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Constraints in allocation of thrusters in a DP simulator

Paweł Zalewski

Maritime University of Szczecin, Faculty of Navigation, Centre of Marine Traffic Engineering
1-2 Wały Chrobrego, 70-500 Szczecin, Poland, e-mail: p.zalewski@am.szczecin.pl

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Abstract

Vessels conducting dynamic positioning (DP) operations are usually equipped with thruster configurations that enable the generation of force and torque. Some thrusters in these configurations are deliberately redundant to minimize consequences of thruster failures, enable overactuated control and increase the safety in operation. On such vessels, a thrust allocation system must be used to distribute the control actions determined by the DP controller among the thrusters. The optimal allocation of the thrusters' settings in DP systems is a problem that can be solved by convex optimization methods depending on the criteria and constraints used. This paper presents a quadratic programming (QP) method, adopted in a DP control model, which is being developed in Maritime University of Szczecin for ship simulation purposes.

Introduction

A Dynamic Positioning (DP) system can be defined as a system that automatically controls a vessel, influenced by external stimuli, in order to maintain her position and heading exclusively by means of active thrust. DP systems divide forces among the ship's thrusters to achieve a resultant force and momentum equal to that set by the control system. Optimization of thrust allocation is based on minimization of the energy usage and thus the requirement for power or fuel, additionally taking into account limitations such as forbidden zones of operation for the thrusters' settings (individually and relative to each other i.e. in opposing thruster pairs).

The optimal allocation of forces generated by thrusters in DP systems is a problem that can be solved by several convex optimization methods depending on the criteria and constraints used (Ruth, 2008; Wit, 2009; Fossen, 2011). In this paper, the quadratic programming (QP) method described in Zalewski (Zalewski, 2016) has been further extended to include real constraints of Multi-Purpose Supply Vessel (MPSV) model propulsion. The elaborated method has been adopted in a DP control model developed at the Maritime University of Szczecin (MUS) for ship simulation purposes.

Generation of forces using thrusters

For DP control, similarly to a ship simulation, a ship's hull can be treated as a rigid body with the centre of gravity (CG) at origin $p = 0 \in \mathbb{R}^2$. Measurements of the position of the vessel are compared with the required position. The difference is fed into an Extended Kalman Filter (EKF) and PID-controller which calculates the resultant force and momentum required to correct the position. The allocation unit controls the thrusters which must generate the component forces of the required resultant one. A model of thrust allocation for a vessel with *i* azimuth thrusters can be built following the geometrical relations presented in Figure 1.

The assumptions in this model are:

- The vessel's position is stabilized at low speed (less than 2 knots or 1 m/s), and the CG (force reference origin) is the fixed rotation centre.

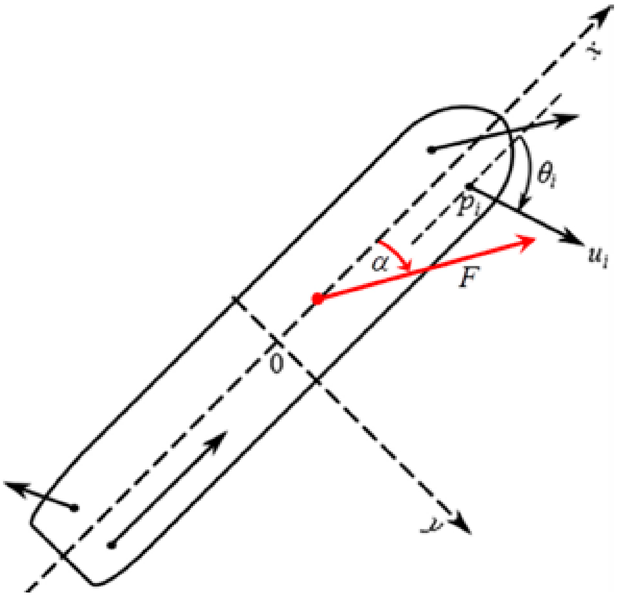

Figure 1. Thrust forces acting on a vessel with *i* azimuth thrusters

- The vessel is of MPSV type (Figure 2) with the following specifics:
  - length overall (LOA) 111 m;
  - beam moulded (B) 21 m;
  - draught, fore & aft 8.5 m;
  - 2 main electric engines rating 2499 kW at 120 rpm;
  - 2 main controllable pitch propellers of outwards revolutions with 2 rudders with maximum deflection angle of 35° and maximum rudder rate of turn of 4° /s, each generating a maximum force of 250 kN;
  - 2 stern azimuth thrusters rating 2800 kW with maximum azimuth rate of turn of 4° /s, each generating a maximum force of 300 kN;
  - 1 bow azimuth thruster rating 1050 kW with maximum azimuth rate of turn of 4° /s, generating a maximum force of 140 kN,
  - 2 bow tunnel thrusters rating 1240 kW, each generating a maximum force of 150 kN.
- There are $n = 7$ component forces of magnitude u_i [kN or tf], acting at $p_i = (p_{xi}, p_{yi})$ [m, m], in direction θ_i [°], $i = 1,2,\ldots,n$ (Figure 3):

$$p_x = [p_{x1}, p_{x2}, \ldots, p_{xn}]^{\mathrm{T}} = \\ = [41, 37.5, 30, -32, -45.7, -45.7, -32]^{\mathrm{T}} \quad (1)$$

$$p_y = [p_{y1}, p_{y2}, \ldots, p_{yn}]^{\mathrm{T}} = \\ = [0, 0, 0, 5.5, 2.7, -2.7, -5.5]^{\mathrm{T}} \quad (2)$$

- The resultant force [kN or tf] is:

$$F = \sqrt{F_x^2 + F_y^2} \quad (3)$$

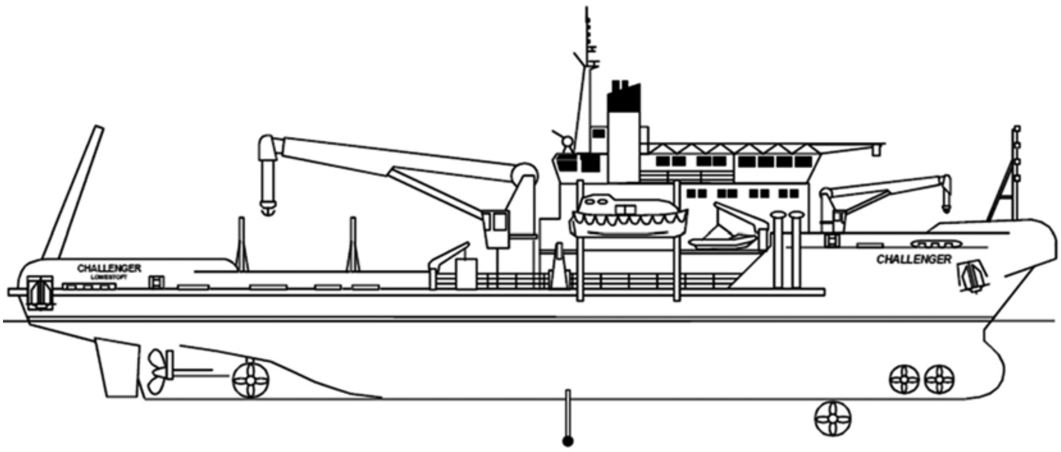

Figure 2. MPSV model used in DP simulator at MUS (source Kongsberg AS)

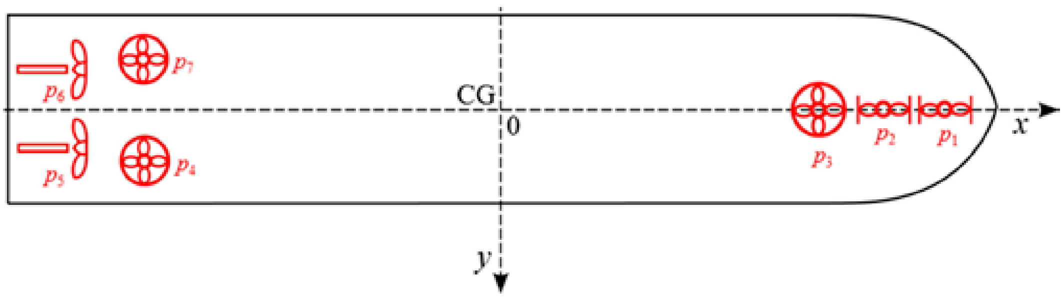

Figure 3. Position of thrusters and propellers in ship-body reference frame

- The resultant longitudinal force (horizontal along ship-body frame) [kN or tf] is:

$$F_x = \sum_{i=1}^{n} u_i \cos\theta_i \quad (4)$$

- The resultant transverse force (vertical across ship-body frame) [kN or tf] is:

$$F_y = \sum_{i=1}^{n} u_i \sin\theta_i \quad (5)$$

- The resultant torque (moment of the resultant force) [kNm or tfm] is:

$$M_z = \sum_{i=1}^{n} \left(p_{yi} u_i \cos\theta_i - p_{xi} u_i \sin\theta_i \right) \quad (6)$$

- The force limits [kN or tf] are:

$$0 \le u_i \le u_{\max i} \quad (7)$$

in [tf]:

$$u_{\max} = \left[u_{\max 1}, u_{\max 2}, \ldots, u_{\max n} \right] = \\ = \left[15.0, 15.0, 14.0, 30.0, 25.0, 25.0, 30.0 \right]^{\mathrm{T}} \quad (8)$$

- The thruster angle limits or allowed zones [°] are:

$$\begin{aligned} &\theta_1 = 90 \text{ or } \theta_1 = 270 \\ &\theta_2 = 90 \text{ or } \theta_2 = 270 \\ &0 \le \theta_3 < 360 \\ &118 \le \theta_4 < 360 \text{ or } 0 \le \theta_4 \le 62 \\ &325 \le \theta_5 < 360 \text{ or } 0 \le \theta_5 \le 35 \text{ or } \theta_5 = 180 \\ &325 \le \theta_6 < 360 \text{ or } 0 \le \theta_6 \le 35 \text{ or } \theta_6 = 180 \\ &298 \le \theta_7 < 360 \text{ or } 0 \le \theta_7 \le 242 \end{aligned} \quad (9)$$

- The energy or fuel usage is strictly dependent on u_i and is assumed to be linearly correlated to:

$$\sum_{i=1}^{n} u_i = u_1 + u_2 + \ldots + u_n \quad (10)$$

The problem to be solved is: find u_i and θ_i that yield the desired resultant force and moment and minimize the fuel or energy requirement. Note that the problem is considered to be 3-DOF (degrees of freedom) or solved in 2-dimensional space. In fact, any movement in the z-direction (up/down) or around the x- or y-axis is ignored because common actuators in offshore vessels do not have the ability to produce thrust in these directions. This clearly reduces the complexity of the problem. The remaining challenge is to rotate the vessel around its fixed rotation centre so as to keep the pivot point steady. The pivot point position in a ship-body reference frame was analysed in Artyszuk (Artyszuk, 2010). To keep it steady while turning on a spot, or with low lateral and forward speeds, the u_i and θ_i must be changed dynamically in response to deviations of the rotation centre from the set-point in the local or global reference frame. This can be done by classical PID, fuzzy, or neural controllers.

QP problem solution

For the thruster allocation problem with variables u_i and θ_i transformed to f_{xi} and f_{yi} (longitudinal and transverse components of forces u_i) the formulation of the objective function and constraints in the form of the QP constrained optimization problem can be given in matrix notation as:

$$\begin{aligned} &\text{minimize} \quad \mathbf{1}\left(f_x^2 + f_y^2\right) \\ &\text{subject to} \quad F_x = \mathbf{1} f_x \\ &\qquad F_y = \mathbf{1} f_y \\ &\qquad M_z = \mathbf{1}\left(p_x \bullet f_y - p_y \bullet f_x\right) \\ &\qquad f_x^2 + f_y^2 \le f_{\max}^2 \\ &\qquad \begin{bmatrix} \sin\theta_{\text{start}} & -\cos\theta_{\text{start}} \\ -\sin\theta_{\text{end}} & \cos\theta_{\text{end}} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} \le \begin{bmatrix} 0^{\mathrm{T}} \\ 0^{\mathrm{T}} \end{bmatrix} \end{aligned} \quad (11)$$

where:

$$\begin{aligned} \mathbf{0} &= [0_1, 0_2, \ldots, 0_n] \\ \mathbf{1} &= [1_1, 1_2, \ldots, 1_n] \end{aligned} \quad (12)$$

$$\begin{aligned} f_x &= [f_{x1}, f_{x2}, \ldots, f_{xn}]^{\mathrm{T}} \\ f_y &= [f_{y1}, f_{y2}, \ldots, f_{yn}]^{\mathrm{T}} \end{aligned} \quad (13)$$

$$\begin{aligned} \theta_{\text{start}} &= [\theta_{\text{start}1}, \theta_{\text{start}2}, \ldots, \theta_{\text{start}n}]^{\mathrm{T}} \\ \theta_{\text{end}} &= [\theta_{\text{end}1}, \theta_{\text{end}2}, \ldots, \theta_{\text{end}n}]^{\mathrm{T}} \end{aligned} \quad (14)$$

$$\begin{aligned} f_{xi} &= u_i \cos\theta_i \\ f_{yi} &= u_i \sin\theta_i \\ u_i^2 &= f_{xi}^2 + f_{yi}^2 \end{aligned} \quad (15)$$

and:

- $\bullet$ – indicates the Hadamard product (elementwise multiplication of matrices or vectors);
- ^{2} – indicates the Hadamard second power;
- $^{\mathrm{T}}$ – indicates matrix transposition;
- $\theta_{\text{start}i}$ – starting angle of the allowed i^{th} thruster azimuth limit;
- $\theta_{\text{end}i}$ – clockwise end angle of the allowed i^{th} thruster azimuth limit;
- F_x, F_y, M_z are designated constraints of the longitudinal and transverse forces and the moment (torque) acting on the ship's hull. If the final constraints calculated by the extended Kalman filter (EKF) of the hydrodynamic model and PID controller are in the form of (see Figure 1):
- F – resultant force;
- α – orientation of the resultant force;
- M_z – resultant momentum;
- $f_{\max}$ – maximum individual thruster force

then:

$$F_x = F\cos\alpha, \quad F_y = F\sin\alpha \tag{16}$$

and the ordinates of the application point of the resultant force F can be calculated as:

$$P_x = M_z / F_y\,, \quad P_y = 0 \tag{17}$$

or

$$P_x = 0\,, \quad P_y = -M_z / F_x \tag{18}$$

Formula (11) has been extended by additional constraints on the thrusters' work sectors (limits of θ_i). These constraints are defined by two additional hyperplanes, limiting the sector angle similarly to the method described in Wit (Wit, 2009). Such a formula can be used directly in the case of azimuth thrusters, but cannot be applied to cases of propeller rudder combination with lateral thrusters. The limits imposed on the ship's propulsion system (9) indicate the nonconvexity in the case of the main propellers working in the reverse mode and lateral thrusters working either to port or starboard. When the propeller is working in reverse mode, a line-shaped thrust region appears, as the rudder cannot generate lift in this situation. This is defined by one equality constraint and two inequality constraints (9) which are added to the problem as a disjunctive thrust region of the propeller/rudder pair. In the case of lateral thrusters there are also two disjunctive thrust regions defined by two equality constraints. The method applied to deal with disjunctive thrust regions of lateral thrusters is to replace the alternative geometrical equalities with conjunctive dual equalities:

$$f_{x1} = 0, \quad f_{x2} = 0 \tag{19}$$

The trick to solving the optimization problem of the propeller/rudder pair, or other thrusters when disjunctive thrust regions are defined, is to first generate all the possible combinations of the thrust regions, picking one disjunctive convex region for each thruster. The total number of combinations can be derived by multiplying the number of disjunctive thrust regions for each thruster.

For example, for 2 propellers and rudders:

(1, 1) the propeller/rudder pairs are both operating in the forward mode:

$$\begin{bmatrix} \sin\theta_{\text{start}5} & -\cos\theta_{\text{start}5} \\ -\sin\theta_{\text{end}5} & \cos\theta_{\text{end}5} \end{bmatrix} \begin{bmatrix} f_{x5} \\ f_{y5} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \sin\theta_{\text{start}6} & -\cos\theta_{\text{start}6} \\ -\sin\theta_{\text{end}6} & \cos\theta_{\text{end}6} \end{bmatrix} \begin{bmatrix} f_{x6} \\ f_{y6} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{20}$$

and the optimization problem (11) considering (19) is modified to:

$$\begin{aligned} &\text{minimize} \quad \mathbf{1}\left(f_x^2 + f_y^2\right) \\ &\text{subject to} \\ &\qquad f_{x1} = 0 \\ &\qquad f_{x2} = 0 \\ &\qquad F_x = \mathbf{1} f_x \\ &\qquad F_y = \mathbf{1} f_y \\ &\qquad M_z = \mathbf{1}\left(p_x \bullet f_y - p_y \bullet f_x\right) \\ &\qquad f_x^2 + f_y^2 \le f_{\max}^2 \\ &\qquad \begin{bmatrix} \sin\theta_{\text{start}} & -\cos\theta_{\text{start}} \\ -\sin\theta_{\text{end}} & \cos\theta_{\text{end}} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} \le \begin{bmatrix} 0^{\mathrm{T}} \\ 0^{\mathrm{T}} \end{bmatrix} \end{aligned} \tag{21}$$

(1, 2) the first propeller/rudder pair is operating in the forward mode, while the second is operating in the reverse mode:

$$\begin{aligned} &\begin{bmatrix} \sin\theta_{\text{start}5} & -\cos\theta_{\text{start}5} \\ -\sin\theta_{\text{end}5} & \cos\theta_{\text{end}5} \end{bmatrix} \begin{bmatrix} f_{x5} \\ f_{y5} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ &f_{x6} < 0 \\ &f_{y6} = 0 \end{aligned} \tag{22}$$

(2, 1) the second propeller/rudder pair is operating in the forward mode, while the first is operating in the reverse mode:

$$\begin{aligned} &f_{x5} < 0 \\ &f_{y5} = 0 \\ &\begin{bmatrix} \sin\theta_{\text{start}6} & -\cos\theta_{\text{start}6} \\ -\sin\theta_{\text{end}6} & \cos\theta_{\text{end}6} \end{bmatrix} \begin{bmatrix} f_{x6} \\ f_{y6} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \tag{23}$$

(2, 2) the propeller/rudder pairs are both operating in the reverse mode:

$$\begin{aligned} &f_{x5} < 0 \\ &f_{x6} < 0 \\ &f_{y5} = 0 \\ &f_{y6} = 0 \end{aligned} \tag{24}$$

For each of these thrust region combinations, the QP problem is formulated and solved. While this happens, the solution corresponding to each combination is stored. After solving all the QP subproblems, the best solution is chosen by comparing the objective costs (value of the minimized goal function) and this is the optimal solution of the main problem.

Implementation in a DP simulator

The algorithms used for solving (11), by applying an interior-point method to a sequence of equality constrained problems, were developed in Matlab with CVX Toolbox (Grant & Boyd, 2013) and afterwards translated to C#.

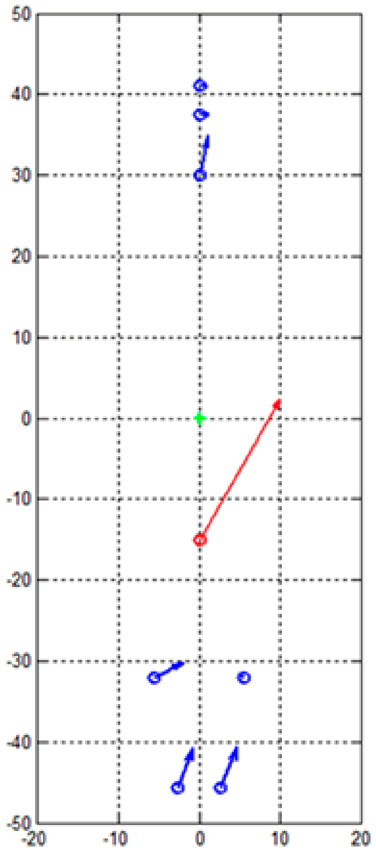

Figure 4. Example of thrust allocation to a MPSV in a DP simulator

The example of thrust allocation within a MPSV, calculated by the model adopted in the DP simulation system of the Maritime University of Szczecin, with the resultant force in a ship-body fixed co-ordinate system, is presented in Figure 4 (corresponding to Figure 1: angles 360° clockwise, x-axis up, y-axis right, ordinates in [m] in a ship-body fixed co-ordinate system from the centre of gravity marked with a green cross, the resultant force is shown in red, the component forces in blue). The allocated thrust corresponds to a resultant force of 20 tf, 30° and torque of 150 tfm (anticlockwise). The detailed numerical values of the component forces are presented in Table 1.

Figure 5 shows a visualisation of the marine environment within the MPSV in the DP Simulation System of the Marine Traffic Engineering Centre at MUS.

Table 1. Numerical data of the allocated thrust shown in Figure 4

F [tf]	α [°]	M_z [tfm]	P_x [m]	P_y [m]
20.00	30.0	–150.00	–15	0
i	u_i [tf]	θ_i [°]		
1	1.0077	90		
2	1.0477	90		
3	5.1598	12.687		
4	0.7676	298		
5	5.4441	21.518		
6	5.3868	21.758		
7	3.9589	62		

Figure 5. DP simulation system with thruster allocation and vision prepared at MUS. From left to right: two operator control stations, vision display, electronic position fixing systems screens and electronic site chart screen

Conclusions

A thrust allocation system must be used to distribute the control actions determined by the DP controller among the thrusters. The allocation problem can be translated to a constrained optimization problem. The quadratic programming (QP) method has been developed for this purpose in the DP ship simulation model implemented in the ship simulator at MUS. The tests proved that the optimization algorithm translated into the C# programming language worked efficiently, using interior-point methods (Boyd & Vandenberghe, 2009) to solve the problem. The system includes extra constraints such as limits to the thrusters' work sectors (forbidden zones) and non-azimuth thrusters. From a critical point of view concerning safety it is also important to take into account actuator limitations such as saturation, wear and tear and rotation time. This problem needs further research following the methodology presented in Ruth and Boyd & Vandenberghe (Ruth, 2008; Boyd & Vandenberghe, 2009).

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