THE HOMOTOPY PERTURBATION METHOD FOR ELECTRICALLY ACTUATED MICROBEAMS IN MEMS SYSTEMS SUBJECTED TO VAN DER WAALS FORCE AND MULTIWALLED CARBON NANOTUBES

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Abstract: This paper presents a summary of a study that uses the Aboodh transformation and homotopy perturbation approach to analyze the behavior of electrically actuated microbeams in microelectromechanical systems that incorporate multiwalled carbon nanotubes and are subjected to the van der Waals force. All of the equations were transformed into linear form using the HPM approach. Electrically operated microbeams, a popular structure in MEMS, are the subject of this work. Because of their interaction with a nearby surface, these microbeams are sensitive to a variety of forces, such as the van der Waals force and body forces. MWCNTs are also incorporated into the MEMSs in this study because of their special mechanical, thermal, and electrical characteristics. The suggested method uses the HPM to model how electrically activated microbeams behave when MWCNTs and the van der Waals force are present. The nonlinear equations controlling the dynamics of the system can be roughly solved thanks to the HPM. The HPM offers a precise and effective way to analyze the microbeam's reaction to these outside stimuli by converting the nonlinear equations into linear forms. The study's findings shed important light on how electrically activated microbeams behave in MEMSs. A more thorough examination of the system's performance is made possible with the addition of MWCNTs and the van der Waals force. With its ability to approximate solutions and characterize system behavior, the HPM is a potent instrument that improves comprehension of the physics at play and facilitates the design and optimization of MEMS devices. The aforementioned method's accuracy is verified by comparing it with published data that directly aligns with Anjum et al.'s findings. We have faith in this method's accuracy and its current application.

Key words: microelectromechanical systems, Aboodh transform, homotopy method, amplitude-frequency relationship, MWCNT-MEMS

1. INTRODUCTION

Microelectronics, mechanical components, sensors, and actuators are all combined in microelectromechanical system (MEMS) technology. For these microsystems, tiny mechanical components like gears, springs, and resonators can be produced by the semiconductor manufacturing industry. Mobile phones, game controllers, lab-on-a-chip systems, and drug delivery systems all incorporate MEMS devices. They are used by airbag sensors, tire pressure monitoring systems, and aircraft navigation and guidance systems. Due to their small size and low power consumption, MEMS devices are perfect for battery-powered electronics. Chips with several functions allow for more intricate and effective system designs. Technology related to material science, design optimization, and packaging is propelling MEMS technology into new domains. MEMS devices will keep having an impact on a wide range of industries and uses [1-6]. In MEMS dynamics, pull-in instability and periodic behavior are well-represented by differential equations. Therefore, it is essential to comprehend modern differential equation solving techniques. It is difficult to solve nonlinear MEMS models analytically. Nonlinear equations without analytical solutions are used in MEMS research [7-8]. Researchers have been working on analytical techniques to deal with MEMS nonlinear oscillation for decades. For these microstructures, classical oscillatory theory has several shortcomings [9–10].

Since these procedures are unable to solve highly nonlinear equations, alternative techniques have been developed and published in the public domain. A number of books [11–13] published in the last ten years have addressed mathematical methods in MEMS applications. Mathematical tools to determine their approximate solutions have been developed extensively, since their exact solutions are difficult to obtain and numerical methods cannot clearly depict the nonlinear frequency-amplitude connection. Additionally, a variety of analytical techniques are employed by researchers to solve N/MEMS oscillators. The residual harmonic balance approach [22], the parameter expansion technique [24], the Adomian decomposition method [25], the variational iteration method (VIM) [14–17], the energy balance method [18], the iteration perturbation technique [19–21], the residual power series method [26], and the frequency formulation tool [27, 28] are a few of these.

An effective technique for analyzing and enhancing the performance of MEMSs is the optimum homotopy equation. The ideal values of different parameters, like beam size, material qualities, and applied forces, can be found to achieve desired system behavior by writing an appropriate optimal homotopy equation. The Euler-Bernoulli beam equation, the equations describing the electrostatic or van der Waals forces, and other governing equations of the MEMS are incorporated into this equation. The goal of the ideal homotopy equation is to minimize an objective function, which may have to do with deflection, stress, power usage, or any

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other relevant performance indicator. Engineers and researchers can improve the design and performance of MEMS devices by finding the ideal values for system parameters by solving the optimal homotopy equation [29–34]. Through the examination of different design configurations and optimization methodologies made possible by this methodology, the field of MEMS is ultimately advanced, and the functionality and efficiency of MEMSs in a variety of applications are improved. Highly nonlinear problems have been solved using the homotopy perturbation method (HPM) [35–40]. This method provides the answer in the form of a series that quickly approaches the approximate answer. This approach's main advantage is that it produces a highly precise answer after just one iteration, making it ideal for usage in real-world scenarios. This advantage motivated us to use this approach in conjunction with the Aboodh transform to solve the oscillatory problem with only two perturbation terms. The Aboodh transform has a close relationship with the Laplace transform and was essentially derived from the Fourier integral. The Aboodh transform is a potent method for figuring out the answers to a lot of partial and ordinary differential equations.

2. ANALYSIS OF THE ABOODH-BASED HOMOTOPY METHOD

The Aboodh transform, which uses the Fourier integral to solve ordinary and partial differential equations, was developed by Khalid Aboodh.

If the function g(t), $t \geq 0$ is piecewise continuous and of an exponential order, then Aboodh transform is defined in this form, as indicated in Aboodh's 2013 paper [37]:

$$A[g(t)] = g(v) = \frac{1}{v} \int_0^\infty g(t) e^{-vt} dt, \qquad (2.1)$$

The Aboodh and inverse Aboodh transforms of some useful functions related to this article are listed in Table 1.

Table 1. Aboodh transforms of some useful functions [37].

$g(t)$	1	t	e^{bt}	$\sin(bt)$
$A[g(t)]$	$\frac{1}{v^2}$	$\frac{1}{v^3}$	$\frac{1}{v^2-bv}$	$\frac{b}{v(v^2+b^2)}$
$g(t)$	$t\sin(bt)$	$\sinh(bt)$	$\cosh(bt)$	$\cos(bt)$
$A[g(t)]$	$\frac{2b}{(v^2+b^2)^2}$	$\frac{b}{v(v^2-b^2)}$	$\frac{1}{(v^2-b^2)}$	$\frac{1}{(v^2+b^2)}$

Aboodh transforms of first and second time derivatives of $g(t)$ are expressed as follows:

$$A[g'(t)] = vg(v) - \frac{g(0)}{v}$$

$$A[g''(t)] = v^2 g(v) - \frac{g'(0)}{v} - g(0)$$

Consider the following nonhomogeneous differential equation in this form:

$$Lq(t) + \omega^2 q(t) + Rq(t) + Nq(t) = g(t) \qquad (2.2)$$

The initial condition of the above equation at time t = 0 is given as follows:

$$q(0) = A, \qquad \dot{q}(0) = 0, \qquad (2.3)$$

where L indicate the linear differential part of second order $(L = \frac{\partial^2}{\partial t^2})$, R is the remaining lesser order linear part, Nq is the nonlinear part, g(t) denotes the inhomogeneous term and ω is the angular frequency of the system.

Now, applying Aboodh transform to both sides of Eq. (2.2), we have the following equation:

$$A[Lq(t)] + \omega^2 A[q(t)] + A[Rq(t)] + A[Nq(t)] = A[g(t)] \qquad (2.4)$$

By utilising the Aboodh transform properties and initial condition given above, Eq. (2.4) can be rewritten as follows:

$$q(v) = \left(\frac{1}{v^2+\omega^2}\right) q(0) + \frac{1}{v(v^2+\omega^2)} q'(0) - \left(\frac{1}{v^2+\omega^2}\right) A[Rq(t)] - \left(\frac{1}{v^2+\omega^2}\right) A[Nq(t)] - \left(\frac{1}{v^2+\omega^2}\right) A[g(t)] \qquad (2.5)$$

The Aboodh inverse transform of Eq. (2.5) is given as follows:

$$A^{-1}(q(v)) = q(t) = Z_0(t) - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[Rq(t)]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[Nq(t)]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[g(t)]\right] \qquad (2.6)$$

where,

$$Z_0(t) = \left(\frac{1}{v^2+\omega^2}\right) q(0) + \frac{1}{v(v^2+\omega^2)} q'(0)$$

According to the standard HPM [42–43], we can expand q(t) in the power of embedding parameter $p \in [0,1]$ as, $q(x,t) = \sum_{n=0}^\infty p^n q_n(t)$ and nonlinear term $Nq(t) = \sum_{n=0}^\infty p^n H_n(t)$.

Here, he's polynomial may be rewritten as follows:

$$H_n = \frac{1}{n!} \frac{d^n}{dp^n} \left[N\left(\sum_{n=0}^\infty p^n q_n(t) \right) \right], \qquad n = 0,1,2,\dots$$

By employing HPM and substituting the value of q(t) and Nq(t) in the above-mentioned Eq. (2.6), we have the following equation:

$$\sum_{n=0}^\infty p^n q_n(t) = Z_0(t) - p\left(A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[R\sum_{n=0}^\infty p^n q_n(t)]\right] + A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[\sum_{n=0}^\infty p^n H_n(t)]\right] + A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[g(t)]\right] \right) \qquad (2.7)$$

$$p^0: z_0(t) = Z_0 \qquad (2.8)$$

$$p^1: z_1(t) = A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[Rq_0(t)]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[H_0(q_0(t))]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[g(t)]\right] \qquad (2.9)$$

$$p^2: z_2(t) = A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[Rq_1(t)]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[H_1(q_1(t))]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right) A[g(t)]\right] \qquad (2.10)$$

As $p \to 1$, we may write the approximate solution as follows:

$$z(t) = \lim_{p \to 1} \sum_{n=0}^\infty p^n q_n(t) = q_0(t) + q_1(t) + q_2(t) \dots \qquad (2.11)$$

Hence, we may use $q_0(t), q_1(t), q_2(t) \dots$ to obtain the ap-

proximate solution of the oscillator system, but we only utilise Eqs. (2.9) and (2.10) to express the solution methodology.

Now, we will rewrite Eq. (2.9) in the following form:

$$z_1(t) = A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right)A[Rq_0(t)]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right)A[H_0(q_0(t))]\right] - A^{-1}\left[\left(\frac{1}{v^2+\omega^2}\right)A[g(t)]\right]. \tag{2.12}$$

3. PROBLEM FORMULATION

In this section, we discuss a doubly clamped microbeam with dimensions L, h, b and ρ, which represent the length, thickness, width and density of the microbeam, respectively, as shown in Figure 1.

The partial differential equation of a motion as the deflection of microbeam by using Euler–Bernoulli beam theory [44] is expressed as follows:

$$EI\frac{\partial^4 W}{\partial q^4} + \rho S\frac{\partial^2 W}{\partial \tau^2} - \left[\tilde{N} + \frac{ES}{2L}\int_0^L\left(\frac{\partial W}{\partial q}\right)dq\right]\frac{\partial^2 W}{\partial q^2} - F(q,\tau) \tag{3.1}$$

where $W(q,\tau)$ represents the function of location q and time τ is the deflection of microbeam. Further, E denotes the Young's modulus, with $S = bh$ and $I = \frac{bh^3}{12}$, indicating the cross-sectional area and moment of inertia along y axis, respectively. $\tilde{N}$ is expressed as axial load, and $F(q,\tau)$ is the force formulated from electrostatic excitation [45].

$$(q,\tau) = \frac{bv^2}{2}\epsilon_v\left[\frac{1}{(d-W)^2} - \frac{1}{(d+W)^2}\right],$$

where ϵ_v denotes a dielectric constant with its values amounting to 8.85 PFm^{-1}, v is the Poisson ratio and d is the gap between a beam and its substrate.

The boundary condition for the doubly clamped beam can be written as follows:

$$W(0,\tau) = W(L,\tau) = 0, \quad \left.\frac{\partial W}{\partial q}\right|_{(0,\tau)} = \left.\frac{\partial W}{\partial q}\right|_{(L,\tau)} = 0 \tag{3.2}$$
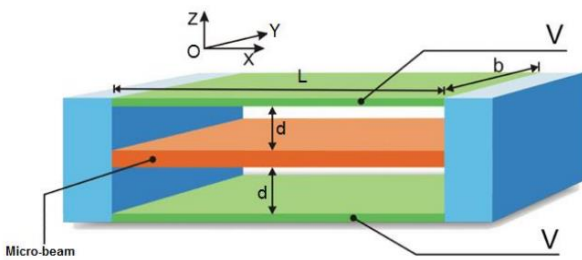

Fig. 1. The model of a doubly clamped electrically actuated microbeam- based MEMS

Nondimensional variables such as location, deflection of the nanobeam and time can be denoted as follows:

$$\eta = \frac{q}{L}, \qquad w = \frac{W}{d}, \quad t = \frac{\tau}{\tilde{T}} \tag{3.3}$$

where $\tilde{T} = \sqrt{\frac{\rho h b L^4}{EI}}$.

By substituting Eq. (3.3) into Eq. (3.1), we obtain:

$$\frac{\partial^4 w}{\partial \eta^4} + \frac{\partial^2 w}{\partial \tau^2} - \left[\alpha + \beta\int_0^1\left(\frac{\partial w}{\partial \eta}\right)d\eta\right]\frac{\partial^2 w}{\partial \eta^2} - \frac{\kappa^2}{4}\left[\frac{1}{(d-W)^2} - \frac{1}{(d+W)^2}\right] = 0, \tag{3.4}$$

where N is the axial load, α denote the aspect ratio and electrostatic v of nondimensional variables are defined as follows:

$$N = \frac{\tilde{N}L^2}{EI}, \alpha = 6\left(\frac{d}{h}\right)^2, \quad v = \frac{24L^4v^2\epsilon_v}{Ed^3h^3} \tag{3.5}$$

The boundary conditions in nondimensional form can be written as follows:

$$w(0,t) = w(1,t) = 0,$$

$$\left.\frac{\partial w}{\partial \eta}\right|_{(0,\tau)} = \left.\frac{\partial w}{\partial \eta}\right|_{(1,\tau)} = 0 \tag{3.6}$$

Now, we employ the discrete Galerkin technique to find the solution of Eq. (3.4). Hence, we write the deflection function $w(\eta,t)$, which denotes the product of two functions.

$$w(\eta,t) = \psi(\eta)q(t) \tag{3.7}$$

where ψ represents the time and trail function, which, as indicated in Lobontiu's 2007 study [11], is expressed as follows:

$$\psi(\eta,t) = 16\eta^2(1-\eta)^2 \tag{3.8}$$

From the governing differential equation, we substitute Eq. (3.7) into Eq. (3.4), multiply by factor $\psi^2(1-w^2)^2$ and then integrate over dimensionless domain to obtain the following equation:

$$\int_0^1 \psi(1-\psi^2q^2)^2\psi''''d\eta + \int_0^1\psi^2(1-\psi^2q^2)^2\ddot{q}d\eta - \int_0^1\psi(1-\psi^2q^2)^2\left(N + \alpha\int_0^1\left(\frac{\partial w}{\partial \eta}\right)d\eta\right)q\psi''d\eta - \int_0^1 v^2\psi^2qd\eta = 0 \tag{3.9}$$

where over-dot indicates the differentiation with respect to time to and prime dot denotes the derivative with respect to time to coordinate variable η. Eq. (3.9) can be rewritten as follows:

$$(g_0 + g_1q^2 + g_2q^4)q'' + g_3q^2 + g_4q'q^3 + g_5q'q^5 + g_6q^7 = 0 \tag{3.10}$$

where coefficients $g_0, g_1, g_2 ......g_6$ can be determined as given in Appendices A–C.

Eq. (3.10) represents the nonlinear ordinary differential equation (ODE) under the given initial conditions.

$$q(0) = A, \qquad q'(0) = 0 \tag{3.11}$$

3.1. Applications

In this article, we address three well-known applications of the higher-order nonlinear problems: the four-order differential equation of the multiwalled carbon nanotube-based microelectromechanical systems (MWCNT-MEMSs), the six-order differential equation of the microbeam subjected to the van der Waals force, and the seven-order differential equation of the doubly clamped electrically actuated microbeam-based MEMS.

3.2. The model of a doubly clamped electrically actuated microbeam-based MEMSs

The structural, electrical, and mechanical properties of the microbeam are usually represented mathematically in MEMSs, as

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shown in Figure 1. This model takes into account the mechanical characteristics of the beam, such as its stiffness and damping, as well as the electrostatic force produced by applying a voltage to the microbeam. To forecast the behavior of the microbeam under various operating conditions, such as changing voltage or temperature, the model's governing equations can be solved analytically or numerically. The concept is frequently applied to the design and optimization of MEMS devices for a range of uses, such as signal processing, actuation, and sensing. The nonlinear differential equation that is discussed in this section serves as an example of numerous oscillatory systems found in nanoscience and engineering [46–48].

$$(c_0 + c_1 q^2 + c_2 q^4)q'' + c_3 q^2 + c_4 q'^{q^3} + c_5 q' q^5 + c_6 q^7 = 0 \tag{3.12}$$

where $c_i (i = 0,1,2,...,7)$ are parameters and dividing c_0 Eq. (3.12) becomes,

$$(1 + g_1 q^2 + g_2 q^4)q'' + g_3 q^2 + g_4 q' q^3 + g_5 q' q^5 + g_6 q^7 = 0 \tag{3.13}$$

where $g_i = \frac{c_i}{c_o}$ for $(i = 0,1,2,...,7)$.

Now, for rewriting Eq. (3.13), we may consider a general nonlinear oscillator equation, which could be expressed as:

$$q''(t) + f(q) = 0$$

where the initial conditions are,

$$q(0) = A, \qquad q'(0) = 0$$

The above equation can be rewritten as:

$$q'' + \omega^2 q + h(q) = 0 \tag{3.14}$$

where ω is the frequency that can be calculated as:

$$h(q) = f(q) - \omega^2 q$$

where

$$h(q) = (1 + g_1 q^2 + g_2 q^4)q'' + (g_3 q - \omega^2)q + g_4 q' q^3 + g_5 q' q^5 + g_6 q^7 = 0.$$

By applying Aboodh transform on both sides of Eq. (3.13), we obtain:

$$q(v) = \left(\frac{1}{v^2 + \omega^2}\right)q(0) + \frac{1}{v(v^2 + \omega^2)}q'(0) - \frac{1}{v^2 + \omega^2}A[(1 + g_1 q^2 + g_2 q^4)q'' + g_3 q + g_4 q^3 + g_5 q^5 + g_6 q^7] \tag{3.15}$$

Now, we apply the Aboodh inverse transform and use the initial conditions $q(0) = A, q'(0) = 0$ in Eq. (3.12) to obtain,

$$q(v) = A\cos wt - A^{-1}\left[\frac{1}{v^2 + \omega^2}A[(1 + g_1 q^2 + g_2 q^4)q'' + g_3 q + g_4 q^3 + g_5 q^5 + g_6 q^7]\right] \tag{3.16}$$

By using Aboodh and inverse properties, we obtain the coefficients of p^0 and p^1 from Eqs. (2.9) and (2.10) as follows:

With the help of the properties of AT and inverse AT, we obtain the coefficients of p_0 and p_1 as follows:

$$p^0 : z_0(t) = A\cos wt \tag{3.17}$$

$$p^1 : z_1(t) = \frac{1}{2\omega}\left[A\omega^2 + \frac{3}{4}g_1 A^3 \omega^2 + \frac{5}{8}g_2 A^5 \omega^2 - g_3 A - \frac{3}{4}g_4 A^3 - \frac{10}{16}g_5 A^5 - \frac{35}{64}g_6 A^7\right]t\sin wt +$$
$$\frac{1}{8\omega^2}\left[-A\omega^2\left(\frac{1}{4}g_1 A^2 + \frac{5}{16}g_2 A^4\right) + \frac{1}{16}g_4 A^4 + \frac{5}{16}g_5 A^5 +$$

$$\frac{21}{64}g_6 A^7\right](\cos wt - \cos 3wt) + \frac{1}{24\omega^2}\left[-A\omega^2\left(\frac{1}{16}g_2 A^4\right) + \frac{1}{16}g_5 A^5 + \frac{7}{64}g_6 A^7\right](\cos wt - \cos 5wt) +$$
$$\frac{1}{48\omega^2}\left[\frac{1}{64}g_6 A^7\right](\cos wt - \cos 7wt) \tag{3.18}$$

The secular-term in Eq. (3.18) is written as follows:

$$A\omega^2 + \frac{3}{4}g_1 A^3 \omega^2 + \frac{5}{8}g_2 A^5 \omega^2 - g_3 A - \frac{3}{4}g_4 A^3 - \frac{10}{16}g_5 A^5 - \frac{35}{64}g_6 A^7 = 0 \tag{3.19}$$

which is similar to first-order frequency determined by Anjum et al. [49]:

$$\omega = \sqrt{\frac{g_3 + g_4 A^2 + g_4 A^4 + g_6 A^6}{1 + \frac{3}{4}g_1 A^2 + \frac{5}{8}g_2 A^4}} \tag{3.20}$$

By substituting Eq. (3.20) in Eq. (3.18), we obtain the first-order approximate analytic solution, which can be expressed as follows:

$$q_{ATHMP} = A\cos wt + \frac{1}{8\omega^2}\left[-A\omega^2\left(\frac{1}{4}g_1 A^2 + \frac{5}{16}g_2 A^4\right) + \frac{1}{16}g_4 A^4 + \frac{5}{16}g_5 A^5 + \frac{21}{64}g_6 A^7\right](\cos wt - \cos 3wt) + \frac{1}{24\omega^2}\left[-A\omega^2\left(\frac{1}{16}g_2 A^4\right) + \frac{1}{16}g_5 A^5 + \frac{7}{64}g_6 A^7\right](\cos wt - \cos 5wt) + \frac{1}{48\omega^2}\left[\frac{1}{64}g_6 A^7\right](\cos wt - \cos 7wt) \tag{3.21}$$

Now, we analyse various applications related to the oscillatory system expressed in Eq. (3.12).

3.3. Case 1 (microbeam is subjected to the van der Waals force)

Fascinating events occur when the van der Waals force acts on a microbeam, which is a tiny beam or structure at the microscale. Because of transient variations in the distribution of electrons between atoms and molecules, there is a weak attractive force known as the van der Waals force. The van der Waals force becomes considerable when the microbeam approaches a surface or another microstructure. The microbeam sticks to the surface or neighboring structures because of this force, which functions as an adhesive. The distance between the microbeam and the surface, the characteristics of the material, and the roughness of the surfaces involved all affect how strong the van der Waals force is. Comprehending and managing the van der Waals force in microscale systems is essential for uses in domains including surface science, nanotechnology, and MEMS. In the study of Qian et al. [22], it is expressed as follows:

$$(h_0 + h_1 q + h_2 q^2 + h_3 q^3)q'' + h_4 + h_5 q + h_6 q^2 + h_7 q^3 + h_8 q^4 + h_9 q^5 + h_{10}q^6 = 0 \tag{3.22}$$

$$q(0) = A, \qquad q'(0) = 0$$

where q is the normalised midpoint deflection of the beam, and q' and q'' denote the first and second derivatives of t, respectively.

The solution of Eq. (3.22) is already found by harmonic balance technique. Eq. (3.22) can be rewritten as follows:

$$(1 + m_1 q + m_2 q^2 + m_3 q^3)q'' + m_4 + m_5 q + m_6 q^2 + m_7 q^3 + m_8 q^4 + m_9 q^5 + m_{10}q^6 = 0 \tag{3.23}$$

The approximate analytic solution of Eq. (3.22) can be obtained by using the Aboodh homotopy method as follows:

$$q_{ATHPM} = -\frac{\Psi_0}{\omega^2} + \frac{1}{\omega^2}\left(\Psi_0 - \frac{\Psi_2}{3} - \frac{\Psi_3}{8} - \frac{\Psi_4}{15} - \frac{\Psi_5}{24} - \frac{\Psi_6}{35}\right.$$
$$\left. + A\right)coswt + \frac{1}{3\omega^2}\Psi_2 cos2wt$$
$$+ \frac{1}{8\omega^2}\Psi_3 cos3wt + \frac{1}{15\omega^2}\Psi_4 cos4wt$$
$$+ \frac{1}{24\omega^2}\Psi_5 cos5wt + \frac{1}{35\omega^2}\Psi_6 cos6wt$$

(3.24)

Moreover, the constituents of the coefficients Ψ_m, (m = 0,1,2, ... 6) may be written in terms of, and the nonlinear frequency of the oscillator is obtained by applying, the Aboodh homotopy perturbation method (ATHPM) in the following form:

$$\omega = \sqrt{\frac{m_5 + 6m_7 A^2 + m_9 A^4}{8 + m_2 A^2}}$$

(3.25)

which is similar to the first-order frequency determined by Anjum et al. [50] with the use of a Laplace method.

3.4. Case II: (MWCNT-MEMSs)

MEMSs based on MWCNTs offer a viable method for producing highly effective and sensitive sensors. The unique mechanical, electrical, and thermal properties of MWCNTs enable the development of MEMS devices with previously unheard-of levels of sensitivity and precision. MWCNT-MEMSs have shown promise for application in a variety of fields, such as aerospace, biology, and environmental monitoring. As technology advances, MWCNT-MEMSs are expected to be utilized more often and have a major influence on the future direction of MEMS devices. The nonlinear vibratory equation [21] in the second case will have the following solution:

$$q'' + k_0 + k_1 q + k_2 q^2 + k_3 q^3 + k_4 q^4 = 0$$

(3.26)

The approximate analytic solution of Eq. (3.26) can be determined by using the Aboodh homotopy method.

$$q_{ATHPM} = \Phi_0 + [\Phi_1 + A]coswt + \Phi_2 cos2wt$$
$$+ \Phi_3 cos3wt + \Phi_4 cos4wt$$

(3.27)

where

$$\omega = \sqrt{k_1 + k_3 A^2}$$

(3.28)

Eqs. (3.27) and (3.28) are also similar to the results obtained with the use of the parameter expansion method [23] for multi-walled nanotube models.

4. CONCLUSION

It has shown to be a useful and successful method to analyze electrically actuated microbeams in MEMSs exposed to van der Waals force and MWCNTs by using the HPM with the Aboodh transformation. The HPM is a potent mathematical method that creates an analytical solution in the form of a convergent series to solve nonlinear differential equations. The precision and convergence of the solutions found are further improved by the Aboodh transformation, which is a variation of the HPM. Researchers have

been able to handle the intricate dynamics of electrically actuated microbeams in MEMS devices by utilizing this mix of techniques. The behavior and performance of microbeams are greatly affected by the presence of van der Waals forces and MWCNTs. The behavior of microscale devices is significantly influenced by van der Waals forces, which result from the interaction of atoms and molecules. On the other hand, MWCNTs are attractive options for a range of MEMS applications due to their distinct mechanical and electrical characteristics. A more accurate depiction of the system's behavior can be obtained by analyzing both of these variables. The outcomes obtained by combining the HPM with the Aboodh transformation have shown that it is possible to forecast electrically activated microbeams in MEMs that are exposed to MWCNTs and van der Waals forces with accuracy. The technique enables a more thorough comprehension of the behavior of the system, including the impact of several parameters on its dynamic response, such as the length, width, and applied voltage of the beam. In terms of computational efficiency and practicality, the HPM with the Aboodh transformation is also superior to other numerical techniques, like finite element methods. For scientists and engineers working on the design and optimization of MEMS devices, this makes it a useful tool.

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