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A CRITICALITY IMPORTANCE-BASED SPARE ORDERING POLICY FOR MULTI-COMPONENT DEGRADED SYSTEMS

OPARTA NA KRYTERIUM KRYTYCZNOŚCI POLITYKA ZAMAWIANIA CZĘŚCI ZAMIENNYCH DO ZDEGRADOWANYCH SYSTEMÓW WIELOELEMENTOWYCH

With the increasing complexity and variety of production systems, more attention is being paid to preventive replacement on multi-component systems. Each component is non-identical and has its own degradation process. In this paper, we propose a criticality importance-based spare ordering policy for a complex system, which consists of multiple series-parallel degrading components. Replacement action is triggered whenever the system reliability drops below a lower threshold and spares for replacement are available. Our policy mainly consists of two steps: (1) determine which components to be replaced; (2) determine when to order spares for components selected. In step 1, when the replacement action is triggered, we select components that most need to be replaced within the system in accordance with the optimum ranking of components until the system meets an upper reliability threshold. In step 2, a spare ordering policy for components selected is made and the optimal spare ordering time is obtained by minimizing the expected replacement cost during the once replacement cycle. Finally, a numerical example is given to illustrate the proposed multi-spare ordering policy. Moreover, the proposed policy is of significance for safety-critical systems such as sub-station automation system, bridge system, nuclear power plants and aerospace equipment.

Keywords: *Multi-component, spare ordering, criticality importance, random lead-time, reliability threshold.*

Wraz ze wzrostem złożoności i różnorodności systemów produkcyjnych, coraz większą uwagę zwraca się na wymianę zapobiegawczą w systemach wieloelementowych. Każdy element takiego systemu jest nieidentyczny z pozostałymi elementami i charakteryzuje się własnym procesem degradacji. W niniejszym artykule proponujemy strategię zamawiania elementów zamiennych dla systemu złożonego składającego się z wielu ulegających degradacji komponentów tworzących strukturę szeregowo-równoległą. Omawiana strategia wymiany opiera się na kryterium krytyczności elementów. Akcja wymiany uruchamiana jest za każdym razem, gdy niezawodność systemu spada poniżej dolnego progu i dostępne są części zamienne. Na proponowaną strategię składają się zasadniczo dwa etapy: (1) określenie elementów wymagających wymiany oraz (2) określenie terminu zamówienia części zamiennych do wybranych elementów. W 1. etapie, po uruchomieniu akcji wymiany, wybiera się komponenty systemu, które najpilniej wymagają wymiany, kierując się optymalnym rankingiem komponentów, do momentu aż system osiągnie górny próg niezawodności. W 2. etapie, opracowuje się politykę zamawiania części zamiennych dla wybranych komponentów oraz określa się optymalny czas zamawiania części zamiennych poprzez minimalizację oczekiwanego kosztu wymiany podczas jednego cyklu wymiany. W artykule przedstawiono przykład numeryczny, który ilustruje proponowaną strategię jednoczesnego zamawiania wielu części zamiennych. Proponowana strategia może znaleźć zastosowanie w systemach o kluczowym znaczeniu dla bezpieczeństwa, takich jak systemy automatyki podstacji, systemy mostowe, elektrownie jądrowe i sprzęt lotniczy.

Słowa kluczowe: *system wieloelementowy, zamawianie części zamiennych, krytyczność, losowy czas realizacji procesu produkcyjnego, próg niezawodności.*

1. Introduction

Maintenance plays an important role in industrial production and system safety, especially in areas where the loss of system failure is large. Various maintenance policies have been developed to improve system safety (or system reliability), reduce system failure and manufacturing cost [1]. In non-repairable systems, preventive replacement (PR) [33, 12, 13, 6, 15, 17] is a policy that occurs when a system is still operating, aiming to renew the system or components. In PR policies, condition-based maintenance (CBM) is a more promising maintenance policy since it emphasizes on combining data-driven reliability models with condition monitoring data. Therefore, CBM has received considerable attention in both academia and industry [2,8]. Most CBM policies are developed under the implicit assumption that at any time there is an unlimited supply of available spares for replacement. However, this assumption is generally unrealistic and unpractical when available spares are limited and/or delivery lead times are much longer. When spares are expensive, scarce, and with higher and random lead times, it is important to consider shortage cost and holding cost. Therefore, proper supply of spares is essential for maintenance [16].

In practice, the performance of a PR policy depends not only on the operating condition of a system but also on the availability of spares. In order to order spares on demand and achieve the minimum maintenance cost, the joint of a system condition and spares ordering is very necessary. Motivated by the idea of joint optimization of maintenance and spare ordering, some spare ordering policies have been extensively researched. Wang, L., et al. proposed a joint optimization of condition-based maintenance and spare ordering management for a single-component system [34]. Chien, Y.H. proposed a spare ordering policy based on the optimal number of minimal repairs with regular lead-time [7]. Louit, D., et al. presented an order policy based on remaining useful life of a component [23]. Godoy, D.R., et al. presented an order policy through graphic technique, which depended on condition-based reliability function and lead-time [11]. Panagiotidou, S. proposed a joint optimization of spares ordering and maintenance policies for multiple identical items [27]. Wang, Z.Q., et al. proposed a condition-based spare ordering policy with random lead-time for a deteriorating system [35]. Chen, X., et al. proposed a joint optimization of replacement and spare ordering for critical rotary component based on collected condition monitoring signals [5]. Cai, J., et al. proposed an appointment policy of spares based on (s,S) policy [4]. Lin, X., et al. proposed a condition based spare parts supply policy that is more efficient on average than a standard, state-independent base stock policy [21]. In the literature, spare ordering policies mainly focus on a single-component system or multiple identical components.

Importance measures (IM) are used in various fields to evaluate the relative importance of various objects such as components in a system [19]. IM would be capable of the needs of selecting components within a complex system. IM is widely used in systems engineering to identify components within a system that more significantly influence the system behavior with respect to reliability, risk and/or safety. The information gathered by the use of IM provides management with useful insights for the safe and efficient operation of a system. IM is valuable in suggesting the most effective way to operation and maintain system status. In general, IM is used to quantify the contribution of individual components of a system to the overall system performance (e.g., reliability, risk, availability) [24,9]. Several IM such as Birnbaum's measure [3], Fussell-Vesely's measure [31,10], risk achievement worth [32], risk reduction worth [20] and criticality importance (CI) [18] for components have been proposed in the past. For more applications, see [19] for an overview about recent advances on IM. Recently, IM provides an efficient tool to solve multi-component maintenance problems [36]. More recently, IM have been applied for maintenance optimization of a multi-component system with complex structure [25,26]. Especially, in the work of Nguyen, K-A., et al. (2017), the authors developed a joint predictive maintenance and inventory strategy for multi-component systems using Birnbaum's structural importance [26]. Whereas Nguyen, K-A., et al. (2017) focused on PR threshold and ordering threshold of each component and IM (Birnbaum's structural importance) is used to reduce the number of decision parameters, we proceed to solve a multi-spare optimal ordering problem for components that most need to be replaced based on system reliability threshold and spares random lead time, and IM is used to select components that most need to be replaced within the system. On the other hand, from the perspective of overall system reliability, multi-spare ordering and replacement can be used as a complementary method to [26]. To the best of our knowledge, studies that investigate multi-spare ordering and replacement for multi-component complex systems are relatively rare. Therefore, it is of great importance to study the multi-spare ordering policy based on system reliability threshold and IM for solving the multi-component replacement of complex systems. After investigating IM, we select two appropriate measures of the importance including CI measure and Birnbaum's measure to quantify the component importance in a complex degraded system.

In recent years, due to the increasing complexity and variety of production systems, more attention should be paid to spares ordering on a complex degraded system composed of many non-identical components. Since each component has its own contribution to the system, it is essential to select the most important components as the replaced objects to ensure the safe and reliable operation of the system. In view of the advantages of IM, this paper attempts to find a policy to solve the issue of multi-spare ordering. Therefore, this paper aims to propose a criticality importance-based spare ordering policy for the system with continuously degrading components. Each component has its own degradation process. Replacement action is triggered whenever the system reliability drops below a lower threshold and spares for replacement are available. Our policy mainly consists of two steps: (1) determine which components to replace though the optimum ranking of components; (2) determine when to order spares for components selected to minimize the expected replacement cost during the once replacement cycle. The main contribution of this paper is to propose a novel multi-spare ordering policy based on CI for the complex degraded system. Under the condition of overall system reliability constraint, the problem of how to select the most needed spares and when to place an order with minimized maintenance cost is solved.

The remainder of this paper is organized as follows. Section 2 describes the problem statement. Section 3 constructs a system reliability model and develops the policy of components selection. Section 4 develops a novel multi-spare ordering policy for a complex system. Section 5 gives a numerical example and performs sensitivity analysis on critical parameters. Finally, Section 6 concludes the study.

Notations and Nomenclatures

PR	Preventive replacement	σ_2	System upper threshold
CBM	Condition-based maintenance	R_i^{π}	Reliability of component i after one maintenance action
IM	Importance measures	R_i^{γ}	Reliability of component i after one replacement action
CI	Criticality importance	$\left\{I_1^{CR}, I_2^{CR}, \ldots, I_m^{CR}\right\}$	Optimum ranking
BI	Birnbaum importance	$\left\{c_{I_1^{CR}}, c_{I_2^{CR}}, \ldots, c_{I_{m_s}^{CR}}\right\}$	Components selected sequence
CDF	Cumulative distribution function	T_r	System PR time
MEMS	micro-electro-mechanical systems	T	Ordering time
X_i	Degradation level of component i	L	Spare lead-time
μ_i	Degradation rate of component i	P_{S1}	CDF of State 1
ε_i	Error term of component i	P_{S2}	CDF of State 2
L_i	Degradation threshold value of component i	$W(t)$	CDF of the lead-time
$R_i(t)$	Reliability of component i	ρ_h	Holding cost per unit time
$F_i(t)$	Unreliability of component i	ρ_s	Shortage cost per unit time
$R(t)$	Reliability of system	C	spares cost
$F(t)$	Unreliability of system	EH	Expected holding time
$I_i^B(t)$	BI of component i	ES	Expected shortage time

$I_i^{CR}(t)$	CI of component *i*	EV	Expected replacement cost
σ_1	System lower threshold	$T^{\bullet}$	Optimal ordering time

2. Problem statement

Consider a complex system consisted of n different components. Each component has its own degradation process. The system reliability is determined by component reliability. In a mission, the system may be maintained many times. The system reliability variation during a mission timespan is depicted in Fig. 1. Under the premise of ensuring safe and reliable operation, how to minimize maintenance costs is a problem that must be solved. The lower and upper threshold values of the system reliability directly affect the execution reliability and maintenance cost of the entire mission. Under normal conditions, the lower threshold is a constant and provided by domain experts. It mainly affects the system safety. However, the upper threshold is a variate. It mainly affects the PR times and the entire maintenance cost in a mission. Considering that each maintenance process is similar, this paper only investigates the first maintenance process. To facilitate the study of a maintenance process, we preset the upper limit as a constant in the first PR action. Replacement action is triggered whenever the system reliability drops below a lower threshold and spares for replacement are available. Therefore, the challenge is to identify an optimal spare ordering policy for most needed components, in order to meet both system reliability constraints and minimum maintenance cost in engineering practice.

To ensure the effectiveness of this study, the following assumptions are used:

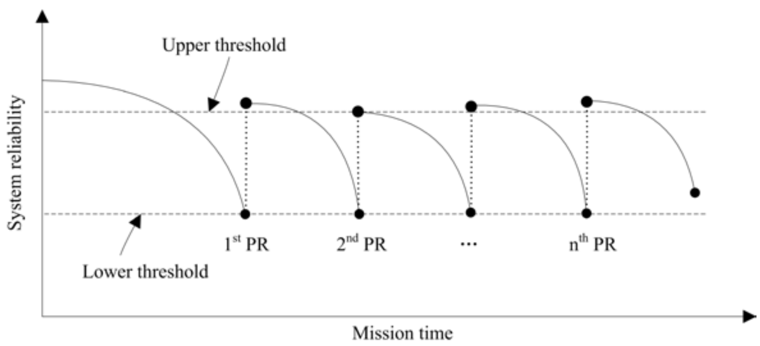

Fig. 1. System reliability variation during a mission timespan

(1) Components are mutually independent and each component is continuously monitored.

(2) Degradation is the only cause of each component failure. The impact of the external environment on the component is not considered, such as artificial destruction and natural disaster.

(3) The system does not degenerate when it is suspended during operation.

(4) Shortage cost per unit time is bigger than holding cost per unit time due to the system shutdown affecting the production progress and custom service negatively.

(5) Spares are supplied by the identical manufacturer.

3. Model statement

3.1. System reliability modeling

The system considered here is a series-parallel system with many different components. In addition, each component has its own contribution to the system and the reliability of the system is measured by the reliability of these components. The failure degree of component is measured by its degradation level. Each component's degradation mechanism follows its own degradation path. Denote the degradation level of component i over time t as $X_i(t;\mu_i,\varepsilon_i)$, where μ_i is degradation rate, and ε_i is error term, i.e., $\varepsilon_i \sim N(0,\sigma_i^2)$. In most cases, $X_i(t;\mu_i,\varepsilon_i)$ is a monotonic function over time t [22,29].

For each component, it fails whenever the degradation level X_i exceeds threshold value L_i. The set of failure threshold values, $L = \{L_1, L_2, ..., L_n\}$, is assumed to be pre-set. Without loss of generality, degradation level is assumed to be monotonically increasing, and reliability of component i at time t is represented by the probability that X_i stays below threshold L_i, that is:

$$R_i(t) = \Pr\{X_i(t;\mu_i,\varepsilon_i) < L_i\} \quad (1)$$

Let $X_i(t;\mu_i,\varepsilon_i) = \mu_i t + \varepsilon_i$, and $R_i(t)$ can be obtained as:

$$R_i(t) = \Pr\{X_i(t;\mu_i,\varepsilon_i) < L_i\} = \Pr\{\mu_i t + \varepsilon_i < L_i\} = \Pr\{\varepsilon_i < L_i - \mu_i t\} = \Phi\left(\frac{L_i - \mu_i t}{\sigma_i}\right), \quad (2)$$

where $\Phi(\bullet)$ is the cumulative distribution function (CDF) of standard normal distribution.

For a series-parallel system composed of m subsystems, with each subsystem containing n_i components, its reliability can be obtained as:

$$R(t) = \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n_i}\left(1 - R_{ij}(t)\right)\right), \quad (3)$$

where $R_{ij}(t)$ is the reliability of each component. It is easy to know that the system contains a total of $n = \sum_{i=1}^{m} n_i$ components.

3.2. Components selection

In view of the advantages of IM, we proposed a method on components selection based on IM. For parallel subsystems, each component within subsystem has the same value of CI. Therefore, for a series-parallel system, we select CI measure and Birnbaum's measure for optimum ranking of components. On the basis of optimum ranking of components, we select components that most need. Next, we first introduce the definitions of Birnbaum's measure and CI.

Birnbaum [3] first introduced the concept of importance in 1969 and it is one of the most widely used reliability importance measures. Analytically, it is defined by:

$$I_i^B(t) = \frac{\partial R(t)}{\partial R_i(t)} = R(t; R_i(t) = 1) - R(t; R_i(t) = 0), \quad (4)$$

where $I_i^B(t)$ is the Birnbaum importance (BI) of component *i*; $R(t)$ is the system reliability at time *t*; $R_i(t)$ is the reliability of component *i* at time *t*; $R(t; R_i(t) = 1)$ is the system reliability at time *t* when the component *i* functions; $R(t; R_i(t) = 0)$ is the system reliability at time *t* when the component *i* fails.

From the definition, the Birnbaum's measure may serve as a good indicator for selecting components that are the best candidates for efforts leading to improving system reliability. However, $I_i^B(t)$ does not depend on the component reliability $R_i(t)$. This is a weakness of Birnbaum's measure. To solve this weakness of Birnbaum's measure, the CI is proposed. The CI includes the component unreliability $F_i(t)$. The CI includes the component unreliability $F_i(t)$. The CI can be defined by:

$$I_i^{CR}(t) = I_i^B(t)\frac{F_i(t)}{F(t)}, \tag{5}$$

where $F(t)$ is the system unreliability at time *t* and $F_i(t)$ is the unreliability of component *i* at time *t*.

As compared with Birnbaum's measure, CI is more suitable for prioritizing maintenance action in complicated systems. However, for parallel subsystems, CI cannot identify the importance of each component. Therefore, to make up for the lack of CI, we adopt CI and BI for optimum ranking for a series-parallel system. The rank of components mainly consists of three steps: (1) we compute CI of each component and rank components according to CI. (2) We compute BI of parallel components within subsystems, and rank components within subsystems according to BI. (3) Only keep the maximum BI of components in each subsystem composed of parallel components and form the optimum ranking. The optimum ranking set of components can be expressed as $\left\{I_1^{CR}, I_2^{CR}, \cdots, I_m^{CR}\right\}$, where $\left\{I_m^{CR}\right\}$ denotes the CI value of component with the maximum BI in mth subsystem.

Secondly, we develop a policy for components selection based on optimum ranking of components. We define σ_1 as reliability threshold of system PR, that is, lower threshold. And we define σ_2 as system reliability upper threshold after replacement. When system reliability drops below a lower threshold, we compute the optimum ranking of components. We select the components that most need to be replaced within the system in accordance with the optimum ranking of components until the system reliability is improved above the upper threshold.

To describe the effect of a replacement action on component reliability, we first assume that a small maintenance action will improve component reliability by an infinitesimal positive shift, that is:

$$R_i^{\pi} = R_i(t-\varepsilon), \tag{6}$$

where $\varepsilon > 0$ denotes infinitesimal reliability transposition due to maintenance. Considering a replacement action is equivalent to performing infinite maintenance until component reliability will be improved to one. On the basis of Eq. (6), a replacement action will improve component reliability by a positive shift of system PR time, that is:

$$R_i^{\gamma} = R_i(t-T_r) \tag{7}$$

Therefore, for a series-parallel system, we denote $\left\{c_{I_1^{CR}}, c_{I_2^{CR}}, \ldots, c_{I_{m_s}^{CR}}\right\}$ as components selected sequence set, that is:

$$\begin{aligned}
&\left\{c_{I_1^{CR}}, c_{I_2^{CR}}, \ldots, c_{I_{m_s}^{CR}}\right\} \\
&= \inf_{\left\{c_{I_1^{CR}}, c_{I_2^{CR}}, \ldots, c_{I_{m_s}^{CR}}\right\}} \left\{ \prod_{i=1}^{m_s}\left(1-\frac{\left(1-R_{I_i^{CR}}^{\gamma}(t)\right)\cdot\prod_{j=1}^{n_i}\left(1-R_{ij}(t)\right)}{\left(1-R_{I_i^{CR}}(t)\right)}\right)\cdot\prod_{i=m_s+1}^{m}\left(1-\prod_{j=1}^{n_i}\left(1-R_{ij}(t)\right)\right) \ge \sigma_2 \,\middle|\, \left\{I_1^{CR}, I_2^{CR}, \cdots, I_{m_s}^{CR}, \ldots, I_m^{CR}\right\}, m_s \le m \right\} \\
&= \inf_{\left\{c_{I_1^{CR}}, c_{I_2^{CR}}, \ldots, c_{I_{m_s}^{CR}}\right\}} \left\{ \prod_{i=1}^{m_s}\left(1-\frac{\left(1-R_{I_i^{CR}}(t-T_r)\right)\cdot\prod_{j=1}^{n_i}\left(1-R_{ij}(t)\right)}{\left(1-R_{I_i^{CR}}(t)\right)}\right)\cdot\prod_{i=m_s+1}^{m}\left(1-\prod_{j=1}^{n_i}\left(1-R_{ij}(t)\right)\right) \ge \sigma_2 \,\middle|\, \left\{I_1^{CR}, I_2^{CR}, \cdots, I_{m_s}^{CR}, \ldots, I_m^{CR}\right\}, m_s \le m \right\}
\end{aligned} \tag{8}$$

where m_s denotes subsystems selected, and $\left\{I_1^{CR}, I_2^{CR}, \cdots, I_{m_s}^{CR}, \ldots, I_m^{CR}\right\}$ denotes the optimum ranking set of components.

4. Multi-spare ordering policy

In this section, we will build replacement cost model to find the optimal ordering time. To this end, we first propose two cases of spare ordering when ordering time occurs before the PR time, that is, the ordered spares are delivered before PR time or after PR time. The detailed spare ordering policy for a system is depicted in Fig. 2 and some parameters are explained as follows, where T_r is system PR time, T is ordering time, and L is spare lead-time. Specifically, the implications of these two cases are summarized as follows.

case1: If spares arrive before system PR time, replacement action is triggered on system PR time and the components selected are replaced by the spares in stock. Let P_1 denote the probability of the current state and corresponding CDF can be shown as:

$$P_1 = \Pr\left\{T + L < T_r\right\} = \int_0^{T_r - T} \mathrm{d}W(t) \tag{9}$$

case2: If spares arrive after system PR time, replacement action is triggered as soon as the spares arrive. Let P_2 denote the probability of the current state and corresponding CDF can be shown as:

$$P_2 = \Pr\left\{T + L > T_r\right\} = \int_{T_r - T}^{\infty} \mathrm{d}W(t) \tag{10}$$

Second, we will build objective function, that is, replacement cost model. The most important task is to express the expected holding time and expected shortage time:

Since holding time occurs in the case1, the expected holding time EH during the once replacement cycle is expressed as:

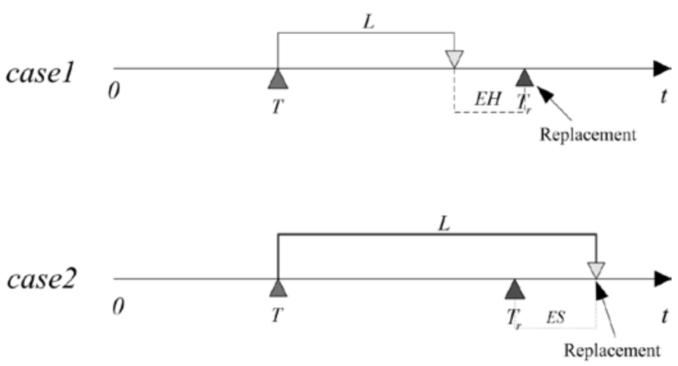

Fig. 2. Possible order-replacement states of one cycle

$$EH = E\left[T_r - T - L\right] = \int_0^{T_r - T} \left(T_r - T - t\right) \mathrm{d}W\left(t\right) \quad (11)$$

Similarly, since the shortage time occurs in the case2, the expected shortage time ES during the once replacement cycle is expressed as:

$$ES = E\left[T + L - T_r\right] = \int_{T_r - T}^{\infty} \left(T + t - T_r\right) \mathrm{d}W\left(t\right) \quad (12)$$

The replacement cost during the once replacement cycle mainly includes spares cost $\sum_{i=1}^{m_s} C_{I_i^{CR}}$, ordering cost C_o , holding cost $\rho_h \cdot EH$ and shortage cost $\rho_s \cdot ES$, that is,

$$EV\left(T\right) = \sum_{i=1}^{m_s} C_{I_i^{CR}} + C_o + \rho_h \cdot EH + \rho_s \cdot ES, \; 0 < T < T_r \quad (13)$$

What we aim is to seek an optimal ordering time $T^{\bullet}$ by minimizing the replacement cost, that is:

$$T^{\bullet} = \min_{T^{\bullet}} \left\{EV\left(T\right)\right\} \quad (14)$$

To facilitate the implementation, the detailed process of spare ordering policy is depicted in Fig. 3. The following content presents detailed steps.

Step1: Building system reliability model. The system reliability is determined by the reliability of each component and can be computed by Eq.(3).

Step2: Selecting components we want to order. We select the most important components using the components selection method in Section 3.2, and replace components selected with spares until the system reliability meets its upper threshold.

Step3: Making spare ordering policy. According to the distribution of the system reliability, components selected and spare lead-time, we consider two cases of spare ordering policy.

Step4: Finding the optimal ordering time. We first build the replacement cost model during the once replacement cycle, and then find the optimal ordering time by minimizing the replacement cost. The optimal ordering time is obtained by search algorithm with given step length.

5. A numerical example

A complex electromechanical system, a typical multi-component system, is the lifeline of the national economy and security. With the advancement of science and technology, and the modern large-scale production, the complex electromechanical system regarded as the key element in manufacturing industry, is developing toward large scale, automation, integration with mechanic, electric, hydraulic and computer technology, while the updating cycle is shorter and shorter. The failure rate is increasing, failure modes are various, and even the disastrous accident happens frequently, which are resulting from multi-function, improved performance, and heavy load of the complex system. Therefore, how to effectively improve the quality and reliability of the complex electromechanical system has become a key proposition and cannot be ignored in the national development strategy. Most complex electromechanical systems can be converted into an equivalent series-parallel system. Therefore, a numerical example of a series-parallel system can provide reference values for the maintenance and reliability of complex systems.

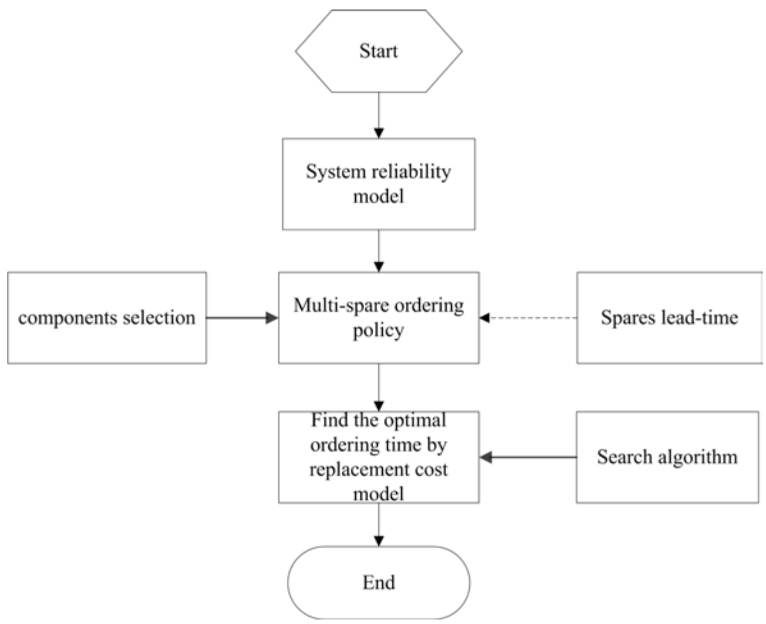

Fig. 3. The detailed process of multi-spare ordering policy

5.1. Specifications of the system and components

To show the implementation procedure of the modeling and analysis proposed in this paper, an illustrative example of a 6-component series-parallel system is used, as show in Fig. 4.

The Fig. 4 shows that the system reliability is:

$$R(t) = R_1(t) \cdot \left(1 - \left(1 - R_2(t)\right)\left(1 - R_3(t)\right)\right) \cdot \left(1 - \left(1 - R_4(t)\right)\left(1 - R_5(t)\right)\left(1 - R_6(t)\right)\right) \quad (15)$$

Many failures can be traced to underlying degradation, such as the wear on rubbing surfaces of a micro-electro-mechanical systems (MEMS) system composed of many non-identical components. In our study, each component follows a linear degradation path. This linear model has been used to characterize the failure mechanism of the wearing process in MEMS [29,30]. According to [29,30], the value of component-specific parameters is summarized in Table 1.

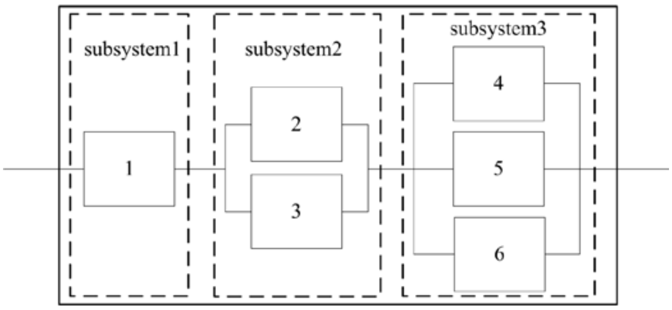

Fig. 4. 6-component series-parallel system

Table 1. Component-specific parameters

c_i	μ_i	ε_i	L_i	C_i
1	1.06	$N(0,1)$	10	0.40
2	1.16	$N(0,1)$	8	0.20
3	1.20	$N(0,2)$	10	0.30
4	1.18	$N(0,2)$	9	0.18
5	1.20	$N(0,3)$	9	0.15
6	1.10	$N(0,3)$	10	0.35

5.2. Ordering decision for the first PR action

5.2.1. Components selection

From the definition of CI, the CI of each component can be obtained as:

$$I_1^{CR}(t)=\frac{\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left(1-(1-R_4(t))(1-R_5(t))(1-R_6(t))\right)\cdot(1-R_1(t))}{(1-R(t))}$$
$$I_2^{CR}(t)=\frac{R_1(t)\cdot(1-R_3(t))\cdot\left(1-(1-R_4(t))(1-R_5(t))(1-R_6(t))\right)\cdot(1-R_2(t))}{(1-R(t))}$$
$$I_3^{CR}(t)=\frac{R_1(t)\cdot(1-R_2(t))\cdot\left(1-(1-R_4(t))(1-R_5(t))(1-R_6(t))\right)\cdot(1-R_3(t))}{(1-R(t))}$$
$$I_4^{CR}(t)=\frac{R_1(t)\cdot\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left((1-R_5(t))(1-R_6(t))\right)\cdot(1-R_4(t))}{(1-R(t))}$$
$$I_5^{CR}(t)=\frac{R_1(t)\cdot\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left((1-R_4(t))(1-R_6(t))\right)\cdot(1-R_5(t))}{(1-R(t))}$$
$$I_6^{CR}(t)=\frac{R_1(t)\cdot\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left((1-R_4(t))(1-R_5(t))\right)\cdot(1-R_6(t))}{(1-R(t))}\quad(16)$$

From Eq. (16), it can be seen that $I_2^{CR}(t)=I_3^{CR}(t), I_4^{CR}(t)=I_5^{CR}(t)=I_6^{CR}(t)$. It is because that parallel components has the same value of CI in subsystems. In order to increase the degree of differentiation, we add the BIs of components, that is,:

$$I_2^{B}(t)=R_1(t)\cdot(1-R_3(t))\cdot\left(1-(1-R_4(t))(1-R_5(t))(1-R_6(t))\right)$$
$$I_3^{B}(t)=R_1(t)\cdot(1-R_2(t))\cdot\left(1-(1-R_4(t))(1-R_5(t))(1-R_6(t))\right)$$
$$I_4^{B}(t)=R_1(t)\cdot\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left((1-R_5(t))(1-R_6(t))\right)$$
$$I_5^{B}(t)=R_1(t)\cdot\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left((1-R_4(t))(1-R_6(t))\right)$$
$$I_6^{B}(t)=R_1(t)\cdot\left(1-(1-R_2(t))(1-R_3(t))\right)\cdot\left((1-R_4(t))(1-R_5(t))\right)\quad(17)$$

PR action would be triggered when the system reliability reaches the lower threshold $\sigma_1=0.70$. Therefore, PR time can be expressed as $T_r=\arg\inf_{T_r}\{t:R(t)\le\sigma_1\}$. In the PR time point, we compute the CIs of components by Eq. (16), and compute the BIs of components with the same value of CI by Eq. (17). On the basis of components selection method proposed above, we would select the components that most need to be replaced within the system in accordance with the optimum ranking of components until the system reliability meets above upper threshold $\sigma_2=0.95$. The specific simulation results are shown in Fig. 5 and Table 2. Table 2 shows components selection sequence and prepares for replacement. Fig. 5 shows the system reliability variation within the first replacement cycle. At the time point of 7.68, the system reliability goes below $\sigma_1=0.70$, i.e., $R(7.68)=0.7000$. After replacement, the system reliability goes above $\sigma_2=0.95$, i.e., $R^{\gamma}(7.68)=0.9685$.

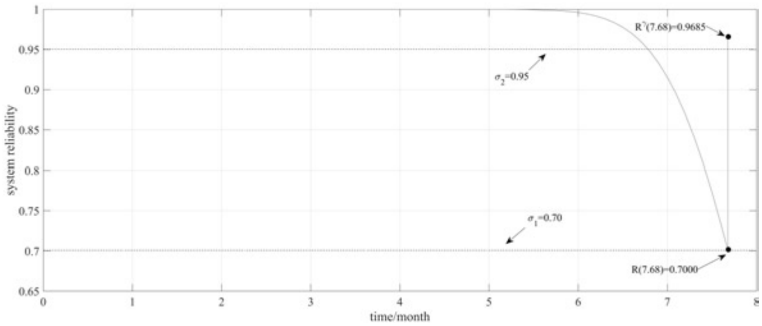

Fig. 5 System reliability variation within the first replacement cycle

Table 2. Components selection sequence

σ_1	0.70
T_r	7.68
Rank of CI	$I_2^{CR}=I_3^{CR}=0.7250, I_4^{CR}=I_5^{CR}=I_6^{CR}=0.1297, I_1^{CR}=0.0759$
Rank of BI with the same value of CI	$I_2^{CR}=I_3^{CR}\left(I_3^B=0.7508, I_2^B=0.2658\right)$ $I_4^{CR}=I_5^{CR}=I_6^{CR}\left(I_6^B=0.2102, I_4^B=0.0752, I_5^B=0.0708\right)$
Optimum Ranking set	$\{I_3^{CR}, I_6^{CR}, I_1^{CR}\}$
σ_2	0.95
Components selected	c_3, c_6

5.2.2. Multi-spare ordering

We need to clarify the values of cost parameters and lead-time parameter before spare ordering. Therefore, the values of cost parameters in spare ordering policy are assumed in Table 3. Assume further that the lead-time for delivering an ordered spares is Normal distributed, which is also one of the most common distributions in spare ordering investigation [28]. In our study, let random variable $X\sim N(\mu_w,\sigma_w^2)$, the CDF of an ordered lead-time is defined as $W(t)=P(X\le t\mid X\ge 0)$. Thus, the random lead-time will not be less than zero. For the purpose of illustration, the mean and standard deviation of the lead-time for delivering an ordered spares is $\mu_w = 2$ and $\sigma_w = 0.3$. Note that $W(t)$ used here are obtained by consulting with supplier of the spares.

As cost parameters and spare lead-time mentioned above, we utilize the proposed replacement cost model to find the optimal ordering time. The optimal ordering time is obtained by search algorithm with given step length as 0.1 and implemented computationally with MATLAB. The change law between ordering time and expected replacement cost is shown in Fig. 6. As seen in Fig. 6, the optimal ordering time is obtained by minimizing the expected replacement cost, i.e., $T^{\bullet} = 5.5, EV = 0.6817$. Considering factors like system preventive maintenance time and random lead-time error, the value of the optimal ordering time is reasonable.

Table 3. The values of cost parameters

C	C_o	ρ_s	ρ_h
$C_3 + C_6$	0.03	0.01	0.005

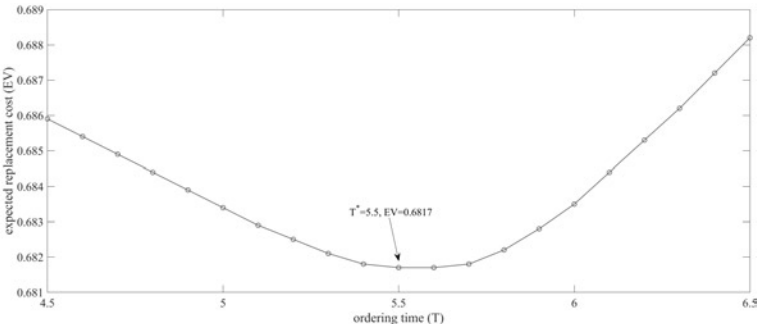

Fig. 6. The change law between ordering time and expected replacement cost

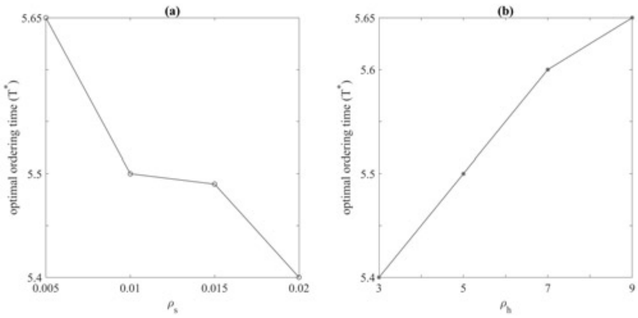

Fig. 7. (a) Sensitivity of optimal ordering time on ρ_s ; (b) Sensitivity of optimal ordering time on ρ_h

5.3. Sensitive analyses of critical parameters

In this section, we provide the sensitivity analysis on some critical parameters in order to verify the applicability of the proposed policy.

We first provide critical parameters on components selection. In Table 4, the influence of σ_1 on the selecting components is studied, where σ_1 takes value from 0.60 to 0.80 with step size 0.10. We can see that the selecting components is affected by σ_1. An explanation for this is that the system reliability needs to replace more components up to the upper threshold as reliability lower threshold decreases. In Table 5, the influence of σ_2 on the selecting components is studied, where σ_2 takes value from 0.90 to 1.00 with step size 0.05. From Table 5, we can find that the number of components selected is growing as σ_2 increases. The reason for this is in that system reliability needs to replace more components up to the upper threshold.

In Table 6, we can find that the optimal ordering time $T^{\bullet}$ deceases as the standard deviation σ_w increases. The reason for this may lie in that the probabilities that the ordered spares can be delivered earlier or later than its mean lead-time will be along with the variation of the standard deviation and these probabilities are highly correlated with the expected shortage time and holding time among the once replacement cycle. In the present case, the optimal ordering time $T^{\bullet}$ decreases since the shortage cost is larger than holding cost. Therefore, the optimal ordering time moves backward.

Table 6. Sensitivity of optimal ordering time on σ_w

σ_w	$T^{\bullet}$	EV
0.1	5.6	0.6806
0.3	5.5	0.6817
0.5	5.4	0.6827

Table 4. The influence of σ_1 on the components selected

σ_1	T_r	Optimum Ranking set	σ_2	Components selected
0.60	7.90	$\{I_3^{CR} = 0.6797, I_6^{CR} = 0.1311, I_1^{CR} = 0.0818\}$	0.95	c_3, c_6, c_1
0.70	7.68	$\{I_3^{CR} = 0.7250, I_6^{CR} = 0.1297, I_1^{CR} = 0.0759\}$	0.95	c_3, c_6
0.80	7.43	$\{I_3^{CR} = 0.7679, I_6^{CR} = 0.1245, I_1^{CR} = 0.0678\}$	0.95	c_3

Table 5. The influence of σ_2 on the components selected

σ_1	T_r	Optimum Ranking set	σ_2	Components selected
0.70	7.68	$\{I_3^{CR} = 0.7250, I_6^{CR} = 0.1297, I_1^{CR} = 0.0759\}$	0.90	c_3
0.70	7.68	$\{I_3^{CR} = 0.7250, I_6^{CR} = 0.1297, I_1^{CR} = 0.0759\}$	0.95	c_3, c_6
0.70	7.68	$\{I_3^{CR} = 0.7250, I_6^{CR} = 0.1297, I_1^{CR} = 0.0759\}$	1.00	c_3, c_6, c_1

To further validate the applicability of the proposed policy, we conduct some sensitivity analysis on the two cost parameters, i.e., shortage cost per unit time and holding cost per unit time. In Fig. 7, they show the impact of ordering time on ρ_s and ρ_h, where ρ_s takes value from 0.005 to 0.02 with step size 0.005 and ρ_h takes value from 0.003 to 0.009 with step size 0.002. We can find that the optimal ordering time decreases as ρ_s increases and the optimal ordering time increases as ρ_h increases. The reason is simply that when the shortage cost is larger, one should place an order earlier. Similarly, when the holding cost is larger, one should postpone ordering.

5.4. Epilog

As mentioned earlier, most complex electromechanical systems can be converted into an equivalent series-parallel system. In practice, we validate the effectiveness of multi-spare ordering policy through a multi-component MEMS system. Further, the proposed method can be extended to more application fields, such as substation automation systems [14], and only needs to meet the following three conditions: (1). A complex system can be converted into an equivalent series-parallel system; (2). The reliability function of each component can be known; (3). Spares have the identical lead time distribution function. In practical application [26], a substation automation system is a complex system consisting of seven non-identical components in series-parallel. Such a system topology can also be found commonly in many industrial plants where various control and supervisory modes exist for different redundancy levels. Moreover, the reliability function of each component is known. Next, according to the multi-component selection method and spares ordering policy proposed in this paper, multi-component selection and multi-spare ordering and replacement can be performed to ensure the overall reliable operation of the system.

6. Conclusion

This study proposes a multi-spare ordering policy based on CI for a complex system with multiple continuously degrading components. According to the approach of components selection, we can select components that most need to be replaced within the system. The method of selecting components aids to recognize the bottleneck of the system and prevent the system from unexpected failure. In addition, the proposed multi-spare ordering policy cannot only identify the most needed components for replacement, but also minimize the expected replacement cost during the once system maintenance. A numerical example shows that the components selected is influenced by the lower threshold and upper threshold. In addition, the optimal ordering time is affected by the standard deviation of spare lead-time, shortage cost per unit time and holding cost per unit time. In one word, experimental results meet our expectations and the proposed multi-spare ordering approach is of significance for safety-critical systems such as substation automation system, bridge system, nuclear power plants and aerospace equipment.

Further work can be achieved by relaxing some assumptions. For example, spares are supplied by the identical manufacturer. In practical, spares are supplied by the different manufacturers. In other words, the lead-time of each spare should be different. In addition, it is significative to study a multi-component ordering policy for a mission. That is to say, in a mission, there may be multiple spare ordering and replacement actions.

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References

1. Ahmad R, Kamaruddin S. An overview of time-based and condition-based maintenance in industrial application. Computers & Industrial Engineering 2012; 63(1): 135-149, https://doi.org/10.1016/j.cie.2012.02.002.
2. Alaswad S, Xiang Y. A review on condition-based maintenance optimization models for stochastically deteriorating system. Reliability Engineering & System Safety 2017; 157: 54-63, https://doi.org/10.1016/j.ress.2016.08.009.
3. Birnbaum Z W. On the importance of different components in a multicomponent system. In multivariate analysis. San Diego: Academic Press, 1969.
4. Cai J, Yin Y B, Zhang L, Chen X. Joint optimization of preventive maintenance and spare parts inventory with appointment policy. Mathematical Problems in Engineering 2017; https://doi.org/10.1155/2017/3493687.
5. Chen X, Xu D, Xiao L. Joint optimization of replacement and spare ordering for critical rotary component based on condition signal to date. Eksploatacja i Niezawodnosc-Maintenance and Reliability 2017; 19(1): 76-85, https://doi.org/10.17531/ein.2017.1.11.
6. Chien Y H, Sheu S H, Zhang Z G, Love E. An extended optimal replacement model of systems subject to shocks. European Journal of Operational Research 2006; 175(1): 399-412, https://doi.org /10.1016/j.ejor.2005.04.042.
7. Chien Y H. Optimal number of minimal repairs before ordering spare for preventive replacement. Applied Mathematical Modelling 2010; 34(11): 3439-3450, https://doi.org/10.1016/j.apm.2010.02.033.
8. de Jonge B, Teunter R, Tinga T. The influence of practical factor on the benefits of condition-based maintenance over time-based maintenance. Reliability Engineering & System Safety 2017; 158: 21-30, https://doi.org/10.1016/j.ress.2016.10.002.
9. Elsayed E A. Reliability engineering. MA: Addison Wesley, 1996.
10. Fussell J B. How to hand-calculate system safety and reliability characteristics. IEEE Transactions on Reliability 1975; 24(3):169-174, https://doi.org / 10.1109/TR.1975.5215142.
11. Godoy D R, Pascual R, Knights P. Critical spare parts ordering decisions using conditional reliability and stochastic lead time. Reliability Engineering & System Safety 2013; 119: 199-206, https://doi.org/10.1016/j.ress.2013.05.026.
12. Grall A, Bérenguer C, Dieulle L. A condition-based maintenance policy for stochastically deteriorating systems. Reliability Engineering & System Safety 2002; 76(2): 167-180, https://doi.org/10.1016/S0951-8320(01)00148-X.
13. Grall A, Dieulle L, Bérenguer C, Roussignol M. Continuous-time predictive-maintenance scheduling for a deteriorating system. IEEE Transactions on Reliability 2002; 51(2): 141-150, https://doi.org/ 10.1109/TR.2002.1011518.
14. Hajian-Hoseinabadi H. Reliability and component importance analysis of substation automation systems. International Journal of Electrical Power & Energy Systems 2013; 49: 455-463, https://doi.org/10.1016/j.ijepes.2010.06.012.
15. Jardine A K S, Tsang A H C. Maintenance, replacement, and reliability: theory and applications. Boca Raton: Taylor & Francis Group, 2013.
16. Kennedy W J, Patterson J W, Fredendall L D. An overview of recent literature on spare parts inventories. International Journal of Production Economics 2002; 76(2): 201-215, https://doi.org/10.1016/S0925-5273(01)00174-8.
17. Kim J, Ahn Y, Yeo H. A comparative study of time-based maintenance and condition-based maintenance for optimal choice of maintenance policy. Structure and Infrastructure Engineering 2016; 12(12): 1525-1536, https://doi.org/10.1080/15732479.2016.1149871.
18. Kuo W, Zuo M J. Optimal reliability modeling: principles and application. New York: John Wiley & Sons, 2003.
19. Kuo W, Zhu X. Importance measures in reliability, risk, and optimization: principles and applications. Chichester: Wiley, 2012.

20. Levitin G, Podofillini L, Zio E. Generalised importance measures for multi-state elements based on performance level restrictions. Reliability Engineering & System Safety 2003; 82(3): 287-298, https://doi.org /10.1016/S0951-8320(03)00171-6.
21. Lin X, Basten R J I, Kranenburg A A, van Houtum G J. Condition based spare parts supply. Reliability Engineering & System Safety 2017; 168: 240-248, https://doi.org/10.1016/j.ress.2017.05.035.
22. Liu B, Liang Z, Parlikad A K, Xie M, Kuo W. Condition-based maintenance for systems with aging and cumulative damage based on proportional hazards model. Reliability Engineering & System Safety 2017; 168: 200-209. https://doi.org/10.1016/j.ress.2017.04.010.
23. Louit D, Pascual R., Banjevic D, Jardine A K S. Condition-based spares ordering for critical components. Mechanical Systems and Signal Processing 2011; 25(5): 1837-1848, https://doi.org/10.1016/j.ymssp.2011.01.004.
24. Meng F C. Some further results on ranking the importance of system components. Reliability Engineering & System Safety 1995; 47(2):97-101, https://doi.org/10.1016/0951-8320(94)00046-Q.
25. Nguyen K-A, Do P, Grall A. Condition-based maintenance for multi-component systems using importance measure and predictive information. International Journal of Systems Science 2014; 1(4): 228-245, https://doi.org/10.1080/23302674.2014.983582.
26. Nguyen K-A, Do P, Grall A. Joint predictive maintenance and inventory strategy for multi-component systems using Birnbaum's structural importance. Reliability Engineering & System Safety 2017; 168: 249-261, https://doi.org/10.1016/j.ress.2017.05.034.
27. Panagiotidou S. Joint optimization of spare parts ordering and maintenance policies for multiple identical items subject to silent failures. European Journal of Operational Research 2014; 235(1): 300-314, https://doi.org/10.1016/j.ejor.2013.10.065.
28. Park Y T, Park K S. Generalized spare ordering polices with random lead time. European Journal of Operational Research 1986; 23(3): 320-330, https://doi.org/10.1016/0377-2217(86)90298-5.
29. Peng H, Feng Q, Coit D W. Simultaneous quality and reliability optimization for microengines subject to degradation. IEEE Transactions on Reliability 2009; 58(1): 98-105, https://doi.org/10.1109/TR.2008.2011672.
30. Peng H, Coit D W, Feng Q. Component reliability criticality or importance measures for systems with degrading components. IEEE Transactions on Reliability 2012; 61(1): 4-12, https://doi.org/10.1109/TR.2011.2182256.
31. Vesely W E. A time-dependent methodology for fault tree evaluation. Nuclear Engineering and Design 1970; 13(2): 337-360, https://doi.org /10.1016/0029-5493(70)90167-6.
32. Vasseur D, Llory M. International survey on PSA figures of merit. Reliability Engineering & System Safety 1999; 66(3): 261-274, https:// doi.org / 10.1016/S0951-8320(99)00011-3.
33. Wang H Z. A survey of maintenance policies of deteriorating systems. European Journal of Operational Research 2002; 139(3): 469-489, https://doi.org/10.1016/S0377-2217(01)00197-7.
34. Wang L, Chu J, Mao W. A condition-based order-replacement policy for a single-unit system. Applied Mathematical Modelling 2008; 32(11): 2274-2289, https://doi.org/10.1016/j.apm.2007.07.016.
35. Wang Z Q, Hu C H, Wang W B, Kong X Y, Zhang W. A prognostics-based spare part ordering and system replacement policy for a deteriorating system subject to a random lead time. International Journal of Production Research 2015; 53(15): 4511-4527, https://doi. org/10.1080/00207543.2014.988892.
36. Wu S, Chen Y, Wu Q, Wang Z. Linking component importance to optimization of preventive maintenance policy. Reliability Engineering & System Safety 2016; 146: 26-32, https://doi.org/10.1016/j.ress.2015.10.008.

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