Influence of the Density of Source Data on a Volume Estimation Using DEM

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Summary

Digital elevation model (DEM) represents an efficient tool for a number of engineering applications. However, decisive for the DEM application is its accuracy, which depends on various factors. The main factors include the surface roughness, the interpolation algorithm, and the accuracy, density and distribution of the source data. This study is devoted to investigating the effect of the source data density on the volume calculation using the grid based DEM. This investigation is provided on the basis of the theoretical surfaces, which are expressed by means of a known mathematical function of the plane coordinates, and also on the experimentally measured surfaces using terrestrial laser scanning. DEMs using data with density from several centimetres to 1 m and using three different interpolation methods were generated and volumes calculated.

Keywords: volume, digital elevation model, density of points, interpolation method, relative error

Introduction

Volume calculation belongs between the common requirements of the construction and mining industry. In order to determine a volume, different methods, whose selection depends on the type and dimensions of an object, character of the border areas, the possibility of the data collection and also on the instrumentation and software, may be used.

One of the possibilities of the volume determination between a topographical surface and a reference plane or a surface represents calculation based on a digital elevation model (DEM). DEM is a mathematical model of a terrain surface, which is modelled using mathematical functions (interpolation functions) on a set of points with known spatial coordinates representing the surface [1]. Decisive for its application is its accuracy, which is in addition to the terrain roughness, influenced by the interpolation function, interpolation methods and the attributes of the source data, namely accuracy, density and distribution [1, 2, 3, 4].

The essential base for the DEM creation is the finite number of points on a surface. For the projects involving small areas or objects, usually surveying methods (conventional method using a total station, methods based on the GNSS and terrestrial laser scanning) are used to collect data on a surface. These methods provide high level of accuracy of the source data but on the other hand, in order to obtain dense representation of a surface is time consuming [5].

The main objective of this paper is assessing the effect of the density of the source data on the DEM based volume calculation. To assess this impact, different approaches may be applied. An absolute evaluation can be made on the theoretical surfaces, whose surface is generated using a known function of the planar coordinates and their exact volume can be calculated using an integral calculus [6, 7, 8], or on the bodies, whose exact volume is known [9]. On the other hand, comparing the results obtained from the measurements with the different densities of the source data or comparison with the results from the different methods provides relative evaluation [10]. Analysis on the theoretical surfaces as well as on the surfaces measured using terrestrial laser scanning is given in this study. Moreover, three different interpolation methods (nearest neighbour, inverse distance squared weighted and Kriging) are examined.

Analysis on the theoretical surfaces

For this purpose, we defined a rectangular area with the dimensions 20 × 30 m and its origin is given the planar coordinates x=0 m and y=0 m (Fig. 1). Subsequently, within this area and on its borders, we created the sets of approximately regularly distributed points with the density from 20 mm up to 1 m (Fig. 1).

In order to create the theoretical surfaces over the defined area, we used two functions for all sets of points:

- theoretical surface A: $z(x, y) = \sqrt{\frac{x \cdot y}{100}}$

- theoretical surface B:

$$z(x,y) = 7 - \sqrt[3]{(x-10)^2 + (y-15)^2}$$

where x and y are the planar coordinates of the points. Based on these sets of the spatial coordinates of points, we finally created the grid DEMs using the nearest-neighbour, the inverse distance squared weighted and Kriging method in the Surfer software. The grid size was adjusted according to the density of the source data [11]. Graphical representation of the contoured maps and DEMs of both theoretical surfaces are shown in figures 2 and 3.

Calculation of the volumes from the generated DEMs was also carried out in the Surfer program using the cross-section method. As a reference plane we chose a horizontal plane with the height 0 m. The correct values of the volumes were determined using the integral calculus:

$$V = \int_{x_{min}}^{x_{max}} \int_{y_{min}}^{y_{max}} z(x,y)\, dy dx\,. \qquad (1)$$

The correct volume between the horizontal plane with height 0 m and the theoretical surface A is

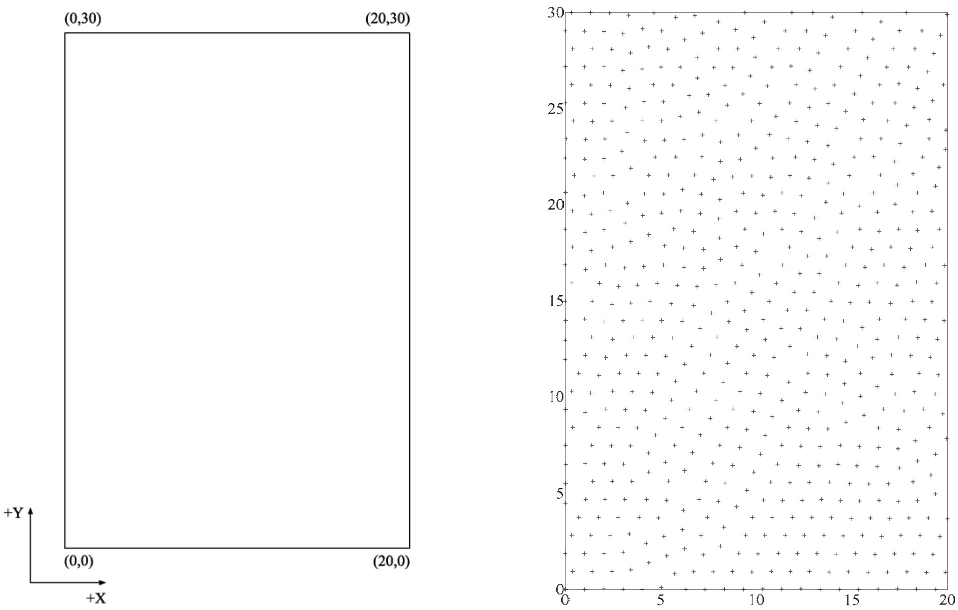

Fig. 1. Defined area (left) and a set of points with the density 1 m (right)

Rys. 1. Obszar określony (z lewej) i zbiór punktów o gęstości 1 m (po prawej)

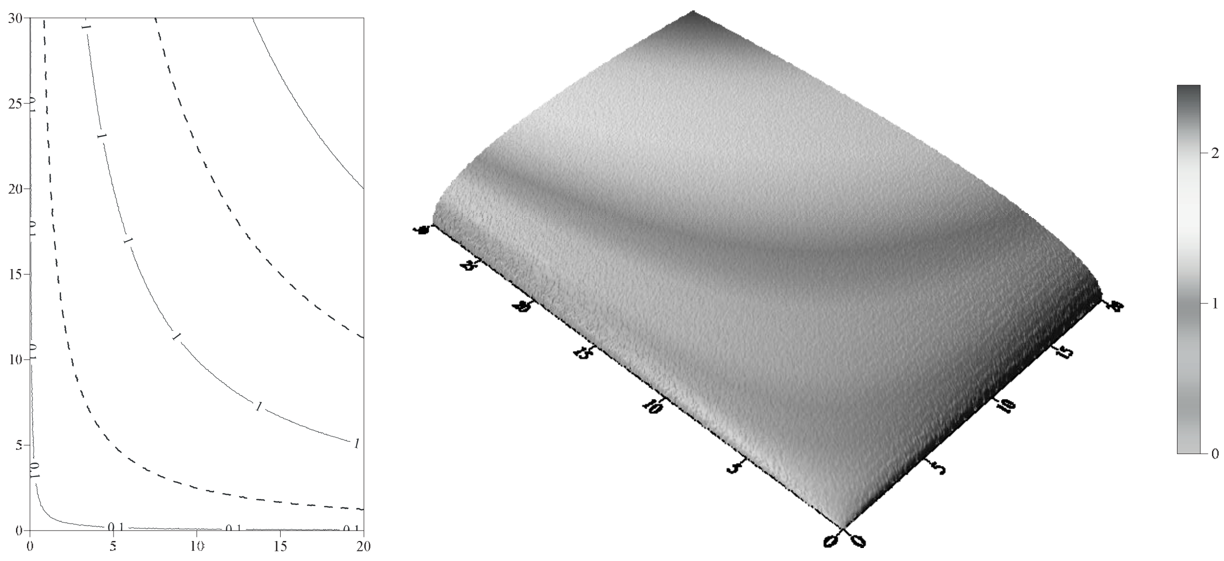

Fig. 2. The contoured map and the DEM visualization of the theoretical surface A

Rys. 2. Wyprofilowana mapa i wizualizacja DEM powierzchni teoretycznej A

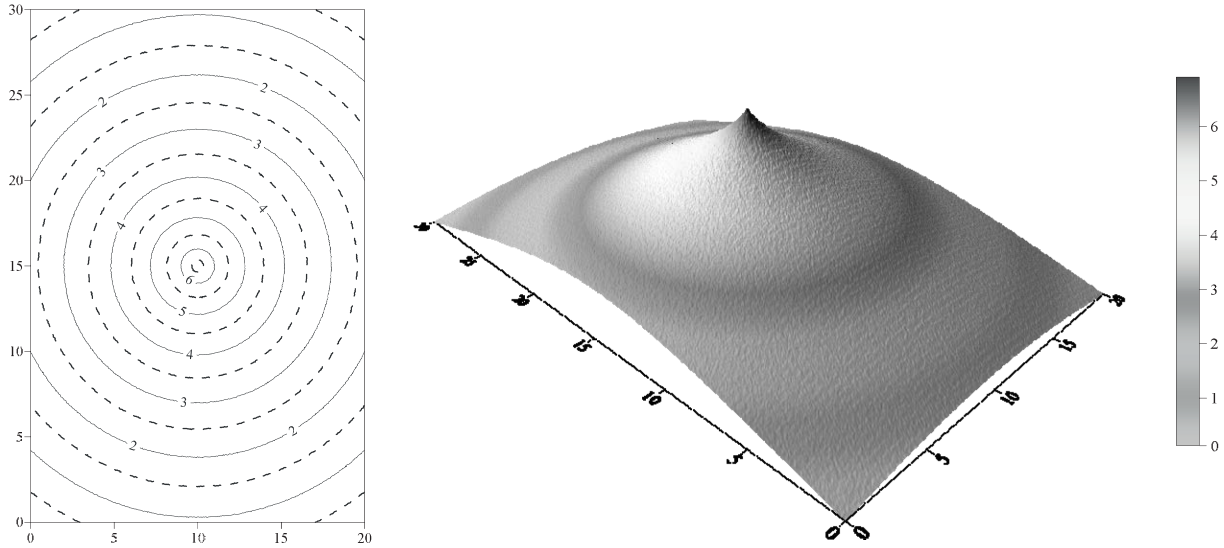

Fig. 3 The contoured map and the DEM visualization of the theoretical surface B

Rys. 3. Wyprofilowana mapa i wizualizacja DEM powierzchni teoretycznej B

653.19 m^3 and the theoretical surface B 1530.20 m^3. To evaluate the effect of the density, relative errors were calculated by means of the following formula:

$$r = \frac{V - V_{DEM}}{V} \cdot 100, \qquad (2)$$

where r is the relative error in [%], V – the volume calculated using the integral calculus, V_{DEM} – the volume determined using the cross-sections from the DEM. The results obtained for both theoretical surfaces are shown in tables 1 and 2.

Table 1. Relative errors (%) for the theoretical surface A

Tabela 1. Błąd względny (%) dla powierzchni teoretycznej A

Density [mm]	Nearest Neighbour	Inverse Distance	Kriging
20	0.00	0.00	0.00
50	0.00	0.01	0.00
100	0.01	0.02	0.01
200	0.04	0.07	0.04
300	0.05	0.12	0.07
400	0.10	0.20	0.10
500	**0.21**	0.26	0.14
750	0.19	0.49	0.29
1000	0.11	**0.76**	**0.41**

Table 2. Relative errors (%) for the theoretical surface B

Tabela 2. Błąd względny (%) dla powierzchni teoretycznej B

Density [mm]	Nearest Neighbour	Inverse Distance	Kriging
20	0.00	0.00	0.00
50	0.00	0.00	0.00
100	0.00	0.00	0.00
200	-0.01	0.01	0.01
300	0.01	0.02	0.01
400	-0.01	0.05	0.02
500	0.11	0.07	0.03
750	0.12	0.11	0.07
1000	0.25	0.35	0.13

Analysis on the experimentally measured surfaces

For the practical measurement and subsequent analysis were chosen two piles of earth in the shape of an irregular truncated cone. The height of both piles of earth was approximately 3.5 m and the diameter of a smaller pile was approximately 15 m and 30 m for a bigger pile.

The Trimble GX scanner with the Trimble accessories was used for scanning (Fig. 4). This system allows scanning to 350 m with speed up to 5000 points per second [12]. We defined the resolution of scanning 20 mm at 50 m and the orientation of the scanner was provided by measuring the Trimble planar targets (Fig. 4). Scanning of the bigger pile of earth (PE-B) was performed from the 9 scanner stations (5001–5009) and to more detailed coverage of the upper part of the pile, 4 stations (5006–5009) were placed directly on its upper surface. The smaller pile of earth (PE-S) was scanned together from the 7 stations (5001–5003, 5007, 5010–5012), from whose 2 stations were also on its upper surface (Fig. 5).

Data from TLS were loaded into the Trimble Realworks software for processing. Because the scanner captures everything within its field of view, the point clouds were filtered from the unwanted ambient vegetation and objects. Thus prepared point clouds were resampled into the datasets of the spatial coordinates of points with densities from 20 mm to 1 m. The other steps of the DEM construction were the same as in the case of the theoretical surfaces (Fig. 6 and 7). The volumes were computed in the Surfer software using the cross-section method with respect to a horizontal plane with the height 136,60 m for the PE-S and 136,30 m for the PE-B. Because the exact volume of both piles was unknown, the relative errors were calculated with respect to the 20 mm DEM volume. The results obtained for both piles are shown in tables 3 and 4.

Fig. 4. The Trimble GX scanner (left) and the Trimble planar target (right)

Rys. 4. Skaner Trimble GX (z lewej) i Trimble planar target (z prawej)

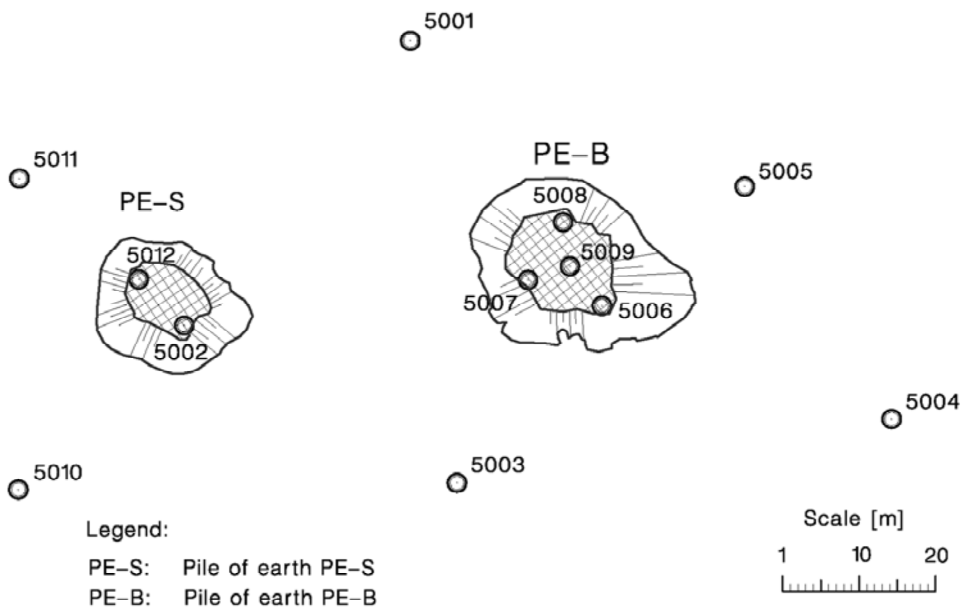

Fig. 5. Layout of the scanner stations

Rys. 5. Otoczenie stacji skanowania

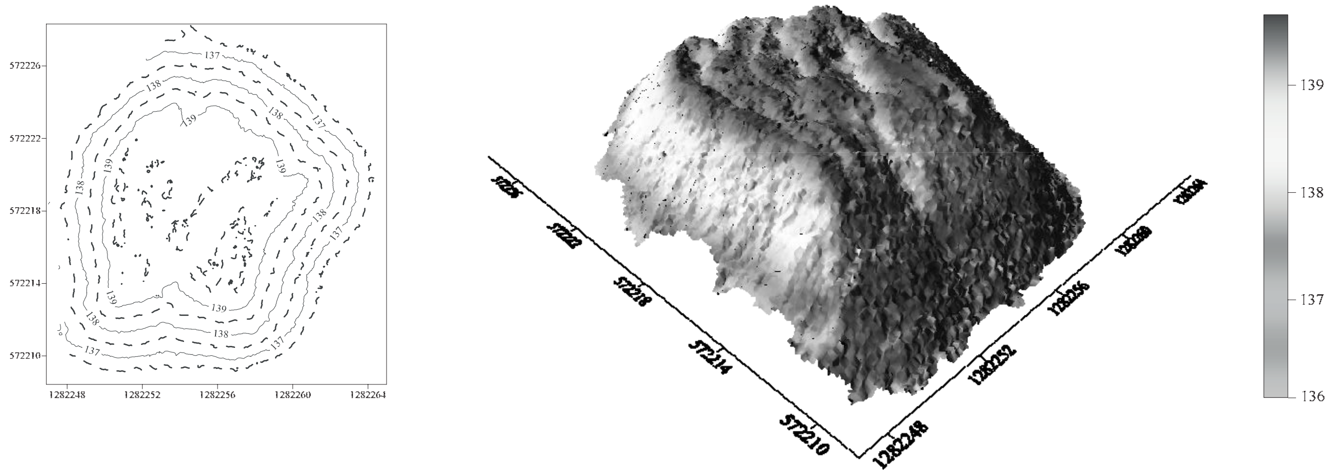

Fig. 6. The contoured map and the DEM visualization of PE-S

Rys. 6. Mapa konturowa i wizualizacja DEM PE-S

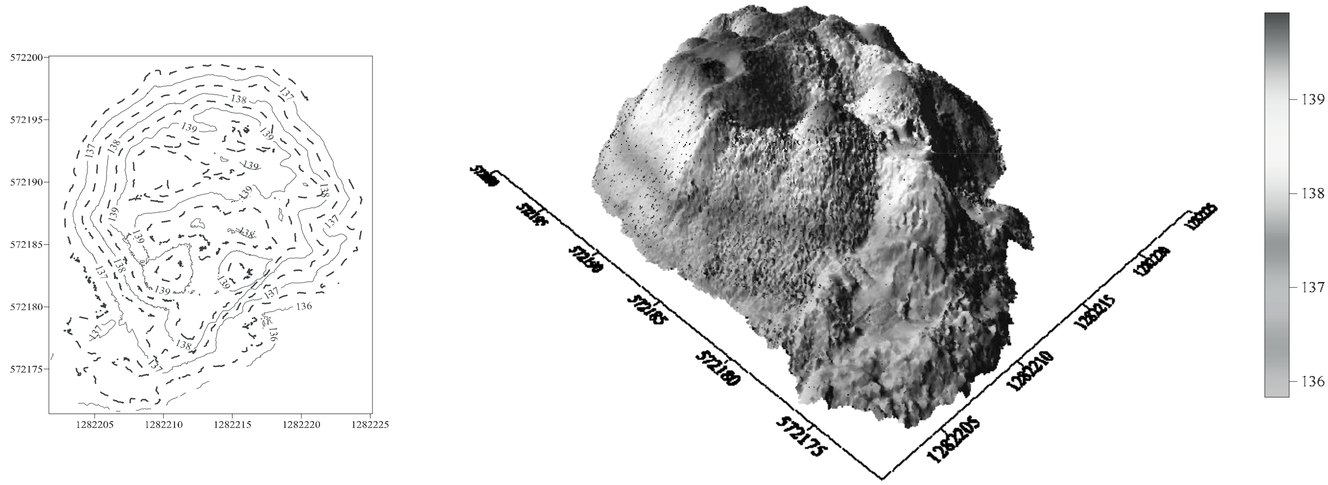

Fig. 7. The contoured map and the DEM visualization of PE-B

Rys. 7. Mapa konturowa i wizualizacja DEM PE-B

Table 3. Relative errors (%) for the PE-S

Tabela 3. Błąd względny (%) dla PE-S

Density [mm]	Nearest Neighbour	Inverse Distance	Kriging
20	–	–	–
50	0.04	0.05	0.04
100	0.09	0.11	0.07
200	0.18	0.26	0.14
300	0.18	0.41	0.21
400	0.24	0.48	0.17
500	0.41	0.70	0.34
750	**0.62**	1.32	**0.47**
1000	0.58	**1.65**	0.10

Table 4. Relative errors (%) for the PE-B

Tabela 4. Błąd względny (%) dla PE-B

Density [mm]	Nearest Neighbour	Inverse Distance	Kriging
20	–	–	–
50	0.02	0.02	0.03
100	0.07	0.06	0.07
200	0.26	0.22	0.20
300	0.15	0.26	0.23
400	0.22	0.42	0.30
500	0.39	0.46	0.34
750	0.56	0.73	0.49
1000	**1.16**	**1.54**	**0.73**

Conclusion

The results obtained within this study have confirmed the influence of the density of source data on the volume determination using the DEM. From the results given in tables 1–4 can be seen that the accuracy of the volume computation decreases with the decreasing density of the source data. For both theoretical surfaces the maximum relative errors are acquired at 1 m grid DEM generated with the inverse distance squared weighted method, 0.76% for the surface A and 0.35% for the surface B. The nearest neighbour method has maximum 0.21% error at 500 mm grid on the surface A and 0.25% at 1 m grid on the surface B, the Kriging method acquires the maximum values at 1 m grid, 0.41% on the surface A and 0.13% on the surface B. The inverse distance method provides the maximum errors in the case of experimentally measured surfaces as well. However, the relative errors exceed 1%, 1.65% for the PE-S and 1.54 for the PE-B. Using the nearest neighbour method, 0.62% relative error at 750 mm grid for the PE-S and 1.16% at 1 m grid for the PE-B is obtained. In the case of the Kriging method, relative errors 0.47% at 750 mm grid for the PE-S and 0.73% at 1 m grid for the PE-B are gained.

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Wpływ gęstości danych źródłowych na określenie objętości przy użyciu DEM

Cyfrowy model terenu (digital elevation model – DEM) stanowi skuteczne narzędzie do wielu zastosowań inżynierskich. Decydującym czynnikiem przemawiającym za DEM jest jej dokładność, która zależy od wielu czynników. Głównymi czynnikami są chropowatość powierzchni, algorytm interpolacji oraz dokładność, gęstość i rozkład danych źródłowych. Niniejszy artykuł jest poświęcony zbadaniu wpływu gęstości danych źródłowych na obliczanie objętości przy użyciu siatki opartej na DEM. Badanie to jest przeprowadzone bazując na fundamentach teoretycznych, które wyrażone są przez funkcję matematyczną współrzędnych płaszczyzny, jak również na eksperymentalnie zmierzonych powierzchni przy użyciu naziemnego skaningu laserowego. DEM używająca danych o gęstości od kilku centymetrów do 1 m oraz stosująca trzy różne metody interpolacji została wygenerowana a objętość obliczona.

Słowa kluczowe: objętość, numeryczny model terenu, gęstość punktów, metoda interpolacji, błąd względny