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Susceptibility of the roll equation to the bifurcation phenomenon depending on the damping coefficient value and form of the roll damping formula

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Abstract

This paper deals with the susceptibility of the roll equation to the bifurcation phenomenon depending on the damping coefficient value and form of the roll damping formula. Generally, the bifurcation phenomenon depends mainly on the shape of the righting arm curve (*GZ* curve), but roll damping also has a significant impact. The commonly used formulas for roll damping are presented, as well as values of the linear equivalent roll damping coefficient, calculated according to the simple Ikeda method. Values of the linear equivalent roll damping coefficient were calculated for a wide spectrum of roll amplitudes and roll frequencies for two ships. The loading conditions for these ships were selected to show different *GZ* curve characteristics. One ship has a softening spring characteristic and the second has a hardening spring characteristic. For these two ships, a number of calculations of roll spectra are presented where the bifurcation phenomenon occurs. Calculations were made for different damping coefficient values and forms of the roll damping formula.

Introduction

Ship rolling is a phenomenon which adversely affects the operations of a ship. Under certain conditions rolling becomes resonant. The amplitude of resonant oscillations can exceed angles of 0.6 rad and sometimes even 0.7 rad. Resonant rolling may be divided into parametric and synchronous oscillations. The first occur in head or following seas, or regions close to them, and generally are caused by the periodic variation of the stability characteristics (excitations) and not as a direct result of external excitation. The phenomenon of parametric rolling can occur for several values of the ratio of the wave period and the natural roll period of the ship. However, its occurrence is most probable in the first instability region (Ince-Strutt diagram) i.e. for a ratio close to 0.5. In turn, synchronous rolling is a direct result of an external force and takes place at beam seas when the period of the excitation is close to the natural roll period. Both phenomena (parametric and synchronous rolling) are included in the list of dangerous scenarios comprised by the second generation intact stability criteria which are still being developed (Belenky, Bassler & Spyrou, 2011; Umeda, 2013).

Parametric resonance has been known about for many years but initially was thought to be radically unlikely for relatively large ships. The situation has changed since 1998 when a post-Panamax container ship experienced extreme parametric rolling which caused massive cargo loss (400 containers were lost and an additional 400 were damaged). Since that time, parametric rolling has been the subject of many studies (France et al., 2003; Shin et al., 2004; Bulian, 2005; Neves & Rodriguez, 2006; Spyrou et al., 2008; Ribeiro e Silva & Guedes Soares, 2013).

Studies on synchronous rolling are not as numerous as those on parametric resonance. Moreover, most papers that can be found were published before the year 2000. The reason for this is probably that synchronous rolling is treated as a phenomenon that is quite simple to interpret and predict. Since resonance occurs when the natural roll frequency is equal to or close to the excitation frequency, avoidance of such situations appears to be fairly simple (i.e. by alternation of the ships heading and/or speed causing a change of wave aspect and frequency). When the natural roll frequency has to be predicted, then most often the simple IMO method (IMO, 2008) is used. The problem is that the IMO method gives a constant value of the roll period and, as a consequence, a constant value of the resonance frequency, meaning they are independent of the roll amplitude. In fact, rolling exhibits significant nonlinearities which often result in a distinct change of natural oscillation period (frequency) when the amplitude increases. A relatively simple method for prediction of the roll period with regards to the amplitude has been developed (Wawrzyński & Krata, 2016a). This method was tested in an extensive study on synchronous rolling (Wawrzyński & Krata, 2016b).

A mathematical model of rolling is a nonlinear, dynamic system. The parameter that determines the nonlinearity of rolling is the restoring moment which usually is described by the righting arms curve (the *GZ* curve). In some nonlinear, dynamic systems, including the rolling of a ship, nonlinear oscillations can appear. A typical phenomenon for nonlinear oscillations is that the resonance frequency depends on the amplitude of the oscillations. A strong deviation of the resonance frequency leads to bistability (sometimes called multivaluedness) which means that for the same value of excitation moment, rolling with two different amplitudes is possible. This possibility refers to the different nature of these two oscillation modes: resonant and non-resonant. It is known that, for a constant value of the excitation moment, the amplitude differs for resonant and non-resonant oscillations.

Since two different amplitudes of rolling are possible, transition between them is also possible, but for a transition to take place an adequate impulse is necessary. This impulse can be a group of two or three higher or lower waves or another external influence. To induce the transition between roll modes (roll amplitudes), a jump exactly to the amplitude of the second mode is not implausible. Achieving a certain amplitude level causes the system to reach the amplitude of resonant or non-resonant oscillations. For each frequency of the bistability region, an analysis of the domains of attraction can be made, where the role of the attractor for each domain is played by the amplitude of the non-resonant or resonant oscillations.

When bistability and jumps are possible then at some frequencies, another phenomenon can be observed – bifurcation. This phenomenon refers to the sudden qualitative change in the solution of the mathematical model due to a small, smooth change made to the value of some parameter (the bifurcation parameter). In a mathematical model of rolling, bifurcation can be observed as a jump in roll amplitude due to a very small change of the excitation frequency or excitation value. An analysis of the bifurcation phenomenon in rolling has previously been performed (Francescutto & Contento, 1999).

The above-mentioned phenomena are not revealed in every rolling system. The occurrence of bistability depends on the nonlinearity of the *GZ* curve, the value of the excitation moment (roll amplitude) and roll damping. Because a very strong nonlinearity can be found in the area of the maximum of the *GZ* curve, most susceptible to bistability and bifurcation phenomena are ships in loading conditions for which the maximum of the *GZ* curve is located at small angles – for an intact ship this can occur at angles in the range of 0.45–0.6 rad. For a maximum located at larger angles, a bigger roll amplitude must be excited for the occurrence of bistability. For extreme, yet realistic, values of roll amplitude close to 0.5–0.7 rad, these phenomena will not occur when the maximum of the *GZ* curve is located at angles above 0.8 rad.

Among the studies describing the problem of the nonlinearity of rolling oscillations of a ship are works of Falzarano and Taz Ul Mulk (Falzarano & Taz Ul Mulk, 1994), Contento at al. (Contento, Francescutto & Piciullo, 1996), Francescutto and Contento (Francescutto & Contento, 1999), Murashige at al. (Murashige, Aihara & Komuro, 1999), Wawrzyński and Krata (Wawrzyński & Krata, 2016a, 2016b). It should also be noted that the occurrence of the bistability phenomenon was confirmed in an experiment (Francescutto & Contento, 1999). Often, works not directly dedicated to rolling can be a very useful source of information on the phenomena which occur during nonlinear oscillations.

The main goal of the research presented in this paper was an analysis of the susceptibility of the roll equation to the bifurcation phenomenon depending on the damping coefficient value and form of the roll damping formula. Generally, the bifurcation phenomenon depends mainly on the shape of the *GZ* curve, but roll damping also has a significant impact. In commonly used mathematical models of ship rolling, roll damping is presented in the form of a constant coefficient or in the form of an expression with one linear term and one or two nonlinear terms. In the course of the presented research, both approaches were tested and, for the latter, values of the damping coefficient for a wide spectrum of amplitudes and frequencies of rolling were calculated with the use of the simple Ikeda method.

Applied model of ship rolling

This research is focused on studying the characteristics of the ship as an object, disregarding the environmental impact on their behavior as far as reasonable. Roll motion is modeled by a single degree of freedom (1-DOF) rolling equation instead of a system of coupled equations describing the complex motions of a ship in a rough sea. The influence of couplings is time-dependent and can distort the results, disabling the extraction of some effects that can be observed in the mathematical model of uncoupled roll motion.

The most commonly used 1-DOF model of rolling is given by the formula:

$$\left(I_x + A_{44}\right)\ddot{\phi} + B_e\,\dot{\phi} + K(\phi) = M_w \cdot \cos(\omega_e t) \tag{1}$$

where I_x denotes the transverse moment of the ship's inertia, A_{44} is the moment of added mass due to water friction on the rolling hull, B_e is the equivalent linear roll damping coefficient, $K(\phi)$ describes the restoring moment e.g. stiffness of the ship, M_w is the exciting moment, ω_e is the wave frequency and t denotes time.

Dividing both sides of formula (1) by $(I_{xx} + A_{44})$, and performing a few transformations, the roll equation becomes:

$$\ddot{\phi} + 2\mu \cdot \dot{\phi} + \frac{g}{r_x^2} GZ(\phi) = \xi_w \cdot \cos(\omega_e t) \tag{2}$$

where μ is the damping coefficient, g is the acceleration due to gravity, r_x is the gyration radius of the ship and added masses (which is assumed to be constant for the purposes of this study), GZ is the righting arm and ξ_w is the exciting moment coefficient.

The restoring moment $K(\phi)$ is nonlinear and is described by the GZ curve. For the purposes of this study, the GZ curve was approximated by a ninth or higher order polynomial with odd powers only:

$$GZ(\phi) = C_1 \cdot \phi + C_3 \cdot \phi^3 + C_5 \cdot \phi^5 + C_7 \cdot \phi^7 + C_9 \cdot \phi^9 + \cdots \tag{3}$$

where C_1 to C_n are the coefficients obtained with the use of the least squares method. The order of approximation function was selected individually for each case of the righting arm curve to get the best fit.

Roll damping and, as a consequence, coefficients B_e in equation (1) and μ in equation (2) depend on the amplitude and frequency of rolling as well as the forward speed of the ship. However, in many studies of ship rolling, constant values of these coefficients are assumed. In such a case, equation (2) is used. To account for the dependence of damping on the amplitude of rolling, equation (2) should be modified to the form (Himeno, 1981):

$$\ddot{\phi} + 2\alpha \cdot \dot{\phi} + \beta \cdot \dot{\phi}\left|\dot{\phi}\right| + \gamma \cdot \dot{\phi}^3 + \frac{g}{r_x^2} GZ(\phi) = \xi_w \cdot \cos(\omega_e t) \tag{4}$$

where α, β and γ are coefficients which can be established for the particular value of the rolling frequency on the basis of the damping, determined during model tests or calculated analytically according to Ikeda's method. In the presented study Ikeda's simple method (Kawahara, 2008; Kawahara, Maekawa & Ikeda, 2009; 2012) was used with one modification; the bilge keel component was calculated according to the full Ikeda method (ITTC, 2011).

After obtaining, from Ikeda's method, the surface of damping coefficients for the considered range of amplitudes and frequencies of rolling, the procedure is further split into two steps. At first, for the considered constant values of roll frequency, the coefficients α, β and γ are fitted fulfilling condition (5) for the full spectrum of considered amplitudes (if this spectrum is too wide, the method gives way to some inaccuracy).

$$\mu(\phi_a, \omega) = \alpha + \frac{4}{3\pi}\phi_a \omega \beta + \frac{3}{8}\phi_a^2 \omega^2 \gamma \tag{5}$$

The following step is performed so that the coefficients α, β and γ take rolling frequency into account. Then, the series for each coefficient (calculated for successive frequencies) should be individually fitted, e.g. by the third order polynomial with roll frequency as an argument:

$$\alpha(\omega) = a_\alpha + b_\alpha \omega + c_\alpha \omega^2 + d_\alpha \omega^3 \tag{6}$$

$$\beta(\omega) = a_\beta + b_\beta \omega + c_\beta \omega^2 + d_\beta \omega^3 \tag{7}$$

$$\gamma(\omega) = a_\gamma + b_\gamma \omega + c_\gamma \omega^2 + d_\gamma \omega^3 \tag{8}$$

The final form of the roll equation with damping dependent on amplitude and frequency of rolling becomes:

$$\ddot{\phi} + 2\alpha(\omega_e)\cdot\dot{\phi} + \beta(\omega_e)\cdot\dot{\phi}\left|\dot{\phi}\right| + \gamma(\omega_e)\cdot\dot{\phi}^3 + \\ + \frac{g}{r_x^2}GZ(\phi) = \xi_w \cdot \cos(\omega_e t) \qquad (9)$$

The damping coefficients, calculated with the use of formulas (5)–(8), are shown in Figures 1 and 2 (right side).

Considered ships

For the purposes of the presented work, two ships were considered. For each ship, only one loading condition was selected so as to obtain one case of the *GZ* curve with softening spring characteristic (LNG carrier) and one with hardening spring characteristic (general cargo ship). The main particulars and values of the initial metacentric height, GM_0, natural roll period and frequency for small rolling amplitudes for the considered ships are shown in Table 1. Drawings in Figure 1A and 2A show surfaces of the damping coefficients, μ, calculated for the ships in the selected loading conditions, according to Ikeda's method, while Figures 1B and 2B show μ values approximated by the coefficients α, β and γ in equation (9) and calculated in the way described above. In both cases, it can be seen that the A and B surfaces are slightly different. The main reason for this is the fact that too wide a spectrum of amplitudes is considered. However, these discrepancies had no

Table 1. Main particulars of ships and parameters of selected loading conditions

No.	Type of vessel	Length L [m]	Breadth B [m]	Draft T [m]	GM_0 [m]	Natural roll period τ [s]	Natural roll frequency ω_n [rad/s]
1	LNG carrier	278.80	42.60	7.50	6.00	13.4	0.469
2	General cargo ship	140.00	22.00	6.00	0.40	27.6	0.228

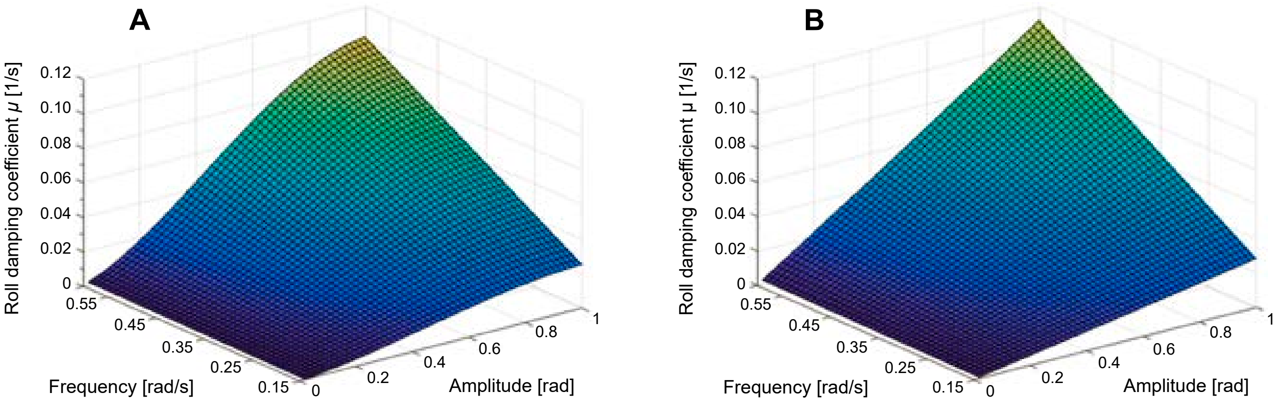

Figure 1. Damping coefficient for the LNG carrier (T = 7.50 m, GM_0 = 6.00 m) calculated according to Ikeda's method (drawing A) and, next, approximated by coefficients $\alpha(\omega)$, $\beta(\omega)$ and $\gamma(\omega)$ in equation (4) (drawing B). Assumed bilge keel dimensions: length = 40 m, breadth = 0.50 m

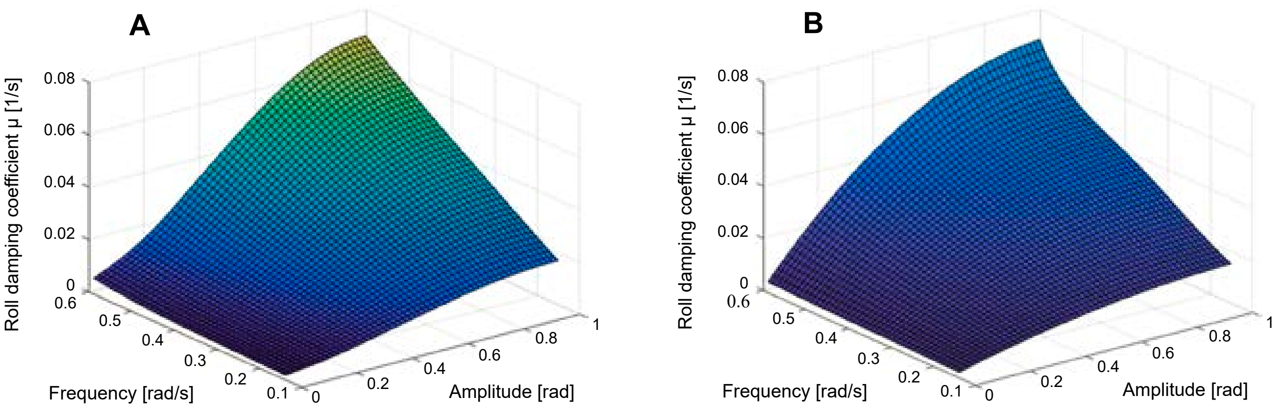

Figure 2. Damping coefficient for the general cargo ship (T = 6.00 m, GM_0 = 0.40 m) calculated according to Ikeda's method (drawing A) and, next, approximated by coefficients $\alpha(\omega)$, $\beta(\omega)$ and $\gamma(\omega)$ in equation (4) (drawing B). Assumed bilge keel dimensions: length = 30 m, breadth = 0.40 m

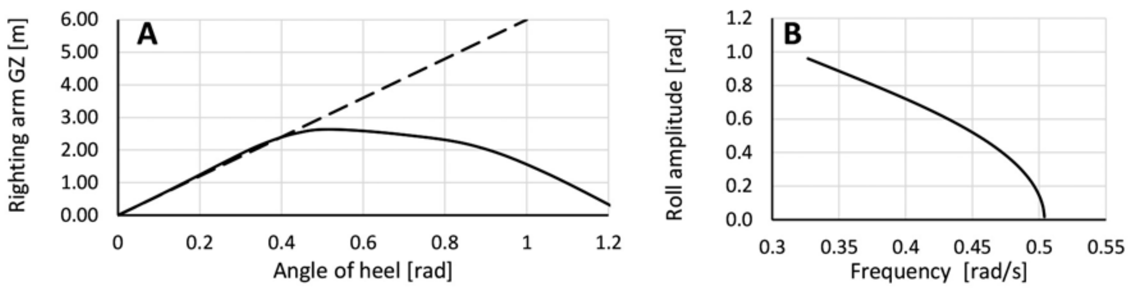

Figure 3. The *GZ* curve (drawing A) and resonance curve (drawing B) for the LNG carrier (T = 7.50 m, GM_0 = 6.00 m). The resonance curve was calculated according to the procedure described in (Wawrzyński & Krata, 2016b)

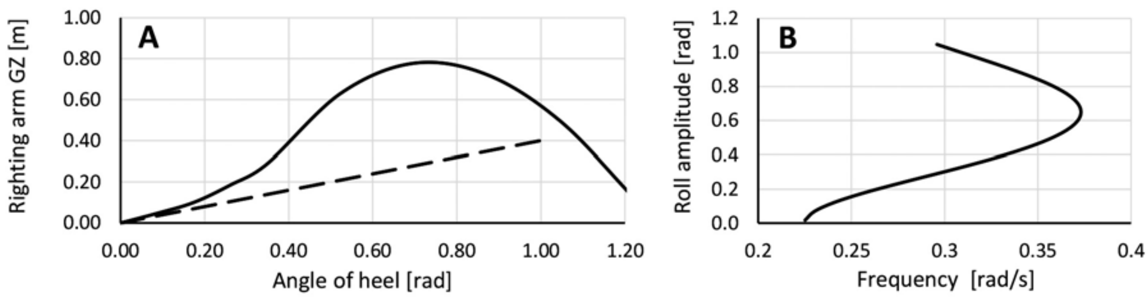

Figure 4. The *GZ* curve (drawing A) and resonance curve (drawing B) for the general cargo ship (T = 6.00 m, GM_0 = 0.40 m). The resonance curve was calculated according to the procedure described in (Wawrzyński & Krata, 2016b)

significant impact on the results of the planned calculations. It should also be noted that Ikeda's method is considered to be fully applicable for amplitudes not bigger than 0.4 rad and does not consider changes to roll damping due to bilge keel emergence and deck submergence. For the latter problem, no exact method is currently known: an attempt at a solution can be found in the literature (Bassler & Reed, 2010).

The *GZ* curve with softening spring characteristic (LNG carrier) shows that the resonance curve (i.e. resonance frequency) deviates in the direction of smaller frequencies with increasing roll amplitude, while for the *GZ* curve with hardening spring characteristic (general cargo ship) this deviation is in the direction of higher frequencies. The *GZ* curves and resonance curves for both considered ships are shown in Figures 3 and 4.

Calculation procedures and results

Calculations were performed according to equations (2) and (9) for the two ships described above. When equation (2) was used, two constant values for the roll damping coefficient were applied: μ = 0.05 1/s and μ = 0.10 1/s. The value of damping coefficient equal to 0.05 can be found in many papers where calculations of a theoretical nature are conducted and, as an average value, seems to be reasonable when looking at Figures 1 and 2. The value of μ = 0.10 1/s is twice as big but is not impossible. Such large values of the damping coefficient can be achieved for ships with bilge keels large enough.

The easiest way to observe the bifurcation phenomenon (if it can occur) is to perform roll simulations in order to obtain the roll spectrum (frequency response curve) for a series of increasing values of the exciting moment coefficient, ξ_w, in equation (2) or (9). Within the theoretical analysis, the value of the exciting moment is often assumed to be constant; however, when research is experimental, e.g. for a constant wave height, its value will be determined by the excitation frequency.

Figure 5 shows the exemplary roll spectra for situations without the occurrence of bifurcation (drawing A) and with bifurcation (drawing B) where the bifurcation is marked by the dotted line. However, the roll spectrum in Figure 5B is incomplete. When the bifurcation phenomenon occurs, to get a full picture of the roll spectrum, it is necessary to perform two rolling simulations with a continuously changing frequency. One simulation is performed for increasing values of the excitation frequency while the second is performed for decreasing values of the excitation frequency. The effect of such simulations is shown in Figure 6. In Figure 6, inside the bistability region, A, marked/limited by two bifurcation frequencies, rolling with two values of amplitude is possible for the same value of excitation.

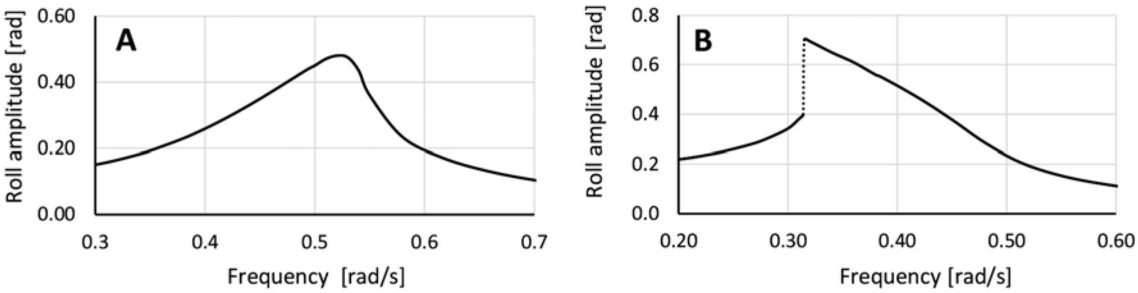

Figure 5. Roll spectrum without bifurcation (drawing A) and with bifurcation (drawing B)

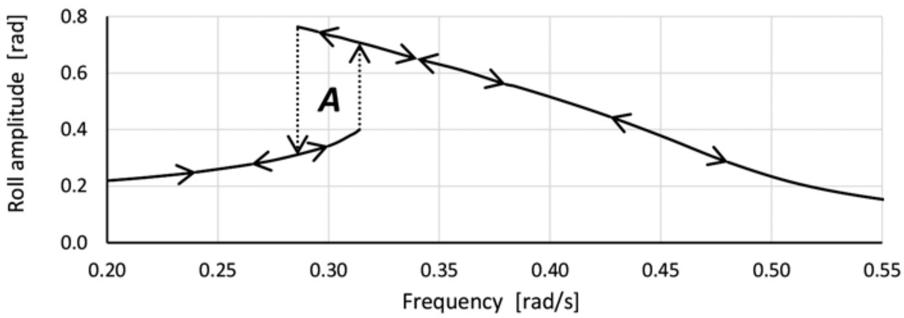

Figure 6. Full roll spectrum for the case with bifurcation occurring

Regardless of the above, to improve the clarity of the drawings, the roll spectra shown in further figures are the results of rolling simulations performed only for cases of increasing values of excitation frequency. Figures 7 to 9 show roll spectra for the LNG carrier and Figure 10 to 12 for the general cargo ship.

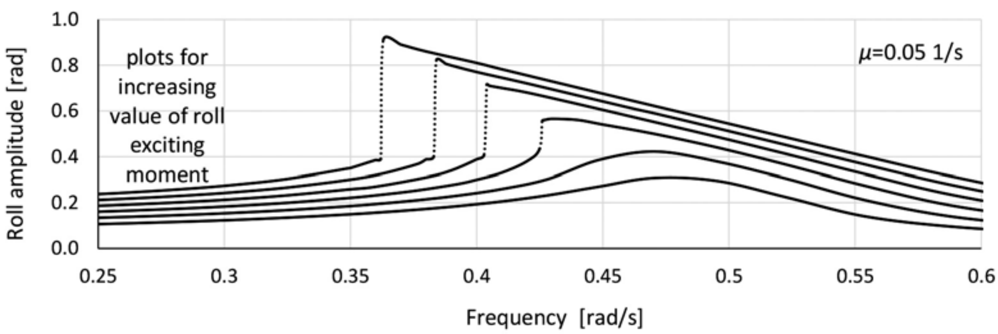

Figure 7. Roll spectra for the LNG carrier (T = 7.50 m, GM_0 = 6.00 m) calculated for a constant value of the damping coefficient μ = 0.05 1/s – equation (2)

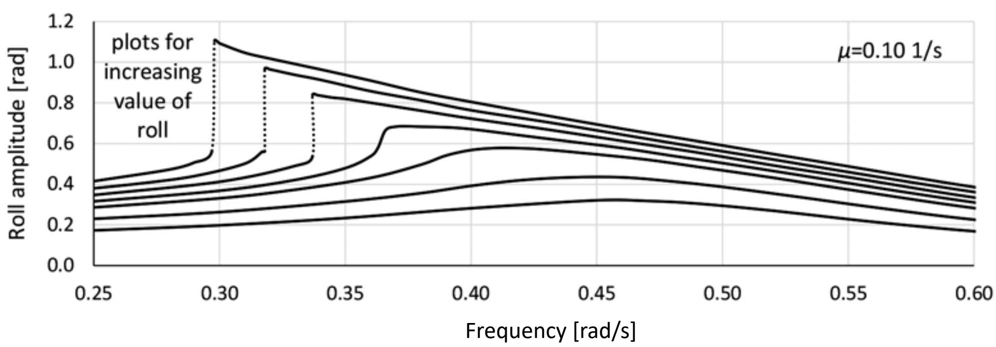

Figure 8. Roll spectra for the LNG carrier (T = 7.50 m, GM_0 = 6.00 m) calculated for a constant value of the damping coefficient μ = 0.10 1/s – equation (2)

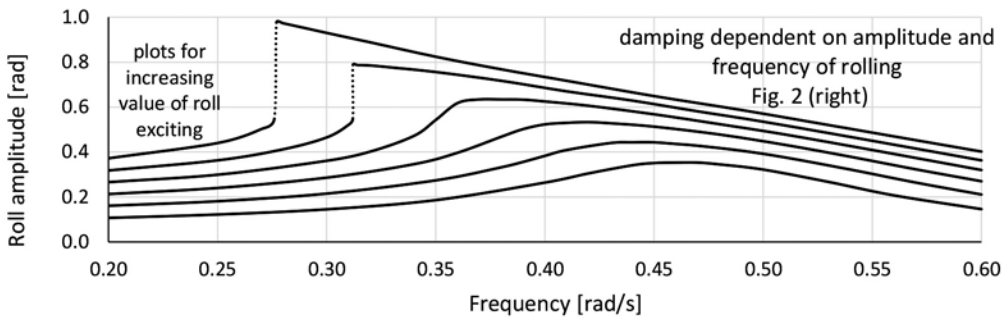

Figure 9. Roll spectra for the LNG carrier (*T* = 7.50 m, GM_0 = 6.00 m) calculated for damping dependent on the amplitude and frequency of rolling – equation (9), with damping in Figure 1 (right drawing

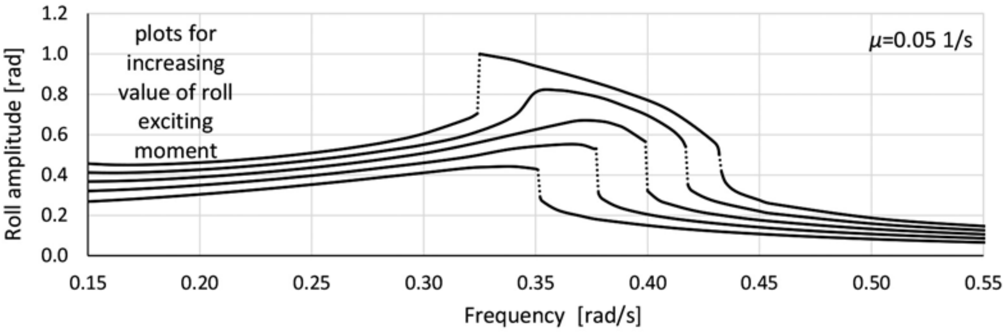

Figure 10. Roll spectra for the general cargo ship (*T* = 6.00 m, GM_0 = 0.40 m) calculated for a constant value of the damping coefficient μ = 0.05 1/s – equation (2)

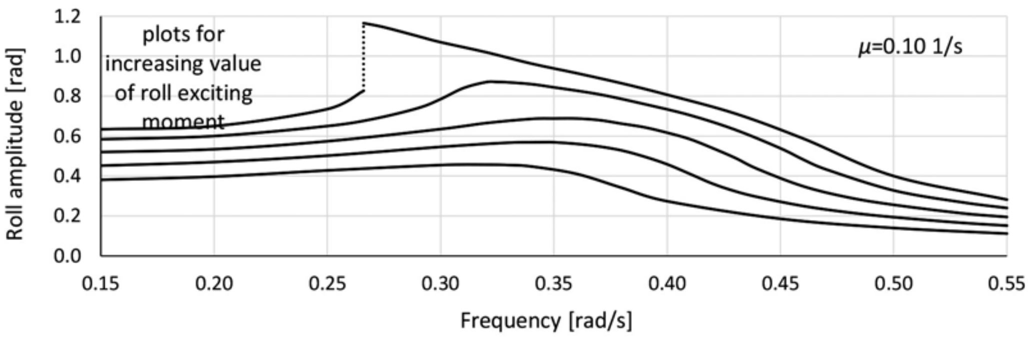

Figure 11. Roll spectra for the general cargo ship (*T* = 6.00 m, GM_0 = 0.40 m) calculated for a constant value of the damping coefficient μ = 0.10 1/s – equation (2)

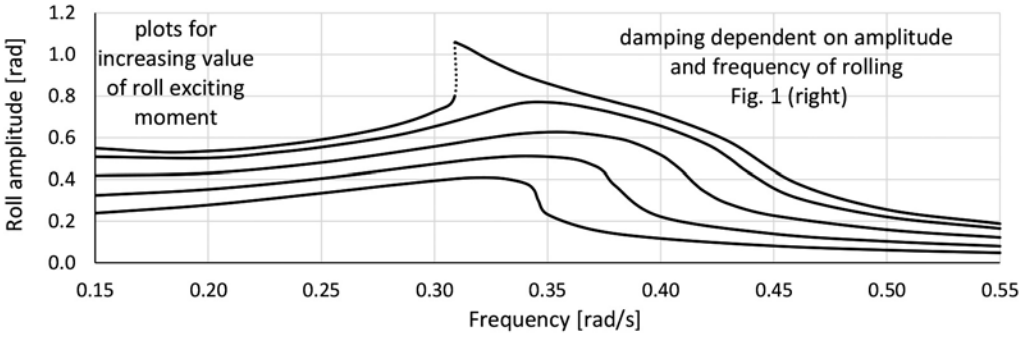

Figure 12. Roll spectra for the general cargo ship (*T* = 6.00 m, GM_0 = 0.40 m) calculated for damping dependent on the amplitude and frequency of rolling – equation (9), with damping in Figure 2 (right drawing)

Discussion and conclusions

When the *GZ* curve has softening spring characteristics the resonance curve always deviates in one direction, that of smaller frequencies (Figure 3). In such cases, the bifurcation phenomenon can be observed only on one side (left) of the roll spectrum (Figures 7 to 9). When the *GZ* curve has hardening spring characteristics (Figure 4) the bifurcation phenomenon can present on both sides of the roll spectrum (Figure 10). The reason for this is the change in *GZ* curve characteristic. Figure 4 shows that the *GZ* curve has hardening spring characteristics only to the heel of about 0.6 rad, beyond this the characteristic becomes softening spring. The resonance curve, which at the beginning deviates in the direction of higher frequencies, after exceeding the amplitude of 0.6 rad changes its direction of deviation (Figure 4, right). Additionally, for the region where the *GZ* curve has a hardening spring characteristic, the bifurcation jumps appear at smaller amplitudes.

Analysis of the drawings in Figures 7 and 8, as well as Figures 10 and 11, allows one to state that the damping increase clearly decreases the susceptibility of the ship roll equation to the bifurcation phenomena. In Figure 7, where the damping coefficient, μ = 0.05 1/s, the bifurcation jumps start at rolling amplitudes close to 0.4 rad, while in Figure 8 (μ = 0.10 1/s), jumps start at rolling amplitudes closer to 0.6 rad. Additionally, it can be seen that the bifurcation region is shifted in the direction of smaller frequencies. In the case shown in Figures 10 and 11, it can be seen that bifurcation jumps on the right side of the roll spectra have disappeared completely. Only on the left side of the roll spectrum do bifurcation jumps remain with the same change characteristics of Figures 7 and 8. Additionally, it is worth pointing out that the bifurcation phenomenon for the *GZ* curve with a hardening characteristic is quite rare. In previous research (Wawrzyński & Krata, 2016b) where seven ships were considered and a summary of 30 different loading conditions, for 16 cases with hardening characteristics of the GZ curve and damping coefficient equal to only 0.05, the bifurcation phenomenon was observed in just 3 cases.

A change from a constant damping coefficient to one dictated by a formula dependent on the amplitude and frequency of rolling (equation (9)) has a similar influence on the roll spectrum as increasing the value of the damping coefficient in equation (2). This is because the value of damping increases with rolling amplitude (Figures 1 and 2). In both considered cases, for frequencies 0.30–0.60 rad/s and amplitudes larger than 0.60 rad, the damping coefficient is larger than 0.05 1/s and may extend to over 0.07 1/s for amplitudes larger than 0.80 rad.

The final conclusions are:

1. An increase of the damping coefficient decreases the susceptibility of the rolling equation to the bifurcation phenomena.
2. Use of the damping formula dependent on amplitude, instead of the constant average value of the damping coefficient, gives a similar effect, i.e. the susceptibility of the rolling equation to bifurcation phenomena decreases.
3. Generally, the rolling equation is more susceptible to bifurcation phenomena when the *GZ* curve has a softening spring characteristic.
4. When the *GZ* curve has a hardening spring characteristic then bifurcations occur only for small values of damping.
5. Increasing the damping value shifts the region of bifurcations towards the lower frequencies and towards higher amplitudes.
6. For large enough roll damping, bifurcation will not occur even in cases of significant nonlinearity of the *GZ* curve.

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