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FERROFLUID FLOW DUE TO A ROTATING DISK IN THE PRESENCE OF A NON-UNIFORM MAGNETIC FIELD

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The flow of a ferrofluid due to a rotating disk in the presence of a non-uniform magnetic field in the axial direction is studied through mathematical modeling of the problem. Contour and surface plots in the presence of *10* kilo-ampere/meter, *100* kilo-ampere/meter magnetization force are presented here for radial, tangential and axial velocity profiles, and results are also drawn for the magnetic field intensity. These results are compared with the ordinary case where magnetization force is absent.

Key words: magnetization force, ferrofluid, rotating disk, magnetic field.

1. Introduction

Ferrofluids are stable suspensions of colloidal ferromagnetic particles of the order of *10nm* in suitable non-magnetic carrier liquids. These colloidal particles are coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, the investigation on them has fascinated the researchers and engineers vigorously for the last five decades. One of the many fascinating features of ferrofluids is the prospect of influencing the flow by a magnetic field [1, 2]. Ferrofluids are widely used in sealing of hard disc drives, rotating *x*-ray tubes under engineering applications. Sealing of the rotating shafts is the most known application of the magnetic fluids. The major application of ferrofluids in the electrical field is that of controlling heat in loudspeakers which makes their life longer and increases the acoustical power without any change in the geometrical shape of the speaker system. Magnetic fluids are used in the contrast medium in *X*-ray examinations and positioning tamponade for retinal detachment repair in eye surgery. Therefore, ferrofluids play an important role in bio-medical applications also. In the presence of a uniform magnetic field, the magnetization characteristics depend on the particle spin but do not depend on the fluid velocity. Convection of ferromagnetic fluids is gaining much importance due to their astounding physical properties.

There are rotationally symmetric flows of incompressible ferrofluids in the field of fluid mechanics, having all three velocity components; radial, tangential and vertical in space different from zero. In such types of flow, the variables are independent of the angular coordinates. Detailed accounts of magneto viscous effects in ferrofluids have been given in the monograph by Odenbach [3]. In the flow of an incompressible ferrofluid, the plate is subjected to the magnetic field $\left[H_r, 0, H_z\right]$ using, Neuringer-Rosensweig model [4]. This model was used by Verma *et al*. [5, 6, 7] for solving the paramagnetic Couette flow, helical flow with heat conduction and flow through a porous annulus. Rosensweig [8] gave an authoritative introduction to the research on magnetic liquids in his monograph and studied the effect of magnetization.

A study of flow within the boundary layer and its effect on the general flow around the body, is given in Schlichting [9]. Karman's [10] rotating disc problem is extended to the case of flow started

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impulsively from rest and also the steady state is solved to a higher degree of accuracy then previously done by a simple analytical method which neglects the resembling difficulties in Cochran's [11] well known solution. The pioneering study of an ordinary viscous fluid flow due to an infinite rotating disc was carried by Von Karman. He introduced the famous transformation which reduces the governing partial differential equations into ordinary differential equations. Cochran obtained asymptotic solutions for the steady hydrodynamic problem formulated by Von Karman. Benton [12] improved Cochran's solutions, and solved the unsteady case. Attia [13] studied the unsteady state in the presence of an applied uniform magnetic field. The steady flow of an ordinary viscous fluid due to a rotating disc with uniform high suction was studied by Mithal [14]. Attia [15] discussed the flow due to an infinite disk rotating in the presence of an axial uniform magnetic field by taking Hall effect into consideration.

Using linear instability analysis, Venkatasubramanian and Kaloni [16] discussed the effects of rotation on the onset of convection in a horizontal layer of ferrofluids rotating about the vertical axis, heated from below and in the presence of a uniform vertical magnetic field. The effect of an alternating uniform magnetic field on convection in a horizontal layer of a ferrofluid within the framework of a quasi-stationary approach is studied by Belyaev [17].

The effect of the magnetic field along the vertical axis on thermo-convective instability in a ferromagnetic fluid saturating a rotating porous medium was studied by Sekar *et al.* [18] by using the Darcy model. Attia [19] studied the steady flow of an incompressible viscous fluid above an infinite rotating disk in a porous medium with heat transfer and also discussed the effect of porosity of the medium on the velocity and temperature distribution. Frusteri and Osalusi [20] examined the laminar convective and slip flow of an electrically conducting Newtonian fluid with variable properties over a rotating porous disk.

In general, magnetization is a function of the magnetic field, temperature and density of the fluid. This leads to convection of the ferrofluid in the presence of the magnetic field gradient. Sunil *et al.* [21] studied the effect of magnetic field-dependent viscosity on thermosolutal convection in a ferromagnetic fluid saturating a porous medium. Sunil *et al.* [22] discussed the influence of rotation on medium permeability and how MFD viscosity affects the magnetization in ferromagnetic fluid heated from below in the presence of dust particles saturating a porous medium of very low permeability using the Darcy model. The effect of MFD viscosity on thermal convection in a ferromagnetic fluid in a porous medium is studied by Sunil *et al.* [23]. Nanjundappa *et al.* [24] studied Benard-Marangoni ferroconvection in a ferrofluid layer in the presence of a uniform vertical magnetic field with magnetic field dependent (MFD) viscosity. Ram *et al.* [25] solved the non-linear differential equations under Neuringer-Rosensweig model for the ferrofluid flow by using power series approximations and discussed the effect of magnetic field-dependent viscosity on the velocity components and pressure profile. The effect of negative viscosity on the ferrofluid flow due to a rotating disk under alternating magnetic field is studied by Ram *et al.* [26-27].

In the present case, we take cylindrical coordinates $r, \theta, z,$ where the z-axis is normal to the plane and this axis is considered as the axis of rotation. The radius of the disk is taken 0.5 meter and the disk is rotating with 4 radian / second. We have studied the effects of magnetization force on the ferrofluid flow due to a rotating disk for different values of the magnetic field intensity of 10 and 100 kilo ampere per meter which is applied in the axial direction.

2. Mathematical formulation of the problem

The constitutive set of equations is as follows:
The equation of continuity

$$\nabla . \boldsymbol{v} = 0 . \tag{2.1}$$

The equation of motion

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}.\nabla)\boldsymbol{v}\right]=-\nabla p+\mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}+\mu\nabla^2\boldsymbol{v}. \tag{2.2}$$

Maxwell's equations

$$\nabla\times\boldsymbol{H}=\boldsymbol{0}, \qquad \nabla.(\boldsymbol{H}+\boldsymbol{M})=0; \qquad \text{with} \qquad \boldsymbol{M}=\chi\boldsymbol{H}. \tag{2.3}$$

Here ρ is the ferrofluid density, $\frac{d}{dt}=\frac{\partial}{\partial t}+\boldsymbol{v}.\nabla$, $\boldsymbol{v}$ is the fluid velocity, p is the pressure, μ is the reference viscosity, μ_0 is magnetic permeability of free space, $\boldsymbol{M}$ is the magnetization, $\boldsymbol{H}$ is the magnetic field intensity, χ is the magnetic susceptibility, t is the time.

Equations (2.1) and (2.2) can be written in the cylindrical form

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0, \tag{2.4}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left[\frac{\partial^2 v_r}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)+\frac{\partial^2 v_r}{\partial z^2}\right]=\left[v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right], \tag{2.5}$$

$$\nu\left[\frac{\partial^2 v_\theta}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right]=\left[v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right], \tag{2.6}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}|\boldsymbol{M}|\frac{\partial}{\partial z}|\boldsymbol{H}|+\nu\left[\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right]=\left[v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right] \tag{2.7}$$

where v_r, v_θ and v_z are velocity components in the radial, tangential and axial direction, respectively. Here, $\nu=\frac{\mu}{\rho}$ is the kinematic variable viscosity.

Here the disk is rotating with uniform angular velocity ω. The boundary conditions for the flow of the ferrofluid due to a rotating disk in the presence of a non-uniform magnetic field used by Schlichting [9] are given as follows

$$\begin{aligned}&\text{at} \quad z=0; \quad v_r=0, \quad v_\theta=r\omega, \quad v_z=0,\\ &\text{at} \quad z\to\infty; \quad v_r\to 0, \quad v_\theta\to 0.\end{aligned} \tag{2.8}$$

Here, v_z does not vanish at $z\to\infty$, but tends to a finite value.

The solution of Eqs (2.5)-(2.7) with the help of (2.8) is obtained using the Finite Element Method. All the units are taken in MKS system and the initial magnetic field is taken *10* kilo-ampere/meter and *100* kilo-ampere/meter.

3. Results and discussions

Figures 1-3 represent the radial velocity profile for different values of the magnetic field intensity. Figure 1 is plotted when the externally magnetic field is not applied. However, in the presence of the magnetic field, the radial velocity increases and depends on the strength of the filed. In Fig.2, *10* kilo/ ampere magnetic field is applied and in Fig.3, *100* kilo/ampere magnetic field is applied. Due to strong magnetic, polarization, the radial velocity depends on the strength of the magnetic field and the effect of the rotation of the disk diminishes. As the magnetic field increases in the axial directions, the fluid is being polarized in the axial directions and depends on the intensity of the magnetic field.

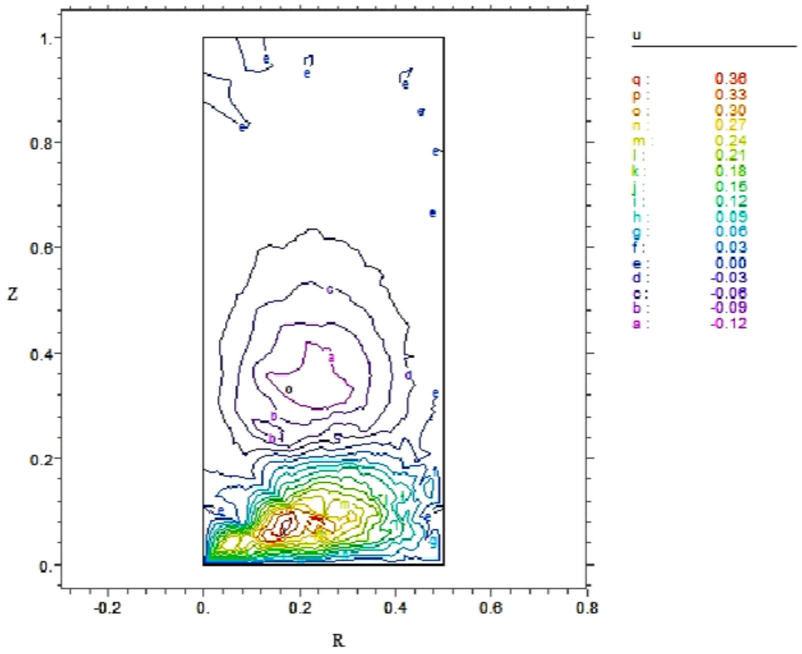

Fig.1. Radial velocity profile for *H=0*.

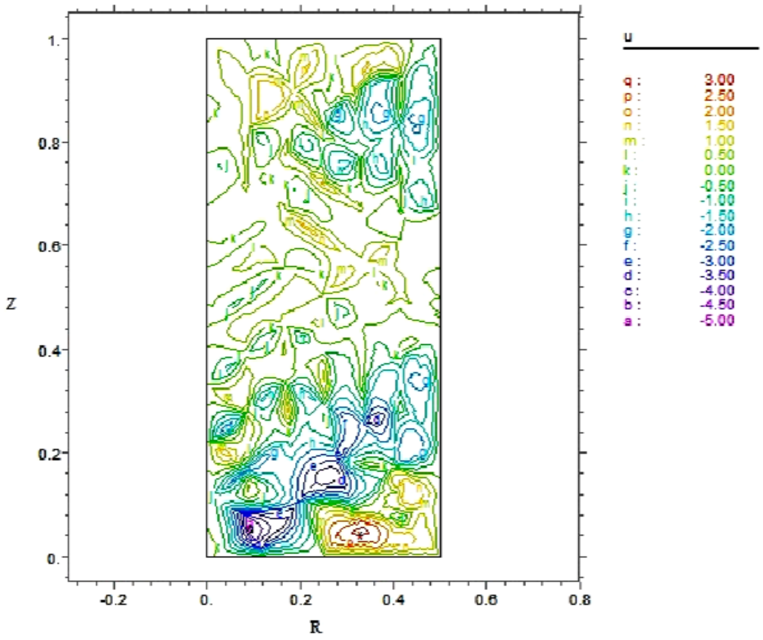

Fig.2. Radial velocity profile for *H=10* kilo/ampere.

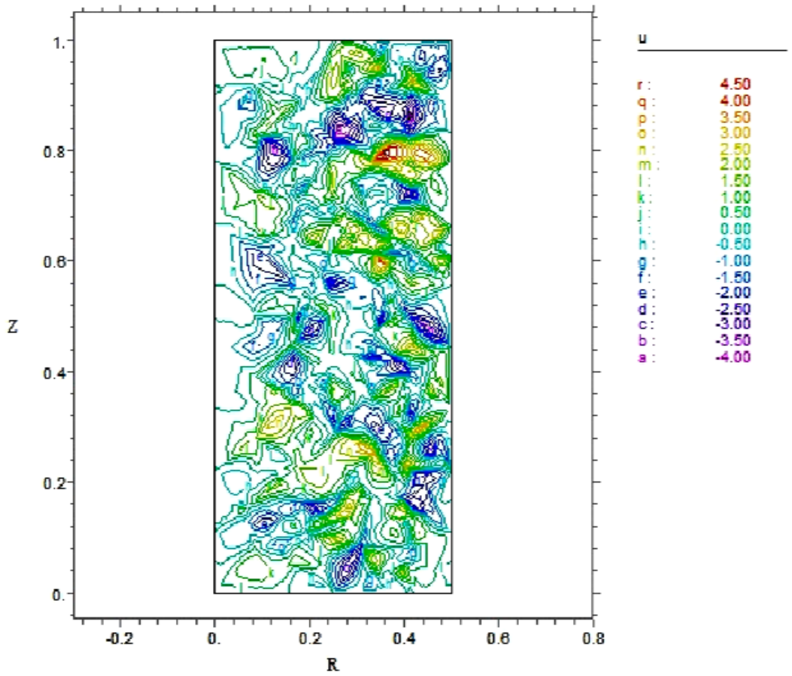

Fig.3. Radial velocity profile for *100* kilo/ampere.

Figures 4-6 represent the tangential velocity profile for different values of the magnetization force. It is clear from the figure that near the disk tangential velocity increases in Fig.4. However, in Figs 5 and 6, the tangential velocity increases along the z direction since the field is being applied in z direction. In Fig.4, the tangential velocity increases along the *z*-axis, however, as far as for large distance along *z*-axis, the tangential velocity reaches the steady state region. In Fig.5, due to the applied magnetic field in the radial direction, the velocity components get large value far from the disk. And, magnetization force is more dominant in Fig.6 in comparison with Fig.5 since the disk is being fixed in *100* kilo/ampere magnetization force.

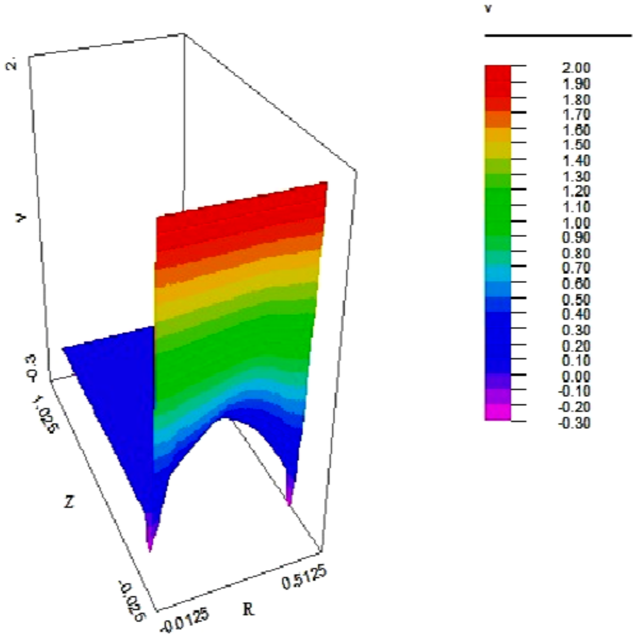

Fig.4. Tangential velocity profile for *H=0*.

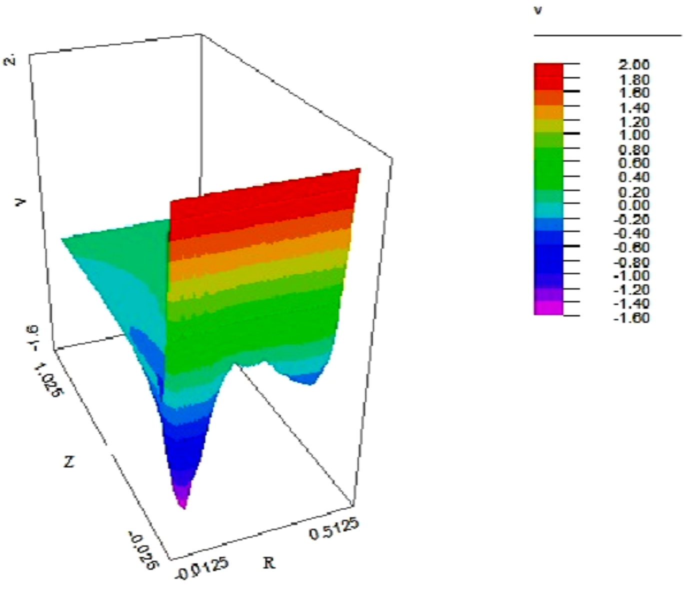

Fig.5. Tangential velocity profile for *H=10* kilo/ampere.

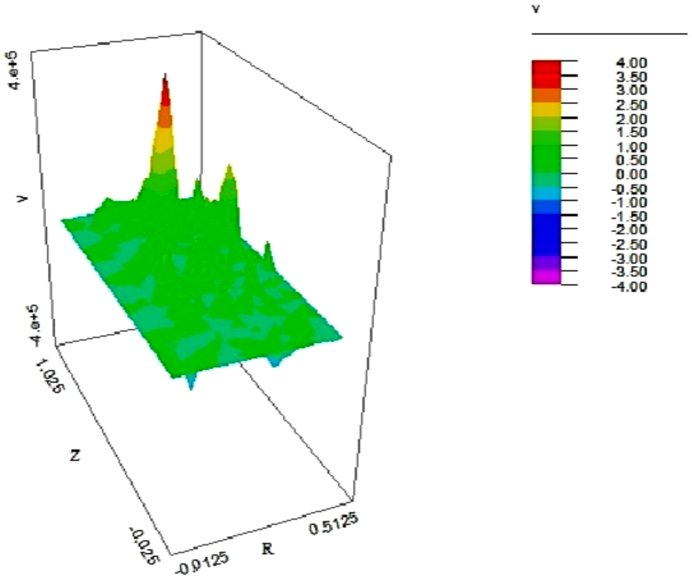

Fig.6. Tangential velocity profile for *H=100* kilo/ampere.

Figures 6-9 depicts the axial velocity profile. The negative values of the axial velocity indicate that the flow of the ferrofluid is towards the disk. Figure 7 shows the vortex flow about the *z* axis and the particles follow the pattern in the flow, however, in case of *10* kilo/ampere magnetization force the path of the particles is disturbed as shown in Fig.8. However, in Fig.9, it is more disturbed in comparison with Fig.8 and the overall axial velocity is being controlled by the magnetization force as per the properties of the ferrofluid. As we increase the magnetic field intensity, it is observed from the results that the fluid velocity depends on the intensity of the field.

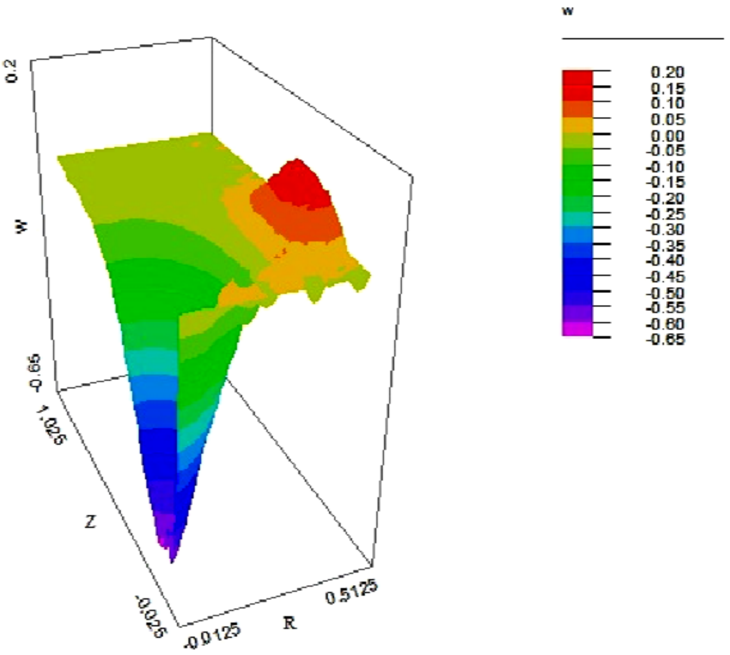

Fig.7. Axial velocity profile for *H=0*.

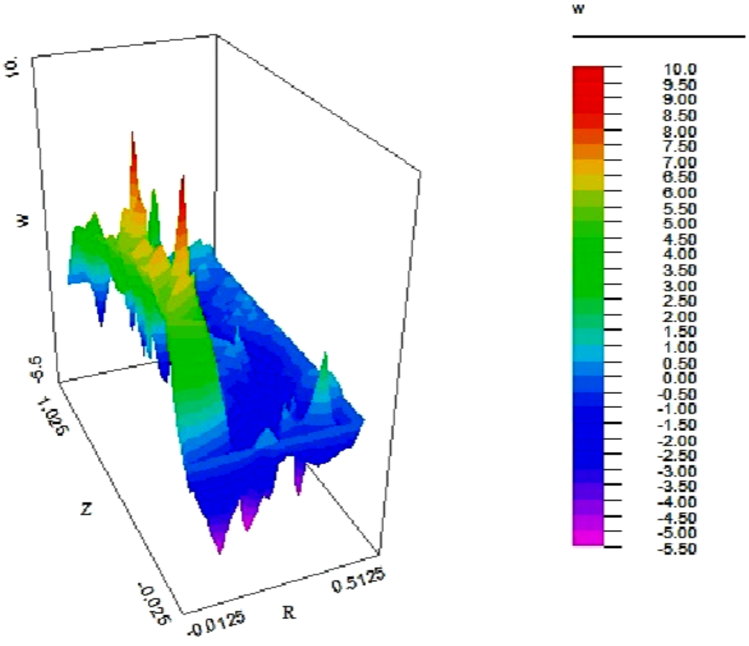

Fig.8. Axial velocity profile for *H=10* kilo/ampere.

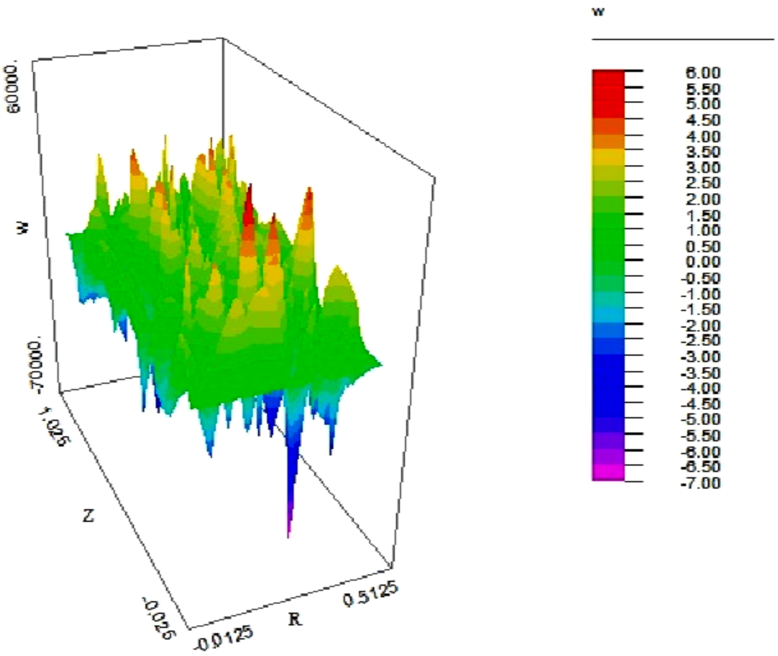

Fig.9. Axial velocity profile for *H=100* kilo/ampere.

Figures 10-11 illustrate the behavior of the magnetic field intensity. Here the disk is being kept at *10* and *100* kilo /ampere magnetization force and that force is being applied in the axial direction. As the fluid moves along the *z*-axis the magnetization force reduces there due to the no-uniform distribution of the magnetization force.

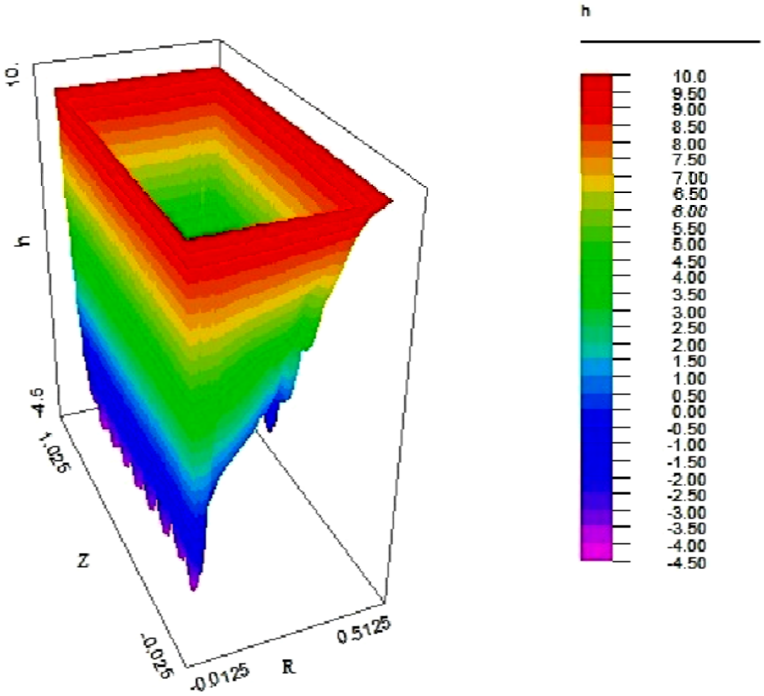

Fig.10. Variation of the magnetic field in the axial direction at *H=10* kilo/ampere.

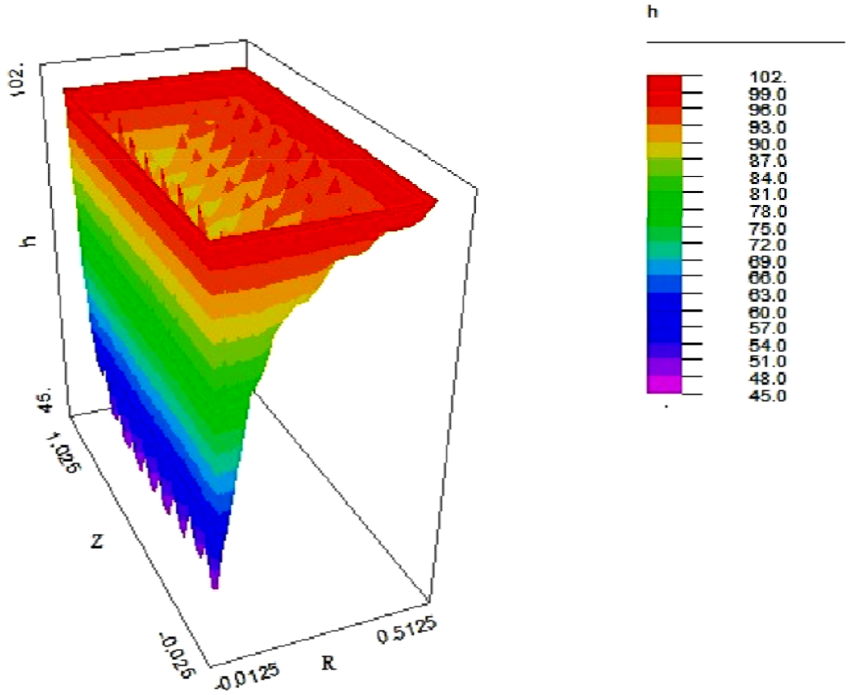

Fig.11. Variation of the magnetic field in the axial direction at *H=100* kilo/ampere.

4. Conclusions

Present results show the effect of the magnetization force on the axisymmetric ferrofluid flow due to a rotating disk. As we increase the intensity of the magnetic field in the direction normal to the disk, the velocity of the magnetic fluid depends much on its intensity. These results indicate also the space application of this fluid since its flow can be controlled by the intensity of the magnetic field. The magnetic field in any particular direction indicates that the fluid is be polarized in the same direction and polarization depends on the strength of the magnetic field. The present results might be applicable where strong changes in the magnetic field produce an electronically controllable signal and electronically controlled damping systems.

Nomenclature

$\boldsymbol{H}$ – magnetic field intensity
$\boldsymbol{M}$ – magnetization
p – fluid pressure
r – distance from the rotational axis
t – time
$\boldsymbol{v}$ – velocity of ferrofluid
μ – reference viscosity of ferrofluid
μ_0 – magnetic permeability of free space
v_r – radial velocity
v_θ – tangential velocity
v_z – axial velocity

ρ – fluid density
χ – magnetic susceptibility
ν – kinematic viscosity of ferrofluid
ω – angular velocity of the disk

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