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ANALYSIS OF A CHEMICALLY REACTIVE MHD FLOW WITH HEAT AND MASS TRANSFER OVER A PERMEABLE SURFACE

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This paper investigates a chemically reactive Magnetohydrodynamics fluid flow with heat and mass transfer over a permeable surface taking into consideration the buoyancy force, injection/suction, heat source/sink and thermal radiation. The governing momentum, energy and concentration balance equations are transformed into a set of ordinary differential equations by method of similarity transformation and solved numerically by Runge-Kutta method based on Shooting technique. The influence of various pertinent parameters on the velocity, temperature, concentration fields are discussed graphically. Comparison of this work with previously published works on special cases of the problem was carried out and the results are in excellent agreement. Results also show that the thermo physical parameters in the momentum boundary layer equations increase the skin friction coefficient but decrease the momentum boundary layer. Fluid suction/injection and Prandtl number increase the rate of heat transfer. The order of chemical reaction is quite significant and there is a faster rate of mass transfer when the reaction rate and Schmidt number are increased.

Key words: heat and mass transfer, chemically reactive MHD flow, permeable surface .

1. Introduction

The hydromagnetic flow and heat transfer over a surface have practical, industrial and engineering applications in the streamlined expulsion of plastic sheets, paper creation, glass blowing, metal turning, drawing plastic film, aerodynamic expulsion of plastic sheets, condensation process of metallic plate in the cooling bath and expulsion of a polymer sheet from a colour. Chamkha [1] solved a general boundary layer problem governing steady, Laminar, hydromagnetic flow with heat and mass transfer over a permeable cylinder moving with a linear velocity in the presence of heat/absorption, chemical reation, suction/injection effects and uniform transverse magnetic field using standard, fully implicit, iterative, tri-diagonal finite difference method. Aziz [2] obtained a similarity solution for a Laminar boundary layer flow over a flat plate with a convective surface boundary condition. Bhattacharyya and Gorla [3] solved the axisymmetric boundary layer flow and heat transfer past a permeable shrinking cylinder subject to surface mass transfer using finite difference method of quasilimearization technique. Bhattacharyya and Layek [4] analyzed the distribution of a reactant solute undergoing first order chemical reaction in the boundary layer flow of an electrically conducting incompressible fluid over a permeable stretching sheet subjected to suction or blowing using finite difference method of quasilinearization technique. Also, Bhattacharyya [5], considered

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the effects of heat source /sink on the steady two dimensional MHD boundary layer flow and heat transfer past shrinking sheet with wall mass suction using finite difference method of quasilinearization technique. Arthur *et al.* [6] investigated the hydromagnetic flow over a flat surface with convective boundary condition and internal heat generation in the presence of chemical reaction using Newton-Raphson Shooting method along with fourth order Runge-Kutta algorithm. Gnaneswara and Sandeep [7] analyzed the heat and mass transfer in Carreau fluid flow over a permeable stretching sheet with convective slip condition in the presence of applied magnetic field, nonlinear thermal radiation, cross diffusion and suction/injection effects using Runge-Kutta and Newton's method. Also, Nayak [8] considered a steady MHD flow of a viscous conducting fluid past a stretched permeable vertical permeable surface with heat generation/absorption, thermal radiation and chemical reaction using Runge- Kutta method based on Shooting technique. Prakash *et al.* [9] examined the hydromagnetic two dimensional boundary layer flow of a non-Newtonian fluid accompanied by heat and mass transfer towards an exponentially stretching sheet in the presence of chemical reaction and thermal radiation using Casson model. Sulochana and Kishor Kumar [10] used shooting technique to analyze the heat and mass transfer in magnetohydrodynamic flow over a stretching sheet in the presence of thermal radiation and chemical reaction. Seth *et al.* [11] used exact solution in closed form and numerical solution to investigate an unsteady hydrodynamic natural convection flow with heat and mass transfer of a viscous incompressible, electrically conducting, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with arbitrary ramped temperature, embedded in a fluid saturated in a porous medium. Also, Hayat *et al.* [12] obtained convergent series solutions for a boundary layer flow of a Nano fluid over power-law stretched surface in the presence of applied magnetic field and chemical reaction with heat and mass convective conditions. Ishak [13] provided a similarity solution for a steady, Laminar, boundary layer flow and heat transfer over a permeable flat plate in a uniform free stream with the surface of the plate heated by convection from a hot fluid. Also, Makinde and Olanrewaju [14] analyzed the effects of thermal buoyancy on the Laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition using fourth order Runge-Kutta iteration scheme. Olanrewaju *et al.* [15] used Shooting iteration technique with sixth order Runge-Kutta integration scheme to analyze the effects of internal heat generation, thermal radiation and buoyancy force on the Laminar boundary layer flow about a vertical plate in a uniform stream of fluid under a convective surface boundary condition

This work extends the work of Aziz [2], Arthur *et al*. [6], Ishak [13], Makinde and Olanrewaju [14] and Olanrewaju *et al.* [15] to include buoyancy force and fluid injection or suction on heat and mass transfer over a permeable surface of a chemically reactive magnetohydrodynamics fluid flow in the presence of heat source and sink and thermal radiation.

2. Formulation of the problem

Consider a steady, two dimensional boundary layer flow of a stream of cold, incompressible, electrically conducting viscous fluid, coupled with heat and mass transfer past a permeable stretching surface. The flow is assumed to be in the direction of x-axis along the plate and y- axis is normal to the plate. A uniform magnetic field B_0 is applied in transverse direction to the flow. The left surface is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f. Also, the left surface of the plate is heated by convection from a viscous fluid at concentration c_f to give rise to a coefficient mass transfer and the right side of the plate is electrically conducting

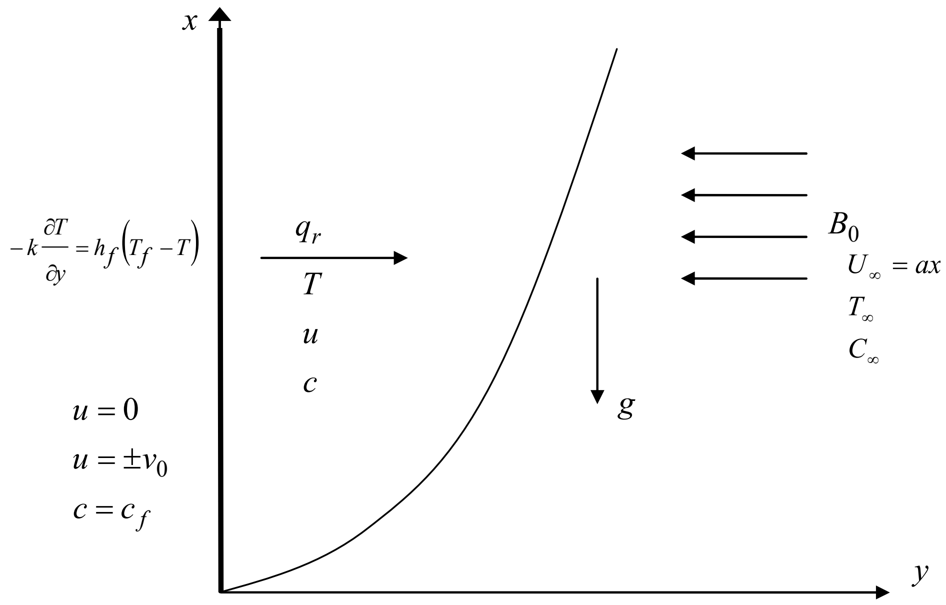

Fig.1. Flow configuration and coordinate system.

The governing continuity, momentum, energy and concentration equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)-\frac{\sigma B_0^2(u-U_\infty)}{\rho}-\frac{\gamma}{\kappa_p}(u-U_\infty), \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\gamma}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2(u-U_\infty)^2}{\rho C_p}+\frac{Q}{\rho C_p}\left(T-T_\infty\right)-\frac{1}{C_p}\frac{\partial q_r}{\partial y}, \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_r\left(C-C_\infty\right)^n \tag{2.4}$$

where u,v,T,C are respectively the x and y .components of the velocity; temperature and concentration components while U_∞, T_∞ and C_∞ are respectively the velocity, temperature and concentration outside the plate, γ is the coefficient of kinematic viscosity, β is the thermal expansion coefficient, σ is the electrical conductivity, g is the acceleration due to gravity, ρ the density, C_p the specific heat at constant pressure, K the thermal conductivity, B_0 is the magnetic strength, D_m is the mass diffusivity, K_r is the reaction rate constant, n is the order of the chemical reaction and q_r is the radiative heat flux.

The corresponding boundary conditions are

$$u\left(x,0\right)=0,\quad v\left(x,0\right)=\pm V_0,\quad -k\frac{\partial T}{\partial y}\left(x,0\right)=h_f\left(T_f-T\left(x,0\right)\right),\quad C\left(x,0\right)=C_f, \tag{2.5}$$

$$u(x,\infty)=U_\infty=ax, \qquad T(x,\infty)=T_\infty, \qquad C(x,\infty)=C_\infty \tag{2.6}$$

where a is a constant. Using Rosseland approximation, the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y} \tag{2.7}$$

where σ^* and K are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.

Assuming that the temperature differences within the flow are sufficiently small, then, Eq.(2.7) can be linearized by expanding T^4 in Taylor series about the free stream temperature T_∞ and neglecting higher-order terms to obtain

$$T^4\approx 4T_\infty^3T-3T_\infty^4\,. \tag{2.8}$$

Then substituting Eqs (2.8) in (2.7) gives

$$q_r=-\frac{16T_\infty{}^3\sigma^*}{3K}\,\frac{\partial^2T^4}{\partial y^2}\,. \tag{2.9}$$

Using the similarity transformation

$$\eta=x^{\frac{1}{2}}y\sqrt{\frac{u_e}{\gamma}}\,,\ u=U_\infty f',\ \ v=\frac{1}{2}\sqrt{\frac{U_\infty}{x}}\gamma\left(\eta f'-f\right),\ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ c(\eta)=\frac{C-C_\infty}{C_f-C_\infty}\,. \tag{2.10}$$

Equation (2.1) is satisfied if we define the stream function ψ as

$$u=\frac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\frac{\partial\psi}{\partial x}\,. \tag{2.11}$$

The governing Eqs (2.2) to (2.4) are transformed into the following ordinary differential equations

$$f'''+\frac{1}{2}ff''+\mathrm{Gr}_x\theta+\left(\left(M_x+Da_x\right)-\left(1-f'\right)\right)=0\,, \tag{2.12}$$

$$\left(1+\frac{4}{3}\mathrm{Ra}\right)\theta''+\mathrm{Br}\left(f''\right)^2+\mathrm{Br}\,M\left(1-f'\right)^2+\frac{1}{2}\mathrm{Pr}\,f\theta'+\mathrm{Pr}\,\lambda_x\theta=0\,, \tag{2.13}$$

$$\phi''+\frac{1}{2}\mathrm{Sc}\,f\phi'-\mathrm{Sc}\,B_x\phi^n=0\,. \tag{2.14}$$

The corresponding boundary conditions take the form

$$f'(0)=0,\quad f(0)=F_{wx},\quad -\theta'(0)=Bi_x\left(1-\theta(0)\right),\quad \phi(0)=1\,, \tag{2.15}$$

$$f'(\infty)=1,\quad \theta(\infty)=0,\quad \phi(\infty)=0 \tag{2.16}$$

where the prime symbol represents the derivative with respect to η.

For the momentum, energy and concentration equations to have similarity solutions, the parameters Gr_x, B_x, M_x, Da_x, λ_x must be made constants.

3. Numerical method for solution

The governing equations and the boundary conditions are solved using Runge-Kuttamethod based on sixth order Shooting technique.

3.1. Particular cases

1. For a chemical reaction of first order when $n=1$ and in the absence of buoyancy force and heat source/sink i.e., when $\mathrm{Gr}=\lambda=F_w=0$, then the results of this paper is the same as Arthur *et al*. [6].
2. For a fluid which is not chemically reactive in the absence of the magnetic field, i.e. when, $\mathrm{Sc}=K=M=\mathrm{Br}=F_w=0$ then the result of this paper will be same as Olanrewaju *et al*. [15].
3. In the absence of heat source/sink, buoyancy force, radiation effects and magnetic field for a non-chemically reactive fluid i.e., when $\lambda=\mathrm{Gr}=\mathrm{Ra}=M=F_w=\mathrm{Sc}=K=\mathrm{Br}=0$, then the result of this paper will be same as Makinde and Olanrewaju [14], Aziz [2] and Ishak [13].

4. Results and discussion

Numerical calculations have been carried out for different values of thermo physical parameters controlling the fluid dynamics in the flow region. Table 1 shows the comparison of Arthur *et al.* [6] with the present work for the numerical values of skin friction coefficient $f''(0)$, local Nusselt number $-\theta'(0)$ together with Sherwood number $-\phi'(0)$ and there was a perfect agreement of result in the absence local Grashof number Gr, internal heat generation λ and injection/suction F_w i.e. for $\mathrm{Gr}=\lambda=F_w=0$. Table 2 shows the comparison of Olanrewaju *et al.* [15] with the present work for the numerical values of skin friction coefficient $f''(0)$, local Nusselt number $-\theta'(0)$ together with plate surface temperature $\theta(0)$ and there was a perfect agreement of result when $\mathrm{Sc}=K=M=\mathrm{Br}=F_w=0$. Table 3 shows the comparison of the present work with the work of Makinde and Olanrewaju [14], Aziz [2] and Ishak [13] for numerical values of $-\theta'(0)$ and $\theta(0)$ and there was a perfect agreement of result when $\lambda=\mathrm{Gr}=\mathrm{Ra}=M=\mathrm{Sc}=K=\mathrm{Br}=F_w=0$ and $\mathrm{Pr}=0.72$.

Table 1. Computations showing comparison of the Emmanuel Maurice Arthur *et al*. [6] for $\boldsymbol{n}=1$, Gr=$\lambda=\boldsymbol{F_w}=0$ and the present work.

							Arthur *et al*. [6]			Present paper		
Pr	Sc	M	Ra	Br	K	Bi	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.72	*0.24*	*0.1*	*0.1*	*0.1*	*0.1*	*0.1*	*0.451835*	*0.068283*	*0.248586*	*0.4518350*	*0.0682832*	*0.2485861*
0.72	*0.24*	*0.1*	*0.1*	*0.1*	*0.1*	*0.1*	*0.451835*	*0.068415*	*0.494321*	*0.4518350*	*0.0684153*	*0.4943214*
0.72	*0.2i*	*0.1*	*0.1*	*0.1*	*0.1*	*0.1*	*0.770792*	*0.064224*	*0.261862*	*0.7707922*	*0.0642241*	*0.2618619*
0.72	*0.24*	*0.1*	*0.1*	*0.1*	*0.1*	*0.1*	*0.451835*	*0.066984*	*0.248586*	*0.4518350*	*0.0669838*	*0.2485861*
0.72	*0.24*	*0.1*	*0.1*	*0.5*	*0.1*	*0.1*	*0.451835*	*0.042658*	*0.248586*	*0.4518350*	*0.0426580*	*0.2485861*

Table 2. Computations showing comparison of the Olanrewaju *et al.* [15] for $n=1$, $\mathrm{Sc}=K=M=\mathrm{Br}=F_w=0$ with the present work.

					Olanrewaju *et al.* [14]			Present paper		
Bi	Gr	Pr	λ	Ra	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1	*0.1*	*0.72*	*0.1*	*0.1*	*0.386316*	*0.066810*	*0.331810*	*0.38694698*	*0.066661*	*0.333390*
10	*0.1*	*0.1*	*0.1*	*0.1*	*0.483261*	*0.213880*	*0.978610*	*0.48431420*	*0.212285*	*0.978771*
0.1	*0.5*	*0.1*	*0.1*	*0.1*	*0.557241*	*0.069730*	*0.302690*	*0.55978647*	*0.069577*	*0.304227*
0.1	*0.1*	*0.1*	*0.6*	*0.1*	*0.298365*	*0.102052*	*-0.020520*	*0.29716417*	*0.102356*	*-0.023564*

Table 3. Computations showing comparison of Makinde and Olanrewaju [14], Aziz [2] and Ishak [13] for $\lambda=\mathrm{Gr}=\mathrm{Ra}=M=\mathrm{Sc}=K=\mathrm{Br}=F_w=0$ and $\mathrm{Pr}=0.72$ with the present work.

	Aziz [2]		Ishak [13]	Makinde and Olanrewaju [14]	Present paper	
Bi	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$\theta(0)$
0.05	*0.0428*	*0.1447*	*0.042767*	*0.0428*	*0.0428*	*0.1447*
0.10	*0.0747*	*0.2528*	*0.074724*	*0.0747*	*0.0747*	*0.2528*
1.00	*0.2282*	*0.7718*	*0.228178*	*0.2282*	*0.2283*	*0.7718*
5.00	*0.2791*	*0.9441*	*0.279131*	*0.2791*	*0.2791*	*0.9442*

Figures 2-17, together with Tabs 4, 5 and 6 shows the computational results showing the effects of various thermophysical parameters on the electrically conducting and *nth* order homogeneous reacting fluid velocity, temperature, concentration as well as skin-friction coefficient, plate temperature, rate of heat and mass transfer over the vertical plate. It was observed from Tab.4 that with more injection of the chemically reactive and electrically conducting fluid into the flow system, there was a corresponding rise in the skin-friction coefficient. The same result was observed by increasing the values of the magnetic parameter, Brinkman and Grashof numbers. In Tab.5, the varying values of the reaction rate parameter increased the rate of mass transfer within the flow system. On the other hand, when the order of the chemical reaction was increased, there was a retardation of the rate of mass transfer. Table 6 shows the computation of the values of plate temperature and the Nusselt number for various values of the thermo physical parameters and for a first order chemical reaction with variations in $\mathrm{Ra}, M, \mathrm{Bi}, \mathrm{Pr}, \lambda$ and Br. Some of the parameters like $\mathrm{Ra}, \mathrm{Bi}, \lambda, M$ and Br increased the plate temperature while F_w and Pr decreased the plate temperature. The rate at which heat is transferred, $-\theta'(0)$ was increased by increasing the values of the Prandtl and Biot numbers as a result of convective heat exchange at the plate surface. There was a retarding effect on the heat transfer rate as Br, Ra and λ were increased due to viscous dissipation.

Table 4. Computation of the values of the coefficient of skin-friction $f''(0)$, $\mathrm{Gr}=0.1$, $\mathrm{Gr}=0.5$ and for various values of F_w and M with $\mathrm{Sc}=0.24$, $n=1$ and $\mathrm{Pr}=0.72$, $\mathrm{Br}=\lambda=\mathrm{Bi}=0.1$.

Br	F_w	M	$f''(0)$	
			Gr=*0.1*	Gr=*0.5*
0.1	*-0.5*	*0.1*	*0.39361058*	*0.65350847*
		0.5	*0.70067376*	*0.90802398*
0.1	*-0.2*	*0.1*	*0.45707988*	*0.68141995*
		0.5	*0.76538977*	*0.94502622*
0.1	*0.1*	*0.1*	*0.53696439*	*0.72717975*
		0.5	*0.83964894*	*0.99484111*
0.1	*0.2*	*0.1*	*0.56665846*	*0.74636625*
		0.5	*0.86636772*	*1.01419416*
0.1	*0.2*	*0.1*	*0.57990202*	*0.81777795*
		0.5	*0.88152265*	*1.09168253*

Table 5. Computation of the values of the rate of mass transfer $-\phi'(0)$ with $n=1$ and $n=2$ for various values of Sc and K with $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=\mathrm{Gr}=\lambda=\mathrm{Bi}=M=0.1$ and $F_w=0.2$.

Sc	K	$-\phi'(0)$	
		$n=1$	$n=2$
0.24	0.2	0.306149262887398	0.282317722057957
	0.5	0.401474188901933	0.351677619285553
0.62	0.2	0.480601350113419	0.439729402647025
	0.5	0.640594758401727	0.556682245239545
2.64	0.2	1.00000022213442	0.902937070038939
	0.5	1.35242007039814	1.16094061749772

Table 6. Computation of the values of the plate temperature $\theta(0)$ and Nusselt number $-\theta'(0)$ for various values of Pr, Ra, M, λ, F_w and Bi with Sc=0.24, $n=1$ and Br = K = 0.1.

Pr	Ra	M	Br	λ	Bi	F_w	$\theta(0)$	$-\theta'(0)$
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.294179000291035	0.0705820999708965
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.197259784726029	0.0802740215273972
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.105635594228629	0.0894364405771371
0.72	0.2	0.1	0.1	0.01	0.1	0.2	0.298728417419719	0.0701271582580281
0.72	0.5	0.1	0.1	0.01	0.1	0.2	0.311607584522124	0.0688392415477876
0.72	0.7	0.1	0.1	0.01	0.1	0.2	0.319524516162399	0.0680475483837601
0.72	0.1	0.5	0.1	0.01	0.1	0.2	0.334401846213312	0.0665598153786687
0.72	0.1	1.5	0.1	0.01	0.1	0.2	0.401694171590128	0.0598305828409872
0.72	0.1	2.0	0.1	0.01	0.1	0.2	0.427892749481842	0.0572107250518158
0.72	0.1	0.1	0.2	0.01	0.1	0.2	0.361953891510507	0.0638046108489493
0.72	0.1	0.1	0.5	0.01	0.1	0.2	0.573874654466193	0.0426125345533806
0.72	0.1	0.1	0.8	0.01	0.1	0.2	0.800059046108015	0.0199940953891985
0.72	0.1	0.1	0.1	0.02	0.1	0.2	0.299897798159221	0.0700102201840779
0.72	0.1	0.1	0.1	0.05	0.1	0.2	0.318922669945282	0.0681077330054718
0.72	0.1	0.1	0.1	0.09	0.1	0.2	0.349732052451735	0.0650267947548265
0.72	0.1	0.1	0.1	0.01	0.2	0.2	0.424417014509244	0.115116597098151
0.72	0.1	0.1	0.1	0.01	1.0	0.2	0.766077997636570	0.233922002363430
0.72	0.1	0.1	0.1	0.01	2.0	0.2	0.865496405782359	0.269007188435282

A. Velocity profiles

Figures 2 to 6 show the influence of some of the controlling parameters on the velocity boundary layer. The fluid velocity was lowered at the plate surface and increased to the free stream value satisfying the far field boundary condition. In Fig.2, the distractive force due to Lorentz force increased as the magnetic parameter increased because there was a very consistent drop in the longitudinal velocity and therefore the momentum boundary layer thickness get thinner. An increase in fluid injection, Grashof number and Brinkman number have the same effects on both the momentum boundary layer and velocity with the magnetic parameter as shown in Figs 3 to 6.

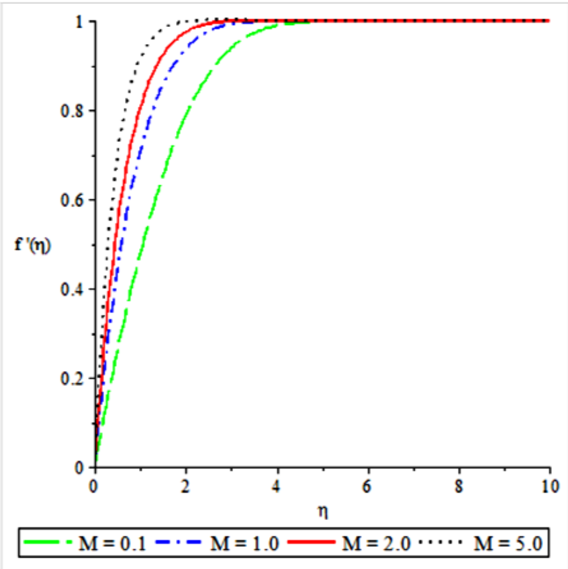

Fig.2. Velocity profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=\mathrm{Gr}=\mathrm{Ra}=\lambda=\mathrm{Bi}=K=0.1$ and $F_w=0.2$.

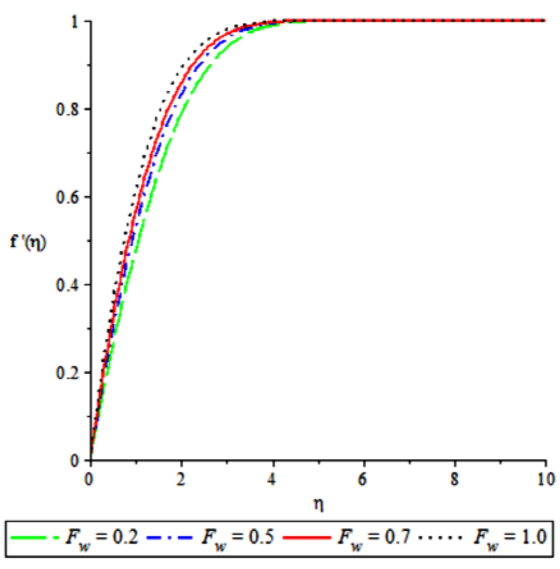

Fig.3. Velocity profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$ and $M=\mathrm{Br}=\mathrm{Gr}=\mathrm{Ra}=\lambda=\mathrm{Bi}=K=0.1$.

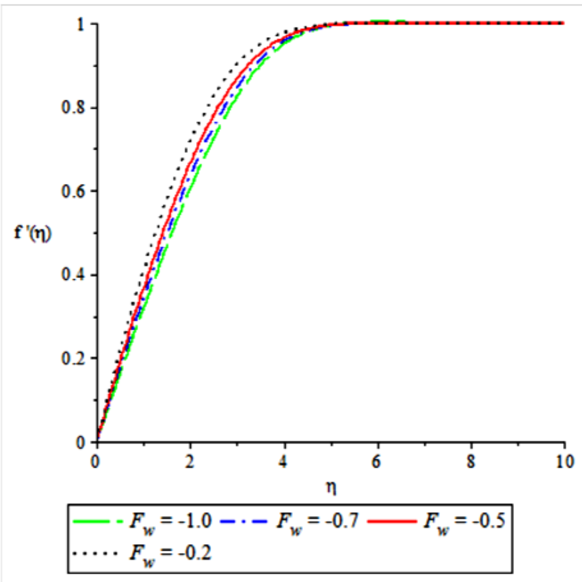

Fig.4. Velocity profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$ and $M=\mathrm{Br}=\mathrm{Gr}=\mathrm{Ra}=\lambda=\mathrm{Bi}=K=0.1$.

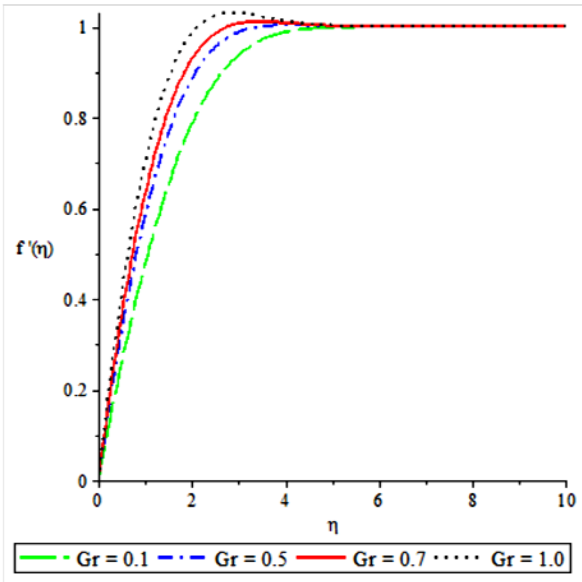

Fig.5. Velocity profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=M=\mathrm{Ra}=\lambda=\mathrm{Bi}=K=0.1$ and $F_w=0.2$.

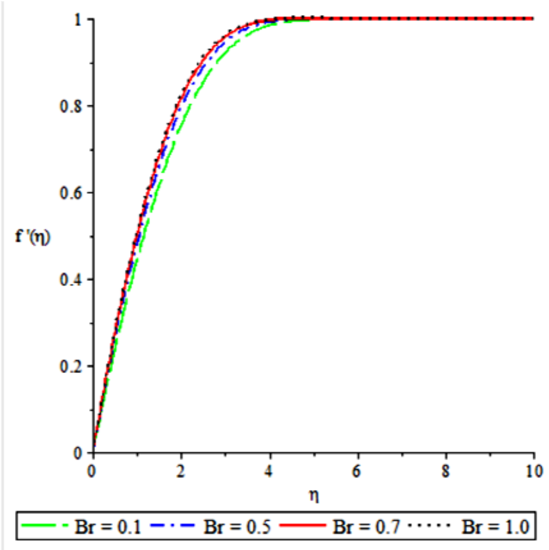

Fig.6. Velocity profiles for $\mathrm{Sc} = 0.24$, $n = 1, \mathrm{Pr} = 0.72$ and $\mathrm{Gr} = M = \mathrm{Ra} = \lambda = \mathrm{Bi} = K = 0.1$.

B. Temperature profiles

The effects of various controlling parameters on the temperature distribution are shown in Figs 7-14. It should be noted that, the temperature reaches its maximum at the permeable plate surface and asymptotically decreases to a minimum zero value far away from the plate, thereby satisfying the boundary condition. Also, increasing the magnetic parameter increases the fluid temperature which in turn increases the thermal boundary layer. This is attributed to the effect of Ohmic heating on the flow system. An increase in the Biot number gave rise to increase in fluid temperature due to the convective heat exchange between the hot fluid at the lower surface of the plate and the cold fluid at the upper surface of the plate resulting in the thickening of the thermal boundary layer. The same reason can be given for increase in the Brinkman number, internal heat generation parameter and radiation parameter but an opposite trend was observed by increasing the Prandtl number and fluid injection and so lowering the rate of thermal diffusion within the boundary layer resulting in a thinning of the thermal boundary layer.

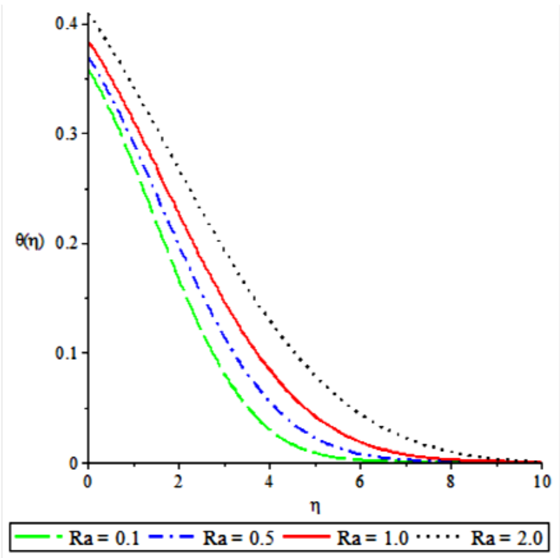

Fig.7. Temperature profiles $\mathrm{Sc} = 0.24$, $n = 1, \mathrm{Pr} = 0.72$, $\mathrm{Br} = \mathrm{Gr} = M = \lambda = \mathrm{Bi} = K = 0.1$ and $F_w = 0.2$.

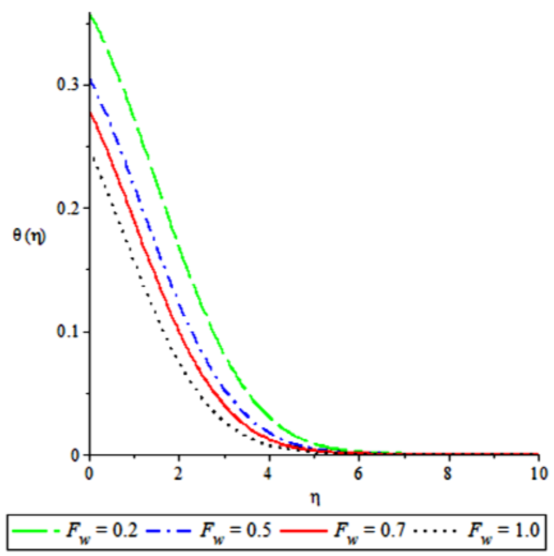

Fig.8. Temperature profiles for $\mathrm{Sc} = 0.24$, $n = 1, \mathrm{Pr} = 0.72$ and $\mathrm{Br} = \mathrm{Gr} = M = \mathrm{Ra} = \lambda = \mathrm{Bi} = K = 0.1$.

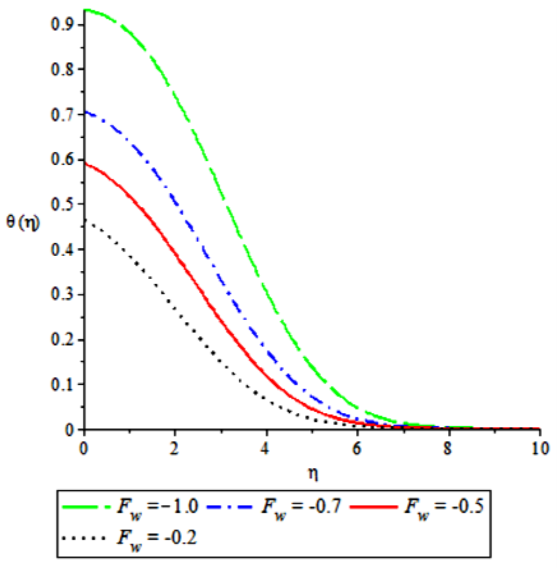

Fig.9. Temperature profiles for Sc=*0.24*, *n=1*, Pr=*0.72*, and $\mathrm{Br}=\mathrm{Gr}=M=\mathrm{Ra}=\lambda=\mathrm{Bi}=K=0$.

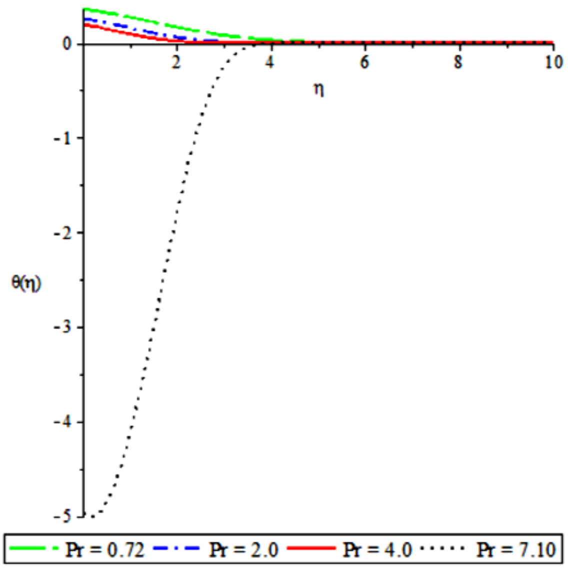

Fig.10. Temperature profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Ra}=\mathrm{Br}=\mathrm{Gr}=M=\lambda=\mathrm{Bi}=K=0.1$ and $F_w=0.2$.

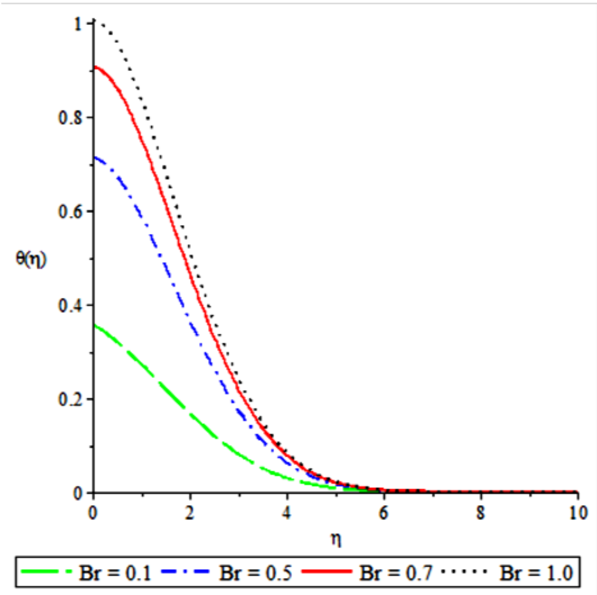

Fig.11. Temperature profiles for Sc=*0.24*, *n=1*, Pr=*0.72*, $\mathrm{Ra}=\mathrm{Gr}=M=\lambda=\mathrm{Bi}=K=0.1$ and $F_w=0.2$.

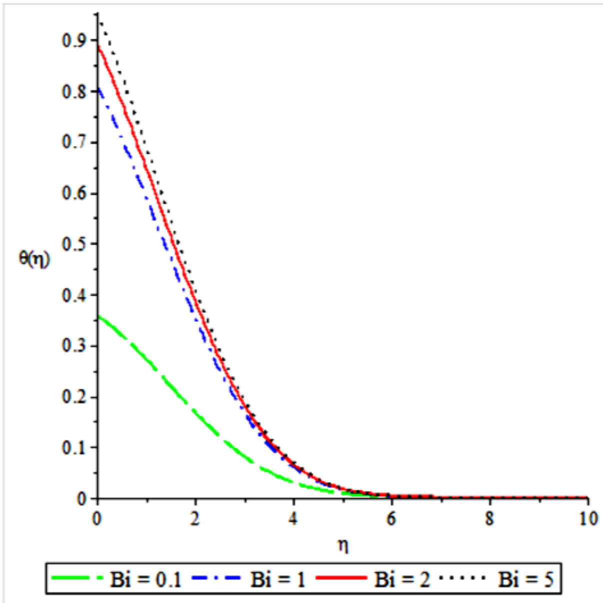

Fig.12. Temperature profiles for Sc=*0.24*, *n=1*, Pr=*0.72*, $\mathrm{Ra}=\mathrm{Br}=\mathrm{Gr}=M=\lambda=K=0.1$ and $F_w=0.2$.

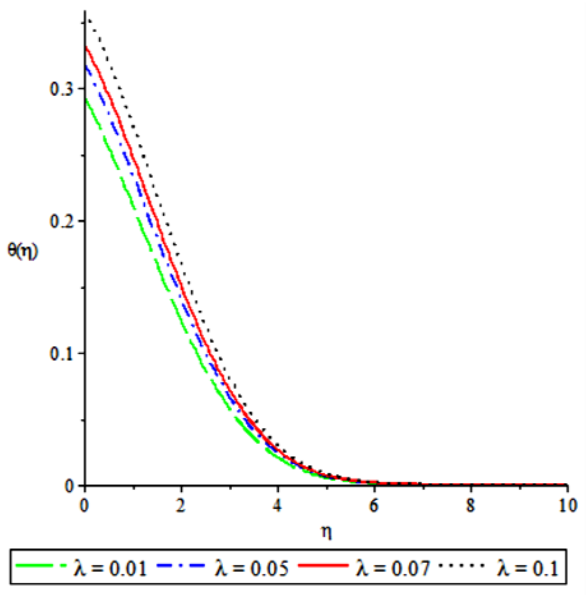

Fig.13. Temperature profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=\mathrm{Gr}=M=\mathrm{Ra}=\mathrm{Bi}=K=0.1$ and $F_w=0.2$.

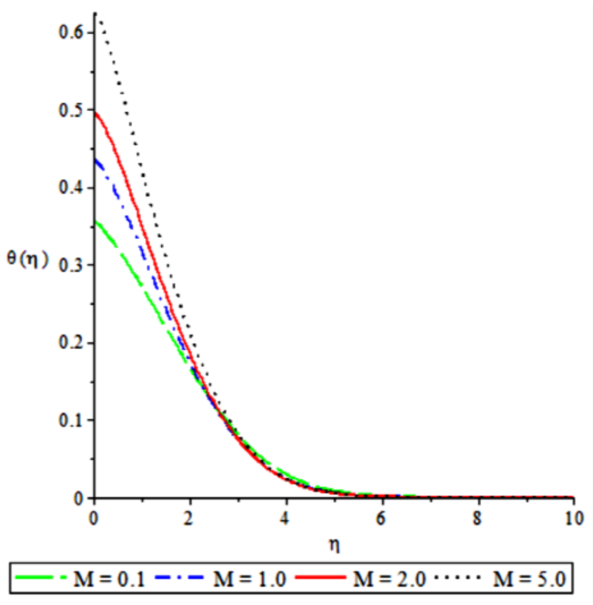

Fig.14. Temperature profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=\mathrm{Gr}=\lambda=\mathrm{Bi}=K=0.1$ and $F_w=0.2$.

C. Concentration profiles

The effects of the controlling parameters on concentration profile were shown in Figs 15-17. The boundary conditions were fulfilled as the graphs indicated maximum concentration at the permeable plate surface and asymptotical decrease to the prescribed free stream value. Fluid injection and suction, Schmidt number and the reaction rate parameters decreased the rate of mass diffusivity. The solutal boundary layer also decreased for all the three controlling parameters.

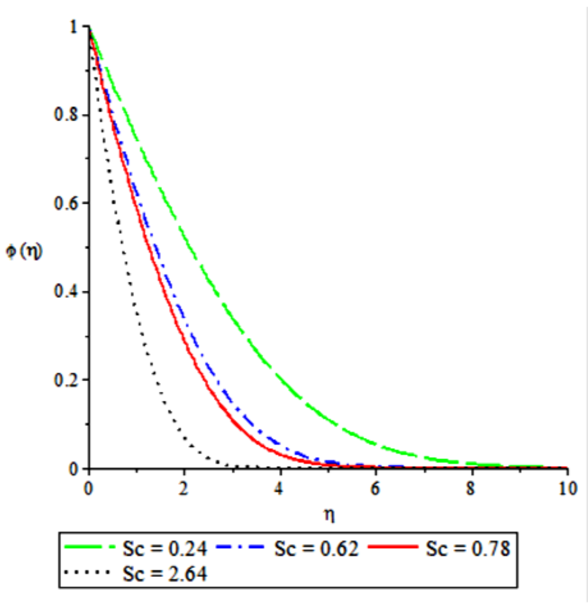

Fig.15. Concentration profiles for $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=\mathrm{Gr}=K=\lambda=\mathrm{Bi}=M=0.1$ and $F_w=0.2$.

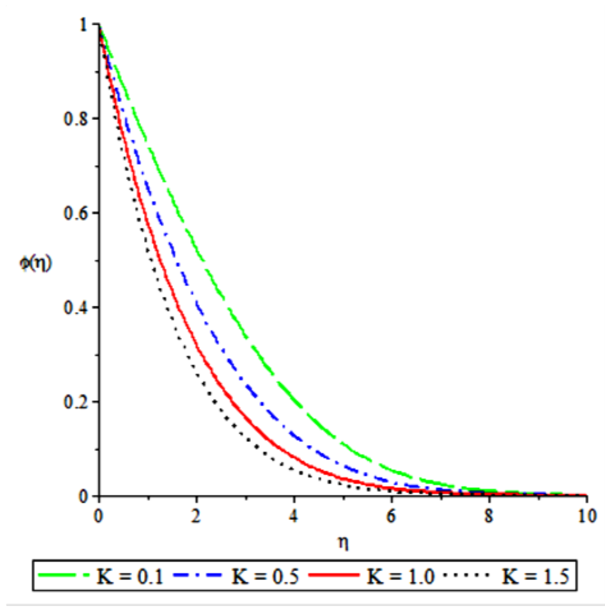

Fig.16. Concentration profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$, $\mathrm{Br}=\mathrm{Gr}=\lambda=\mathrm{Bi}=M=0.1$ and $F_w=0.2$.

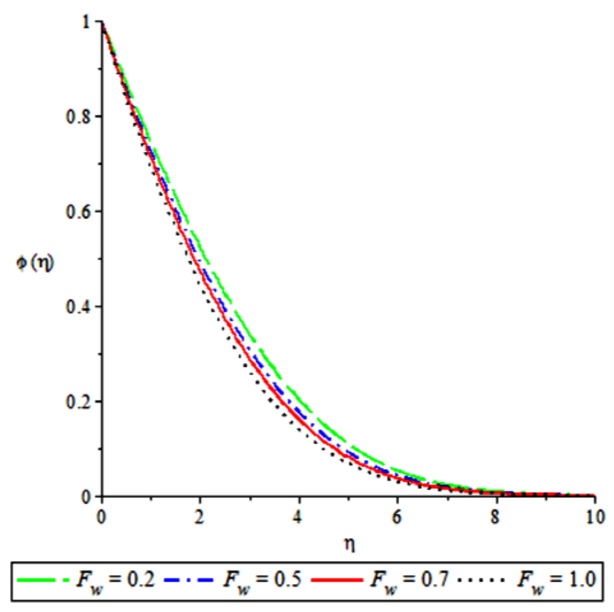

Fig.17. Concentration profiles for $\mathrm{Sc}=0.24$, $n=1$, $\mathrm{Pr}=0.72$ and $\mathrm{Br}=\mathrm{Gr}=\lambda=\mathrm{Bi}=M=K=0.1$.

Conclusively, this work examined the analysis of a steady, two-dimensional, chemically reactive MHD flow of heat and mass transfer of a viscous, incompressible, chemically reactive and electrically conducting fluid flow over a permeable surface. The similarity equations were obtained and solved numerically using Runge-Kutta method based on Shooting technique. Numerical results were presented, illustrated and analyzed graphically with all the controlling thermo physical parameters in the velocity, temperature and concentration profiles. Then, it was noted that:

- increase in radiation parameter, magnetic parameter, Brinkman number and internal heat generation parameter increase the plate surface temperature but decrease the rate of heat transfer.
- the plate temperature decreases with fluid suction/injection and increases with Prandtl number
- fluid suction/injection and Prandtl number increase the rate of heat transfer;
- all the embedded parameters in the momentum boundary layer equations increase the skin friction coefficient;
- there is a faster rate of mass movement when the reaction rate parameter and Schmidt number are increased;
- fluid suction/injection has significant effect on the Skin-friction coefficient, Nusselt number and Sherwood number;
- the order of the chemical reaction is quite significant;
- all the embedded parameters in the momentum boundary layer equations decrease the momentum boundary layer.

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Nomenclature

Bi – Biot number
Br – Brinkman number
B_0 – uniform magnetic field or magnetic field strength $\left[Am^{-1}\right]$
C – specific concentration of the fluid inside the plate/boundary layer $\left[molm^{-3}\right]$

C_f – concentration from viscous fluid that give rise to coefficient mass transfer $\left[molm^{-3}\right]$
C_∞ – specific concentration outside the plate/boundary layer $\left[molm^{-3}\right]$
c_p – specific heat at constant pressure $\left[J / kgK\right]$
D_m – mass diffusivity
F_w – suction/injection
$f''(0)$ – skin friction coefficient
Gr – Grashof number
g – acceleration due to gravity $\left[ms^{-2}\right]$
h_f – heat transfer coefficient $\left[Wm^3K^{=1}\right]$
K – mean absorption coefficient
K – thermal conductivity coefficient $\left[Wm^{-1}k^{-1}\right]$
K_r – reaction rate constant $\left[mol. / s\right]$
M – magnetic parameter
MHD – magnetohydrodynamics
n – order of the chemical reaction
Pr – Prandtl number
Ra – radiation parameter
q_r – radiative heat flux $\left[Wm^{-2}\right]$
Sc – Schmidt number
T – fluid temperature inside the plate/boundary layer $[K]$
T_f – hot fluid temperature $[K]$
T_∞ – fluid temperature outside the plate/boundary layer $[K]$
U_∞ – velocity outside the plate or free stream velocity $\left[ms^{-1}\right]$
u and v – velocity components along x and y $\left[ms^{-1}\right]$
x and y – Cartesian coordinates $[m]$
β – thermal expansion coefficient
γ – coefficient of the kinematic viscosity
$-\theta'(0)$ – Nusselt number
λ – internal heat generation parameter
ρ – fluid density $\left[kgm^3\right]$
σ – electrical conductivity $\left[Sm^{-1}\right]$
σ^* – Stefan-Boltzman constant
$-\varphi'(0)$ – Sherwood number
ψ – stream function $\left[m^2s^{-1}\right]$

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