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SELECTED PROBLEMS RELATING TO THE DYNAMICS OF BLOCK-TYPE FOUNDATIONS FOR MACHINES

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Abstract: Atypical but real practical problems relating to the dynamics of block-type foundations for machines are considered using the deterministic approach and assuming that the determined parameters are random variables. A foundation model in the form of an undeformable solid on which another undeformable solid modelling a machine is mounted via viscoelastic constraints was adopted. The dynamic load was defined by a harmonically varying signal and by a series of short duration signals. The vibration of the system was investigated for the case when stratified ground (groundwater) occurred within the side backfill was present. Calculation results illustrating the theoretical analyses are presented.

Keywords: block – type foundations dynamics, stratified soil

1. INTRODUCTION

For machines most commonly block-type foundations are used. There is an extensive literature on their dynamics, covering both theoretical and practical problems (Lipiński, 1980; Gazetas, 1983; Klasztorny at all, 1978; Major, 1962). But there are relatively few publications devoted to atypical problems, such as the change in the stiffness of the ground as a result of its gel injection (Tschebatarioff, 1964), freezing (Stevens, 1975), cementation (Chlang and Chae, 1972), underground water (Siva Reddy i in., 1970) and damage due to the use of unconventional solutions (Mironowicz, 1991). Such publications contribute to the knowledge on the dynamics of block-type foundations and so serve both theory and engineering practice. This means that they are worthy of continuation, as noted by, among others, the authors of

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(Gazetas, 1983; Novak, 1989). The present study contributes to the above field, dealing with selected natural and forced vibration problems for a system being a calculation model of a machine foundation sunk in the ground and bearing a machine mounted on it. One, emerging from practice, problem is considered (Novak, 1989; Gazetas, 1983; Braja, 1984):

The presence of stratified soil within the foundation's side backfill; more precisely the presence of groundwater up to height h_d (fig. 1) and the occurrence of a stratum frozen down to depth h_g. Practically, this applies to cases when the foundation is located outdoors, which a real possibility.

2. FORMULATIONS AND THEORETICAL SOLUTIONS

2.1. EQUATION OF VIBRATION – GENERAL FORMULATION

The system shown in fig. 1 is considered. It consists of two undeformable solids, the lower of which is a model of a block-type foundation while the upper one is a model of a machine mounted on it. The foundation is sunk in the ground at depth *h*. A case when the ground is homogenous and a case when it consists of two different layers with respectively height h_g and h_d are considered. There is a set of vibration dampers (viscoelastic constraints).

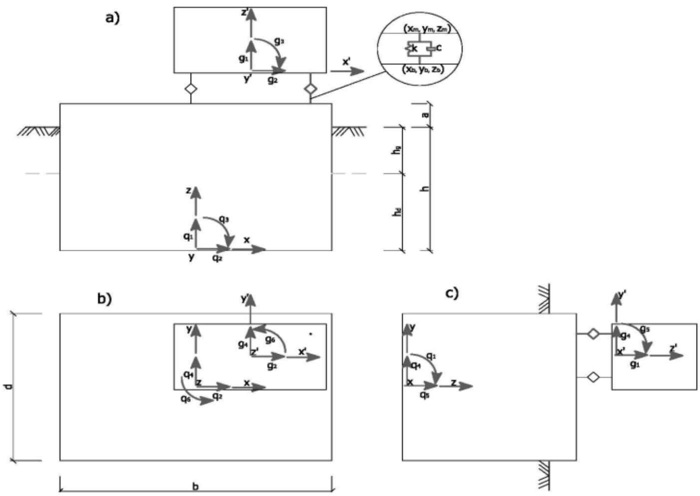

Fig. 1. Model of block-type foundation sunk in ground and machine resting on foundation: a) front view, b) top view, c) side view

The vibration of the system is described in the generalized coordinates basis

$$\mathbf{q} = [\mathbf{q_b}, \mathbf{g_m}]^{\mathrm{T}} \tag{1}$$

where (fig. 1): $\mathbf{q_b} = [q_1, \ldots, q_6]^{\mathrm{T}}$, $\mathbf{g_m} = [g_1, \ldots, g_6]^{\mathrm{T}}$
The equation of vibration has this form

$$\mathbf{B}\,\ddot{\mathbf{q}} + \mathbf{C}\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{F}\ \mathrm{f}(t)\,. \tag{2}$$

The foundation-machine system inertia matrix has the block form

$$\mathbf{B} = \begin{bmatrix} \mathbf{B_b} & \mathbf{0} \\ \mathbf{0} & \mathbf{B_m} \end{bmatrix}. \tag{3}$$

Block-type foundation inertia matrix $\mathbf{B_b}$ in basis $\mathbf{q_b}$, has this well-known form

$$\mathbf{B_b} = \begin{bmatrix} m & 0 & -S_{yz} & 0 & S_{xz} & 0 \\ 0 & m & S_{xy} & 0 & 0 & -S_{xz} \\ -S_{yz} & S_{xy} & J_y & 0 & -D_z & -D_x \\ 0 & 0 & 0 & m & -S_{xy} & S_{yz} \\ S_{xz} & 0 & -D_z & -S_{xy} & J_x & -D_y \\ 0 & -S_{xz} & -D_x & S_{yz} & -D_y & J_z \end{bmatrix}_{(b)} \tag{4}$$

where: m – the mass of the foundation block, S_{xy} – a static moment of the foundation block mass relative to a plane defined by axes $x\,y$, D_x – a moment of deviation of the foundation block mass relative to planes intersecting along axis x, J_x – a moment of inertia of the foundation block mass relative to axis x.

Inertia matrix $\mathbf{B_m}$ of the solid being the machine model has also form (4) with subscript $_{(m)}$ used instead of subscript $_{(b)}$.

The foundation-machine system stiffness matrix is written in basis $\mathbf{q}$ as follows

$$\mathbf{K} = \mathbf{K_w} + \mathbf{K_g}. \tag{5}$$

Soil subbase stiffness matrix $\mathbf{K_g}$ is formulated assuming the same elastic half space model soil parameters – as in (Wolf, 1975). Assuming the ground to be homogenous, one gets

$$\mathbf{K_g} = \begin{bmatrix} \mathbf{K_g}(\mathbf{q_b}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{6}$$

where:

$$\mathbf{K_g}(\mathbf{q_b}) = \begin{bmatrix} K_v & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{K_{xz}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K_{yz}} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & K_t \end{bmatrix}, \mathbf{K_{xz}} = \mathbf{K_{yz}} = \begin{bmatrix} K_h & K_{hr} \\ K_{hr} & K_r \end{bmatrix},$$

$$K_v = 4\,G_o r_o (1 - n)^{-1}\left(1 + 0{,}54\,h\,r_o^{-1}\right), \quad K_h = 8\,G_o r_o (2 - n)^{-1}\left(1 + h\,r_o^{-1}\right),$$

$$K_h = 8\,G_o r_o (2 - n)^{-1}\left(1 + h\,r_o^{-1}\right), \quad K_t = (16/3)\,G_o r_o^3\left(1 + 2{,}67\,h\,r_o^{-1}\right),$$

$$K_r = 8\,G_o r_o^3 (3(1 - n))^{-1}\left(1 + 2{,}3\,h\,r_o^{-1} + 0{,}58\left(h\,r_o^{-1}\right)^3\right), \quad K_{hr} = (h/3)K_h .$$

The above formulas hold true for $h / r_o \leq 2$.

The symbols stand for: h – the depth of foundation of the block, r_o – the effective radius of the circular foundation footing, ν – the Poisson ratio of the ground, G_o– the shear modulus of the ground.

Stiffness matrix $\mathbf{K_w}$, which follows from the vibration insulation, is defined by the relation

$$\text{grad}\,E_w(\mathbf{q}) = \text{grad}\,(\,½\mathbf{q}^T\mathbf{K_w}\mathbf{q}) = \mathbf{K_w}\mathbf{q} \tag{7}$$

where:

$$E_w = ½\sum_i k_i \left(u_{mi} - u_{bi}\right)^2 \tag{8}$$

E_w – the potential energy accumulated in the vibration insulation,
u_{mi} – the local displacement of the machine in the location and along the direction of the constraint with stiffness,
k_i, u_{bi} – the local displacement of the foundation block in the location and along the direction of the constraint with stiffness,
k_i, k_i – the stiffness of the vibration damper.

The first term in formula (11) is an exemplary matrix $\mathbf{K_w}$ for a flat system.

In accordance with the Voigt-Kelvin hypothesis, the damping matrix is written as

$$\mathbf{C} = \kappa_1\mathbf{K_w} + \kappa_2\mathbf{K_g} \tag{9}$$

where κ_1, κ_2 – dimensional structural damping parameters.

Two types of dynamic load $f(t)$ typical for dynamic machine interaction, i.e. a sum of harmonic loads and a series of short-term loads, are considered. Hence

$$f(t) = (A_s \sin p \cdot t + A_c \cos p \cdot t)\,. \tag{10}$$

2.2. CASES OF STRATIFIED SOIL WITHIN SIDE BACKFILL

Practically such cases occur when groundwater is present in a stratum with thickness h_d or when a frozen soil layer with thickness h_g is present (fig. 1). In these circumstances stiffness matrix (6) needs to be corrected, which in the case of groundwater can be written in the simplified form (the symbols in brackets represent the height of the side backfill)

$$\mathbf{K_g} = \mathbf{K_o}(h) - \mathbf{K_o}(h_d) + \mathbf{K_w}(h_d) \tag{11}$$

where: $\mathbf{K_g}$ – the corrected soil subbase stiffness matrix; $\mathbf{K_o}$ – the stiffness matrix for naturally deposited soil (without groundwater); $\mathbf{K_w}$ – the stiffness matrix for soil in groundwater environment $0 < h_d < h$.

3. NATURAL VIBRATION PROBLEM AND FORCED VIBRATION PROBLEM

3.1. SOLUTION OF NATURAL VIBRATION PROBLEM FOR STRATIFIED SOIL

Assuming that the system parameters are deterministic, the equation for natural frequencies has this well-known form

$$\det(\mathbf{K} - \lambda \mathbf{B}) = \mathbf{0}$$

where $\lambda = \omega^2$, and by solving the equation one determines the spectrum of angular natural frequencies

$$\{\boldsymbol{\omega}\} = \text{diag}\,(\omega_1,\ \omega_2,\ \omega_3,\ \ldots).$$

When the selected parameters, e.g. G_o, G_w, h_d are random (random variables treated as discrete sets), then using the realizations set method one can calculate the expected value and variance λ_i $(\,i = 1, 2, 3, \ldots)$ from the relations

$$\mathrm{E}[\lambda_{\mathrm{i}}] = \sum_{j}\sum_{k}\sum_{l}\lambda_{\mathrm{i}}\left(G_{oj},\ G_{wk},\ h_{dl}\right)\ \mathrm{P}\left(G_{oj},\ G_{wk},\ h_{dl}\right) \tag{12}$$

$$\sigma_{\lambda\mathrm{i}}{}^{2} = \sum_{j}\sum_{k}\sum_{l}(\lambda_{\mathrm{i}}\left(G_{oj},\ G_{wk},\ h_{dl}\right) - \mathrm{E}[\lambda_{\mathrm{i}}]\ )^{2}\ \mathrm{P}\left(G_{oj},\ G_{wk},\ h_{dl}\right) \tag{13}$$

where: G_o – the shear modulus for naturally deposited soil (outside the groundwater zone or the frozen zone); G_w – the shear modulus for soil in the groundwater environment; P – probability density function.

3.2. SOLUTION OF FORCED VIBRATION PROBLEM

If the dynamic load is harmonic as in (10), then the solution of equation (2) can be written in the form

$$\mathbf{q} = \mathbf{q}_{\mathrm{s}}\mathrm{sinp}\ t + \mathbf{q}_{\mathrm{c}}\mathrm{cosp}t. \tag{14}$$

The solution can be obtained from the relation

$$\mathbf{q}_{\mathbf{o}} = \mathbf{M}^{-1}\mathbf{F}_{\mathbf{o}} \tag{15}$$

where: $\mathbf{M} = \begin{bmatrix} \mathbf{K} - \mathrm{p}^2\mathbf{B} & -\mathrm{p}\mathbf{C} \\ \mathrm{p}\mathbf{C} & \mathbf{K} - \mathrm{p}^2\mathbf{B} \end{bmatrix}$, $\mathbf{q}_{\mathbf{o}} = \begin{bmatrix} \mathbf{q}_{\mathrm{s}} \\ \mathbf{q}_{\mathrm{c}} \end{bmatrix}$, $\mathbf{F}_{\mathbf{o}} = \begin{bmatrix} \mathbf{F}\mathrm{A}_{\mathrm{s}} \\ \mathbf{F}\mathrm{A}_{\mathrm{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{\mathrm{s}} \\ \mathbf{F}_{\mathrm{c}} \end{bmatrix}$.

When load parameters A_{s}, A_{c}, p are random variables, p being continuous, the expected value and the correlation matrix for solution q can be presented in the form

$$\mathrm{E}\left[\mathbf{q}\right] = \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{E}[\left(\mathbf{q}_{\mathrm{s}}\sin \mathrm{p}\ t + \mathbf{q}_{\mathrm{c}}\cos \mathrm{p}\ t\right) | \mathrm{p}\big]\ \mathrm{F}\left(p\right)\ \mathrm{d}p \tag{16}$$

$$\mathbf{K}_{\mathrm{gg}}\left(t_1,\ t_2\right) = \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{E}[\big(\mathbf{q}_{\mathrm{s}}\sin \mathrm{p}\ t_1 + \mathbf{q}_{\mathrm{c}}\cos \mathrm{p}\ t_1)(\mathbf{q}_{\mathrm{s}}{}^{\mathrm{T}}\sin \mathrm{p}\ t_2 + \mathbf{q}_{\mathrm{c}}{}^{\mathrm{T}}\cos \mathrm{p}\ t_2) | \mathrm{p}\big]\ \mathrm{F}\left(p\right)\ \mathrm{d}p \tag{17}$$

where $\mathrm{F}\left(p\right)$ is a function defining the distribution of probability p in interval $<\mathrm{a},\mathrm{b}>$.

4. NUMERICAL ANALYSIS

4.1. INTRODUCTION

The results presented in this section illustrate the above theoretical formulations. More extensive numerical analyses are needed to draw more general conclusions concerning the problems considered here.

The model of the foundation with the machine mounted on it, shown in figure 1 is considered. It is a system of two cuboidal solids joined together by a set of viscoelastic constraints, with the bottom solid (the foundation model) sunk in the ground at depth h = 4 m. The bottom solid is a cuboid with dimensions: b = 6 m and the other dimensions – 4 m. The top solid is a cube with the side of 1.6 m and it is centrally located on the bottom solid and joined with the latter by means of 4 viscoelastic constraints with a height of 0.1 m and a stiffness of 12 MN/m, located in the corners. The density of the bottom solid is $2400\,\mathrm{kg/m^3}$, and that of the upper solid is $2500\,\mathrm{kg/m^3}$. The soil shear modulus is $\mathrm{G_o} = 15\,\mathrm{MPa}$, and the Poisson ratio is $\nu = 0{,}3$.

4.2. ANALYSES OF EIGEN PROBLEM FOR STRATIFIED SOIL; $\mathbf{h} = \mathbf{h_d} + \mathbf{h_g}$

4.2.1. PRESENCE OF GROUNDWATER TO HEIGHT $\mathrm{h_d}$ – DETERMINISTIC PROBLEM

Figure 2 shows the variation in frequency $\omega_i\left(i = 1,\ldots,6\right)$ depending on height h_d when there is no machine on the foundation. Shear modulus $\mathrm{G_o}$ of watered ground is assumed to amount to $G_w = w_w \cdot G_o$. w_w is changing in range 0,2 ÷ 0,75. The read off values of ω_i are presented in table 1.

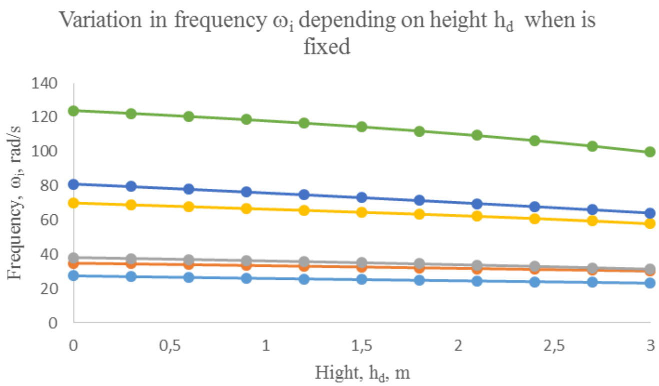

Fig. 2. Variation in frequency ω_i depending on height $\mathrm{h_d}$ when $\mathrm{w_w}$ is fixed

Table 1. The range of changing of natural frequencies

Height, *h* m	Frequency, ω_i, rad/s					
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
min (0)	27.49287	34.89198	38.17748	70.0195	81.06571	123.946
max (3)	23.24242	30.26729	31.38613	57.8689	64.17313	99.61586

It appears that the variation is rather low, particularly for the initial ω_i.

4.2.2. PRESENCE OF GROUND WATER TO HEIGHT h_d (FROZEN SOIL TO DEPTH h_g) – RANDOM PROBLEM

In order to examine the influence of the random characteristics of parameters h_d, h_g, the above flat foundation model.

Figures 3 and 4 show the variation in expected value $E(\lambda_i)$ and standard deviation σ_i of response $\lambda_i = \omega_i^2$ when groundwater is present up to height h_d. The following were assumed: $G_w = w_w \cdot G_o$, $w_w = 0,2 \div 0,75$, $h_d = 1 \div 3$ m, a normal distribution of random variable h_d with $\sigma_h = 0,333$. Similar $E(\lambda_i)$ and σ_i results were obtained for the case when frozen ground occurs to depth h_g, assuming: $G_z = w_z \cdot G_o$, $w_z = 5 \div 15$, $h_g = 0 \div 0,7$ m, normal distribution h_g with $\sigma_g = 0,117$. The expected values and standard deviation of λ_i for the extreme values of w_w, w_z are shown in table 2. One can notice a slight influence of w_w in the case of frequencies ω_1, ω_2 and a strong influence in the case of ω_3.

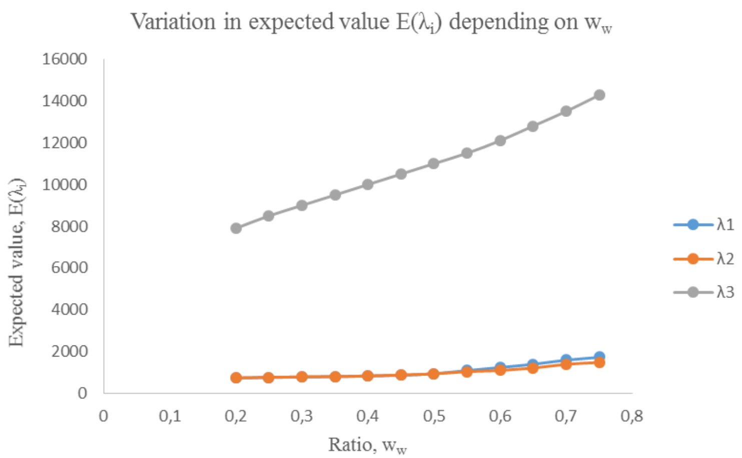

Fig. 3. Variation in expected value $E(\lambda_i)$ depending on w_w

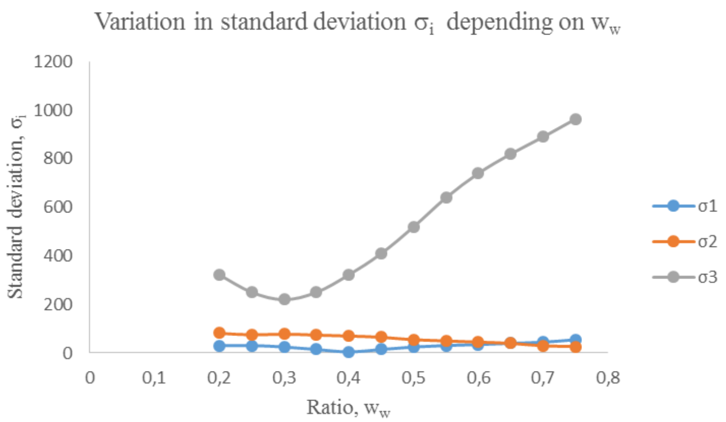

Fig. 4. Variation in standard deviation σ_i depending on w_w

Table 2. The range of changing of expected value and deviation of eigenvalue

	Expected value, $E(\lambda_1)$	Expected value, $E(\lambda_2)$	Expected value, $E(\lambda_3)$	Standard deviation, σ_1	Standard deviation, σ_2	Standard deviation, σ_3
$W_w = 0.20$	742.01	751.36	7903.2	31.18	81.5	322.27
$W_w = 0.75$	1735.78	1488.98	14283.16	55.08	26.81	963.74
$W_z = 5$	2013.02	2308.21	21281.43	93.13	160.23	1746.51
$W_z = 15$	2716.15	3420.42	35276.08	325.95	499.42	6162,29

5. CONCLUSION

This study is devoted to atypical practical engineering problems relating to the dynamics of block-type foundations for machines. Firstly, these are cases when stratified soil occurs within the side backfill, i.e. groundwater extending up to a certain height. A vibration equation which takes into account the above phenomena has been formulated. Two types of dynamic loads most common in engineering practice, i.e. harmonic loads and a series of short-term loads, were considered. Solutions of the natural vibration equation and the forced vibration equation have been formulated for the deterministic range and under the assumption that the selected parameters are random variables. Exemplary results of numerical analyses are reported. It emerges from them that the analyzed phenomena may cause significant changes in the dynamic responses of the system. However, much more extensive numerical analyses need to be carried out in order to draw more general conclusions.

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