Determination of Moving Chord Length for Determining the Curvature of In-Service Railway Track

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Summary

The paper deals with the still unexplained issue of the choice of chord length, which will be the most beneficial when determining the horizontal curvature of railway track with the use of the moving chord method. In the railway track – with an incorrect choice of the chord length – the horizontal track deformations and coordinate measurement error may cause irregular curvature diagrams, which will be difficult to interpret. The study analysed three test geometric layouts adapted to the speeds of 80 km/h, 120 km/h and 160 km/h (the radii of circular curves determined as a result of the curvature estimation performed were approximately 410 m, 880 m and 1480 m, respectively). The lengths of the moving chord in the range of 10÷50 m were considered. On the basis of the conducted analysis, it was unequivocally demonstrated that the chord length used to determine the curvature in the railway track should depend on the value of the radius of the circular curve. Approximate lengths l_c were proposed depending on the range of the R_{CA} radius. The adaptation of the moving chord method to the adopted measurement procedure presented in this paper and the way of using the obtained curvature diagram, provide the appropriate application basics.

Keywords: railway track, curvature of the track axis, the moving chord method, applications

1. Introduction

The shape of the railway track axis in the horizontal plane and the vertical plane is determined by basic geometric parameters determined by appropriate measurements. The measurement methods currently in use are similar across different railway administrations [1–8]. Classic surveying methods use distance and angle measurements with total stations and geodetic calculus to determine the position of the track axis in a spatial system, in reference to the geodetic control network. These systems involve the relative movement of a total station and a prism focusing a laser beam along the track sections to be measured, using purpose-made survey vehicles or adapted vehicles in the form of trolleys pushed by the operator.

New opportunities for engineering inventories are being created by the development of satellite measurements and the increased accuracy of measurements based on Global Navigation Satellite System (GNSS) technology, using active geodetic networks (e.g. Real Time Network (RTK) reference stations [9–11]). This solution does not require the use of a railway ground geodetic control network, as the measurement systems work with networks of reference stations. Modern methods of mobile measurement using multi-sensor devices for determining the position of the track axis are also used in the railway industry. For this purpose, GNSS receivers, Inertial Navigation System (INS) devices [12] supporting satellite measurements and visual methods, the so-called Terrestrial Laser Scanning (TLS) [13], are used, which are able to generate the course of the track axis with a certain – varying – accuracy. The possibility of using systems consisting of satellite receivers mounted on different types of vehicles is currently being investigated [14–18].

As a result of surveying of the railway track, sets of coordinates of relevant measurement points are obtained. Since the primary purpose of the measurements carried out is to determine the geometric parameters of the measured route, appropriate calculation algorithms must be applied (referring – for example – to the principles of the analytical design method [19–22]). The first solution is to use the obtained measurement data to determine the occurring curvature of the geometric layout.

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Currently, the most commonly used tool for assigning track points to sections of defined geometry is based on a diagram of horizontal arrows related to a chord stretched along the track. This requires an additional measurement process, and while the diagram shows the values of the measured arrows, it does not specify the directions in which these arrows are measured. The baseline is the chord directions, which are constantly changing.

The papers [23–25] presented relevant analyses relating to the proposed new method for determining the curvature of a track axis, referred to as the "moving chord method". They concerned the application of this method to model geometric layouts (described by mathematical equations). The paper [26] addressed the issue of its use for estimating the horizontal curvature of the axis of a railway track in service on the basis of Cartesian coordinates, obtained through direct measurements carried out.

2. Moving chord method for the estimation of track axis curvature

As a result of surveying, Cartesian coordinates of points of the railway route are determined in the appropriate state spatial reference system. In Poland – with regard to plane coordinates – the PL-2000 system [27] is in force, created on the basis of a mathematically unambiguous assignment of points on the GRS 80 reference ellipsoid [28] to corresponding points on the plane according to the Gauss-Krüger projection [29].

The first step in analysing the measurement results is to visualise the course of the measured route in the horizontal plane. For this, it is necessary to know the eastern plane coordinates Y and the northern plane coordinates X of the relevant measurement point in the PL-2000 system. In the case under consideration, the basis for the diagram of the relationship $X(Y)$ is the set of measured and appropriately corrected coordinates Y_i and X_i. However, the proper identification of the track axis is provided by the corresponding diagrams relating to the length parameter L_i. In the case of the horizontal plane, these are curvature diagrams $\kappa(L)$, while for the vertical plane, they are ordinate height diagrams $H(L)$. Thus, in order to create the possibility for further analysis, it is necessary to switch to a linear system, i.e. to determine the distances (variable L) of the individual measurement points from the selected starting point i_0.

From a practical point of view, it will be most beneficial to transfer the measurement data to the local x, y coordinate system. This operation will consist of moving the starting point of the PL-2000 system to the selected point $O(Y_0, X_0)$; sometimes it may additionally be advisable to rotate this system by an appropriate angle β.

From the definition of curvature, it is necessary to operate on the angles of inclination of the tangent to the geometric layout. If a mathematical notation of the curve in question is available, this obviously poses no problem. However, in a real, mostly deformed railway track, it is very difficult to determine the position of straight tangents. In contrast, the situation is completely different for stretched chords, whose position is always unambiguously determined. Therefore, the idea emerged that, when determining the curvature of the track axis, one should not operate with tangents but with corresponding virtual chords. The assumption was made that, for the small track sections under consideration, the geometric elements are parallel to each other, while the points of tangency are projected perpendicularly onto the centre of the given chord. Figure 1 shows a schematic diagram of the curvature determination by the proposed method of changing the chord angles; this method has been referred to as the "moving chord method" [23].
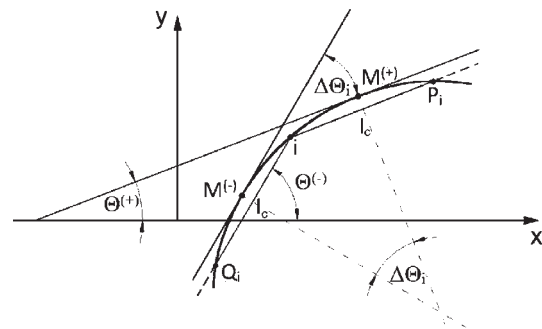

Fig. 1. Schematic diagram of the curvature determination using the moving chord method [author's own elaboration]

To determine the track axis at point i, two chords of length l_c must be derived from this point – forward and backward – and the coordinates of their points of intersection with the given curve, i.e. points P_i and Q_i respectively, must be determined. Since the coordinates of point i are known, the equations of the two chords, the values of the tangents and the angles of inclination can be easily determined. The curvature κ_i is determined from the following formula:

$$\kappa_i = \frac{\Delta\Theta_i}{l_c}, \qquad (1)$$

in which l_c is the length of the virtual chord, while the angle $\Delta\Theta_i$ results from the difference in the angles of the chords converging at point i, i.e:

$$\Delta\Theta_i = \Theta_i^{(+)} - \Theta_i^{(-)}. \qquad (2)$$

The procedure requires the coordinates of the curve in question in the Cartesian coordinate system (either analytically or discretised), as the values of the $\Theta_i^{(+)}$ and $\Theta_i^{(-)}$ angles result from the slope coefficients of the straight lines describing the two chords.

The paper [23] presents a verification of the proposed method for determining horizontal curvature on an explicitly defined elementary geometric track layout, consisting of a circular curve and two symmetrically aligned transition curves (of the same type and length), calculated according to the principles of the analytical design method [19]. Several geometric examples were considered for different train speeds, while also varying the types of transition curves used and the route turning angle. A complete correspondence between the curvature diagrams obtained and the diagrams underlying the corresponding geometric solution was obtained. This was the case for both circular curve sections and transition curve regions.

It was also pointed out that the proposed method offers great potential for application. The practical aspect of the presented considerations may become apparent when the geometric characteristics of the track axis determined by the measurements are not known and the primary objective becomes the determination of these characteristics. In this situation, the described method ideally corresponds to the assumptions of mobile satellite measurements, as it provides track axis coordinates in the Cartesian system, in a very large number and in a very short time.

The paper [24] addressed two important specific issues: the influence of chord length on the obtained values of horizontal curvature and the possibility of determining the position of boundary points between geometric elements. The variants analysed resulted from the type of transition curves used. For the model systems, the effect of chord length in the range of 5 to 20 m on the determined curvature values was found to be negligible. At the same time, the precision of the determination of the nature of the curvature and its consistency with the theoretical course on the transition curves is noteworthy. At the same time, it was shown that by using the moving chord method it is possible to determine the position of the boundary points between the geometric elements, with the required chord length having to be adapted to the type of transition curve.

The paper [25] considers not only the problem of determining curvature in the horizontal plane, but also in the vertical plane, indicating the versatility of the method in question. Here, the focus is on the computational basis of this method, concerning the formation of the angles of the moving chord. It was found that for a circular curve in the horizontal plane, the values of the moving chord angles depend on the curve deflection angle, the radius of the curve and the

length of the chord, while the variation of the angles of inclination itself depends on the length of the chord and the radius of the curve. In the case of a circular curve in the vertical plane, the values of the angles of the moving chord are much smaller than is the case in the horizontal plane, which is related to the range of curve radii used. As in the horizontal plane, the value of the difference in the angles of the moving chord is determined by the radius of the vertical curve and the length of the chord.

After clarifying the basic issues concerning the proposed method, subsequent works began to explain its application aspects. The paper [26] addressed the use of the moving chord method for estimating the horizontal curvature of the axis of a railway track in service on the basis of Cartesian coordinates, obtained through direct measurements carried out. The curvature diagrams obtained were less regular, oscillatory, but this did not prevent the basic geometric parameters of the measured layout from being estimated from them. It was shown how to adapt the moving chord method to the adopted measurement procedure and how to use the obtained curvature diagram in practice. In addition to determining the value of the radius of the circular curve, the lengths of the transition curves and the location of the characteristic points can be determined from this diagram; this provides the method in question with a suitable implementation basis.

In the paper [30], a detailed procedure for the estimation of horizontal curvature was presented with a view to its practical application in an operational railway track. Since in the measurements used we are considering a geometric layout with unknown characteristics, and it is not possible to operate with mathematical notation, the basic problem concerns the determination of the coordinates of the ends of the two chords by interpolation carried out in appropriate intervals. However, as it turns out, it is possible to carry out the whole procedure without having to resort to numerical methods. The procedure presented here is based on the use of the given calculation formulae and is sequential.

At this point, it should be clarified that in maintenance, identification of track geometric layouts based on curvature estimation is only possible for the horizontal plane. In the vertical plane, where there are very large radii of curves rounding the bends of the longitudinal profile, the curvature values turn out to be too small in relation to the measurement error present.

This article focuses on an as yet unexplained crucial issue – the selection of the chord length that will be most beneficial in a particular situation. Formally, this is of little relevance for the moving chord method. As shown in the work [24], for model systems, the ef-

fect of the chord length used (in the range of 5 to 20 m) on the determined curvature values proved to be insignificant. However, in an in-service railway track, due to track deformation and measurement error, the situation can be quite different. This is evidenced, for example, by the irregular curvature diagrams shown in the work [31], where a chord of $l_c = 7$ m was used. Three test geometric layouts adapted for speeds of 80, 120 and 160 km/h were then analysed. The Cartesian coordinates of the individual measuring points were determined at intervals of approximately 5 m, and the maximum error of this operation was ±25 mm.

3. Test section adapted to a speed of 80 km/h

Figure 2 shows the route of the test section adapted to 80 km/h in the local coordinate system. As can be seen, the test section consists of two straight sections of track connected by a curve with unknown characteristics. Beyond this statement, Figure 2 does not explain much. In order to be able to fully identify a given geometric layout, an estimation of the occurring curvature of the track axis must be carried out using the moving chord method.
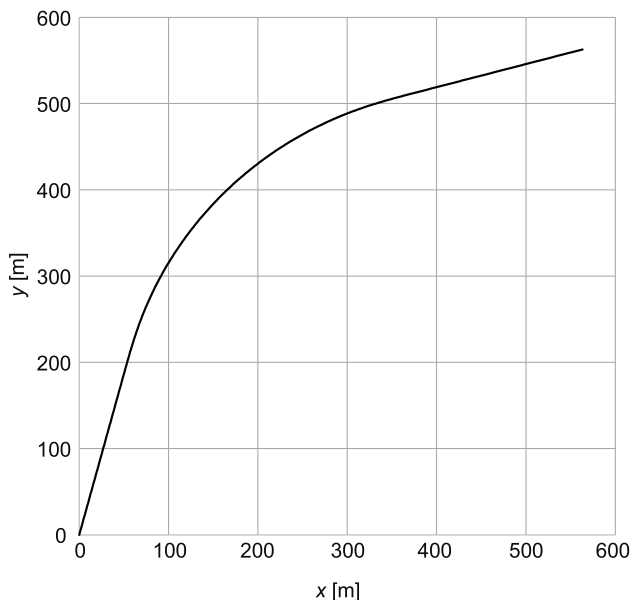

Fig. 2. Routing on the test section of the track adapted to a speed of 80 km/h in the local coordinate system [author's own elaboration]

In the case under consideration, virtual chords were used, projected from a given point forward and backward, with lengths of 10 m, 20 m and 30 m. Figure 3 shows the curvature diagrams obtained. Each diagram $\kappa(L)$ consists of elements of two types:
- segments oscillating within a horizontal course, which describe a curvature of a fixed value (equal to zero on straight sections of track and non-zero on circular curves),
- segments oscillating within linearity (i.e. lines inclined to the L-axis), which describe the varying curvature present in the transition curves.

From these diagrams, the value of the radius of the circular curve and the lengths of the transition curves can be determined, as well as the location of the so-called segmentation points (which lie at the junctions of the straight sections with the transition curves and the transition curves with the circular curve).

The curvature of the track on straight sections is assumed to be zero, and the disturbances in the curvature diagram that occur there are the result of existing deformations and measurement error. From the marked range of curvature values undoubtedly belonging to the circular curve, the arithmetic mean $\overline{\kappa_{CA}}$ is determined; its inverse determines the value of the radius:

$$R \cong \frac{1}{\overline{\kappa_{CA}}} \qquad (3)$$

Table 1 gives the relevant numerical characteristics relating to the circular curve. The following designations are adopted therein (as well as in Tables 4 and 7):
l_c – length of the chord,
$\overline{\kappa_{CA}}$ – average value of curvature on a circular curve,
R_{CA} – radius of circular curve resulting from the mean value of curvature,
σ_κ – value of the standard deviation of curvature on a circular curve,
s_κ – percentage relationship of the standard deviation of curvature to the mean value.

Table 1

Numerical characteristics for curvature estimation on a circular curve for a test section of track adapted to $V = 80$ km/h

l_c [m]	$\overline{\kappa_{CA}}$ [rad/m]	R_{CA} [m]	σ_κ [rad/m]	s_κ [%]
10	−0.00243235	411.125	0.00028569	11.746
20	−0.00244140	409.600	0.00006643	2.721
30	−0.00244067	409.723	0.00002626	1.076

[Authors' own elaboration].

Transition curves occur on both sides of the circular curve. They can be easily identified on the $\kappa(L)$ diagram – the ordinates of the curves there oscillate within linearity. In order to determine the linear coordinates of the start (L_{PKP}) and end (L_{KKP}) points of the transition curves, it is necessary to determine the coefficients of the least squares approximation lines
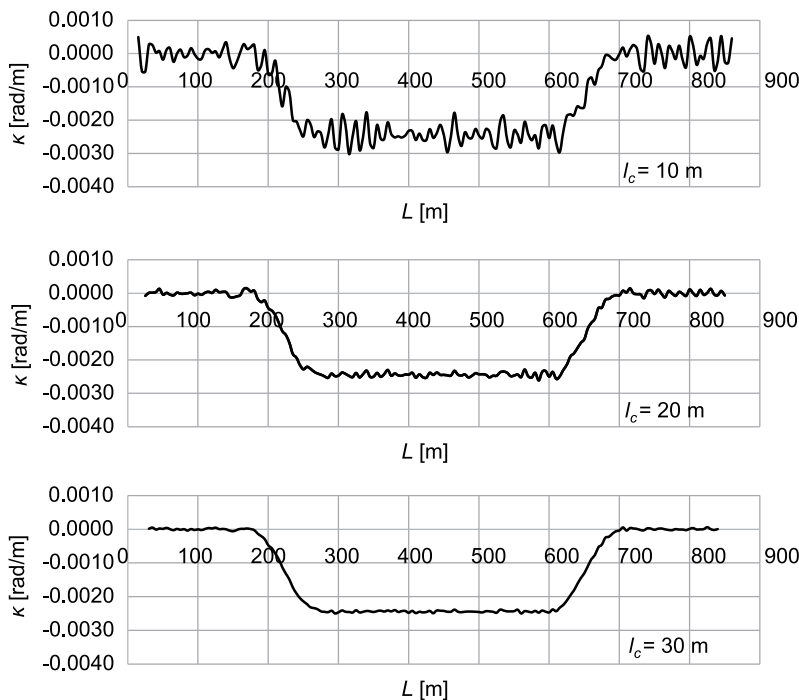
Fig. 3. Curvature diagrams along the length of the test section of the track adapted to a speed of 80 km/h obtained using chord lengths l_c = 10 m, 20 m and 30 m [author's own elaboration]

describing the regions of the $\kappa(L)$ diagram with varying curvature values. The lines in the form:

$$\kappa(L) = a + bL, \qquad (4)$$

determine the linear coordinates L of their points of intersection with the curvature diagrams on the straight sections of the track (where curvature $\kappa = 0$) and the circular curve sections (where curvature $\kappa = \kappa_{CA}$). The length of the transition curve is directly derived from the values of the respective L_{PKP} and L_{KKP} coordinates determined:

$$l_{KP} = \left| L_{KKP} - L_{PKP} \right|. \qquad (5)$$

Figure 4 shows the effects of identifying the transition curve *KP*1 located to the left of the geometric layout in Figure 2, and Table 2 shows the corresponding design characteristics. The following designations are used in Table 2 (as well as in Tables 3, 5, 6, 8 and 9):

   a, *b* – coefficients of the least squares approximation line describing curvature of the track axis on the transition curve,

   L_{PKP} – linear coordinate of the start point of the transition curve,

   L_{KKP} – linear coordinate of the end point of the transition curve,

   l_{KP} – length of the transition curve.

Figure 5 shows the effects of identifying the transition curve *KP*2 located to the right of the geometric layout in Figure 2, and Table 3 shows the corresponding design characteristics.

The following observations emerge from the analysis:
- from the point of view of determining the curvature of a circular curve, the use of a chord with a length of l_c = 30 m proved to be the most favourable, l_c = 20 m may also be allowed; the use of l_c = 10 m should be considered unacceptable,
- the above is also valid for the identification of the *KP*1 transition curve,
- for the identification of the *KP*2 transition curve, the use of l_c = 20 m has proven to be most favourable (l_c = 30 m may also be allowed); however, a chord of l_c = 10 m is of little use.

Table 2

Numerical characteristics for the identification of the transition curve *KP*1 for the test section of the track adapted to V = 80 km/h

l_c [m]	a [rad/m]	b [rad/m²]	L_{PKP1} [m]	L_{KKP1} [m]	l_{KP1} [m]
10	0.00765151	–0.00003916	195.406	257.524	62.118
20	0.00739782	–0.00003879	190.739	253.686	62.947
30	0.00729227	–0.00003823	190.747	254.589	63.842

[Authors' own elaboration].

Table 3

Numerical characteristics for the identification of the transition curve *KP*2 for the test section of the track adapted to V = 80 km/h

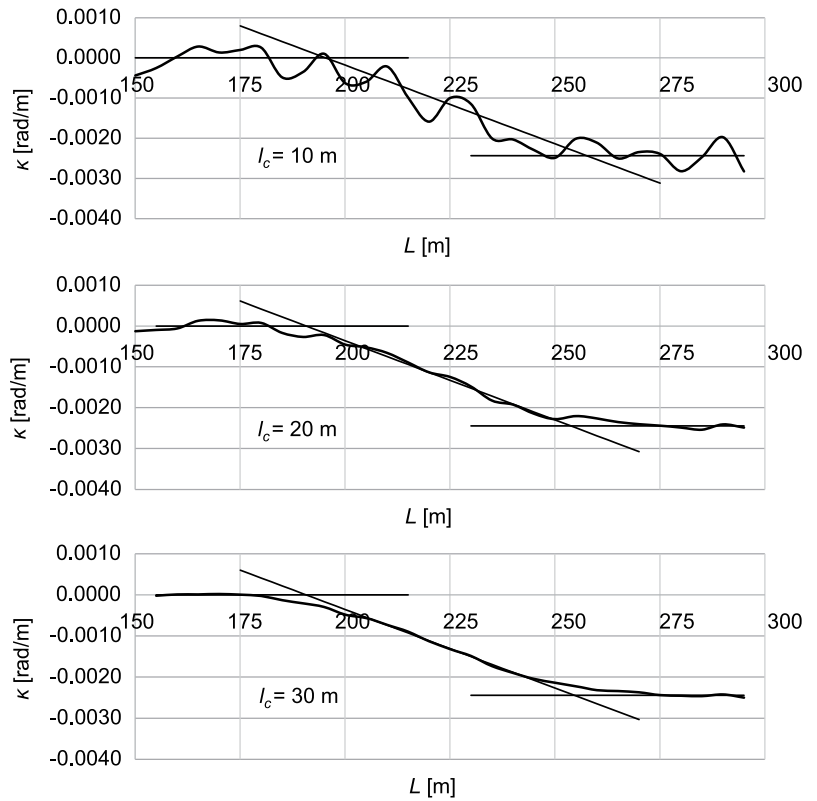| l_c [m]	a [rad/m]	b [rad/m²]	L_{PKP2} [m]	L_{KKP2} [m]	l_{KP2} [m]
10	–0.02367021	0.0003451	685.829	615.353	70.476
20	–0.02440522	0.0003561	685.340	616.781	68.559
30	–0.02338751	0.0003403	687.352	615.622	71.731

[Authors' own elaboration].

Fig. 4. Identification of the *KP*1 transition curve on a test section of track adapted to a speed of 80 km/h (curvature diagrams obtained using chord lengths l_c = 10 m, 20 m and 30 m) [author's own elaboration]
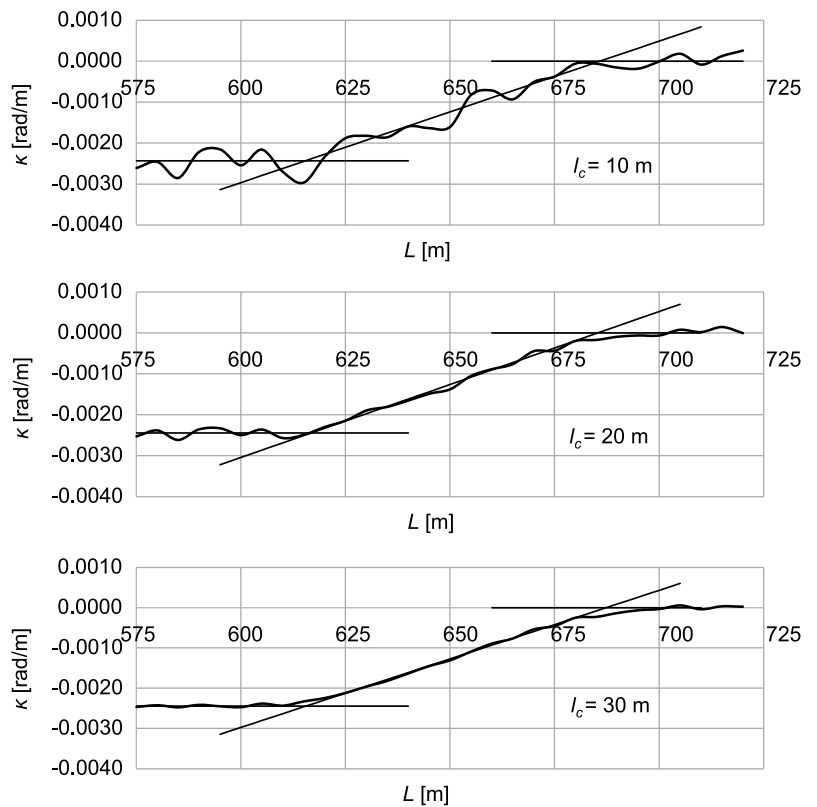

Fig. 5. Identification of the *KP*2 transition curve on a test section of track adapted to a speed of 80 km/h (curvature diagrams obtained using chord lengths l_c = 10 m, 20 m and 30 m) [author's own elaboration]

In summary, one might argue that, for a circular curve radius $R \cong 400$ m, it proves most favourable to adopt a virtual chord length of $l_c = 20$ m (or alternatively $l_c = 30$ m). From the given values, the shorter chord more accurately determines the length of the transition curve.

4. Test section of a track adapted to a speed of 120 km/h

Figure 6 shows the routing on a test section of the track adapted to 120 km/h. In the given case, virtual chords with lengths of 10 m, 20 m, 30 m and 40 m were used. Figure 7 shows the resulting curvature diagrams $\kappa(L)$ and Table 4 shows the corresponding numerical characteristics relating to the circular curve.
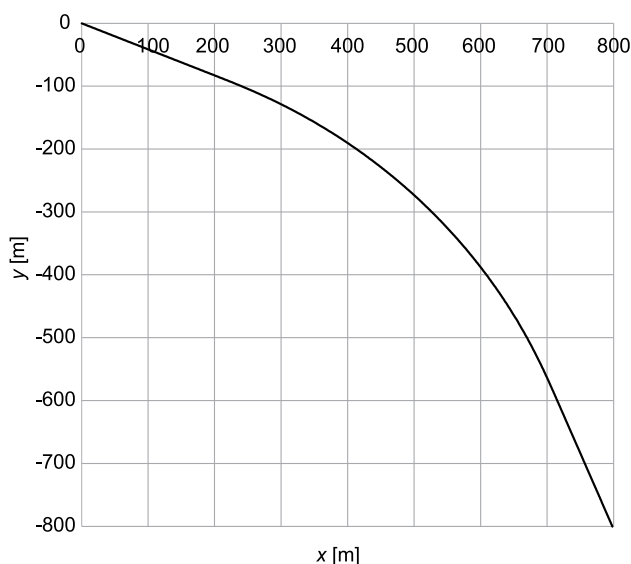

Fig. 6. Routing on the test section of the track adapted to a speed of 120 km/h in the local coordinate system [author's own elaboration]

Table 4

Numerical characteristics for curvature estimation on a circular curve for a test section of track adapted to $V = 120$ km/h

l_c [m]	$\overline{\kappa_{CA}}$ [rad/m]	R_{CA} [m]	σ_κ [rad/m]	s_κ [%]
10	−0.00113388	881.930	0.00025615	22.590
20	−0.00113669	879.746	0.00006268	5.514
30	−0.00113694	879.557	0.00002689	2.365
40	−0.00113681	879.653	0.00001609	1.416

[Authors' own elaboration].

Figure 8 shows the effects of identifying the transition curve $KP1$ located to the left of the geometric lay-

out in Figure 6, and Table 5 shows the corresponding design characteristics.

Table 5

Numerical characteristics for the identification of the transition curve $KP1$ for the test section of the track adapted to $V = 120$ km/h

l_c [m]	a [rad/m]	b [rad/m²]	L_{PKP1} [m]	L_{KKP1} [m]	l_{KP1} [m]
10	0.00225219	−0.00001151	195.601	294.077	98.476
20	0.00241139	−0.00001212	199.038	292.861	93.823
30	0.00244596	−0.00001228	199.165	291.741	92.576
40	0.00234201	−0.00001185	197.565	293.463	95.898

[Author's own elaboration].

Figure 9 shows the effects of identifying the transition curve $KP2$ located to the right of the geometric layout in Figure 6, and Table 6 shows the corresponding design characteristics.

Table 6

Numerical characteristics for the identification of the transition curve $KP2$ for the test section of the track adapted to $V = 120$ km/h

l_c [m]	a [rad/m]	b [rad/m²]	L_{PKP2} [m]	L_{KKP2} [m]	L_{KP2} [m]
10	−0.01359770	0.00001391	977.437	895.931	81.506
20	−0.01208245	0.00001230	982.005	889.620	92.385
30	−0.01237697	0.00001261	981.138	891.012	90.126
40	−0.01196196	0.00001217	989.465	891.026	93.408

[Authors' own elaboration].

The following observations emerge from the analysis:
- for the identification of the curvature of a circular curve, the use of a chord length of $l_c = 40$ m proved to be most favourable, $l_c = 30$ m may also be allowed; the use of $l_c = 10$ m should be considered unacceptable,
- for the identification of the $KP1$ transition curve, the use of $l_c = 30$ m has proven to be most favourable, $l_c = 40$ m and $l_c = 20$ m may also be allowed; the use of $l_c = 10$ m is unacceptable,
- when identifying the $KP2$ transition curve, the same arrangements apply as for the $KP1$ transition curve.

In summary, one might argue that, for a circular curve radius $R \cong 900$ m, it proves most favourable to adopt a virtual chord length of $l_c = 30$ m (or alternatively $l_c = 40$ m). As before, the shorter chord better determines the length of the transition curve.
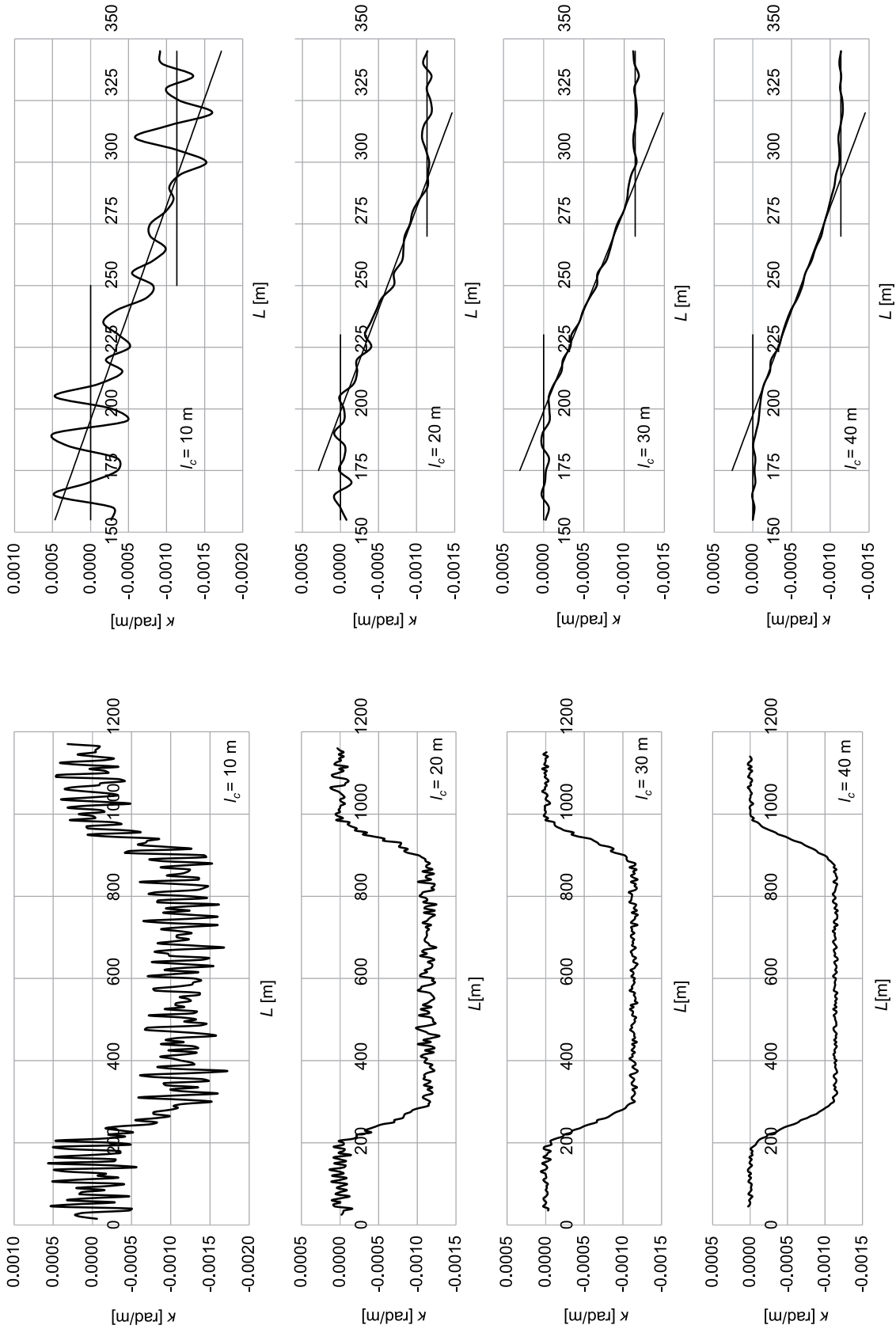
Fig. 8. Identification of the *KP1* transition curve on a test section of track adapted to a speed of 120 km/h (curvature diagrams obtained using chord lengths l_c = 10 m, 20 m, 30 m and 40 m) [author's own elaboration]

Fig. 7 Curvature diagrams along the length of the test section of the track adapted to a speed of 120 km/h obtained using chord lengths l_c = 10 m, 20 m, 30 m and 40 m [author's own elaboration]
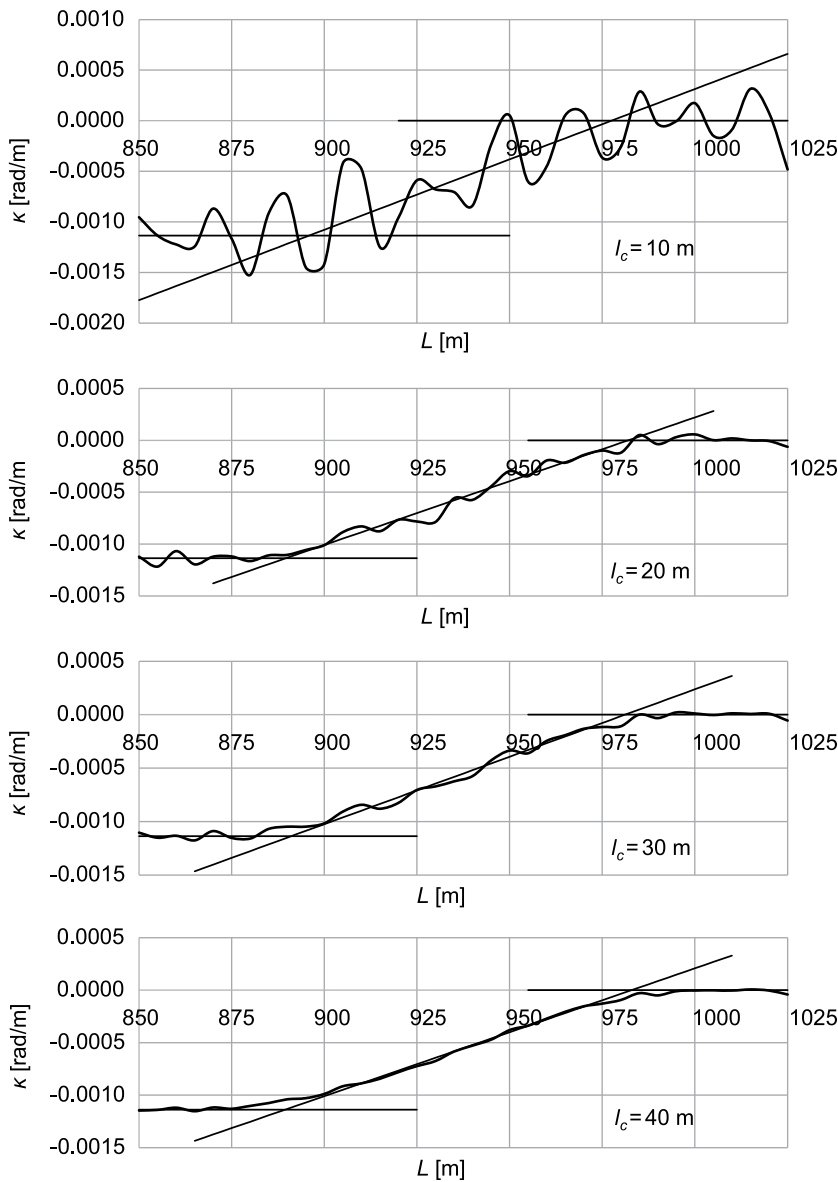
Fig. 9. Identification of the *KP*2 transition curve on a test section of track adapted to a speed of 120 km/h (curvature diagrams obtained using chord lengths l_c = 10 m, 20 m, 30 m and 40 m) [author's own elaboration]

5. Test section of a track adapted to a speed of 160 km/h

Figure 10 shows the routing on a test section of the track adapted to 160 km/h. In the given case, virtual chords with lengths of 20 m, 30 m, 40 m and 50 m were used. Figure 11 shows the resulting curvature diagrams $\kappa(L)$ and Table 7 shows the corresponding numerical characteristics relating to the circular curve.

Table 7

Numerical characteristics for curvature estimation on a circular curve for a test section of track adapted to V = 160 km/h

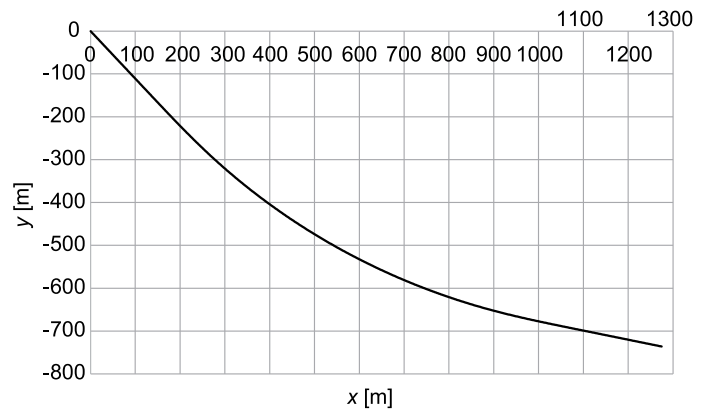| l_c [m]	$\overline{\kappa_{CA}}$ [rad/m]	R_{CA} [m]	σ_κ [rad/m]	s_κ [%]
20	0.00067629	1478.657	0.00006646	9.827
30	0.00067591	1479.496	0.00002858	4.228
40	0.00067596	1479.371	0.00001685	2.493
50	0.00067602	1479.252	0.00001021	1.511

[Authors' own elaboration].

Fig. 10. Routing on the test section of the track adapted to a speed of 160 km/h in the local coordinate system [author's own elaboration]
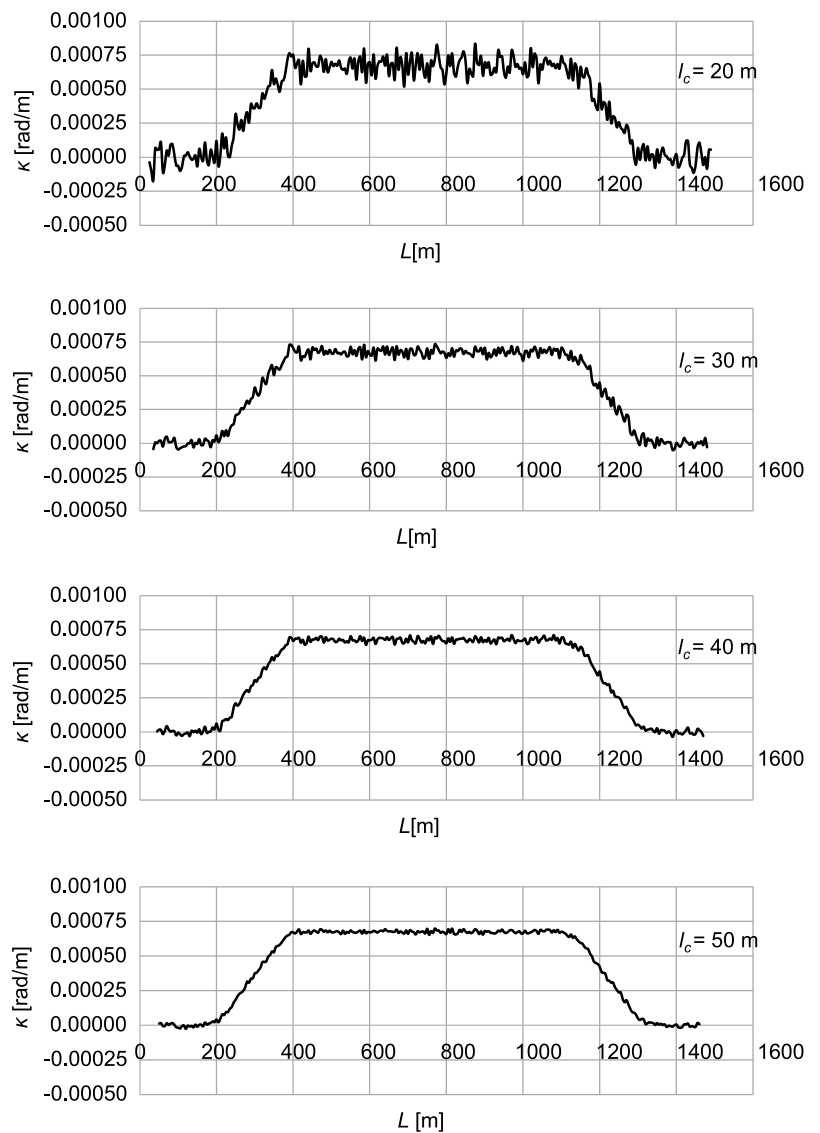

Fig. 11. Curvature diagrams along the length of the test section of the track adapted to a speed of 160 km/h obtained using chord lengths l_c = 20 m, 30 m, 40 m and 50 m [author's own elaboration]

Figure 12 shows the effects of identifying the transition curve *KP*1 located to the left of the geometric layout in Figure 10, and Table 8 shows the corresponding design characteristics.

Figure 13 shows the effects of identifying the transition curve *KP*2 located to the right of the geometric layout in Figure 10, and Table 9 shows the corresponding design characteristics.

Table 8

Numerical characteristics for the identification of the transition curve *KP*1 for the test section of the track adapted to *V* = 160 km/h

l_c [m]	a [rad/m]	b [rad/m²]	L_{PKP1} [m]	L_{KKP1} [m]	l_{KP1} [m]
20	−0.00081388	0.00000395	206.187	377.516	171.330
30	−0.00071376	0.00000358	199.475	388.371	188.896
40	−0.00069521	0.00000352	197.300	389.137	191.837
50	−0.00070185	0.00000355	197.966	388.646	190.680

[Authors' own elaboration].

Table 9

Numerical characteristics for the identification of the transition curve *KP*2 for the test section of the track adapted to *V* = 160 km/h

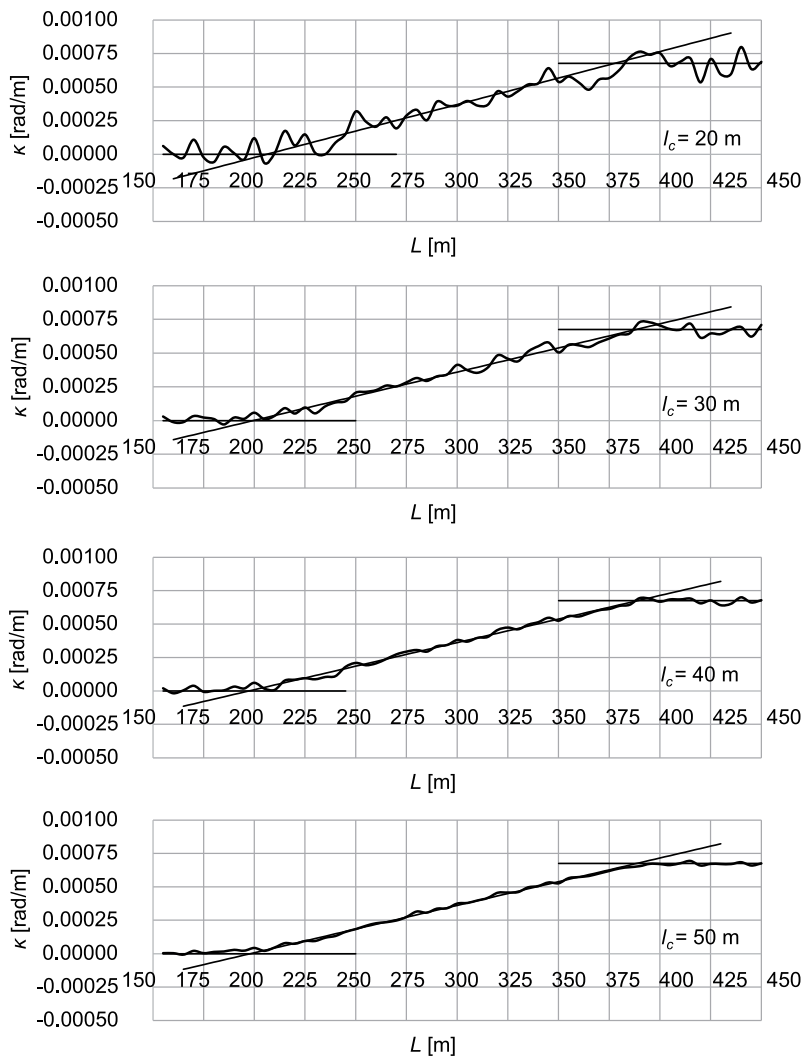| l_c [m]	a [rad/m]	b [rad/m²]	L_{PKP2} [m]	L_{KKP2} [m]	L_{KP2} [m]
20	0.00429970	−0.00000326	1317.037	1103.456	213.581
30	0.00501263	−0.00000382	1311.621	1134.762	176.860
40	0.00495725	−0.00000378	1311.992	1133.091	178.901
50	0.00483857	−0.00000368	1314.097	1130.499	183.598

[Authors' own elaboration].

Fig. 12. Identification of the *KP1* transition curve on a test section of track adapted to a speed of 160 km/h (curvature diagrams obtained using chord lengths l_c = 20 m, 30 m, 40 m and 50 m) [author's own elaboration]
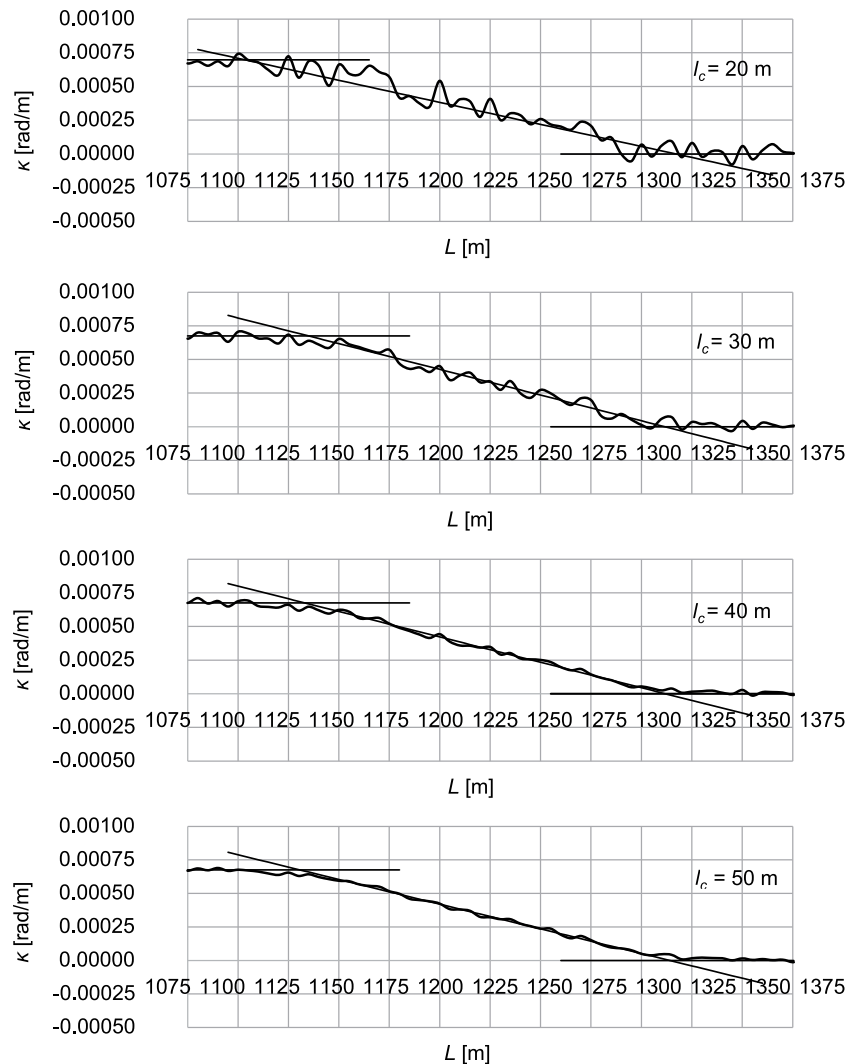
Fig. 13. Identification of the *KP*2 transition curve on a test section of track adapted to a speed of 160 km/h (curvature diagrams obtained using chord lengths l_c = 20 m, 30 m, 40 m and 50 m) [author's own elaboration]

The following observations emerge from the analysis:

- for the identification of the curvature of a circular curve, the use of a chord length of l_c = 50 m proved to be most favourable, l_c = 40 m may also be allowed; the use of l_c = 20 m should be considered unacceptable,
- for the identification of the *KP*1 transition curve, the use of l_c = 50 m has proven to be most favourable, l_c = 40 m and l_c = 30 m may also be allowed; the use of l_c = 20 m is unacceptable,
- for the identification of the *KP*2 transition curve, the use of l_c = 40 m has proven to be most favourable, l_c = 50 m and l_c = 30 m may also be allowed; the use of l_c = 20 m is unacceptable,

In summary, one might argue that, for a circular curve radius $R \cong 1500$ m, it proves most favourable to adopt a virtual chord length of l_c = 50 m (or alter-

natively l_c = 40 m). As before, the shorter chord better determines the length of the transition curve.

6. Recommendations on the accepted length of the virtual chord

On the basis of the analyses carried out, it must be concluded that using a virtual chord length of l_c = 10 m is not a reasonable solution. For large circular curve radii, a chord of l_c = 20 m may also be of little use. The chord length adopted should undoubtedly depend on the value of the radius of the circular curve. The following approximate lengths of l_c can be proposed depending on the range of values of the R_{CC}:

- for $R_{CA} \leq 600$ m l_c = 20 m,
- for $600 < R_{CA} \leq 1000$ m l_c = 30 m,
- for $1000 < R_{CA} \leq 1400$ m l_c = 40 m,
- for $R_{CA} > 1400$ m l_c = 50 m.

However, nothing prevents the use of other, intermediate l_c values as well, following the suggestions mentioned above, as this will most often not have a significant effect on the accuracy of the achieved curvature of the track axis. In this respect, the moving chord method is very flexible.

It is also possible to adopt a different (i.e. shorter) chord when determining the curvature along the length of the transition curve than is the case for a circular curve. During the analysis for the test section $V = 80$ km/h, a chord of $l_c = 30$ m would be most favourable for the identification of the curvature of the circular curve and a chord of $l_c = 20$ m for the identification of the curvature of the transition curve. For the test section $V = 120$ km/h, a chord $l_c = 40$ m for the circular curve and $l_c = 30$ m for the transition curves would be suitable, while the appropriateness of using a chord $l_c = 40$ m for the identification of transition curves on the test section $V = 120$ km/h should be considered (the curvature of the circular curve must be determined here using a chord $l_c = 50$ m).

7. Conclusions

The basis for determining the position of straight sections of the railway track and sections located in a curve, as well as for determining the corresponding geometric parameters, is the knowledge of the curvature of the track axis. The works [23–25] present the concept of a new method for determining curvature (called the "moving chord method") and its verification on an unambiguously defined elementary geometric layout of tracks.

The papers [26, 30, 31] discuss the procedure for estimating the curvature of a track axis by the moving chord method using Cartesian coordinates obtained by direct measurements carried out. It was shown that values of circular curve radius and lengths of transition curves, as well as the location of so-called segmentation points (i.e. boundary points between straight sections, transition curves and circular curves) can be determined from the curvature diagram.

This article focuses on the issue of selecting the chord length that will be most beneficial in a particular situation Although for the model systems, as shown in the work [24], the effect of the applied chord length on the determined curvature values turned out to be insignificant, in the operated railway track, due to track deformations and measurement error, the situation may differ significantly. This is evidenced, for example, by the irregular curvature diagrams shown in the paper [31]. The study analysed three test geometric layouts adapted to the speeds of 80 km/h, 120 km/h and 160 km/h (the radii of circular arcs determined as a result of the curvature esti-

mation performed were approximately 410 m, 880 m and 1480 m, respectively). The lengths of the moving chord in the range of 10÷50 m were considered.

On the basis of the conducted analysis, it was unequivocally demonstrated that the chord length used to determine the curvature in the railway track in service should depend on the value of the radius of the circular curve. The virtual chord should not be too short; for example, using a chord length of $l_c = 10$ m is not a reasonable solution. A chord $l_c = 20$ m may also be of little use for large circular curve radii; in such cases, a chord $l_c = 50$ m should be considered. Indicative lengths of l_c have been suggested, depending on the range of values of R_{CA}, but nothing prevents the use of other, intermediate values of l_c, as this will usually not have a significant impact on the accuracy of the achieved curvature of the track axis. In this respect, the moving chord method is very flexible. It is also possible to adopt a different (i.e. slightly shorter) chord when determining the curvature along the length of the transition curve than is the case for a circular curve. Often, the position and length of the transition curve can be determined more accurately from the curvature diagram obtained using a shorter chord.

The issues presented in the article, concerning the adaptation of the moving chord method to the adopted measurement procedure and the use of the obtained curvature diagram, provide a suitable application basis for the described method. Implementation of the presented procedure should significantly improve the process of identification of geometric layouts of the track in the horizontal plane.

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