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Bond´s work index estimation using non-standard ball mills

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Abstract: Ore concentrators seek the stability of processes by feeding blends of rocks with average hardness and ore content. Therefore, large amounts of samples must be characterized in a short time. The Bond Work Index (Wi) is a common technique for the estimation of hardness and energy requirement for comminution using ball mills. However, this technique is time-consuming (close to 5 hours) and liable to experimental errors. This work contributes to obtaining new models for rapid Bond Work Index estimation using non-standard dimensions mills. This was done by proposing grinding tests using four types of ores and four mills of different dimensions, including the standard Bond ball mill (BBM). For all tests it was kept constant: (a) critical speed (91%), and (b) mill charge by volume (10.5%), varying the amount of fresh feed according to its density. The results showed that using the non-standard mills (between 20 and 35 cm in diameter), the Bond´s model constants (α=0.23; β= 0.82, and γ = 44.5), are unable to predict the Work Index properly. Therefore, these constants must be recalculated using linear models based on mill diameter. With the models proposed for α, β, and γ, the Bond Work Index (kWh/t) can be rapidly estimated (less than 2 hours) and show a high accuracy for mills of non-standard dimensions (R^2= 0.96).

Keywords: Bond´s Work Index, rocks grinding, laboratory scale ball mills, ore characterization, non-standard ball mills

1. Introduction

Unit operations are a key factor in mineral processing, especially those related to particle size reduction. Once the ore is obtained from the mine, it is transported to the concentration plant, where the rocks are reduced in size to the micrometric scale for later concentration (Wills, 2015). This task is performed with specialized machinery such as: crusher, ball mills, autogenous, semi-autogenous mills, among others. Such equipment requires high energy consumption; in some cases, the energy consumption for particle size reduction represents about 50% of the total energy consumption of a concentrator plant (Lynch, 1977; Napier-Munn et al., 1999; Wills, 2015) Therefore, the study of energy applied to mineral processing remains a topic of great relevance. The relationship between energy and change in particle size has been studied with the following differential equation (Charles, 1957; Wills, 2015; García et al. 2021).

$$dE = -k\frac{dx}{x^n} \tag{1}$$

where dE is the specific energy differential, k is a constant, x is the particle size, n is the order of the equation, and dx is the particle size change differential.

By integrating Equation (1), when the order of equation (n) takes numerical values of 1 and 2, the equations of Kick (1885) and Rittinger (1867) are obtained, respectively. Both equations are insufficient to predict the energy required for grinding in industrial-scale ball mills. While Bond (1952) employing n =1.5 proposed that the energy used would be a function of the change in particle size (Equation 2).

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$$E = 2k \cdot \left[\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}}\right] \quad (2)$$

where F_{80} is the particle size at 80% through the size distribution in the feed, and P_{80} is the particle size at 80% through the size distribution in the grinding product, while the constant $2k$=$10Wi$.

Bond's Work Index (Wi) is a property of minerals, which relates the resistance that these oppose to break by the effect of the specific energy applied to the rupture, starting from an indefinite original size, to reach a product with a P_{80} close to 100 µm (Bond, 1952). The specific energy (E) can be estimated by the relationship between the energy consumed [power (P) · time (t)], and the mass of the ground ore ($\dot{m}$).

$$E = \frac{P \cdot t}{\dot{m}} \quad (3)$$

Therefore, to estimate the net power (P), the Bond´s Work Index is required for design an industrial grinding station (see Equation 4).

$$P = Wi \cdot \left[\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right] \cdot \dot{Q} \quad (4)$$

where $\dot{Q}$ is the mass flow of ore to be processed ($\dot{m}/t$).

Bond (1952) proposed a standard procedure to determine the Wi in the laboratory assuming a circulating load of 250%. The standard procedure employs a ball mill with specific dimensions (Bond Ball Mill (BBM), 30.5 cm in diameter and 30.5 cm in length without lifters) and constant operating conditions during the test, such as: volume filling level (10.5%; of this percentage, 70% steel balls and 30% ore), critical rotation speed 91% (70 rpm) and gradient of steel balls (43 balls of 3.70 cm, 67 balls of 3.00 cm, 10 balls of 2.54 cm, 71 balls of 1.90 cm, 94 balls of 1.55 cm, representing a balls load of 20.18 kg). Additionally, Bond proposed the following equation to determine the laboratory Work Index:

$$Wi = \frac{\gamma}{{p_i}^{\alpha} \cdot gpr^{\beta} \cdot \left[\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right]} \quad (5)$$

where γ = 44.5 is parameter related to the mass of balls in pounds (since 44.5 lb = 20.18 kg). p_i represents the opening of the reference mesh (usually the mesh 100# or 149 µm), and gpr is the mass of "fine" particles produced by each ball mill spin or rotations (particles below the reference mesh) when a circulating load of 250% is reached. In addition, Bond proposes values of α =0.23 and β = 0.82.

The criteria used by Bond to determine the numerical values of the constants (α and β) in Equation (5) are unknown. It has been mentioned that Bond determined these parameters based on extensive experimentation with industrial mills of 2.44 m in diameter, operating at 250% of circulating load due to his work in the company Allis-Chalmers (Bond, 1961). Additionally, multiple errors have been reported in the formulation of Equation (5) due to inadequate dimensional analysis, thus considered a "semi-empirical" model. Despite this, it is the most used technique for the estimation of hardness and energy for comminution.

The standard methodology proposed by Bond to determine the Work Index (Wi) consists of a series of laboratory-scale and batch-based dry milling and sieving. The ore to be characterized is firstly prepared (F_{80} below 3350 µm). Using the reference mesh and adjusting for each grinding cycle the number of ball mill spins (initially 100 rotations). This to find the mass of fines produced by spin or revolution (gpr), which represent a circulating load of 250%. This is on the premise that more ball mill spins will produce finer particles, and therefore less circulating load (circulating load=coarse particles/fines particles). Usually after eight grinding cycles the process stabilizes at circulating load values close to 250%. The test ends by determining the granulometric analysis of the fine fraction of the last cycle to obtain the P_{80}. The experimental values of gpr, P_{80} and F_{80} are replaced in Equation (5) to obtain the laboratory Work Index.

The standard Bond test has disadvantages, one of which is the large number of cycles required to reach the circulating load of 250%. Therefore, it is a test that takes between 5 and 6 hours to perform, this is not feasible for an adequate large ore deposits characterization. This increases the number of experimental errors. For example, the inadequate classification between coarse and fine affects the estimation of the grams produced per revolution (gpr), making the test unnecessarily prolonged.

Other authors have proposed reduced methodologies for estimating the Work Index, performing only two grinding and classification cycles, considering a first order grinding kinetics (Magdalinovic, 1989; Ahmadi et al., 2009, Chakrabarti, 2013; Todorovic et al. 2017; and Chitalov et al., 2019).

Bond ball mills are popular and widely available in the raw material processing industry. But laboratory mills of non-standard dimensions are more common in several mining units; therefore, some concentrator plants cannot estimate the standard Bond´s Work Index. Some authors have proposed the simulation approach in determining the work index (Lira et al., 1990; Tavares et al., 2007 and Menéndez-Aguado et al., 2013). In the present article, two grinding test strategies where employed: the standard procedure proposed by Bond (1961) and the reduced procedure proposed by Ahmadi et al. (2009). Both were used to estimate the Bond Work Index (Wi), for four different ores and four mills of different dimensions, including the standard Bond ball mill. This to find the empirical correlations that allow to estimate the value of the constants α, β and γ, with the use of mills of non-standard dimensions. The information generated in this research is original and useful for mining operations that want to quickly estimate Bond's Work Index, for large ore deposits hardness characterization, especially where Bond ball mill is not available.

2. Materials and methods

2.1. Ore characteristics

Fig. 1 (a) shows the ore used in the determination of the Work Index with the standard methodology (Bond, 1961), the reduced methodology (Ahmadi et al., 2009), and with the use of standard and non-standard ball mills. The ores used for these tests were obtained from several mining operations in Zacatecas, Mexico. Ore A (28.75% Si, 7.50% Al, 3.1% K, 2.13% Fe and 1.30% Ca; apparent density 1.70 g/cm^3) consists of a mixture of minerals of which quartz and aluminosilicate abound. Ore B (0.95% Pb, 1.46% Zn and 9.94% Fe; apparent density 1.56 g/cm^3) is a polymetallic primary sulphide mineral (pyrite, sphalerite, and galena). Ore C (5.74% Pb, 2.96% Zn and 17.92% Fe; apparent density 2.23 g/cm^3) is a massive, sulphured ore with high iron content and ore D (31.5% Si and 12.2% Fe; apparent density 1.82 g/cm^3) contains native gold and silver. On the other hand, Fig. 1(b) shows the particle size distribution of each ore with which the work index was evaluated. In all tests the size F_{80} is below 2000 μm.

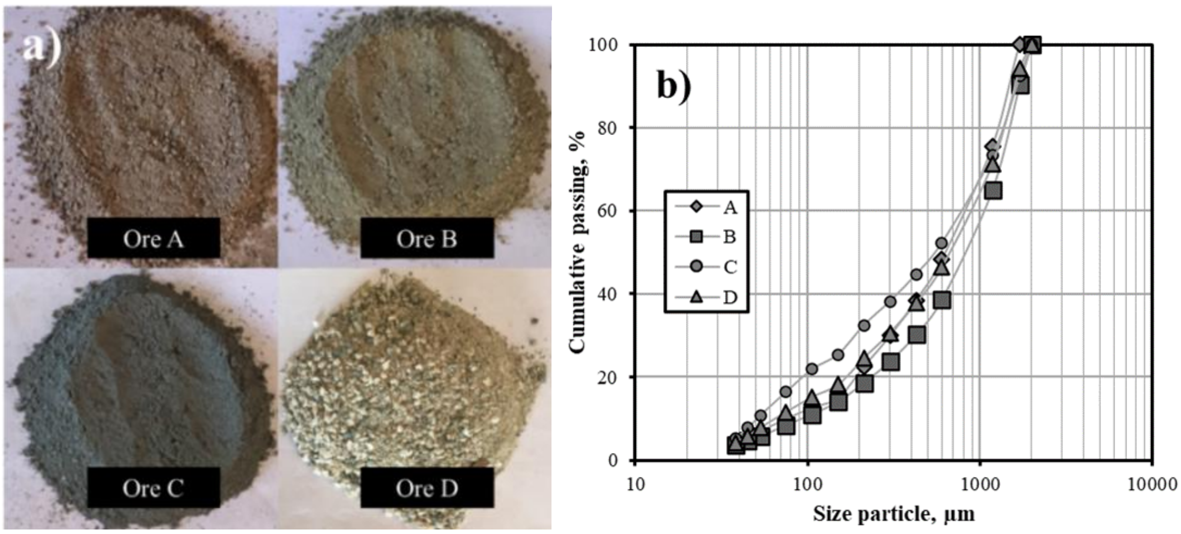

Fig. 1. Ores for the work index test: a) type and b) feed size

2.2. Ball mills and operation conditions

Fig. 2(a-d) shows the ball mills used in the determination of the Work Index (standard mill BBM and non-standard mills BM-1 to BM-3, all ball mills without lifters). Bond standard ball mill (Fig 2(a); BBM) has a total volume of 22284 cm^3, is fed with 700 cm^3 of ore and 1638 cm^3 of steel balls. This corresponds to a filling charge of 10.5% by volume, of which 30% is ore and 70% steel balls (loaded with the gradient). On the other hand, in Fig. 2(b-d), the non-standard ball mills can be seen, that were mounted on rotating rollers with controlled speed.

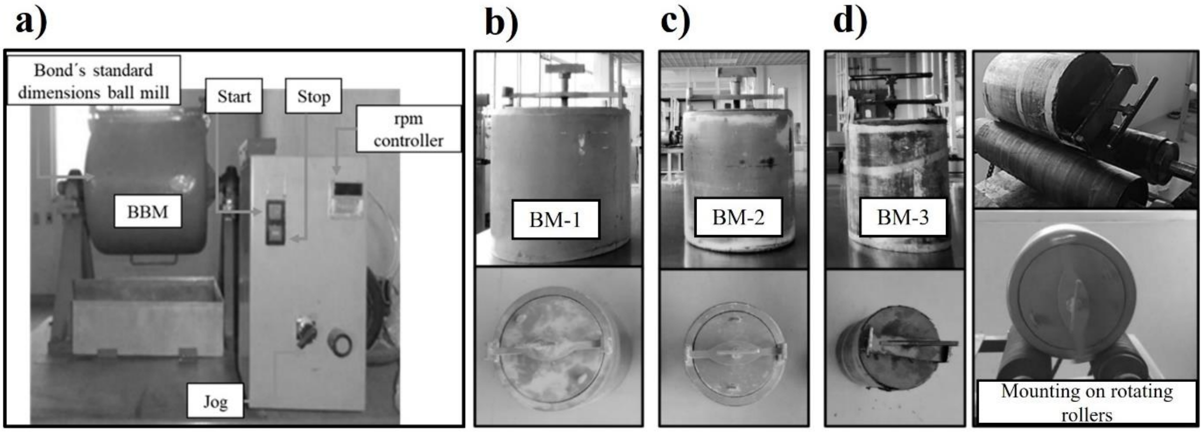

Fig. 2. Ball mills used in determining the working index. a) standard bond mill, and c-d) non-standard mills

Operating conditions of non-standard mills were matched to standard Bond ball mill operating conditions based on: (a) the filling volume for each mill used, (b) the ore density and (c) the critical speed (calculated from the diameter of the mill and using a speed controller). More detail is seen in the following equations.

Rotational velocity (for 91% critical speed), % $$\frac{76.6}{\sqrt{ball\ mill\ diameter/31.5}} * 0.91 \quad (6)$$

Ball mill total volume, cm^3 $$\pi \cdot \left[\frac{ball\ mill\ diameter}{2}\right]^2 \cdot ball\ mill\ length \quad (7)$$

Ball mill fill charge by volume (10.5%), cm^3 $$Ball\ mill\ total\ volume \cdot 0.105 \quad (8)$$

Ball mill fill charge (iron balls), cm^3 $$Ball\ mill\ fill\ charge\ by\ volume \cdot 0.700 \quad (9)$$

Ball mill fill charge (ore), cm^3 $$Ball\ mill\ fill\ charge\ by\ volume \cdot 0.300 \quad (10)$$

Ore charge, g $$Ball\ mill\ fill\ charge\ (ore) \cdot ore\ density \quad (11)$$

Table 1 summarizes the equivalent operating conditions of the standard mill and non-standard mills. The required mass for each test and each mill was calculated by considering the load volume and the density of the ore used (e.g., use of the BM-1 and ore A: 1061cm^3 ·1.70 g/cm^3 = 1803.70 g).

In the case of filling with steel balls, for all tests with non-standard dimensions, mono size balls of 2.54 cm in diameter were used (2475 cm^3/ 8.58 cm^3≈288 steel balls, to minimize the interstitial spaces). The weight of the balls in small mills also affects the grain size of the product. Although tests with non-standard mills did not use a balls gradient. This is justified since the comparative study is based on the filling volume (ore and balls). Additionally, there are parameters that affect to a greater extent the performance of discontinuous grinding in small mills (between 20 to 35 cm in diameter), these are: critical speed, grinding time (or ball rotations) and the mass of ore to be ground. These variables were controlled in this investigation. Additionally, the optimal size of balls for non-standard mills was calculated with the Allis Chalmers methodology (Bond, 1961).

2.3. Methodologies in Bond´s Work Index estimation

In Fig. 3, an algorithm for the easy estimation of the standard Work Index using the Bond procedure (with BBM) and with the non–standard mills (BM-1, BM-2, and BM-3) is presented. In this figure, $700cc\_wt$ is the mass of the ore in 700 cubic centimetres to the ball mill feed. When a ball mill with non–standard dimensions is used, this fresh feed changes (as can be seen in Table 1), varying the mass in the ore feed as a function of its density.

Before feeding the mineral to the mill, the feed granulometry must be obtained and then the

Table 1. Rotation speed and filling charge in standard and non-standard ball mills

			(a)				(b)	(c)
Ball mill type	Ball mill name	Dimensions (Diameter x Length), cm	Ball mill total volume, cm^3	Ball mill fill charge by volume (10.5%), cm^3	Ball mills fill charge (ore), cm^3	Ball mills fill charge (iron balls), cm^3	Ore density, g/cm^3	Rotational velocity (91% critical speed), rpm
Standard	BBM	30.5 x 30.5	22284	2340	702	1638	A: 1.70 B: 1.56 C: 2.23 D: 1.82	70
Non-standard	BM-1	35.0 x 35.0	33674	3536	1061	2475	A: 1.70 B: 1.56 C: 2.23 D: 1.82	65
Non-standard	BM-2	26.0 x 32.0	16990	1784	535	1249	A: 1.70 B: 1.56 C: 2.23 D: 1.82	76
Non-standard	BM-3	20.3 x 28.8	9321	979	294	685	A: 1.70 B: 1.56 C: 2.23 D: 1.82	86

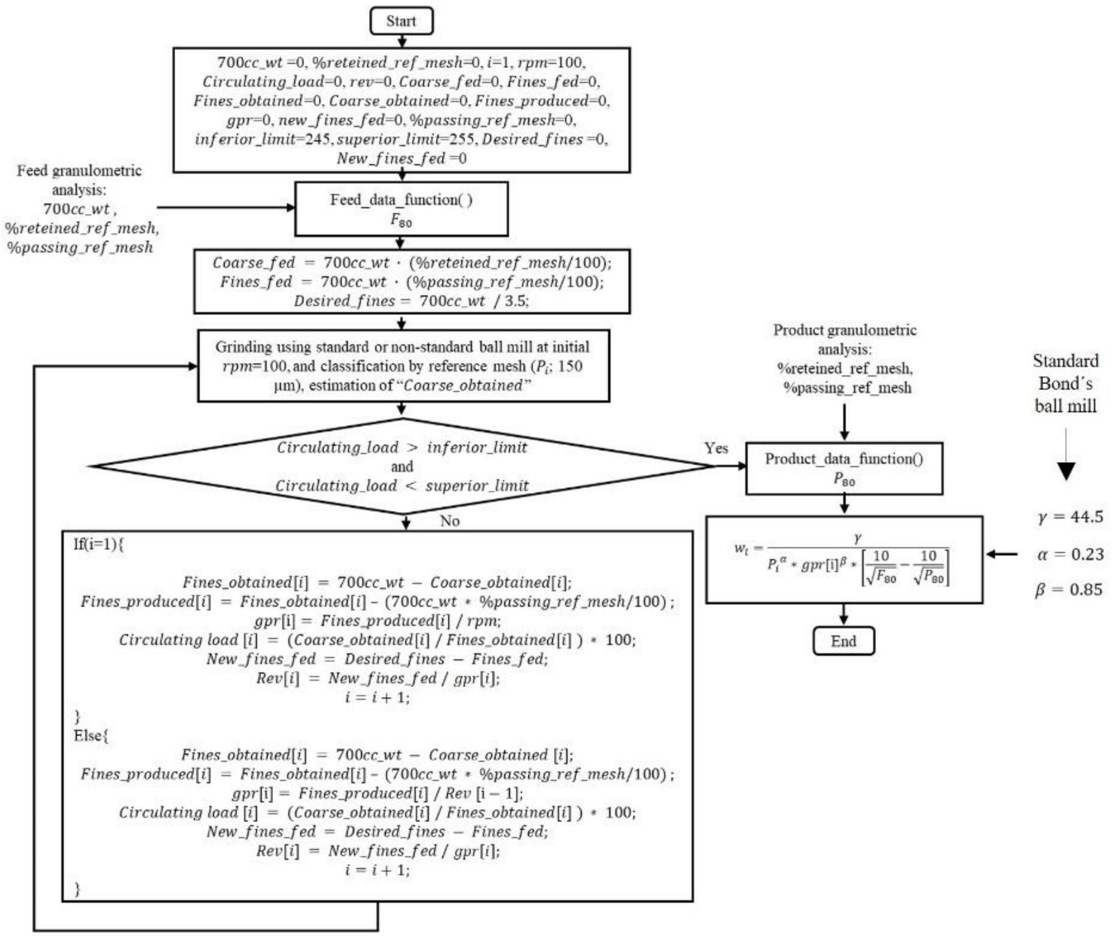

Fig. 3. Algorithm for standard Bond´s Work Index determination

retained and passing percentage through the reference mesh can be computed ($\%reteined\_ref\_mesh$ and $\%passing\_ref\_mesh$). $Desired\_fines$ is the amount of fines sought to obtain a desired circulating load (250%). Once the granulometric analysis in the feed is obtained, the first grinding can be started, starting with 100 revolutions (rpm). after the first grinding cycle, the classification is carried out using the reference mesh (Pi) to obtain the mass retained above the reference mesh ($Coarse\_obtained[i]$) and the through mass of the reference mesh after the first milling cycle ($Fines\_obtained[i]$). The new fines produced by the first grinding cycle ($Fines\_produced\ [i]$) represents the mass of new product obtained below the reference mesh (Pi), this value divided by the initial revolutions (rpm = 100), will provide the initial value of grams per revolution in the first cycle ($gpr[i]$). $Circulating\_load\ [i]$ will represent the circulating load in the first cycle. The mass of mineral retained in the reference mesh ($Coarse\_obtained\ [i]$) is supplemented with fresh load to obtain the same feed mass ($700cc\_wt$) in the second cycle. $Rev[i]$ represents the new ball mill revolutions in the second grinding cycle ($i = i + 1$), the grinding and classification cycles continues until a circulating load is 250% plus an error which define the permissible limits ($inferior\_limit$ and $superior\_limit$). It is worth mentioning that to calculate the subsequent mill spins, the New_fines_fed and the grams produced by the revolution $gpr[i]$ must be considered (See Fig. 3; $Rev[i] = New\_fines\_fed\ /gpr[i]$). Finally, Equation 5 is used for Work Index estimation.

The reduced methodology consists of carrying out two mineral grindings and estimating the grams per revolution (grp) in a similar way to that proposed in the standard methodology (Ahmadi et al. 2009).

To estimate the parameters of Equation (5) with the results obtained with a non-standard mill, non-linear regression was used, by the least squares method, making use of the following equation:

$$min \sum_{i=1}^{n}[(Wi_{BBM} - Wi_{NSBM})^2] \tag{12}$$

where n is the number of tests of this study, Wi_{BBM} is the Work Index of each ore obtained with the Bond mill (BBM), and Wi_{NSBM} is the Work Index obtained with the standard methodology with the use of mills BM-1, BM-2, and BM-3. The following equation was used to estimate the absolute percentage error:

$$\%error = \left|\frac{Wi_{BBM} - Wi_{NSBM}}{Wi_{BBM}}\right| \cdot 100 \tag{13}$$

where Wi_{BBM} is the Work Index obtained with the Bond methodology as the true value and Wi_{NSBM} is the Work Index obtained with non-standard mills as the value of the approximation.

3. Results and discussion

3.1. Estimation of the Work Index using the standard and reduced methodology with the standard Bond ball mill (BBM)

Table 2 presents the work index determined by the standard procedure (Bond, 1960) and the reduced procedure (Ahmadi and Shahsavari, 2009), using the Bond ball mill (BBM).

Table 2. Comparison of the work index determined in BBM by the standard and reduced procedure

Ore type	Wi_{BBM} Standard procedure (Bond), kWh/t (Mg)	Wi_{BBM} Reduced procedure (Ahmadi), kWh/t (Mg)	Error, %
A	12.3	11.8	4.1
B	15.9	16.5	3.8
C	18.7	19.7	5.3
D	16.5	17.2	4.2

In this table, the reduced procedure presents good accuracy in the estimation of the Work Index since it presents a maximum error of 5.3%. The reduced method allows the test to be performed in a time not exceeding 2 hours. For the search of the values of the constants (α, β and γ) when using non-standard mills, all the data presented in Table 2 were involved. This to obtain the Bond´s constants that can be used for both methodologies (standard and reduced), when using mills with non-standard dimensions.

3.2. Estimation of the Work Index using the standard and reduced methodology with the Bond mill (BBM) and with the non-standard mills (BM-1, BM-2, and BM-3)

Table 3 shows the estimation of the Work Index with non-standard mills (Wi_{NSBM}) and the use of the constants defined in the standard procedure (α = 0.23, β = 0.85, and γ = 44.5). In this table it can be observed that, the precision in the estimation of the Work Index is directly related to the diameter of the ball mill used. There is less error in the estimation of the Work Index with the largest mill (BM-1, maximum error 15.3%) in which the constants obtained with the standard procedure can be functional because, the diameter of BM-1 is close to the diameter of the standard mill (30.5 cm). On the other hand, the greatest error was manifested with the non-standard mill with the smallest diameter (BM-3; maximum error 68.3%). This behaviour can be observed, both for the tests carried out with the standard method and the reduced method. Therefore, regardless of the method employed, the Bond equation parameters are not suitable for their use in calculating Work Index with non-standard laboratory ball mill. Even though grinding conditions such as mill fill and critical speed, were like those proposed in the standard Bond method.

Fig. 4 (a–d) shows the circulating load obtained experimentally in each grinding and classification cycle for all ores and using four ball mills with the standard procedure.

Table 3. Work Index estimation (kWh/t) with the standard and reduced methodology, using standard and non-standard mill using α = 0.23, β = 0.85, and γ = 44.5

		BM-1		BM-2		BM-3	
Ball mill diameter, cm		35.0		26.0		20.5	
Ore type	procedure	Wi_{NSBM}	Error, %	Wi_{NSBM}	Error, %	Wi_{NSBM}	Error, %
A	Standard	13.7	11.4	10.1	17.9	3.9	68.3
A	Reduced	13.6	15.3	9.5	19.5	3.8	67.8
B	Standard	16.8	5.7	13.7	13.8	5.4	66.0
B	Reduced	16.3	1.2	13.2	20.0	5.5	66.7
C	Standard	18.8	0.5	16.1	13.9	7.1	62.0
C	Reduced	19.4	1.5	16.0	18.8	7.0	64.5
D	Standard	17.2	4.2	13.8	16.4	5.6	66.1
D	Reduced	17.0	1.2	13.8	19.8	5.6	67.4

In this figure, it can be identified that in the first cycle a low circulating load is produced, in some cases less than 250%, this is because, in the initial condition of 100 rpm, a large amount of fines is generated (particles smaller than the reference mesh). For the second cycle, the number of revolutions is decreased, decreasing the amount of fines produced and obtaining a much higher circulating load. As of the 3rd cycle, this value begins to decrease until it reaches the desired circulating load (250%), obtained in the 8th cycle. Some authors (Bond, 1961; Mosher et al., 2001, Ozkahraman, 2005) mention that after the cycle where a circulating load of 250% is reached, another 3 cycles must be carried out to ensure the process stabilization. However, it has been identified that this cycle's stabilization depends on the experimental procedure and the adequate preparation of the samples (sieving).

In some cases (when a reference mesh finer than no. 100 is used), the obstruction of the mesh prevents the stabilization of the cycles (Makhija et al., 2016, Nikolic et al., 2021). In Fig. 4, it is evident that, with the modification of the grinding operating conditions, adjusting the filling of ore and balls according to the volume of the mill, it is possible to reproduce the fracture mechanics (Austin, 1971 and Austin et al., 1971; Herbst et al., 1980) defined as the probability of particle selection (Si) and particle breakage (Bij), that occurs with the use of a standard dimensioned Bond ball mill (BMM), a graphic representation of both functions are presented in Fig. 5.

In Fig. 5 context, more revolutions will represent more grinding time, thus, more fine particles will

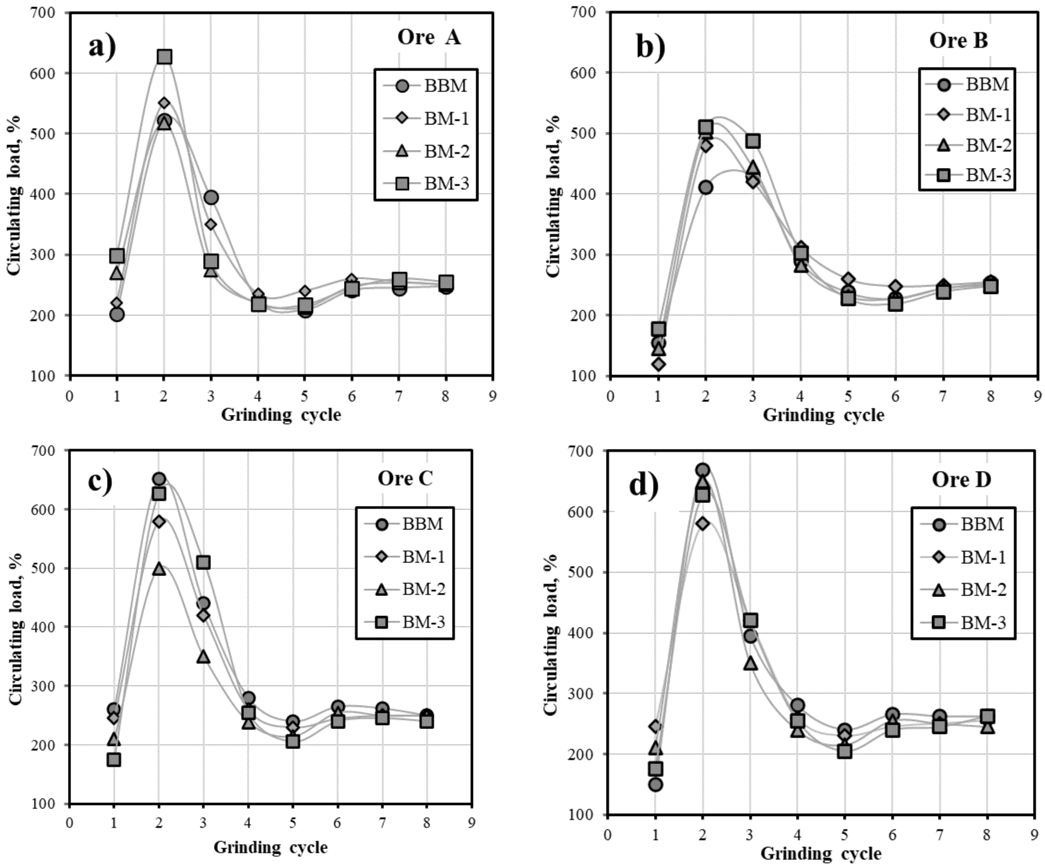

Fig. 4. Circulating load for each grinding cycle using the standard methodology, BBM and non-standard mills: a) ore A, b) ore B, c) ore C, and d) ore D

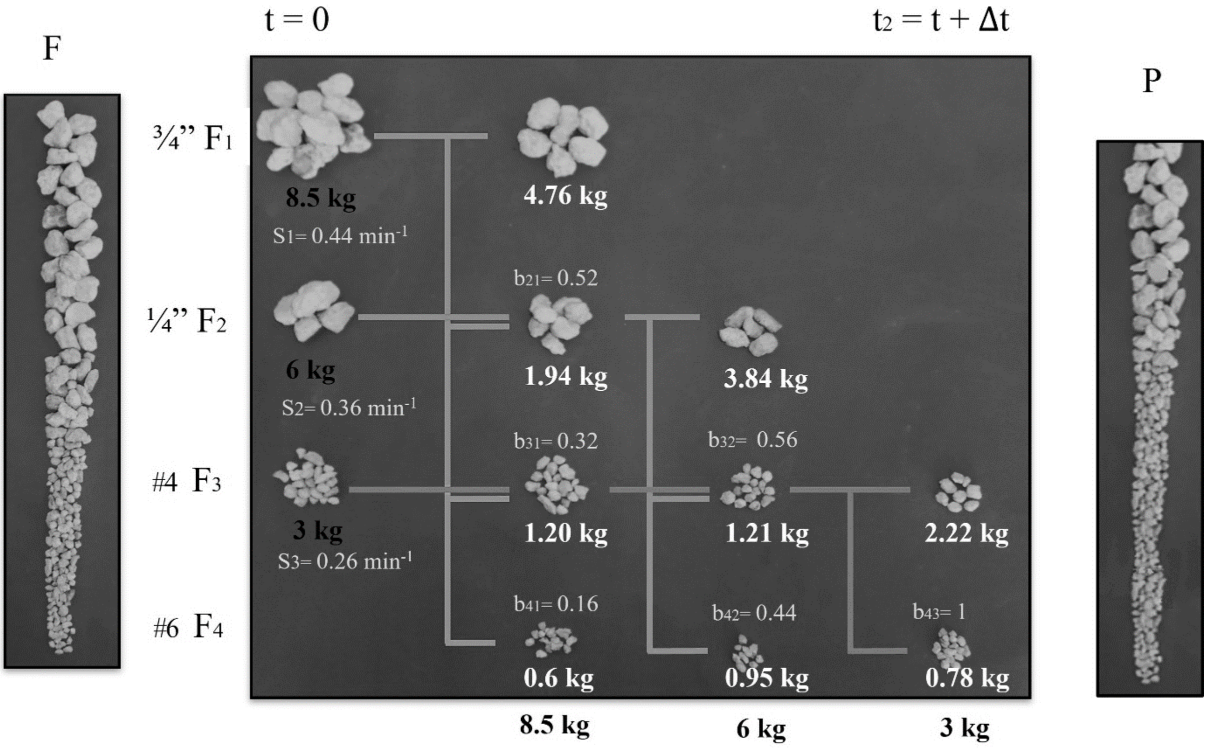

Fig. 5. Mass balance including the fracture mechanics

be produced (Bond´s cycles are in linear grinding behavior; this means more grinding time represents more fines produced). Even though the mechanics of particle fracture is similar, to find precise values of the Work Index, the constants (α, β, and γ) must be recalculated based on the diameter of the mill used, as expressed in the analysis of the Table 3.

Fig. 6(a) shows the grams of fines per revolution in the eighth grinding cycle (gpr in the 8th cycle), as a function of the Work Index obtained for each ore, using the BBM and non-standard ball mills. In this figure it can be observed that the production of grams per revolution increases with the ores with a higher Work Index, they also increase with the use of larger diameter mills (affected by the ore filling charge). When using the smallest ball mill (BM-3), it is required to feed the least amount of ore mass and steel balls into the mill to obtain a constant 10.5% fill volume. Feeding less mass to the mill generates the fewest grams per revolution since there will be less mass of coarse particles that can be selected for the breaking process and generation of fine particles (related with Fig. 5).

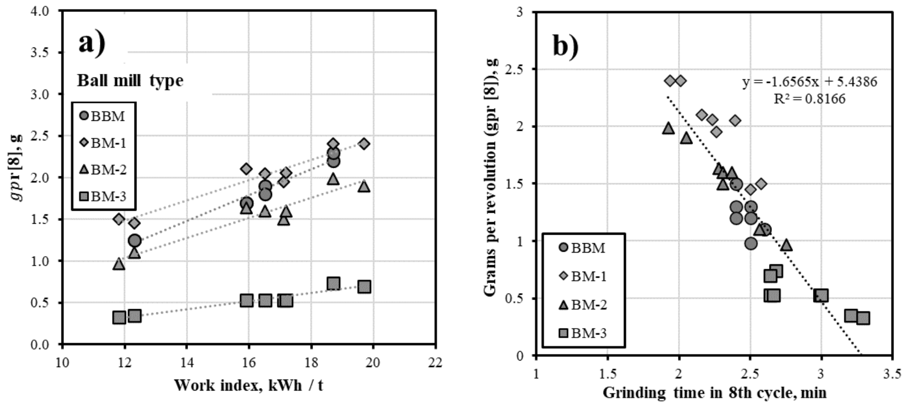

Fig. 6. Grinding products. a) grams per revolution in the 8th. Cycle and b) grams per revolution as a function of the grinding time of the last cycle

On the other hand, with the use of the largest diameter mill (BM-1), which exceeds the BBM diameter, the greatest amount of fines generated per revolution (gpr) can be seen. In the standard Bond procedure, each cycle is fed to the mill with fresh charge and coarse ore (ore retained on the reference mesh) from the previous cycle, therefore, for each cycle, coarser ore is fed. Coarse particles fed will have a greater probability of being selected for a fracture process, thus generating a greater quantity of fine grams per revolution. Fig. 6(b) shows the relationship between the generation of grams of fines per revolution and the grinding time in the eighth cycle, for each mill and type of ore (the speed of rotation is different for each mill, see Table 1). In this figure, there is a relationship between the grams of fines produced per revolution of the mill and the time spent in the last grinding cycle. With the larger mill (BM-1), less grinding time is required to produce the grams per revolution needed to obtain a circulating head of 250%. This is because the test is carried out under conditions of a greater mass of ore and a greater number of steel balls. Even though using the largest mill represents less grinding time. It should be considered that, by involving a greater mass of ore in the test, the sieving time will be much longer to make an adequate estimate of the grams of fines produced. Therefore, for the rapid estimation of the Work Index, the use of smaller diameter mills is recommended, in which the grinding time is slightly increased, but by decreasing the amount of mass used, the sieving time will be reduced.

3.3. Estimation of the Bond´s parameters: α, β and γ

Fig. 7 (a-d) presents the parameters α, β, and γ calculated with each mill diameter, using least squares nonlinear regression (Equation (12) and the coincidence between Wi_{NSBM} and Wi_{BBM} (Fig. 7(e)).

Fig. 7(a-c) shows that the three parameters are adjusted to linear models that have the diameter of the mill as a dependent variable (exhibiting a connection factor greater than $R^2 = 0.90$). On the other hand, Fig. 7(d) shows an extrapolation of the trend lines presented in Fig. 7(a-c) for mill diameters greater than the studied in the present study (up to a mill with a diameter of 2.44 m). As appeared in the introduction, no clear information has been found on how Bond could propose the values of the constants (α, β, and γ). Fig. 7(d) shows that the parameters obtained in this research coincide with those proposed by Bond (1961). From the above, it can be assumed that Bond used a similar scheme, where the reference mills were 2.44 m, 0.7 m, and 0.30 m. It is worth noting that the constants converge on the 0.7m mill which could be used to define the constant $2k = 10Wi$.

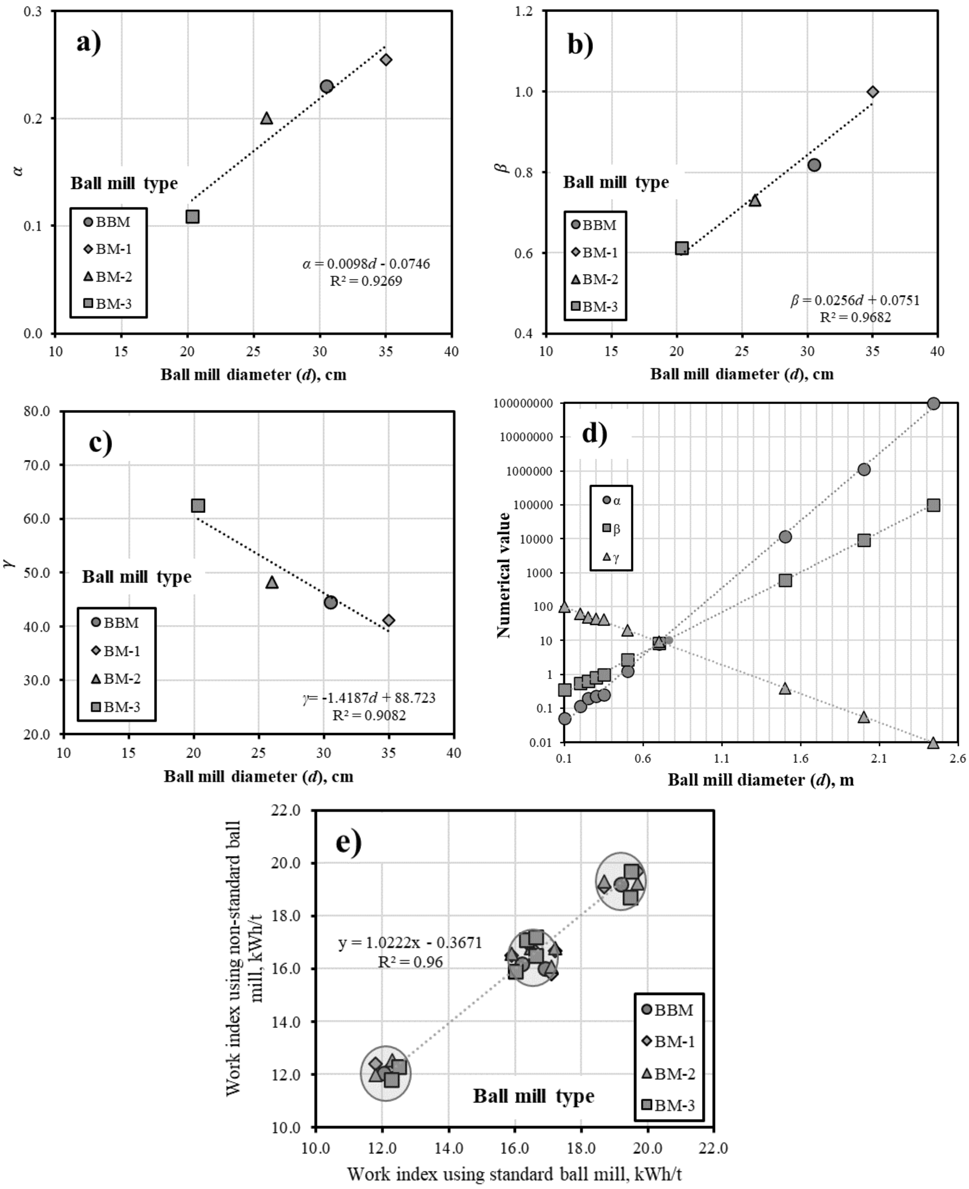

Fig. 7. Calculation of parameters a) α, b) β, c) γ, d) extrapolation of the study parameters, and e) correlation between Wi_{NSBM} and Wi_{BBM}

Finally, Fig. 7(e) shows the certainty that exists between the work index obtained with the standard (Wi_{BBM}) and non-standard ball mills (Wi_{NSBM}) with the standard and reduced methodology, using the parameters computed for each ball mill. In this figure it can be observed an adequate prediction of the Work Index presented with a correlation R^2=0.96. This is adequate since at least 50% of the time invested in determining the Work Index was reduced.

4. Conclusions

This article shows that it is possible to calculate the Bond´s work index in non-standard dimension ball mills. This with the use of new constants from the Bond´s model, calculated from the diameter of the mill. Minerals with different hardness were used in order to estimate parameters that are representative of the grinding of sulphides. It is shown that, matching the grinding operating conditions in Bond test (e.g., ball mill filling by volume, fresh feed influenced by density, and the critical speed), the fracture mechanics is reproducible in non-standard mills, with which it is possible to obtain a valid approximation of the grams that pass the reference mesh per revolution (gpr). For the rapid determination of the Work Index, the reduced methodology of Ahmadi (2009) can be used with the use of a ball mill of non-standard dimensions.

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