Power System State Estimation using Dispersed Particle Filter

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Abstract:
The paper presents a new approach to particle filtering, i.e. Dispersed Particle Filter. This algorithm has been applied to the power system, but it can also be used in other transmission networks. In this approach, the whole network must be divided into smaller parts. As it has been shown, use of Dispersed Particle Filter improves the quality of the state estimation, compared to a simple particle filter. It has been also checked that communication between subsystems improves the obtained results. It has been checked by means of simulation based on model, which has been created on the basis of knowledge about practically functioning power systems.

Keywords: *particle filter, power system, state observer, state estimation, dispersed estimation*

1. Introduction

The Power System State Estimation (PSSE) problem is relatively old, because it has over 40 years, and the first idea of state estimation in power system has been proposed by Fred Schweppe in 1970 [21]. But PSSE is still used also in more advanced calculations, such as Optimal Power Flow (OPF). In this case, correct state vector estimation directly affects final solution.

PSSE is also important in terms of energy security, to prevent so-called "blackouts" – this is the highest degree of failure of the power system, in which many villages and towns can be without access to electric power. To prevent such accidents, current control of the results obtained in the PSSE calculations is needed.

A lot of different algorithms have been created so far in order to solve the problem of PSSE, such as Weighted Least Squares (WLS), varieties of Kalman Filter (Extended Kalman Filter (EKF) [11], Unscented Kalman Filter (UKF) [24]). In [23] authors presented the use of a different estimator than typically used in the WLS method, and apart from the typical Newton method they suggested the use of Primal-dual Interior Point Method (PDIPM). In [4], authors proposed the variety of Particle Filter (PF) as a state observer in relatively small power system.

In this article a new algorithm, i.e. Dispersed Particle Filter (DPF) has been proposed. It involves the use not just one, but several different PF instances to run in parallel for different parts of the power system (each instance can be carried out in another computational unit, which can be placed in another area of power system). This approach is consistent with indicated in [10] need of PSSE based on data from different control centres.

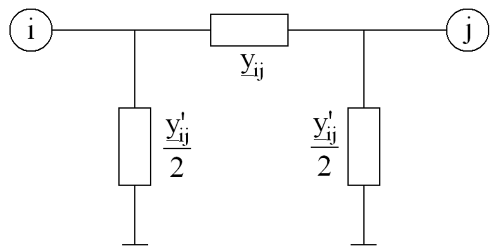

Fig. 1. Branch in network between i-th and j-th buses

The second Section is devoted to the power system and is followed by Section presenting basic information about particle filter, while the fourth Section describes simulations that have been carried out, presenting results and the conclusions. Section no. 5 summarizes the entire article.

2. Power System

Power system has been selected as object. Network is composed of B buses (nodes) and L branches (lines) that connect specified buses. Branch scheme has been shown in Fig. 1, where $\underline{y}'_{ij}/2$ is a half total line charging susceptance [25], whereas $\underline{y}_{ij}$ is a line admittance, which can be expressed by the equation

$$\underline{y}_{ij} = \frac{1}{\underline{Z}_{ij}} = \frac{1}{R_{ij} + \mathrm{j}X_{ij}} \ . \tag{1}$$

Based on $\underline{y}_{ij}$ and $\underline{y}'_{ij}/2$ admittance matrix $\underline{\mathbf{Y}}$ can be created accordingly to equations

$$\underline{Y}_{ij} = -\underline{y}_{ij} \qquad i \neq j \ , \tag{2}$$

$$\underline{Y}_{ii} = \sum_{\substack{j=1 \\ j\neq i}}^{B} \left(\frac{\underline{y}'_{ij}}{2} + \underline{y}_{ij} \right) \ . \tag{3}$$

Set of all voltages U and angles δ at the buses unambiguously describe state of the power system

$$\begin{aligned} \mathbf{x} &= [x_1 \ \ x_2 \ \ \ldots \ \ x_{2B-1}]^T \\ &= [U_1 \ \ \ldots \ \ U_B \ \ \delta_2 \ \ \ldots \ \ \delta_B]^T \ , \end{aligned} \tag{4}$$

because based on them it is possible to calculate all the other values in power system. But there is a problem with the angles, because the only thing that can be calculated is the difference between them. Therefore, one reference node must be chosen, in which angle is always equal to 0 (in expression (4) first node has been chosen as reference, hence δ_1 is always equal to 0, and this angle is not included in the state vector). Accordingly, the state vector is limited to $2B-1$ variables.

The measured values in the network are power injections in the nodes, power flows through the branches and voltage values in the buses. The last type of measurement (voltage magnitude) is special, because it is directly state variable. In other cases, the measured values can be expressed by the equations

$$P_i(\mathbf{U},\boldsymbol{\delta}) = P_i = \\ = \sum_{j=1}^{B} U_i U_j Y_{ij} \cos\left(\delta_i - \delta_j - \mu_{ij}\right), \quad (5)$$

$$Q_i(\mathbf{U},\boldsymbol{\delta}) = Q_i = \\ = \sum_{j=1}^{B} U_i U_j Y_{ij} \sin\left(\delta_i - \delta_j - \mu_{ij}\right), \quad (6)$$

$$P_{ij}(\mathbf{U},\boldsymbol{\delta}) = P_{ij} = U_i^2 Y_{ij} \cos\left(-\mu_{ij}\right) \\ - U_i U_j Y_{ij} \cos\left(\delta_i - \delta_j - \mu_{ij}\right), \quad (7)$$

$$Q_{ij}(\mathbf{U},\boldsymbol{\delta}) = Q_{ij} = U_i^2 Y_{ij} \sin\left(-\mu_{ij}\right) \\ - U_i U_j Y_{ij} \sin\left(\delta_i - \delta_j - \mu_{ij}\right) + U_i^2 \frac{y'_{ij}}{2}, \quad (8)$$

where (5-6) are the power injections (active and reactive), while (7-8) are the power flows. It should be noted that the P_{ij} and P_{ji} are two different power flows (as well as Q_{ij} and Q_{ji}), and first index specifies the node where the measurement is made. Y_{ij} and μ_{ij} values are given in admittance matrix based on

$$\underline{Y}_{ij} = Y_{ij} \cdot \exp\left(\mathrm{j}\mu_{ij}\right). \quad (9)$$

For more information about the power system, references [1, 18, 25] are recommended.

3. Particle Filter

The principle of operation is based on Bayesian filtering, and the PF is one of possible implementation of the Bayes filter [3]

$$\overbrace{p\left(\mathbf{x}^{(k)}|\mathbf{Y}^{(k)}\right)}^{\text{posterior}} = \\ = \frac{\overbrace{p\left(\mathbf{y}^{(k)}|\mathbf{x}^{(k)}\right)}^{\text{likelihood}} \cdot \overbrace{p\left(\mathbf{x}^{(k)}|\mathbf{Y}^{(k-1)}\right)}^{\text{prior}}}{\underbrace{p\left(\mathbf{y}^{(k)}|\mathbf{Y}^{(k-1)}\right)}_{\text{evidence}}}, \quad (10)$$

where $\mathbf{x}^{(k)}$ is vector of state variables in k-th time step, $\mathbf{y}^{(k)}$ is vector of measurements, whereas $\mathbf{Y}^{(k)}$ is a set of output vectors from the beginning of simulation to k-th time step.

Posterior probability density function (PDF) is represented by the set of particles in PF, where each particle is composed of the value $\mathbf{x}^i$ (vector of state variables) and the weight q^i. Therefore it can be written that the set of particles corresponds to the posterior PDF. With higher number of particles N, this approximation is more accurate

$$p\left(\mathbf{x}^{(k)}|\mathbf{Y}^{(k)}\right) \overset{N\to\infty}{=} \hat{p}\left(\mathbf{x}^{(k)}|\mathbf{Y}^{(k)}\right) = \\ = \sum_{i=1}^{N} q^{i,(k)} \delta\left(\mathbf{x}^{(k)} - \mathbf{x}^{i,(k)}\right), \quad (11)$$

where $\delta(\cdot)$ is a Dirac delta.

In [12] the authors point out that the prior should be chosen so that as many particles as possible were drawn in the area where likelihood has significant values. Prior can be written as [2]

$$p\left(\mathbf{x}^{(k)}|\mathbf{Y}^{(k-1)}\right) = \int p\left(\mathbf{x}^{(k)}|\mathbf{x}^{(k-1)}\right) \\ p\left(\mathbf{x}^{(k-1)}|\mathbf{Y}^{(k-1)}\right) d\mathbf{x}^{(k-1)}, \quad (12)$$

where $p(\mathbf{x}^{(k)}|\mathbf{x}^{(k-1)})$ is a transition model and $p(\mathbf{x}^{(k-1)}|\mathbf{Y}^{(k-1)})$ is posterior from previous time step.

First who suggest something what today can be considered as a particle filter was Norbert Wiener, and it was as early as in first half of the twentieth century [22]. However, only a few decades later, the power of computers made it possible to implement these algorithms. Algorithm proposed in [9] by Gordon, Salmond and Smith, named by them Bootstrap Filter (BF), is considered as the first PF. The operation principle of the algorithm is presented below.

Algorithm 1 (Bootstrap Filter)

1) Initialization. Draw N initial particle values from the PDF $p(\mathbf{x}^{(0)})$. Set iteration number $k=1$.
2) Prediction. Draw N new particles based on transition model: $\mathbf{x}^{i,(k)} \sim p(\mathbf{x}^{(k)}|\mathbf{x}^{i,(k-1)})$.
3) Actualization. Calculate weights of particles based on equation $\tilde{q}^{i,(k)} = p(\mathbf{y}^{(k)}|\mathbf{x}^{i,(k)})$.
4) Normalization. Normalize particle weights so that their sum be equal to 1

$$q^{i,(k)} = \frac{\tilde{q}^{i,(k)}}{\sum_{j=1}^{N} \tilde{q}^{j,(k)}}. \quad (13)$$

5) Resampling. Draw N new particles based on posterior PDF obtained in steps 2–4.
6) End of iteration. Calculate estimated vector value, increase number of iteration $k = k+1$, go to step 2.

BF algorithm belongs to a class of Sequential Importance Resampling (SIR) algorithms and is one of its simpler implementation. Of course, also more complicated algorithms are proposed in the literature, for example Auxiliary PF [20], Rao-Blackwellised PF [8] or the Gaussian PF [13]. However, for purpose of this article, BF algorithm and its modifications were used – results which have been obtained in previous studies [14, 16] are satisfactory. In addition, the simplicity of implementation with such a complex problem, which is the PSSE, led to the choice of algorithm proposed in [9].

Resampling (5 step in Algorithm 1) also can be made in several different ways. In algorithm the stratified resampling has been used. In order to learn more about the resampling, see [6, 17, 19].

To find more information about particle filter, references [2, 7, 15] are recommended.

3.1. Dispersed Particle Filter

Network composed of 16 buses has been proposed, as shown in Fig. 2, so there are $2B - 1 = 31$ state variables. To use DPF the whole power system has been divided into 3 parts – PS_1, PS_2 and PS_3. This division is one of many possible, as well as the number of subsystems in this example. The main purpose in this article was to show how the division of the system into smaller parts affects on obtained results.

For the DPF implementation the assumption has been made that only measurements inside the subnet are available. However, for a good modeling, border lines and nodes at their ends are also required. For example, in the first subsystem there are 9 modeled nodes (1, 2, 3, 4, 5, 6 and additionally 7, 8 and 13) and the number of state variables in PS_1 is 17 (in the other two subsystems there are 8 nodes, and 15 state variables – reference node must exist in each subnet).

As one can see, the number of state variables in each subsystem has been decreased and this should has positive influence on estimation quality.

4. Simulations

16-nodal system has been proposed for simulations (see Fig. 2). The numbers in circles indicate numbers of the nodes and values of R, X and $y'/2$ are, respectively, line resistance, line reactance and half of the total line charging susceptance. The double circles represent the location of the generators, single circles are the loads. There are also locations and types of measurements – the gray squares mark measurements of the power flows, while the grey circles mark the measurements of power injections and voltage values.

One simulation, which consists of 100 time steps has been prepared and has been used for all calculations.

Simulation computations for all subsystems have been made not in parallel, but sequentially one by one.

4.1. Simulation Results

The first simulations have been performed for the simple PF algorithm. The results have been shown in Fig. 3. The simulation for each number of particles N has been repeated 100 times with different values of the seed of random number generator. The value D, which is shown in the graph, was calculated based on mean square errors (MSE) of each of the state variables (there are 31 state variables in whole power system). This can be written as

$$D = 10^6 \cdot \sum_{i=1}^{31} (\mathrm{MSE}_i)^2 \,. \tag{14}$$

Thanks to this, estimation quality of whole system can be represented by one coefficient.

Next the another approach has been proposed, in which simultaneously three different particle filters operate, each in a different subsystem, thus creating Dispersed Particle Filter. The assumption has been made that the individual subnets does not communicate with each other and does not exchange any information. Values of the state variables in each node were obtained based on the estimated values in the subnets. The simulation results have been shown in Fig. 4.

As before, the graph shows the averaged results of D, based on 100 different simulations for each value of N. Significant improvement is visible, both in terms of the mean and standard deviation.

In the third approach the exchange of data between subnets has been implemented. For the border branch information about the power flows was passed to another subsystem, and was taken as additional measurements. For example, in PS_3 one of such border branch is the line (1,13). Information, which was sent from PS_1 to PS_3, was the estimated values of $P_{1,13}$ and $Q_{1,13}$. Both of these values were regarded in PS_3 as another measurements. Similar information was transferred from the PS_3 to the PS_1, but this time the values $P_{13,1}$ and $Q_{13,1}$. The results have been shown in Fig. 5.

In the last approach the impact of the additional measurement (transferred as in the previous case) – voltage magnitude in bus – has been checked. The results have been shown in Fig. 6.

All results (excluding the standard deviation) have been shown in Fig. 7, for comparison.

Based on obtained results, one can see that the use of DPF significantly improves the quality of estimation, in comparison to the standard PF algorithm. As it can be seen, performance improves even for the DPF without any communication between subnets. This can be explained by the fact that in the case of standard PF particles have to move in a 31-dimensional space (the number of state variables). In the case of DPF number of particles was smaller, but the number of dimensions of the state vector was also decreased – to 17 (for the PS_1) and 15 (for the PS_2 and PS_3). Similar conclusions can be found in [5] (case without communication).

The results obtained for DPF with additional measurements of power flows and voltage are very similar to those in which the voltage measurement is not passed. This is understandable, because in the values of the power flows are already contained information about state variables in this node.

5. Summary

The article presents a new approach to particle filtering in the problem of Power System State Esti-

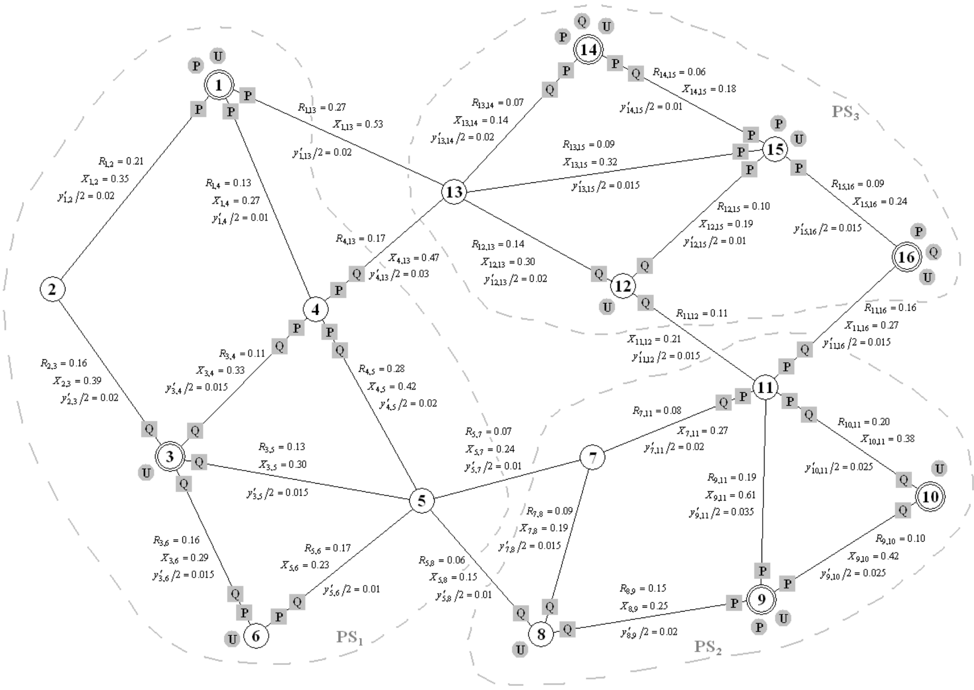

Fig. 2. Power system used in simulations composed of 16 buses, with marked measurements

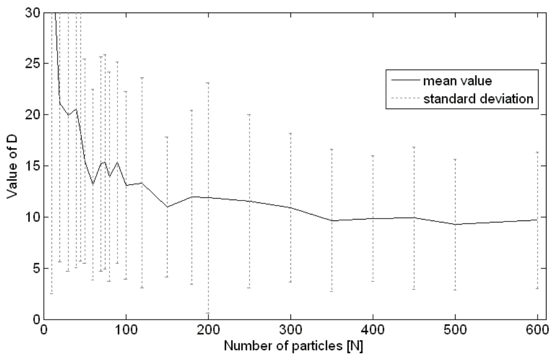

Fig. 3. Mean D values for standard PF

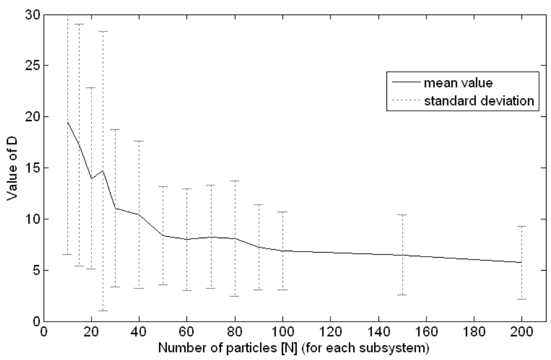

Fig. 4. Mean D values for DPF without any communication between subsystems.

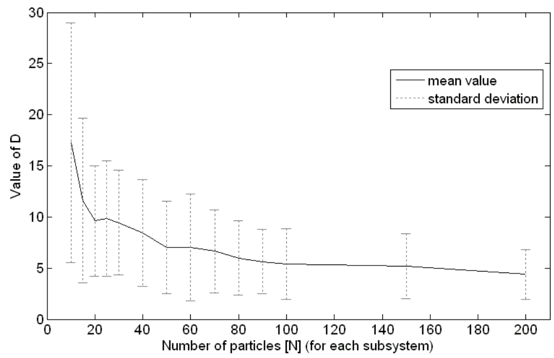

Fig. 5. Mean simulation results for DPF with additional measurements of power flows

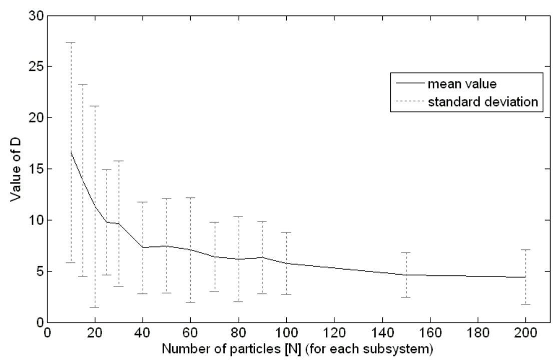

Fig. 6. Mean simulation results for DPF with additional measurements of power flows and voltages

mation – Dispersed Particle Filter. In each configuration of communication, results obtained by the DPF were better than for the standard PF. This is because power system division causes reduction of state vector length, and improvement of estimation quality is observed with decrease of object order. The best results have been obtained for cases with additional measurements.

Further studies on the DPF are being planned, in-

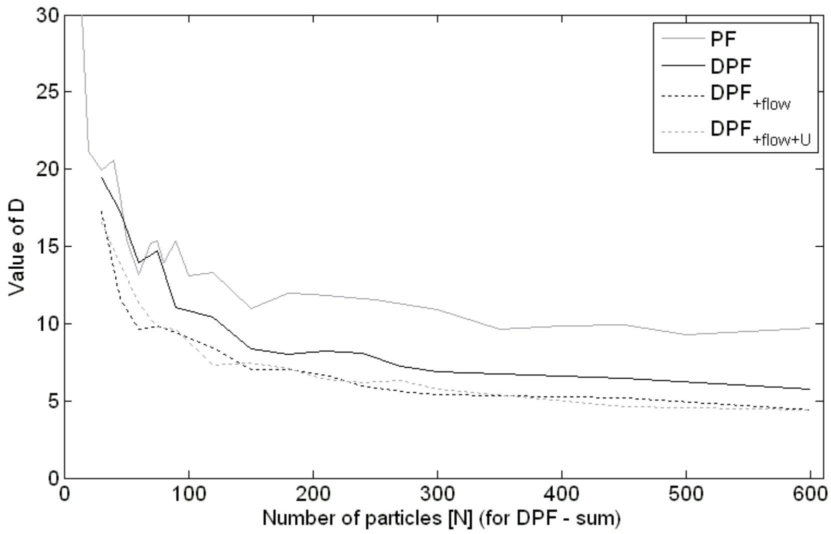

Fig. 7. Simulation results for different algorithms.

cluding the impact of the number of subsystems on simulation results. For a smaller number of state variables into subsystem simulation results should be better, hence the best division will be probably one subsystem for every bus.

There are also plans for FPGA algorithm implementation and verification of the proposed algorithm performance of parallel computing for several computational units.

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