=\frac{\tau^2\sqrt{\pi\tau}}{5}\left\langle \operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)\left\{1-\frac{2}{3}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\left[1-2\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\right]\right\}+\frac{\zeta}{2\sqrt{\tau}}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)\left[1-\frac{2}{3}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\right]\right\rangle, \tag{38}$$

$$L_8(\zeta,\tau)=\frac{\tau^3\sqrt{\pi\tau}}{7}\left\langle\left\{1-\frac{2}{5}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2+\frac{4}{15}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^4\left[1-2\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\right]\right\}\operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)+\left[1-\frac{2}{5}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\left[1-\frac{2}{3}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\right]\right]\frac{\zeta}{2\sqrt{\tau}}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)\right\rangle. \tag{39}$$

Substituting the functions $L_k(\zeta,\tau), k=2,4,6,8$ (36)–(39) to the formula (33) we determined:

$$\begin{aligned}T_m(\zeta,\tau)=\frac{\tau\sqrt{\tau}}{\tau_m}\Bigg\langle&\operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)\left\{\frac{4}{3}\left[1+\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2\right]-\frac{\tau^2}{\tau_s^0\tau_m}\left[\frac{8}{105}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^6+\frac{16}{21}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^4+\frac{58}{35}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2+\frac{16}{35}\right]\right\}-\\&-\left(\frac{\zeta}{2\sqrt{\tau}}\right)\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right)\left\{\frac{2}{3}-\frac{\tau^2}{35\tau_s^0\tau_m}\left[\frac{4}{3}\left(\frac{\zeta}{2\sqrt{\tau}}\right)^4+12\left(\frac{\zeta}{2\sqrt{\tau}}\right)^2+19\right]\right\}\Bigg\rangle, \zeta\ge 0,\ 0\le\tau\le\tau_m.\end{aligned} \tag{40}$$

In analogy to the above, function $T_s(\zeta,\tau)$ (31) was written in the form:

$$\begin{aligned}T_s(\zeta,\tau)=\frac{1}{\sqrt{\pi}}\Bigg\{&\frac{1}{\tau_s^0\tau_m\tau_s}[\tau^3L_2(\zeta,\tau-\tau_m)-3\tau^2L_4(\zeta,\tau-\tau_m)+3\tau L_6(\zeta,\tau-\tau_m)-L_8(\zeta,\tau-\tau_m)]+\\&+\left(2-\frac{2\tau}{\tau_s}+\frac{\tau_m-2\tau}{\tau_s^0}\right)L_2(\zeta,\tau-\tau_m)+\left(\frac{2}{\tau_s^0}+\frac{2}{\tau_s}\right)L_4(\zeta,\tau-\tau_m)\Bigg\}, \zeta\ge 0,\ \tau_m\le\tau\le\tau_s.\end{aligned} \tag{41}$$

Taking into account in equation (41) function $L_k(\zeta,\tau-\tau_m)$, $k=2,4,6,8$ (36)–(39), we obtained:

$$\begin{aligned}T_s(\zeta,\tau)=&\sqrt{\tau-\tau_m}\left(2-\frac{2\tau}{\tau_s}+\frac{\tau_m-2\tau}{\tau_s^0}+\frac{\tau^3}{\tau_s^0\tau_m\tau_s}\right)\operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)+\\&+(\tau-\tau_m)\sqrt{\tau-\tau_m}\left(\frac{2}{3\tau_s^0}+\frac{2}{3\tau_s}-\frac{\tau^2}{\tau_s^0\tau_m\tau_s}\right)\left\{\left[1-2\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^2\right]\operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)+\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)\right\}+\\&+\frac{3\tau(\tau-\tau_m)^2\sqrt{\tau-\tau_m}}{5\tau_s^0\tau_m\tau_s}\left\{\operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)\left[1-\frac{2}{3}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^2+\frac{4}{3}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^4\right]+\frac{\zeta}{2\sqrt{\tau-\tau_m}}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)\left[1-\frac{2}{3}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^2\right]\right\}+\\&+\frac{(\tau-\tau_m)^3\sqrt{\tau-\tau_m}}{7\tau_s^0\tau_m\tau_s}\left\{\operatorname{ierfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)\left[1-\frac{2}{5}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^2+\frac{4}{15}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^4-\frac{8}{15}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^6\right]+\right.\\&\left.+\frac{\zeta}{2\sqrt{\tau-\tau_m}}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)\left[1-\frac{2}{5}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^2+\frac{4}{15}\left(\frac{\zeta}{2\sqrt{\tau-\tau_m}}\right)^4\right]\right\}, \quad \zeta\ge 0,\ \tau_m\le\tau\le\tau_s.\end{aligned} \tag{42}$$

Thus, dimensionless temperature in a semi–space was determined from the formula (30), based on functions $T_m(\zeta,\tau)$ (40) and $T_s(\zeta,\tau)$ (42).

Numerical analysis

The input dimensionless parameters used to conduct the numerical analysis were: distance from heated surface ζ, time τ, time of pressure increase τ_m and time of braking with constant pressure τ_s^0. Calculations were carried out with $\tau_s^0=1$. Then, substituting the dimensionless time of pressure increase τ_m, from the formula (21) dimensionless braking time τ_s was determined. In order to conduct comparative analysis of the temperature obtained with different time of pressure increase τ_m, total amount of thermal energy directed to the brake disc should have constant value**.** In considered case, taking into account the function form $q^*(\tau)$ (14), (15), it is equal to:

$$Q^*=\int_0^{\tau_s}q^*(\tau)d\tau=\int_0^{\tau_m}\frac{\tau}{\tau_m}\left(1-\frac{\tau^2}{2\tau_s^0\tau_m}\right)d\tau+\int_{\tau_m}^{\tau_s}\left[1-\frac{1}{\tau_s^0}\left(\tau-\frac{1}{2}\tau_m\right)\right]d\tau=-\frac{\tau_m^2}{8\tau_s^0}-\frac{\tau_m}{2}+\frac{\tau_m\tau_s}{2\tau_s^0}-\frac{\tau_s^2}{2\tau_s^0}+\tau_s. \tag{43}$$

Substituting to the formula (43), assumed value of parameter $\tau_s^0=1$ and relation (21), was found $Q^*=0{,}5.$ It means that, with assumed input values the amount of heat absorbed by the disc is constant and independent of the time of the contact pressure increase τ_m.

Evolution of the dimensionless temperature T^* on few depths ζ is shown in Fig. 2a. At the beginning of the process, to the time moment $\tau=\tau_m$, the temperature rapidly increases, and its distribution is similar to the linear. Further, the temperature increase is gentler. After reaching the maximum value, the temperature monotonically decreases until the moment of standstill. Maximum value of the dimensionless temperature $T^*_{\max} = 0{,}53$ is achieved in time moment $\tau = 0{,}57\tau_s$ on the heated surface $\zeta = 0$. The time to reach maximum temperature value increases with increasing distance $\zeta > 0$ from friction surface to the center of the disc (delay effect). Increasing the depth ζ, the temperature monotonically decreases, which is the most rapid in moment $\tau = 0{,}57\tau_s$ (Fig. 2b). Effective depth of the heat penetration, i.e. distance from the contact surface, on which the temperature achieved 5% of the maximum value on heated surface is equal to $\zeta \cong 1$. It confirms the correctness of adopted parameters (20).

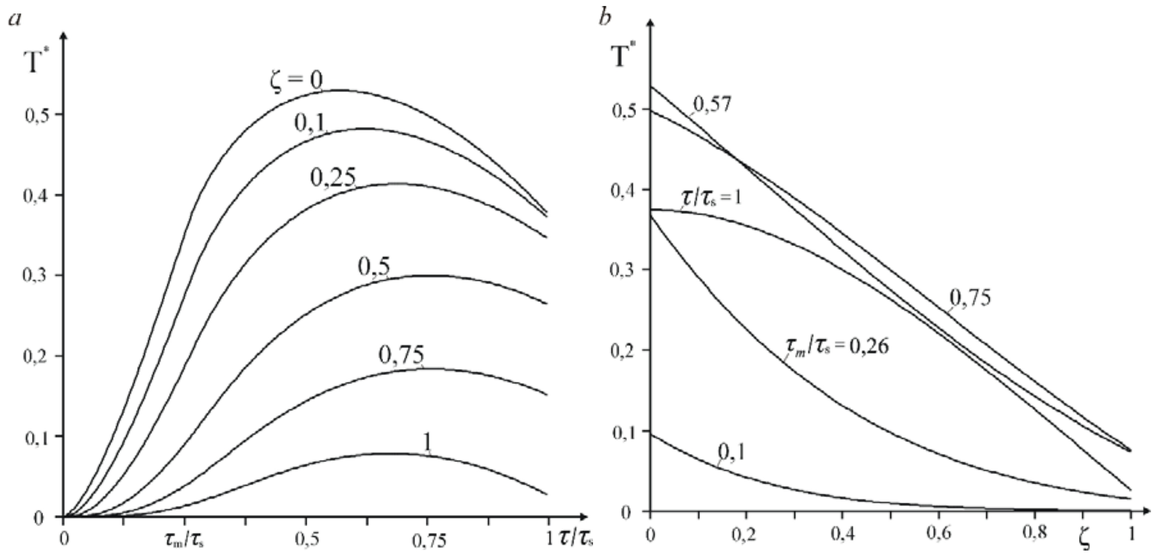

Fig. 2. Relation of the dimensionless temperature T^* with: *a* – dimensionless time τ on different distances from friction surface ζ; *b* – dimensionless depth ζ in selected moments of dimensionless time τ with $\tau_m = 0{,}3$, $\tau_s = 1{,}15$.

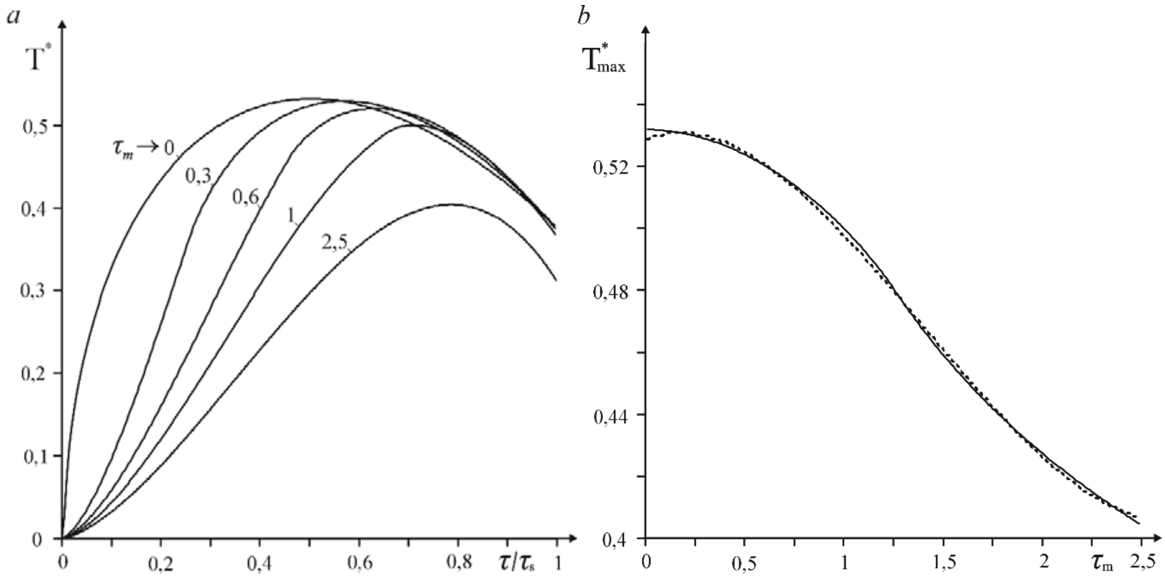

Fig. 3. a – evolution of the dimensionless temperature T^* on heated surface $\zeta = 0$ in selected values of dimensionless time of contact pressure increase τ_m; b – relation between dimensionless maximum temperature $T^*_{\max}$ and dimensionless time of pressure increase τ_m, obtained by calculations (solid line) and by approximation (44) (dashed line).

The dimensionless temperature curves T^* on friction surface of the disc with different time of pressure increase τ_m are presented in Fig. 3a. With $\tau_m \to 0$ the pressure during braking is constant and the temperature increase, at the beginning of the braking process, is the most rapid. Increasing the time of linear pressure distribution τ_m, the temperature increase became gentler, and the time to reach maximal temperature has a higher value. Relation between dimensionless maximum temperature $T^*_{\max}$ and the time of pressure increase τ_m in the range of values $0 \le \tau_m \le 2{,}5$ is shown in Fig. 3b. This figure also presents approximation of this relation, using the following function:

$$T^*_{\max}(\tau_m) = 0{,}0167\tau_m^3 - 0{,}0703\tau_m^2 + 0{,}0224\tau_m + 0{,}53, \quad 0 \le \tau_m \le 2{,}5. \qquad (44)$$

Increasing the time of linear pressure distribution causes monotonically decrease of maximal temperature.

Conclusions

The mathematical model of the frictional heating of the brake disc during single braking was proposed. The one–dimensional thermal problem of friction with linear increasing of the contact pressure was formulated. Analytical solution to the problem was achieved using Duhamel's theorem. Numerical analysis of the obtained solution was conducted. Based on results, the following conclusions were determined:

1 - from the beginning of braking to the moment of the nominal value of contact pressure attain $\tau = \tau_m$, the temperature increase is violent and has almost linear distribution. Then, the temperature increases more slowly to reach the maximum value, and next, cooling of the friction surface occurs to the standstill;

2 - total amount of the dimensionless thermal energy absorbed by the disc does not depend on the time of contact pressure increase τ_m;

3 - maximal temperature is achieved on friction surface of the disc during braking with constant pressure. Increasing the time of linear pressure distribution causes decrease of maximal temperature;

4 - based on numerical analysis the engineering equation (44) was proposed. It describes relation between maximal temperature and the time of pressure increase. It could be helpful during estimation of the maximum value of the temperature in a brake disc, thereby reducing the time and costs of the temperature calculations of braking systems designing.

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