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A study of transcritical carbon dioxide cycles with heat regeneration

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Abstract The paper presents an efficiency analysis of two transcritical CO_2 power cycles with regenerative heaters. For the proposed cycles, calculations of thermal efficiency are given for selected values of operating parameters. It was assumed that the highest working temperature and pressure are in the range from 600 to 700 °C and 40 to 50 MPa, respectively. The purpose of the calculations was optimization of the pressure and mass flows in the regenerative heaters to achieve maximum cycle efficiency. It follows that for the assumed upper CO_2 parameters, efficiency of 51–54% can be reached, which is comparable to the efficiency of a supercritical advanced power cycle considered by Dostal.

Keywords: CO_2 thermodynamic cycle; Near-critical region; Heat regeneration; Thermal efficiency

Nomenclature

a, g	–	mass flow fraction
c_p	–	specific heat at constant pressure, J/kgK
h	–	specific enthalpy, J/kg
l	–	unit work, J/kg
p	–	pressure, Pa

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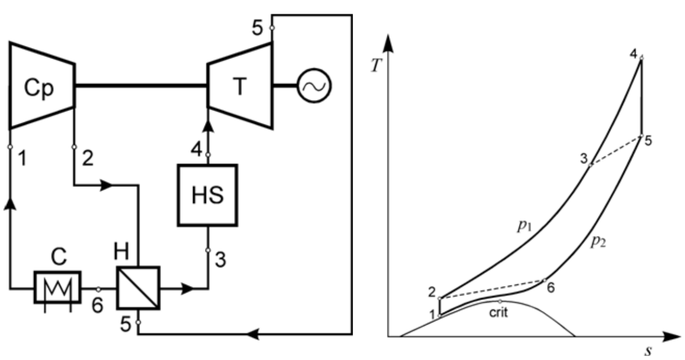
q	–	heat transferred per unit mass, J/kg
s	–	specific entropy, J/kgK
T	–	temperature, °C
T_m	–	temperature of maximum enthalpy difference (see Fig. 5a), °C
η	–	efficiency

Subscripts and superscripts

c	–	cycle
P	–	pump
sat	–	at saturation
T	–	turbine

1 Introduction

Over the last 40 years many power (and refrigeration) cycles have been proposed and analysed, in which carbon dioxide was used as a working fluid. The first was by Feher [1] who considered supercritical regenerative thermodynamic power cycle (called supercritical Brayton cycle, Fig. 1) which offered higher efficiency than the Rankine cycle. Important advantages of the CO_2 in this application are: moderate critical pressure, chemical stability, virtually no corrosive or toxic action, abundance and low cost of acquisition.



a) b)

Figure 1. Supercritical Brayton cycle: a) schematic chart, b) $T-s$ diagram with the cycle principal points marked: HS – heat source, T – turbine, H – recuperator, C – precooler, Cp – compressor.

The increase of the efficiency of the supercritical Brayton cycle was attributed to the reduction of compression work due to the high density of the carbon dioxide (CO_2) close to the critical point. However it was found that in such cycle a large irreversibility occurred due to the heat transfer from the turbine exhaust stream of low specific heat (points 5-6 in Fig. 1) to the high-specific-heat compressor stream (points 2-3 in Fig. 1). This irreversibility is connected with the so-called pinch-point problem. The pinch-point may occur due to the strong temperature and pressure dependence of the specific heat of both streams in the recuperator. At this point the temperature difference between both streams is lowest or even zero in the limit. It means that for the case of thermodynamic processes which occur in the vicinity of the critical point it is necessary to evaluate the local temperature difference between both streams in order to avoid pinch-point problem. It is known, e.g., Trela [2], that this problem is more likely to occur if the pressure difference between both streams in the recuperator is large. Szewalski [3] proposed a new supercritical water steam cycle using Feher idea. He showed that in that cycle the efficiency could be higher by about 6% compared to convectional cycles at the same temperature limits. Sedler [4] considered the pinch-point problem for a regenerator heat exchanger in the supercritical water steam cycle. This exchanger transferred heat from a subcritical pressure stream to the supercritical one. He showed that pinch-point problem had to be taken into account because the temperature differences between the streams at inlet and exit of supercritical regenerator were higher than those encountered in a common subcritical conditions. Due to this effect the cycle efficiency increase was not so high as could be expected.

The idea of Feher cycle was later further developed by Angelino [5] who proposed the so called compound cycles in which additional compressors appeared in the supercritical carbon dioxide cycle in order to diminish the pressure and temperature differences between the streams and in this way to reduce the irreversibility losses in the recuperator. Dostal [6] carried out systematic study on supercritical carbon dioxide cycle for next generation nuclear reactor. He showed that for the initial temperature 550 °C and pressure 20 MPa the cycle efficiency achieved 45.3% and the cost of the power plant would be reduced by about 18% compared to the conventional Rankine cycle. The efficiency and the cost of the power plant depend on the initial thermodynamic parameters. For temperature of 650 °C efficiency is about 50% and the plant cost reduces by about 24%. If the turbine inlet temperature was assumed at 700 °C then calculated thermal efficiency of the cycle raised to about 53% which also yields cost savings.

The increasing consumption of fossil fuels has led to an environmental problems such as global warming, ozone depletion and atmosphere pollution. A solution of these problems is seen mainly in application of low-grade waste heat and solar energy. When utilizing these energy sources the problem arises concerning the type of thermodynamic cycle and working fluids which should to be used. Very popular is application of the Rankine cycle with organic fluids (ORC) [7–9]. However Chen *et al.* [10] showed that carbon dioxide transcritical power cycle has a slightly higher output and efficiency than the ORC. The transcritical cycle means that a part of the cycle is located in supercritical region while the rest of it is subcritical. The better efficiency of carbon dioxide transcritical cycle follows from the physical properties of the working medium. The CO_2 has more potential in utilizing the energy from low-grade heat sources. This is due to the character of carbon dioxide's temperature profile (Fig. 2a) in the supercritical region which can provide a better match to the heat source temperature glide than other fluids operating in Rankine cycle (Fig. 2b). Thus, for considered case the so-called 'pinching', which commonly appears for Rankine cycle, will not be encountered in a heat exchanger working with carbon dioxide.

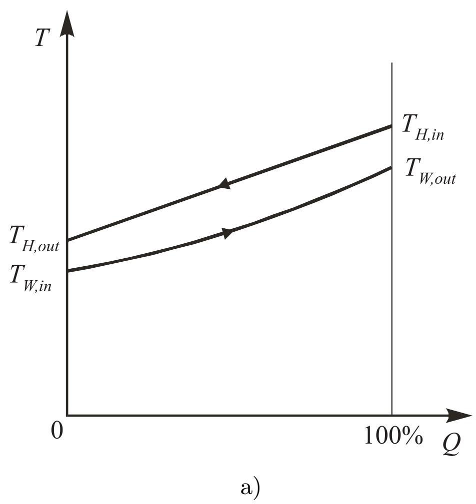


Figure 2. Temperature profiles in heat exchangers working with near-critical fluids: a) transcritical CO_2 cycle, b) ORC cycle. The profiles depict changes of the heat source temperature, T_H, and the working fluid temperature, T_W, as a function of relative transferred heat, Q.

The application of solar energy to power generation, to heat production and refrigeration have received considerable attention. Numerical studies, e.g., [11,12], demonstrate that transcritical carbon dioxide cycles are also suitable for utilization of the solar energy.

In this paper we consider in more detail the features of two transcritical cycles of carbon dioxide with condensation and heat regeneration, which can be applied for power generation from fossil fuels or nuclear reactors. In order to reduce the irreversible losses related to the pinch-point, the idea of Angelino [5] was used with additional compressors in supercritical part of the carbon dioxide cycle.

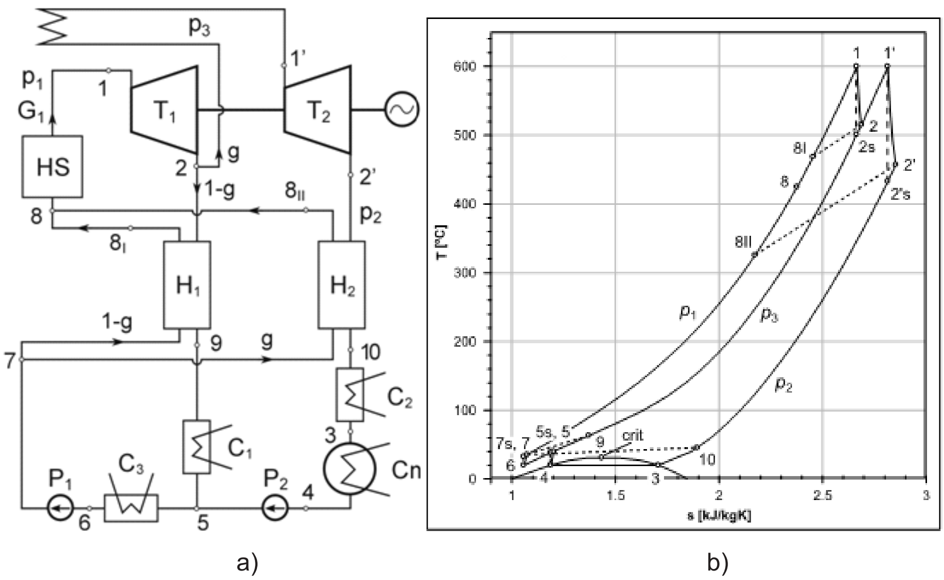
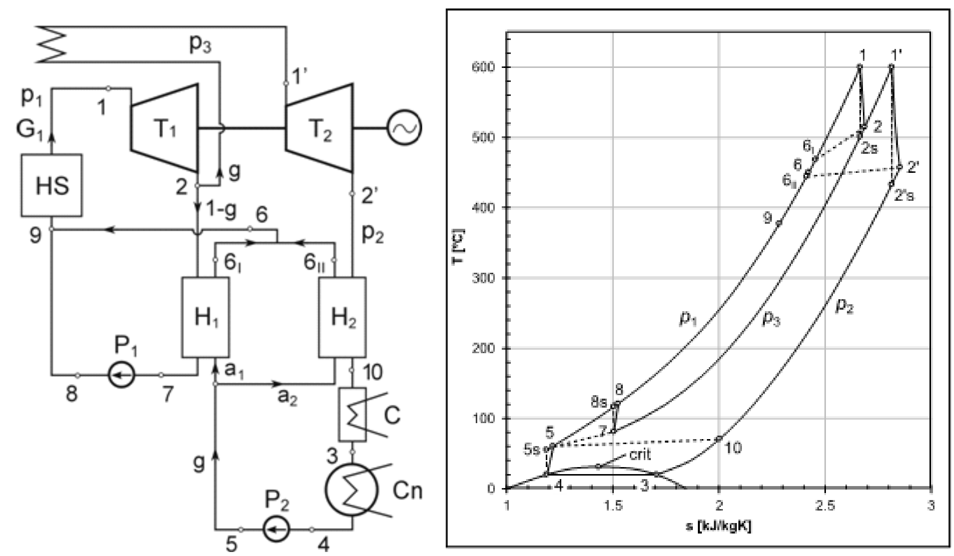


Figure 3. Transcritical CO_2 power cycle, version No. 1: a) schematic view, b) T-s diagram with the cycle principal state points marked. Designations: HS – heat source, T – turbine, H – regenerative heater, C – cooler, Cn – condenser, P – pump.

Both proposed cycles are based on two turbine units. Their simplified schematic views and T-s diagrams are shown in Figs. 3 and 4. The thermal energy, which may come from nuclear reactor or fossil fuel, is supplied to the cycles in the heat source units (HS). The working fluid leaving the high-pressure turbine T_1 is divided into two streams, from which one passes a superheater and then is directed to a low-pressure turbine T_2. The other CO_2 stream is used in regenerative heater H_1. The heat of stream leaving the turbine T_2 is also used to regeneration in a heater H_2, from where the CO_2 flows through a cooler to condenser.



a) b)

Figure 4. Transcritical CO_2 power cycle, version No 2: a) schematic view, b) T-s diagram with the cycle principal state points marked. Designations as in Fig. 1.

Condensed CO_2 returns to the HS unit but its path depends on the cycle version. In cycle No. 1 (Fig. 3), supercritical stream compressed by the pump P_2 is mixed at point 5 with bleed stream $(1-g)$ where, thanks to the cooler C_1, both flows have equal temperatures. The mixed stream is further cooled in C_3 and pumped to the regenerative heaters (regenerators) H_1 and H_2 by the pump P_1. In the point 7 ahead of the heaters, the stream is again divided in two, in the same proportions as it is done at the point 2 downstream of the turbine T_1 outlet. In this way both streams in H_1 have equal flow rates of $1-g$ and, consequently, in H_2 both flow rates equal to g.

In the cycle No. 2, shown in Fig. 4, the flow organisation downstream of the pump P_2 is different, so the intercoolers C_1 and C_3 could be removed. Thus, the CO_2 condensate downstream of P_2 is divided into flows a_1 and a_2, which enter the regenerative heaters H_1 and H_2 directly. Leaving the heaters, the two flows are mixed back at point 6 and then mixed with the bleed stream $(1-g)$ at point 9. The purpose of pumps P_1 and P_2 is to raise the CO_2 pressure to the value p_1 prevailing in the HS unit.

The aim of this paper is determination of the optimal thermal efficiency of the two proposed transcritical CO_2 cycles. This is achieved by a selection of a proper interstage pressure p_3 and proper division of the CO_2 flow between two regenerative heaters for the assumed turbine inlet parameters.

2 Thermal efficiency of the CO_2 power cycle No. 1

A schematic of the proposed transcritical CO_2 cycle in the version No. 1 is shown in Fig. 3. Its thermodynamic parameters in characteristic state points as well as thermal efficiency can be calculated for a given turbine inlet parameters (state 1). In addition, the lowest temperature in the cycle (states 3, 4, and 6), turbine and pump efficiencies, (η_T, η_P), and the mass flow fraction, g, directed to the superheater should also be specified.

Internal work of turbines T_1 and T_2 per unit mass of the working fluid equals to

$$l_{T1} = \eta_T \left(h_1 - h_{2s}\right) = h_1 - h_2 \ , \tag{1}$$

$$l_{T2} = \eta_T \left(h_{1'} - h_{2's}\right) = h_{1'} - h_{2'} \ , \tag{2}$$

respectively. Similarly, the work of pumps P_1 and P_2 per unit mass is

$$l_{P1} = \frac{1}{\eta_P} \left(h_{7s} - h_6\right) = h_7 - h_6 \ , \tag{3}$$

$$l_{P2} = \frac{1}{\eta_P} \left(h_{5s} - h_4\right) = h_5 - h_4 \ . \tag{4}$$

Therefore, the combined work of both turbines related to unit of mass (1 kg) of the working fluid at the turbine T_1 inlet will amount to

$$l_T = l_{T1} + g l_{T2} \ , \tag{5}$$

and the work of pumps sum up to

$$l_P = l_{P1} + g l_{P2} \ . \tag{6}$$

According to (5) and (6), net work output of the cycle per unit mass results from a definition

$$l_c = l_T - l_P = l_{T1} - l_{P1} + g \left(l_{T2} - l_{P2}\right) \ . \tag{7}$$

Heat input to the cycle related to unit of mass (1 kg) of the working fluid at the turbine T_1 inlet is a sum of enthalpy change in the HS unit and the superheater

$$q_c = h_1 - h_8 + g \left(h_{1'} - h_2\right) \ , \tag{8}$$

where enthalpy h_8 results from the energy balance for a mix of two streams leaving regenerators H_1 and H_2, which can be written in the form

$$(1-g)\,(h_{8I}-h_8) = g\,(h_8 - h_{8II}) \ , \tag{9}$$

hence

$$h_8 = (1-g)\,h_{8I} + g h_{8II} \ . \tag{10}$$

Enthalpies h_{8I} and h_{8II} follow from the heat balances for two regenerators:

$$(1-g)\,(h_{8I}-h_7) = (1-g)\,(h_2 - h_9) \ , \tag{11}$$

$$g\,(h_{8II}-h_7) = g\,(h_{2'} - h_{10}) \ , \tag{12}$$

namely

$$h_{8I} = h_2 - h_9 + h_7 \ , \tag{13}$$

$$h_{8II} = h_{2'} - h_{10} + h_7 \ . \tag{14}$$

In relations (11)–(14), enthalpies h_9 and h_{10} appear. Determining their values, one should take into account the anomalies (pinch point) of heat transfer in near-critical region [2,13] originating mainly from large variations of the constant-pressure specific heat, c_p, of the working fluid in the regenerative heaters. As a result of c_p variations, the minimum temperature difference, ΔT_{min}, at pinch point may appear somewhere inside the regenerator. This is unusual because typically, in subcritical heat exchangers, the ΔT_{min} appears at the exchanger inlet or outlet.

To determine where the minimum temperature difference appears for the fluids transferring heat in the exchanger (regenerator) at a given (constant) pressures, one should examine enthalpy difference between two isobars versus fluid temperature, Fig. 5. Temperature, T_m, at which enthalpy difference reaches maximum, sets a reference state for the ΔT_{min} sought after. If

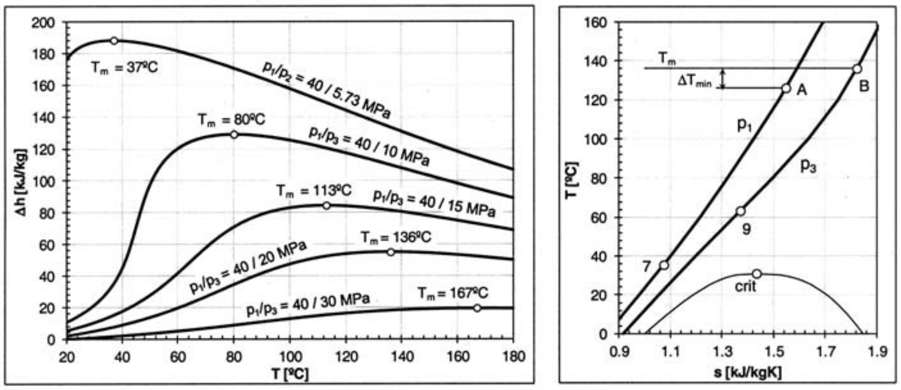
$$T_m > T_7 + \Delta T_{min} \tag{15}$$

then the minimal temperature difference appears somewhere inside the regenerative heater. In the cycle No. 1, this is the case for the heater H_1 and the following balance should be used to calculate the enthalpy h_9:

$$(1-g)\,(h_B - h_9) = (1-g)\,(h_A - h_7) \ , \tag{16}$$

where, according to designations introduced in Fig. 5b, $h_A = h(p_1,\, T_m - \Delta T_{min})$, $h_B = h(p_3,\, T_m)$. Therefore

$$h_9 = h_B - h_A + h_7 \ . \tag{17}$$



a) b)

Figure 5. Determination of a location in the heat exchanger where the temperature difference between the fluids drops to a minimum: a) enthalpy difference Δh as a function of temperature T, calculated for two streams of CO_2 transferring heat at pressures p_1 and p_3 or p_2; maximum value of Δh for each pair of p_1/p_3 or p_1/p_2 appears at temperature T_m; b) determination of state points A and B, for which the temperature difference equals to predetermined minimum value of ΔT_{min} (example for the heater H_1 of the cycle No. 1, shown in Fig. 3).

The condition (15) is usually not fulfilled for the heater H_2 so the minimal temperature difference ΔT_{min} appears at the heater end, between the points 7 and 10. Hence, the temperature T_{10} sought for the balance (14) can be set as

$$T_{10} = T_7 + \Delta T_{min} \ . \tag{18}$$

The above relations suffice to calculate the cycle net work output, l_c, (7) and input heat, q_c (8), and to determine the cycle efficiency defined as

$$\eta \equiv \frac{l_c}{q_c} = \frac{l_{T1} - l_{P1} + g\,(l_{T2} - l_{P2})}{h_1 - h_8 + g\,(h_{1'} - h_2)} \ . \tag{19}$$

It can also be concluded from the above balances and definitions that the cycle efficiency, η, depends primarily on turbine inlet parameters and the flow division between the superheater and regenerator.

2.1 Limiting cases

Before we proceed to the analysis of efficiency for the CO_2 cycle No. 1 working under different pressures p_3 and flow fractions g, it is worthwhile to think of limiting cases encountered when $p_3 = p_1$, $p_3 = p_2$, $g = 0$ or $g = 1$. Let's examine them in detail:

$p_3 = p_1$, $0 < g < 1$:

This condition means that turbine T_1 is removed from the cycle and the working fluid leaving the HS unit is immediately divided between the superheater and the regenerator H_1. States 1 and 2 coincide, as well as 4 and 6 or 5 and 7. The cooler C_3 and the pump P_1 are redundant. In efficiency calculations relations (1)–(19) remain valid, although $l_{T1} = l_{P1} = 0$.

$p_3 = p_1$, $g = 0$

This case has no practical meaning because in this situation the cycle would be lacking turbines and the working CO_2 would be directed through the regenerator to the cooler. The numerator in (19) becomes zero so efficiency $\eta = 0$.

$p_3 = p_1$, $g = 1$

In this case, there is no turbine T_1, regenerator H_1 and the cooler C_1 in the cycle. Whole of the CO_2 is superheated and fed to the turbine T_2. Then, the pump P_2 compresses it to the state 7 right away so the cooler C_3 and the pump P_1 are not needed. The cycle efficiency can be determined with the use of relations (1)–(19) where it will turn up that $l_{T1} = l_{P1} = 0$, $h_5 = h_7$, $h_{5s} = h_{7s}$.

$p_3 = p_2$, $0 \leq g \leq 1$

With the pressure p_3 so low, there is no turbine T_2 ($l_{T2} = 0$) in the cycle and the flow fraction g of the CO_2 stream at the turbine T_1 outlet is directed straight to the regenerator H_2 through the superheater. The efficiency, η, can be calculated according to (1)–(19) and its value is independent of g.

$p_1 < p_3 < p_2$, $g = 1$

The entire outflow from the turbine T_1 is fed to the superheater so the cycle is lacking the regenerator H_1 and cooler C_1. Therefore $h_8 = h_{8II}$ and the balances (9)–(13) could be omitted in the efficiency calculations.

$p_1 < p_3 < p_2$, $g = 0$:

In this case the useful work is generated by the turbine T_1 only. The cycle has not the superheater, turbine T_2, regenerator H_2, cooler C_2, condenser and pump P_2. When $p_3 > p_{cr}$ there is also no condensation of the CO_2. Otherwise, the cooler C_3 should be replaced by a condenser. Efficiency, η, can be calculated according to the relations given above with $h_8 = h_{8I}$.

3 Thermal efficiency of the CO_2 power cycle No. 2

Principal thermodynamic parameters and efficiency of cycle No. 2 (shown in Fig. 4) could be determined in a similar way as described above for the version No. 1, provided that the inlet turbine parameters (state 1) are known. The lowest cycle temperatures (states 3 and 4), turbine and pump efficiencies, (η_T, η_P), and the mass flow fraction g should also be given. Additionally, the flow distribution between regenerators should be predetermined in a way that condition

$$g = a_1 + a_2 \tag{20}$$

would be fulfilled. The symbols a_1 and a_2 denote the flow fractions directed to the regenerators H_1 and H_2, respectively.

Calculations of the cycle No. 2 efficiency may start from evaluation of internal work for the turbines T_1 and T_2, which per unit of mass (1 kg) of working fluid amounts to

$$l_{T1} = \eta_T \left(h_1 - h_{2s}\right) = h_1 - h_2 \ , \tag{21}$$

$$l_{T2} = \eta_T \left(h_{1'} - h_{2's}\right) = h_{1'} - h_{2'} \ . \tag{22}$$

The work of pumps P_1 and P_2 per unit mass equals to

$$l_{P1} = \frac{1}{\eta_P} \left(h_{8s} - h_7\right) = h_8 - h_7 \ , \tag{23}$$

$$l_{P2} = \frac{1}{\eta_P} \left(h_{5s} - h_4\right) = h_5 - h_4 \ . \tag{24}$$

Combined work of both turbines related to unit of mass (1 kg) of the working fluid at the T_1 inlet is the same as in the cycle No. 1, that is

$$l_T = l_{T1} + g l_{T2} \ , \tag{25}$$

but the combined work of pumps in the No. 2 cycle will be

$$l_P = (1 - g)\, l_{P1} + g l_{P2} \ . \tag{26}$$

So, the net work output of this cycle per unit mass is calculated as

$$l_c = l_T - l_P = l_{T1} - l_{P1} + g \left(l_{T2} + l_{P1} - l_{P2}\right) \ . \tag{27}$$

The heat input to the cycle for unit of mas (1 kg) of the CO_2 at the T_1 inlet is a sum of enthalpy change in the HS unit and superheater

$$q_c = h_1 - h_9 + g\,(h_{1'} - h_2)\ , \tag{28}$$

where enthalpy h_9 results from the heat balances for three mixing streams flowing out the regenerators H_1 and H_2. The heat balances for these regenerators take the form

$$a_1\,(h_{6I} - h_5) = (1-g)\,(h_2 - h_7)\ , \tag{29}$$

$$a_2\,(h_{6II} - h_5) = g\,(h_{2'} - h_{10})\ . \tag{30}$$

Hence it follows that the enthalpy of the heated stream at the H_1 outlet equals to

$$h_{6I} = \frac{1-g}{a_1}\,(h_2 - h_7) + h_5\ , \tag{31}$$

and corresponding enthalpy at H_2 outlet is

$$h_{6II} = \frac{g}{g-a_1}\,(h_{2'} - h_{10}) + h_5\ . \tag{32}$$

Now, taking into consideration the mixing balance at the point 6,

$$a_1\,(h_{6I} - h_6) = (g-a_1)\,(h_6 - h_{6II})\ , \tag{33}$$

and at the point 9,

$$g\,(h_6 - h_9) = (1-g)\,(h_9 - h_8)\ , \tag{34}$$

the value of enthalpy h_9 can be determined as

$$h_9 = a_1 h_{6I} + (g-a_1)\,h_{6II} + (1-g)\,h_8\ . \tag{35}$$

Considering the operating parameters of the cycle No. 2 it should be noted that the division of the flow g between the regenerators H_1 and H_2 is constrained, as the temperature difference between the heat-transferring streams should not fall below the presumed value of ΔT_{min}. Therefore, in particular the following conditions should be fulfilled:

$$T_2 - T_{6I} > \Delta T_{min}\ , \tag{36}$$

$$T_{2'} - T_{6II} > \Delta T_{min}\ , \tag{37}$$

$$T_7 - T_5 > \Delta T_{min} \ . \tag{38}$$

The place, at which the minimal temperature difference ΔT_{min} appears in the regenerator depends on variations of the specific heat, c_p, value along the respective isobars and it does not always occur at the end of the exchanger. To check this, a method described above for the cycle No. 1 could be applied. Under typical operating conditions, the reference temperature T_m (see Fig. 5b) appears inside of the regenerator H_1, so the condition (15) will also apply here but in the form

$$T_m > T_5 + \Delta T_{min} \ , \tag{39}$$

in view of different enumeration of the cycle state points. Therefore, enthalpy h_7 follows from the balance

$$a_1 \left(h_A - h_5\right) = \left(1 - g\right) \left(h_B - h_7\right) \ , \tag{40}$$

where, according to designations introduced in Fig. 5b, $h_A = h(p_1, T_m - \Delta T_{min})$, $h_B = h(p_3, T_m)$, namely

$$h_7 = h_B - \frac{a_1}{1-g} \left(h_A - h_5\right) \ . \tag{41}$$

Under typical working conditions, the minimal temperature difference ΔT_{min} in the regenerative heater H_2 appears at its end (states 5 and 7), so the temperature T_{10} required in (32), may be set as

$$T_{10} = T_5 + \Delta T_{min} \ . \tag{42}$$

With these terms in mind, the efficiency of the cycle No. 2 can be evaluated as a ratio of the net work output (27) to the heat input (28).

4 Cycle efficiency calculations

The efficiency of the two CO_2 cycles considered here was evaluated for several selected values of the pressure p_1 and temperature T_1. The calculations were performed for various values of interstage pressure p_3 and different fractions g dividing the flow between the superheater and regenerator H_1. For the cycle No. 2, the influence of the flow fraction a_1, defining the flow partition between the regenerators H_1 and H_2, was also studied. The temperature in the condenser was assumed to be equal to $T_3 = T_4 = 20$ °C, so

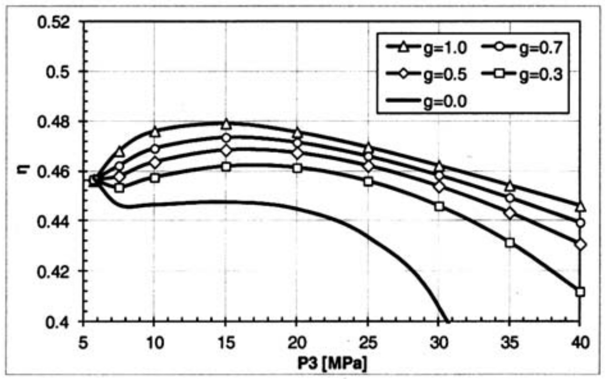


Figure 6. The efficiency, η, of the transcritical CO_2 cycle in version No. 1, working with the turbine input pressure $p_1 = 40$ MPa and temperature $T_1 = 600$ °C, calculated as a function of interstage pressure p_3 for selected values of the flow fraction g.

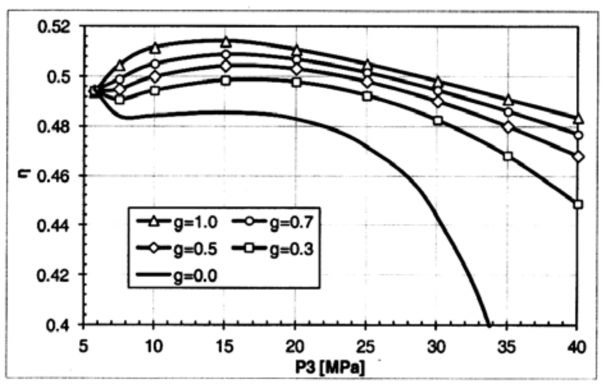


Figure 7. The efficiency, η, of the transcritical CO_2 cycle in version No. 1, working with the turbine input pressure $p_1 = 40$ MPa and temperature $T_1 = 700$ °C, calculated as a function of interstage pressure p_3 for selected values of the flow fraction g.

$p_2 = p_{sat}(T_3) = 5.729$ MPa. Furthermore, it was assumed that the turbine efficiency is $\eta_T = 0.9$ and the pump efficiency $\eta_P = 0.85$.

The efficiency of transcritical cycle No. 1 (Fig. 3) was calculated according to relations (1)–(19). In the first series of calculations it was assumed that $p_1 = 40$ MPa, $T_1 = 600$ °C. The results of these calculations for different pressures p_3 from the interval $[p_1, p_2]$ and for selected values of the flow fraction g are shown in Fig. 6. The best cycle efficiency is achieved when $g = 1$, that is when the entire outlet flow from the turbine T_1 is directed to the superheater which renders the regenerator H_1 and the condenser C_1 redundant. In such conditions the maximum value of efficiency, η, is 0.479 for $p_3 \approx 14$ MPa. The increase of the temperature T_1 to 700 °C results in general efficiency rise by about 0.04, Fig. 7. Again, the maximum value of η is obtained for $p_3 \approx 14$ MPa; now $\eta_{max} = 0.514$. Increasing the pressure p_1 to 50 MPa, while maintaining $T_1 = 600$ °C, also gives a slight increase of the efficiency compared to the primary variant of 40 MPa/600 °C. In this case $\eta_{max} = 0.488$ for the pressure $p_3 \approx 16$ MPa, Fig. 8.

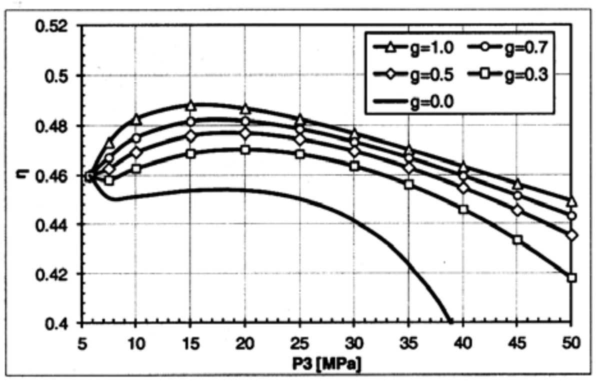


Figure 8. The efficiency, η, of the transcritical CO_2 cycle in version No. 1, working with the turbine input pressure $p_1 = 50$ MPa and temperature $T_1 = 600$ °C, calculated as a function of interstage pressure p_3 for selected values of the flow fraction g.

Similar calculations of the efficiency were also done for the second version of the transcritical cycle. The Eqs. (20)–(42) were used for this purpose with the principal working parameters the same as in the version No. 1 discussed above. However, for the No. 2 cycle, the changes of an additional parameter influencing the efficiency should also be taken into consideration, that is of the flow rate fraction a_1. As already mentioned, the conditions (36)–(38) impose significant limitations on the ranges of a_1 and g from which valid values of efficiency, η, will result. For a chosen value of the fraction a_1 there is a narrow range of g, for which the conditions (36)–(38) are satisfied under some extent of the pressure p_3 values. Figure 9 shows an example of the limits imposed on the g and p_3 by the conditions (36)–(38) for several selected values of a_1, when $p_1 = 40$ MPa, $T_1 = 600$ °C, $T_3 = T_4 = 20$ °C,

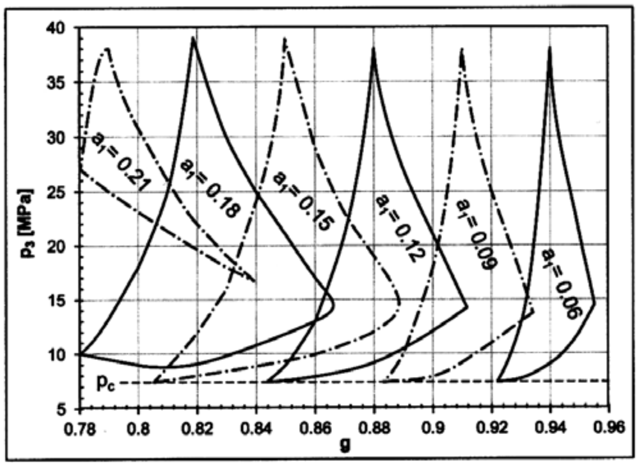


Figure 9. Boundaries of the regions within which, for the indicated flow rate fraction a_1, the values of the flow fraction g and pressure p_3 satisfy the restrictions imposed by the conditions (35)–(37) on the minimum temperature difference in the regenerators of the cycle No 2. The limits shown are valid for $p_1 = 40$ MPa, $T_1 = 600$ °C, $T_3 = T_4 = 20$ °C, $\eta_T = 0.9$, $\eta_P = 0.85$.

$\eta_T = 0.9$, $\eta_P = 0.85$. As it is seen, the widest range of acceptable pressures p_3 is achieved when $a_1 \approx (1-g)$. This is the case when the flow rates of the hot and cold streams in the regenerative heater H_1 are the same or similar.

Efficiency analysis of the No. 2 cycle started from a series of calculations for the live CO_2 parameters set at $p_1 = 40$ MPa, $T_1 = 600$ °C with condenser temperature $T_3 = T_4 = 20$ °C and turbine and pump efficiencies $\eta_T = 0.9$, $\eta_P = 0.85$, respectively. Three values of the flow fraction a_1 were selected: 0.1, 0.15 and 0.2, for which diagrams of efficiency η were prepared as a function of the pressure p_3 with flow fraction g as an additional parameter. The results are shown in Fig. 10. In the analyzed cases, the maximum of the efficiency occurs at $p_3 \approx 13$ MPa, falling within the limits of $0.491 \div 0.503$ depending on the value of g. For the working parameters assumed here, the absolute maximum efficiency of 0.508 is reached when $a_1 = 0.19$ and $g = 0.79$.

Similar efficiency calculations were done for the temperature T_1 raised to 700 °C. With the change of T_1, the value of a_1 was shifted from 0.2 to 0.17, because for the first value the region of solutions satisfying the conditions (36)–(38) is very small. As expected, the increase of the temperature

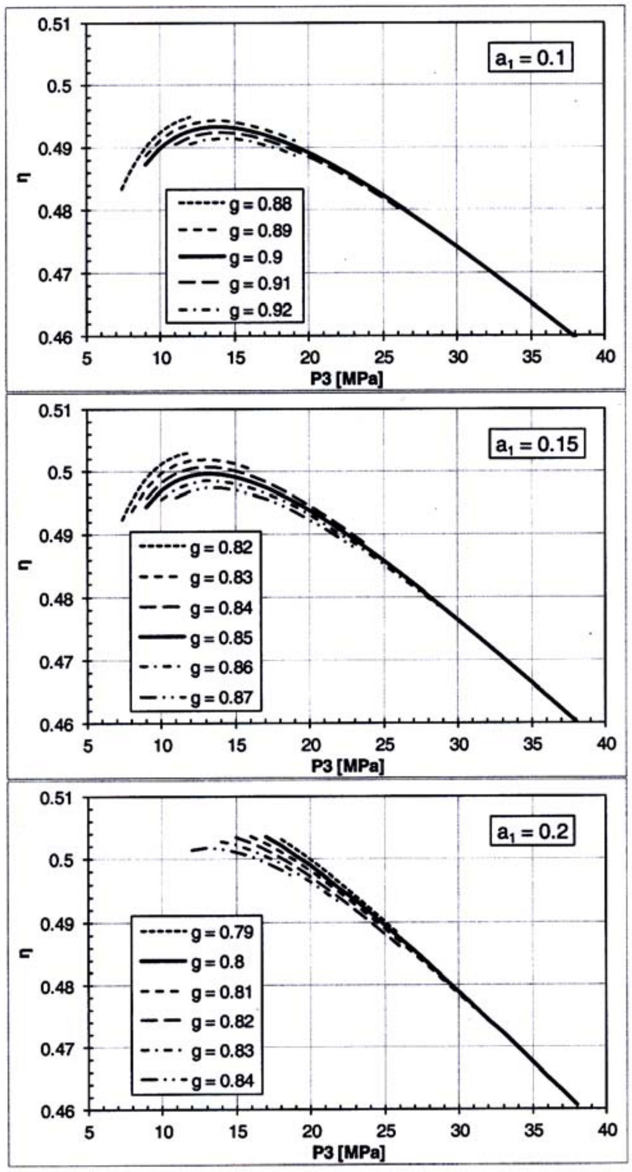


Figure 10. The efficiency, η, of the transcritical CO_2 cycle version No. 2 working with the turbine input pressure $p_1 = 40$ MPa and temperature $T_1 = 600$ °C, calculated as a function of interstage pressure p_3 for selected values of the flow fractions a_1 and g.

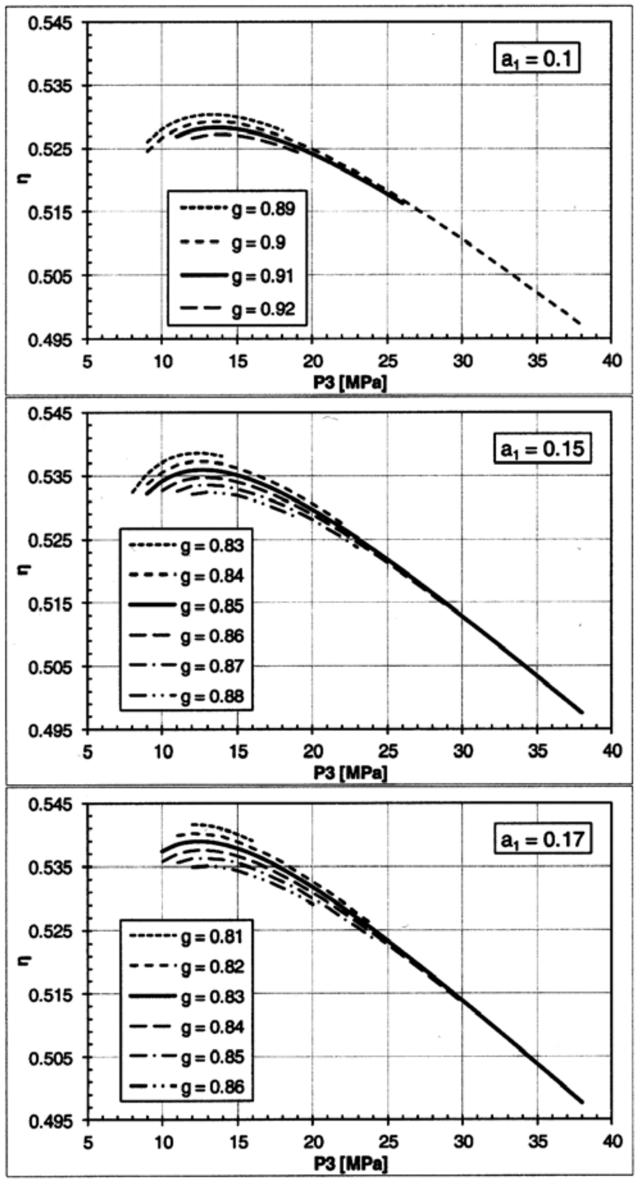


Figure 11. The efficiency, η, of the transcritical CO_2 cycle version No. 2 working with the turbine input pressure $p_1 = 40$ MPa and temperature $T_1 = 700$ °C, calculated as a function of interstage pressure p_3 for selected values of the flow fractions a_1 and g.

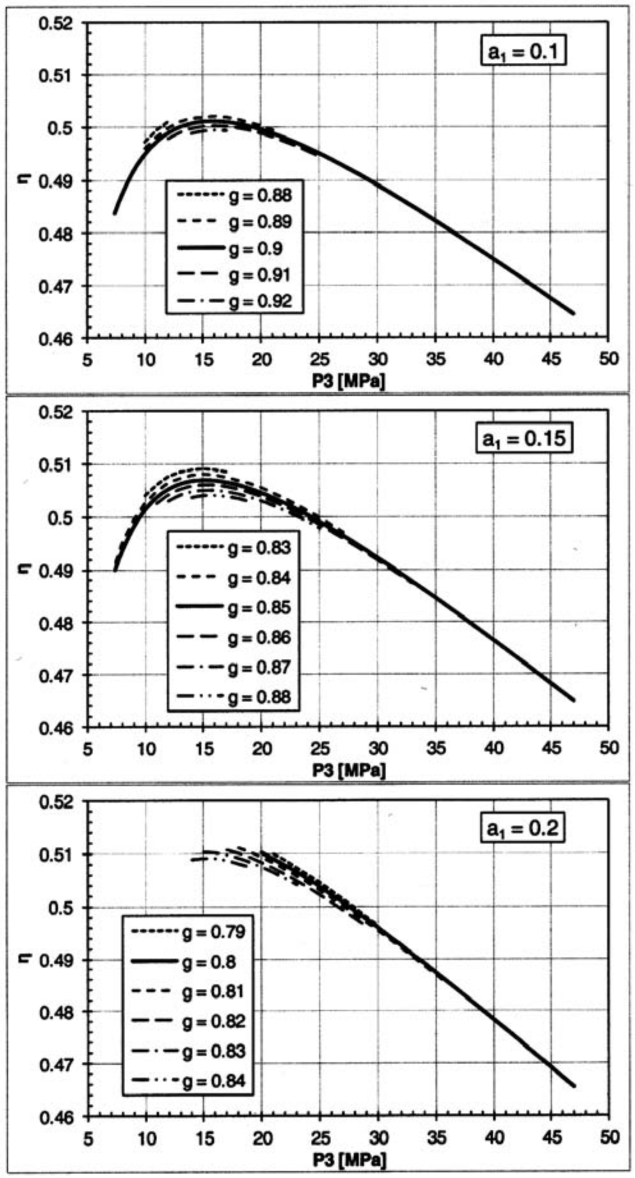


Figure 12. The efficiency, η, of the transcritical CO_2 cycle version No. 2 working with the turbine input pressure $p_1 = 50$ MPa and temperature $T_1 = 600$ °C, calculated as a function of interstage pressure p_3 for selected values of flow fractions a_1 and g.

resulted in an overall increase of the cycle efficiency, Fig. 11. The largest value of the efficiency, η, equal to 0.542, was noted for $a_1 = 0.17$, $g = 0.81$ and $p_3 = 12$ MPa.

The next series of calculations was performed for the pressure p_1 increased to 50 MPa, with other working conditions the same as in the first calculations. The results are shown in Fig. 12. In comparison with the basic set of parameters (i.e., $p_1 = 40$ MPa, $T_1 = 600$ °C), a slight increase of efficiency is observed, but not as large as in the former case, when the temperature T_1 was raised to 700 °C. The largest values of η appear for $p_3 \approx 15$ MPa, with the absolute maximum of 0.514 for $a_1 = 0.19$ and $g = 0.79$.

5 Conclusions

In this work two transcritical carbon dioxide cycles have been considered in respect to the cycles' efficiency. This was done with the strong variation of physical properties of carbon dioxide in the near-critical region taken into account, particularly concerning the constant-pressure specific heat. Also, a procedure was developed to find the pinch-point in the supercritical regenerators.

Comparing the obtained results for the two proposed variants of transcritical CO_2 power cycles, it is seen that the cycle No. 2 promises higher thermal efficiency, exceeding 50% when the highest working temperature $T_1 = 600$ °C. When this temperature is raised to 700 °C, the maximum cycle efficiency may reach 54%. The maximum efficiency of the cycle No. 1 is lower. For this variant, best efficiency is predicted for the special case, when the regenerative heater H_1 and the associated coolers C_1, C_3 and pump P_1 are removed. Then, the maximum efficiency for $T_1 = 600$ °C reaches 48%, and exceeds 51% for $T_1 = 700$ °C. In particular, the calculated efficiency values for the cycle No. 2 are close to or slightly higher than the efficiency of supercritical carbon dioxide cycle considered by Dostal [6].

It is worth to note that our results were obtained assuming the pump efficiency $\eta_p = 0.85$ while Dostal used the value $\eta_p = 0.89$. If the pump efficiency in our calculations would be equal to the Dostal's value then the cycle No. 2 efficiency would increase further by about 0.5%.

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