

# Usefulness of Nonlinear Interpolation and Particle Filter in Zigbee Indoor Positioning

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**Abstract:** The key to fingerprint positioning algorithm is establishing effective fingerprint information database based on different reference nodes of received signal strength indicator (RSSI). Traditional method is to set the location area calibration multiple information sampling points, and collection of a large number sample data what is very time consuming. With Zigbee sensor networks as platform, considering the influence of positioning signal interference, we proposed an improved algorithm of getting virtual database based on polynomial interpolation, while the pre-estimated result was disposed by particle filter. Experimental result shows that this method can generate a quick, simple fine-grained localization information database, and improve the positioning accuracy at the same time.

**Keywords:** Fingerprint positioning algorithm; RSSI; ZigBee sensor networks; Polynomial interpolation; Particle filter

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## 1. Introduction

With the increasing emergence about Internet of Things Technology, using wireless sensor network technology to build an indoor positioning system, has become more mature. RSSI (receive signal strength indication) can be used to provide positioning information in Zigbee wireless network, along with low power consumption, saving hardware costs and other advantages (Si Haifei et al., 2011). Therefore, the RSSI-based indoor positioning algorithm has been paid more and more attention to (Tang Wen-Sheng et al., 2008). In this paper, the RSSI-based fingerprint database positioning (FP) algorithm, which is unrelated with distance information, is used to estimate the rover positioning (Yin J et al., 2005). However, the existing fingerprint database algorithms only collect sampling data RSSI directly, and do not consider the

nonlinear characteristics of RSSI, which lead to workload at offline stage, and can't effectively deal with the phenom of abrupt about information data caused by noise. In this case, we use a non-linear mehod to describe the RSSI-Distance curves, taking into account the interference of rover itself on the signal strength, as well as the influence of node numbers on the approximation (Chen Jiaqi and Yan Zi, 2012). We adopt a partition interpolation method to optimize offline stage of traditional fingerprint database algorithm, save a certain amount of work, and improve work efficiency. At positioning stage of the algorithm, we introduce particle filter (PF) technology to process the initial estimate of the rover coordinates smoothly.

## 2. Fingerprint positioning algorithm

RSSI-based fingerprint matching algorithm is proved to have become the main method of today's indoor positioning algorithm in related research conclusions. Fingerprint database positioning algorithm process includes training offline phase and positioning online phase. Algorithm execution process is shown in Figure 1.

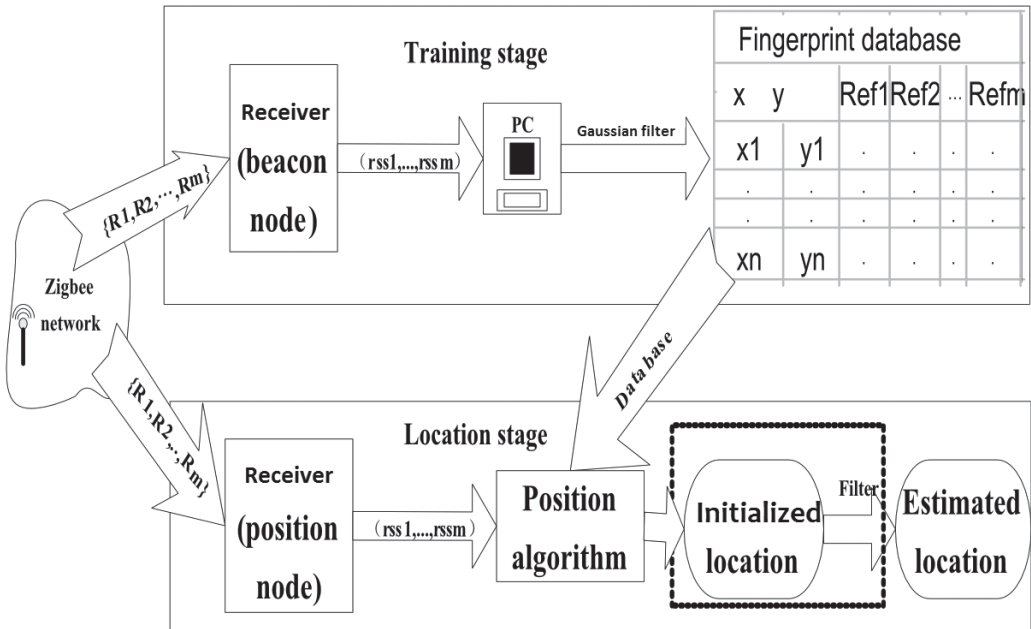


Fig. 1. Execution process of Fingerprint database algorithm

Since interference of weather changes in the indoor environment and natural factors such as air humidity what is unavoidable, the establishment of electromagnetic space propagation model is more difficult. In such case, the training phase selects

actual measurement method to build a database, which has advantage of high precision and easy implementation (Liu Xiaokang et al., 2012). Within the positioning area, we process RSSI data, which was collected from n fingerprint information samples (RP, experimental uniformly distributed within the region). Using some of Gaussian filter technique ( $\delta = 0.2$ ), and saving in vector form like  $R_i = \{\overline{RSS}_1^i, \overline{RSS}_2^i, \dots, \overline{RSS}_m^i\}$  ( $\overline{RSS}_{1-m}^i$  denote RSSI that was obtained from m transmitting nodes after processing at the i-th sampling points), and at the same time storing the corresponding coordinate values in the database together (Neal Patwari et al., 2007).

We use the nearest neighbor (NN) algorithm to estimate the position of the rover node at positioning stage (Hang Guo et al., 2011). When the rover goes into positioning areas, it will receive real-time node RSSI value from the m transmit nodes ( $r_i = \{rss_1^i, rss_2^i, \dots, rss_m^i\}$ ). Taking Euclid distance to characterize degree of proximity between rover node and the information sampling node, and selecting the sampling node, which has the least Euclid distance ( as positioning of user) can be calculated as follows:

$$D_n = \sqrt{\sum_{i=1}^m (RSS_i - rss_i^t)^2} \quad (1)$$

While traditional FP algorithm has two shortcomings as follows:

(1) Fingerprint database must have valid high precision, and need to get enough data at each RP, which causes an increase in the workload of the training phase. Therefore, using a suitable interpolation method to build granular database can reduce work at training phase, and also improve the positioning accuracy.

(2) Under complex indoor environments, the signal strength of the data collected will become invalid data because of interference and other reasons. Currently the fingerprint database algorithms can not effectively prevent such interference by harmful data on the positioning results, so effective treatment for initial estimation results at positioning stage of algorithm is required.

### 3. Construction of fingerprint database

#### 3.1. Improved fingerprint database algorithm based on Regional interpolation

Under multi-node network platform, the signal strength of the transmission node exist "cross mutation" phenomenon, this RSSI value as a function of distance is not a simple linear relationship, shown in Figure 2. Therefore, we choose nonlinear polynomial interpolation algorithm when consider how to refine the fingerprint database.

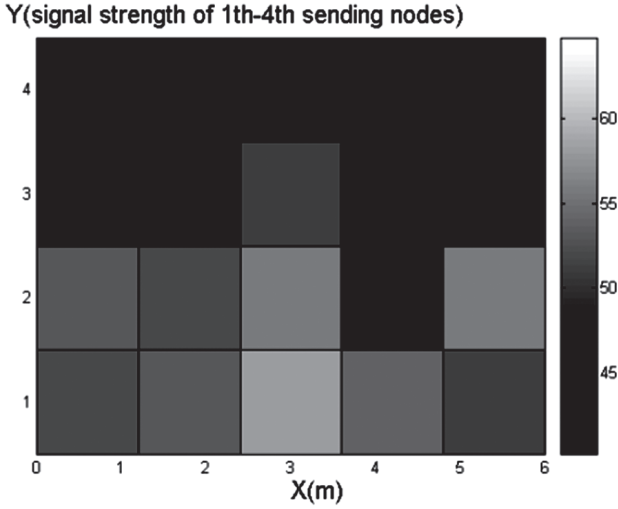


Fig. 2. Relationship between RSSI and Distance

Lagrange interpolation and Newton interpolation are two expressions that are common to see in polynomial interpolation method.

Selecting the distance between information sampling point and the sending node as the independent variable, and the RSSI collected from sending node at sampling points as the dependent variable, the formula can be written as follows:

Lagrange equation:

$$L_n(x) = \sum_{i=0}^n y_i l_{n,i} \tag{2}$$

Among upper Equation,

$$l_{n,i}[x] = \frac{(x - x_0) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)}$$

Newton equation:

$$f(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + \dots + (x - x_0)(x - x_1) \dots (x - x_n)f[x_0, x_1, \dots, x_n] \tag{3}$$

Among upper Equation,

$$f[x_0, x_1, \dots, x_n] = \frac{(f[x_1, \dots, x_k] - f[x_0, \dots, x_{k-1}])}{(x_k - x_0)}$$

In addition, from the above two equations we can see that Lagrange polynomial interpolation formula is simple and easy to implement. However, when the number of nodes changes, the interpolation basis function will change, so the operation is inconvenient and Newton interpolation method can effectively solve above problems. The more the number of nodes, the better the fit of the function is. We take interpolation under two cases respectively:

Method (1): Does not compute new interpolated point into the interpolation points, using simple Lagrange method.

Method (2): Considering the function approximation, the interpolated point into the calculation of the new interpolation points for the convenience of calculation, using the Newton interpolation.

### 3.2. Design of interpolation scheme

Under the actual environment, where rover receiver is in the positioning area, users have different interference on each sending nodes by themselves at different positions. Solid points shown in Figure 3, are the sampling points of information by actual measurement, and the hollow points are the virtual information points obtained by interpolation algorithm. When the rover is facing to AP01 at position A, it has very small interference on AP01 at this time; while the object moves to position B, the rover has a sideways of influence on AP01. If we take the RSSI value of actual measurement points around position B into calculating the RSSI value of position A, it will have bad influence on estimating the real RSSI value of position A. The more nodes, the stronger ability of interpolation to the real value. Therefore we use a limited area interpolation program based on threshold  $\theta$  of the angle, as shown below.

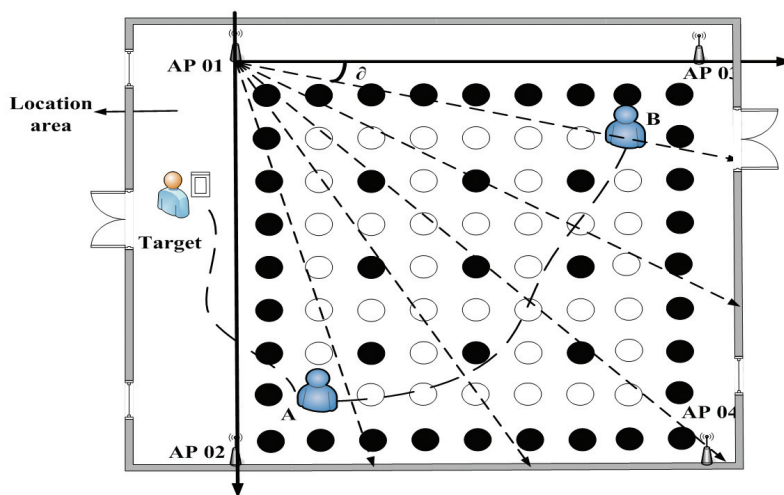


Fig. 3. Interpolation scheme defined area

Interpolation process is described below:

(1) Establish a single coordinate system at each AP respectively, and every information sample point has follow sets of angles:

$$a(k) = \{a(1), a(2), \dots, a(m)\} \tag{4}$$

Separate flat of 90 degree into equal parts based on the set angle value, the measured information sample points at the dividing line belong to two intervals. Every set of divided parts is expressed as below.

$$A(k) = \{A(1), A(2), \dots, A(n)\} \tag{5}$$

(2) Select the distance between information sampling point and transmission node as the independent variable, and the received signal strength indicator is dependent variable, which is from sending node at sampling points. Use equation (2) or equation (3) to computer the RSSI value of virtual information points.

The approximation effect of interpolation algorithm will get better and better by increase of interpolation points, but the degree of Lagrange interpolation basis function is also improved. So result will be numerical instable, because the calculated values will fluctuate largely, and will have great deviation with the real value. Such phenomenon is called “Runge” phenomenon, as shown in Figure 4 (a).

So we can set constraints as follows:

$$R_i = \{R_{real} \mid R_{real} \in [R_{min}, R_{max}]\} \tag{6}$$

Among upper Equation,  $R_{min}(R_{max})$  is the min (max) value of RSSI by testing in positioning area.

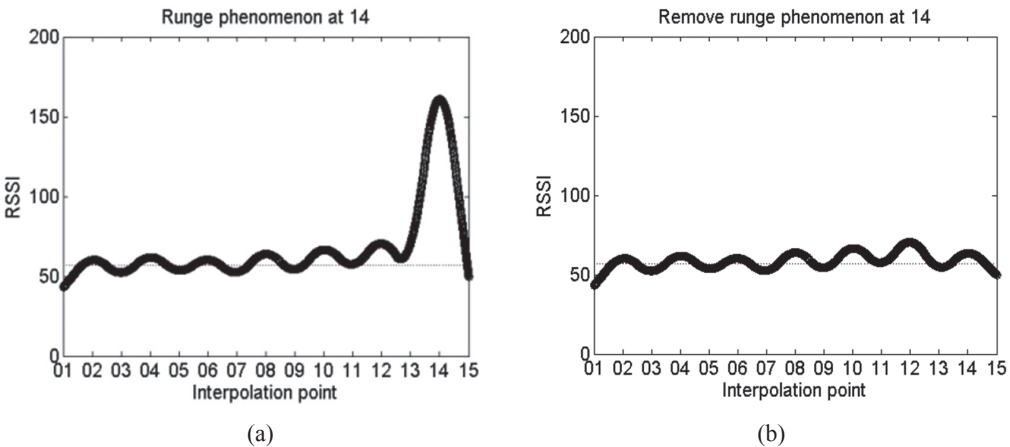


Fig. 4. “Runge” phenomenon in polynomial interpolation

We can reduce the degree of polynomial function by arraying the interpolation points, executing interpolation by groups, and calculating the average value of each group obtained from interpolation. For example, if there are  $p$  measurement information points in the angular space, select  $q$  ( $q \in (2, p]$ ) points from  $p$  points for polynomial interpolation. If the calculated values are not beyond the range in constraints requirement (equation (6)), then reduce  $q$  by every time until  $q$  changes to the 2. Calculate the average value of obtained  $C_p^q$  estimated values, until the RSSI values are within the desired range. So we can remove “Runge” phenomenon, as shown in Figure 4(b).

#### 4. Particle filter algorithm

The received RSSI value is non-linear changing in indoor environment, and does not obey the standard Gaussian distribution. In this paper, we use particle filtering technique to deal with problem of irregular distribution of RSSI at positioning stage of fingerprint database algorithm. The improved fingerprint database algorithm is shown in Figure 5, and the filtering process is within the box.

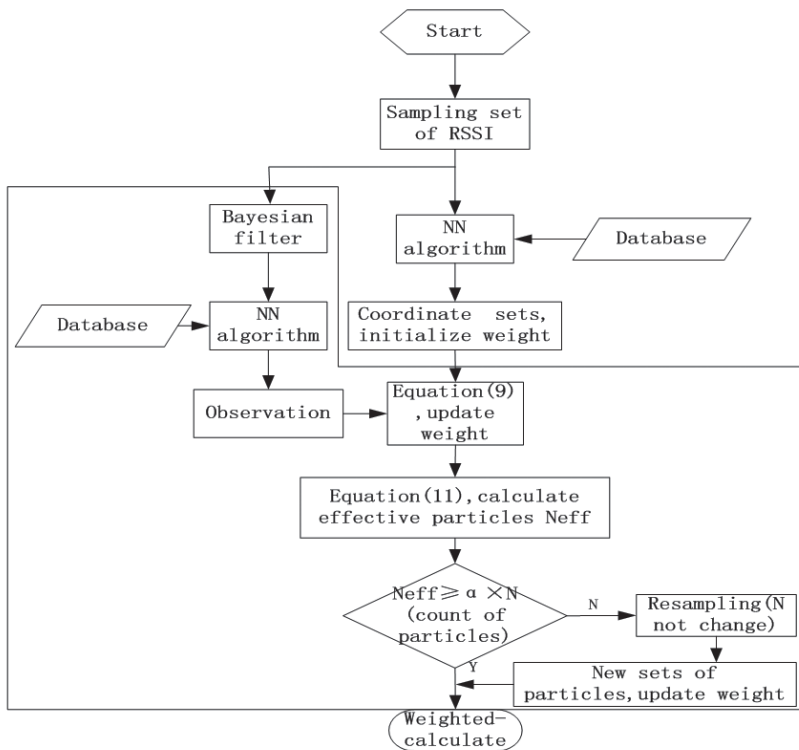


Fig. 5. The flowchart of improved fingerprint database algorithm

We set the state space and observation equations of the moving rover, respectively, as the following format:

$$S_t = f(S_{t-1}, v_t, a_t, \Delta t) + e_t \tag{7}$$

$$O_t = h(S_t, v_t, a_t, \Delta t) + w_t \tag{8}$$

In upper equations, the function  $f$  and  $h$  are described as the state value and measurement value, which changed followed by time;  $v_t$  and  $a_t$  are respectively speed and acceleration information of moving rover at time  $t$ ;  $e_t$  and  $w_t$  are the process noise.

Using importance sampling algorithm to get  $N$  groups multi-dimensional vector data about RSSI values at time  $t$ , we can obtain the coordinates sampling set  $\{S_t\}_{i=1}^N$  through fingerprint database algorithm, initializing each sample weights. Meanwhile we use Gaussian filter ( $\delta = 0.25$ ) to deal with the original RSSI sample data, and match the results by filtering with fingerprint database to get coordinates values as the state observations. But there will be some coordinates of sampling points at this time, which are discrete distribution, and being significantly different from most sampling points (Figure 6(a)).

In response to upper phenomenon, we can use particle filter technology to process data. We take Bayesian estimation to estimate posterior probability density function of the original coordinates of samples, and use classical Monte Carlo methods to change the process of integration to summation of limited sampling point in Bayesian estimation algorithm. Then we can obtain minimum variance estimation of states (Crisan Dan and Del Moral P., 1999).

Firstly, we use updating weights equation to update the weight of particles in samples recursive at time  $t$ , just like below:

$$w_t^i = w_{t-1}^i \frac{p(O_t | S_t^i) p(S_t^i | S_{t-1}^i)}{q(S_t^i | S_{t-1}^i, O_t)} \tag{9}$$

$q(S_t^i | S_{t-1}^i, O_t)$  is known as an easy sampling probability density function.

We choose  $q(S_t^i | S_{t-1}^i, O_t) = p(S_t^i | S_{t-1}^i)$ . For the convenience of calculation, we can assume  $p(O_t | S_t^i)$  as Gaussian distribution. So we can obtain the following formula (10) from using the character of distribution of the sample state by upper assumption.

$$w_t^i = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x_t^i - x_{t,fp})^2 + (y_t^i - y_{t,fp})^2}{2\sigma^2}\right) \tag{10}$$

$(x_{t,fp}, y_{t,fp})$  is observation, and  $\sigma$  is the standard deviation between coordinate value of samples (obtained from formula (1) ) and observation.



However, considering time increasing in the traditional method of particle filter, the particles with important weights has a small number, and the distribution is more concentrated situation. Gordon proposed resampling (SIR) method to solve the problem of particle degradation (Crisan Dan and Del Moral P., 1999). In SIR method we can normalize the particle weights, which are obtained in the previous step, and arrange them in descending order, and calculate the effective particles by the equation (11).

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^N w_i^2} \quad (11)$$

If  $N_{\text{eff}} > \partial \times N$  ( $\partial$  is effective factor,  $\partial \in (0,1)$ ), we do nothing about resampling; otherwise we do not change total number  $N$  of the samples, and discard particles with small weights. Then we copy particles of samples with large weights, and obtain a new set of particles of samples  $new\{S_i\}_{i=1}^N$  at last.

The distribution of new particles of samples acquired by SIR particle filter algorithm is more concentrated, and we remove effectively the points which are far away from the true position of rover, as shown in figure 6(b).

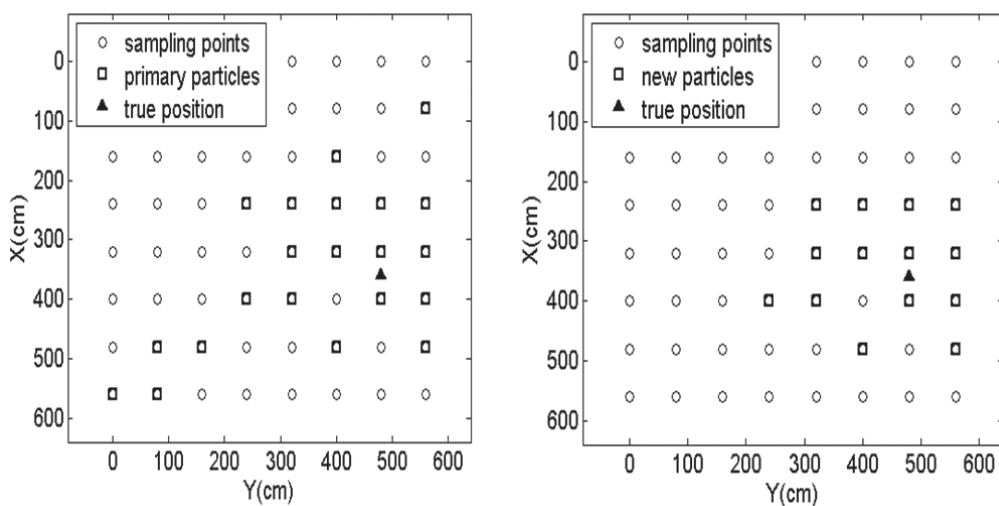


Fig. 6. Distribution of compared particles before and after filtering

At last we take the new obtained coordinates of the particles of samples into weighted calculating to estimate the final coordinates of rover.

## 5. Experiment results

### 5.3. Test Environmental Conditions

We used Zigbee CC2530 (Texas Instruments) chips to build wireless communication network, and to realize communication between nodes. The experimental size of area is 8 meters\*7.4 meters; with calibrated 81 sampling points in the region respectively. It includes 41 solid points for sampling points of measured information (the distance of interval is 1.6 meters except boundary of positioning area), and 40 virtual information points obtained by interpolation (distance of interval change to 0.8 meters after thinning), and four sending nodes (arranged in four corners of the test area, the vertical distance of 2.3 meters). The nodes layout is shown in Figure 7.

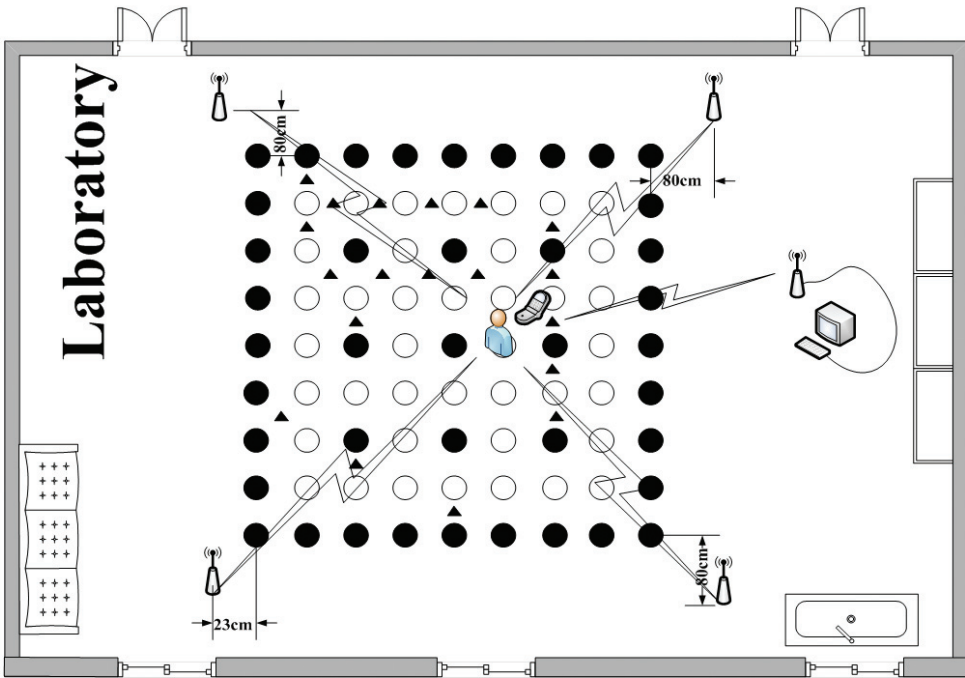


Fig. 7. Experiment localization and node layout

In order to minimize the effect of weather and other natural factors on test results, all of the training and test data of RSSI were collected on the same day. Meanwhile we collected 160 sets of each solid point at stage of building, totally  $160 \times 41 = 6560$  sets of data.

#### 5.4. Evaluation of test results

We selected 18 points (solid triangle points in Fig. 7) for positioning test and collected 25 sets of data for each point, totally  $18 \times 25 = 450$  sets of data. Effective particle factor was set to 0.8, and angle  $\vartheta$  was sequentially taken to the value of  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ . The range of RSSI changes from 40 to 80. Positioning error of Lagrange interpolation method and Newton interpolation method is shown in Figure 8.

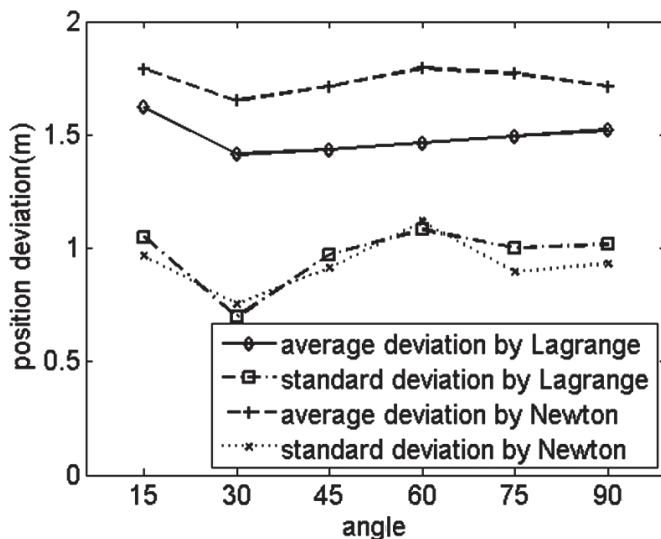


Fig. 8. The mean deviation and standard deviation of different angles for Lagrange and Newton methods

As we can see in upper figure, in the method (1) positioning result is better than in the method (2). When the threshold angle is  $30^\circ$ , the positioning error is the smallest which is about 1.42 m or less; and is also the most stable, while the standard deviation is less than 0.7 m. This is because the latter takes newly interpolation points into aculating other interpolation points, what causes the error accumulation, although it has better expansivity.

Considering the accuracy, we preferred to choose method (1), named Lagrange Interpolation, and  $\vartheta$  is set to  $30^\circ$ . We compared the positioning results under following four conditions: (1) traditional fingerprint database algorithm; (2) improved positioning algorithm with Lagrange interpolation; (3) fingerprint database algorithm combined with traditional particle filter (without interpolation); (4) under interpolation case, results got from the fingerprint database algorithm combined (with traditional particle filter).

Table 1 shows the average positioning accuracy obtained by optimized algorithm (condition (4)). Accuracy was increased by 29.3%, and got 81 nodes (evenly distributed)

in positioning area (which had only 41 nodes before). Using this interpolation method, 49.4% of workload was saved in establishing database at the same time, comparing to the traditional fingerprint database algorithms (condition (1)).

Tab. 1. Contrast of positioning results under 4 cases (unit: m)

	Max deviation	Min deviation	Average deviation	Standard deviation
(1) (samples: 41)	3.5668	0.9260	2.0009	0.9480
(2)(samples: 81)	3.5124	0.8944	1.9317	0.7391
(3) (samples: 41)	3.3947	0.2860	1.6490	0.7159
(4)(samples: 81)	2.9798	0.1844	1.4152	0.6961

The positioning error of each static test points is shown in Figure 9.

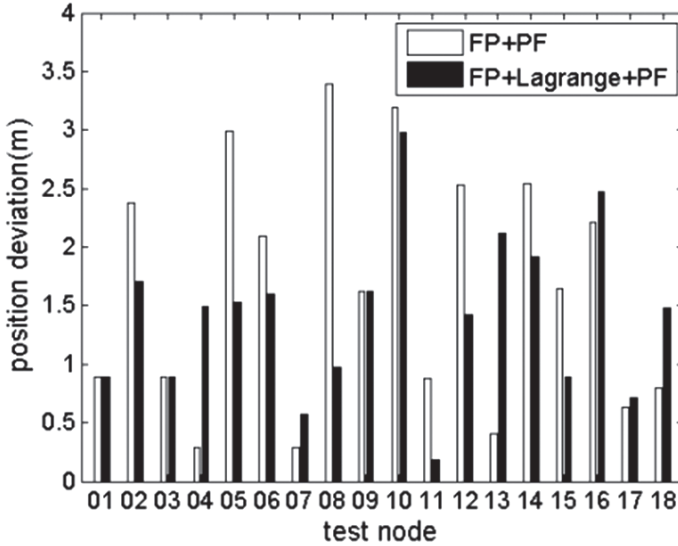


Fig. 9. The positioning errors of 18 test points

As shown in Figure 9, most of the test points have better accuracy using proposed optimized algorithm than traditional fingerprint database algorithms. Therefore, the overall accuracy has been greatly improved.

Figure 10 shows the positioning error of cumulative distribution function (CDF) using optimized algorithm and traditional algorithm.

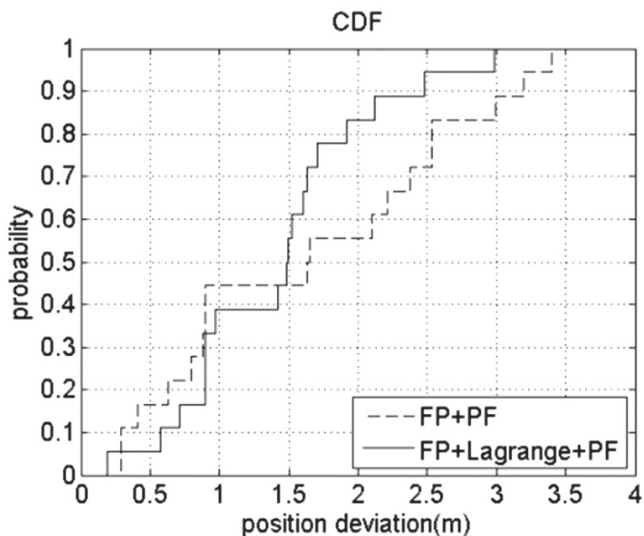


Fig. 10. CDF curve of positioning errors

## 6. Conclusions

In this paper, we considered the shortcoming that the traditional fingerprint database algorithm need much work at training offline stage, and also the characteristics of nonlinear between signal strength and distance in wireless sensor networks. We used non-linear Lagrange interpolation method to improve the fingerprint database algorithms. At the same time, we took into account the effect of moving objects on RF signal and the influence of number nodes on function approximation. We proposed an interpolation scheme based on certain angle area. On the other hand, particle filter is used to deal with harmful RSSI data at positioning stage of the algorithm. Optimized generation algorithm can quickly build a fine-grained fingerprint database, and also improve the positioning accuracy.

Improved fusion algorithm is simple enough and suitable to be applied in other fields, such as military applications, hospitals and warehouses.

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## Wykorzystanie nieliniowej interpolacji oraz filtru cząsteczkowego w pozycjonowaniu wewnątrz pomieszczeń przy użyciu technologii Zigbee

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## Streszczenie

Kluczem do algorytmu pozycjonowania wykorzystującego metodę fingerprinting jest ustanowienie skutecznej bazy danych na podstawie informacji z radiowych nadajników referencyjnych przy wykorzystaniu wskaźnika mocy odbieranego sygnału (RSSI). Tradycyjna metoda oparta jest na przeprowadzeniu kalibracji obszaru lokalizacji na podstawie wielu punktów pomiarowych i otrzymaniu dużej liczby próbek, co jest bardzo czasochłonne.

Z wykorzystaniem sieci czujników Zigbee jako platformy, biorąc równocześnie pod uwagę wpływ zakłóceń sygnału radiowego, autorzy zaproponowali ulepszony algorytm budujący wirtualną bazę danych na podstawie interpolacji wielomianowej, a wstępnie przybliżony wynik został poddany filtracji cząsteczkowej. Wynik eksperymentu pokazuje, że wykorzystana metoda doświadczalna może szybko generować prostą bazę danych informacji lokalizacyjnych, równocześnie poprawiając dokładności pozycjonowania wewnątrz pomieszczeń.