Comparative Study between Several Direction of Arrival Estimation Methods

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Abstract—In this paper a comparative study, restricted to one-dimensional stationary case, between several Direction of Arrival (DOA) estimation algorithms of narrowband signals is presented. The informative signals are corrupted by an Additive White Gaussian Noise (AWGN), to show the performance of each method by applying directly the algorithms without pre-processing techniques such as forward-backward averaging or spatial smoothing.

Keywords—array processing, Direction of Arrival, geolocalization, propagation, smart antenna, spectral analysis.

1. Introduction

In array signal processing Direction of Arrival estimation (DOA) [1], [2] stands for estimating the angles of arrivals of received signals by an array of antennas. It is considered an important processing step in many sensors systems, i.e., radar, sonar, Measure Electronic Surveillance (MSE), submarine acoustics, geodesic location, optical interferometry, etc.

There are many types of DOA algorithms that have been proposed during the past four decades such as conventional spectral-based, subspace spectral-based and statistical methods. Beamforming techniques [3]–[7] are straightforward and require low computational power but these methods have low resolution [8]. That leads to introduction of subspace-based algorithms [9]–[11] that use the eigendecomposition of output data covariance matrix in order to obtain the so-called signal subspace or noise subspace. However these methods become limited in case of larger number of array sensors, many fast algorithms for DOA have been proposed in recent years such as the propagator method (PM) [12]–[14] without eigendecomposition with low computational load. Unfortunately, this method is only suitable to the presence of white Gaussian noise, and its performance will be degraded in spatial nonuniform colored noise. To overcome this problem, a modified PM algorithm has been proposed with different computation method for the propagation operator [15]. It is only obtained by the partially cross-correlation of array output data which makes it suitable for the case of spatially nonuniform colored noise due to using the off-diagonal elements of array covariance matrix.

This paper presents a comparative study that is restricted to one-dimensional stationary case (azimuth) between several DOA estimation algorithms of narrowband signals [16] that are corrupted by uniform Additive White Gaussian Noise (AWGN). The performance of each method is evaluated by applying directly the algorithms on Uniform Linear Array (ULA) without pre-processing techniques such as forward-backward averaging of the cross correlation of array output data R or spatial smoothing. The authors choose the key factor for this evaluation to be the Signal to Noise Ratio (SNR) of the environment surrounding the ULA and the radiating sources while the number of snapshots constant is maintained.

1.1. Problem Statement

Typical smart antenna architecture of base station can be divided into the following functional blocks as shown in Fig. 1 [16]. Radio signals arriving at the array antennas are conversed from analog to digital form by downconversion and sampling operations, next summation of the digitized signals over all array elements produces single stream output for further processing.
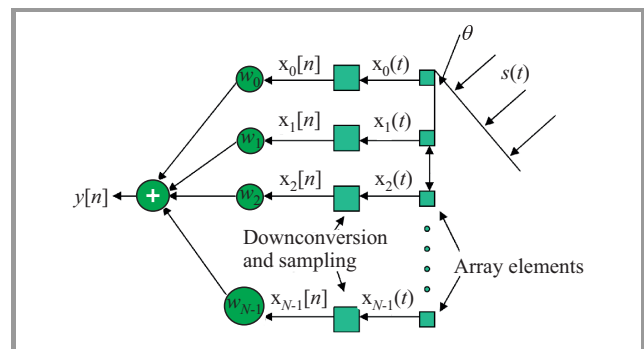

Fig. 1. Typical front-end architecture of base station receiver.

Let's consider an array of N elements receiving P signals such that each element of the array contains zero mean Gaussian noise, the output array is given by:

$$y[t] = \sum_{k=1}^{N} w_k x_k[t], \qquad (1)$$

where:

$$x(t) = A(\theta)s(t) + N(t) \qquad (2)$$

$x[t] = [x_1(t), \ldots, x_N(t)]^T$, $A(\theta) = [a(\theta_1), \ldots, a(\theta_p)]$ are the received array data and the array manifold matrix respectively, $s(t) = [s_1(t), \ldots, s_p(t)]^T$ and $N(t) = [n_1(t), \ldots, n_N(t)]^T$ stand for the source waveform vector and sensor noise vector, respectively. In Eq. (2)

$$a(\theta_i) = \left[1, \, e^{\frac{j2\pi d}{\lambda} \sin(\theta_i)}, \ldots, e^{\frac{j2\pi d(N-1)}{\lambda} \sin(\theta_i)} \right]^T$$

is the steering vector, and d is the distance between elements of the Uniform Linear Array (ULA), λ is the wavelength of the propagating signals, θ_i is the angle of arrival of the ith source and $(.)^T$ denotes the transposition of matrix.

The array signal waveform is considered as stationary process therefore the $N \times N$ correlation matrix can be defined as:

$$R = E\left[(X(t) - m_x(t)).(X(t) - m_x(t))^H \right], \qquad (3)$$

where $(.)^H$ denotes the conjugate transposition of matrix.

In this study it is assumed that:

- the signals and the additive Gaussian noise are stationary and ergodic zero mean complex valued random processes,

- the signals sources are not correlated,

- the set of P steering vectors is linearly independent and the P signal sources are statistically independent of each other,

- the number of sources P is known and the number of sensors N satisfies the condition $N \geq 2P + 2$.

Under those assumptions the cross correlation matrix is given by:

$$\begin{aligned} R &= E\left[A(\theta)S(t)S^H(t)A^H(\theta) \right] + E\left[(N(t)).N^H(t) \right] \\ &= A(\theta)R_{ss}A^H(\theta) + \sigma^2 I_N, \end{aligned} \qquad (4)$$

where $R_{ss} = E\left[S(t)S^H(t) \right]$ is $P \times P$ source signal covariance matrix, σ^2 is the noise variance and I_N stands for an $N \times N$ identity matrix.

In practice, the exact covariance matrix R is unavailable and must be estimated from the received data. The forward-only estimate of covariance matrix is given by:

$$\hat{R}_{xx} = \frac{1}{K} \sum_{k=1}^{K} XX^H. \qquad (5)$$

In the Section 2 different algorithms for DOA estimation are presented.

2. DOA Algorithms

2.1. Beamforming Techniques

The beamforming techniques are based on scanning all possible angles in the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and measuring the output power of the array such that the power spectrum has peak when the given angle is the direction of arrival of one of the incoming signal. The output signal $y(t)$ is computed using a weight vector w with the received data x:

$$y(t) = w^H x(t). \qquad (6)$$

Given N spanshots, the total output power of an array is:

$$\begin{aligned} P(w) = \frac{1}{N} \sum_{n=1}^{N} |y(t_n)|^2 &= \frac{1}{N} \sum_{n=1}^{N} w^H x(t_n) x^H(t_n) w \\ &= w^H \hat{R}_{xx} w. \end{aligned} \qquad (7)$$

Based on the Eq. (7) two main techniques have been developed.

2.2. Bartlett Method

Also known as method of averaged periodograms [3], Bartlett method computes the power spectrum as follows. Let $w = a(\theta)$ be the steering vector with arbitrary scanning angle:

$$a(\theta) = \left[1, \, e^{j\mu}, \ldots, e^{j(N-1)\mu} \right],$$

$$\mu = \frac{-2\pi f_c}{c} d \sin\theta,$$

where f_c is the carrier frequency of the incoming narrowband signals, c is the speed propagation of the wave signals and d stands for distance between array sensors.

The weight vector is normalized as the following:

$$w = \frac{a(\theta)}{\sqrt{a^H(\theta)a(\theta)}}, \qquad (8)$$

and the spatial spectrum is then given by:

$$P(\theta) = P_{bart}(\theta) = \frac{a^H(\theta)\hat{R}_{xx}a(\theta)}{a^H(\theta)a(\theta)}. \qquad (9)$$

The weight vector w can be considered as spatial filter, which has been matched to the incoming signal, the array weighting equalizes the delays experienced by the signal on various sensors to combine their respective contributions.

2.3. Capon Beamformer

Capon beamformer is an enhanced version of the Bartlett method, when the sources to be located are closer than the beamwidth, The Bartlett method fails in separating the sources, for this purpose Capon in [4] proposed the maximum likelihood method to solve the for Minimum Variance

Distortion Response (MVDR) of an array such that it maximizes the signal to interference ratio:

$$\min \left(P(w) \right) \qquad \text{subject to} \qquad w^H a(\theta) = 1 .$$

The resulting weight vector is given by:

$$w = w_{Capon} = \frac{\hat{R}_{xx}^{-1} a(\theta)}{a^H(\theta) \hat{R}_{xx}^{-1} a(\theta)} \qquad (10)$$

Replacing the weight vector w in the Eq. (7) yields to the power spectrum:

$$P(\theta) = P_{Capon}(\theta) = \frac{1}{a^H(\theta) \hat{R}_{xx}^{-1} a(\theta)} . \qquad (11)$$

2.4. Linear Prediction

The linear prediction method [5] is widely used in spectral analysis and speech processing. It is based on the concept of minimizing the mean output signal power of the array elements subject to constraint that the weight on a selected element in ULA is unity. The array weight vector is given by:

$$w = \frac{\hat{R}_{xx}^{-1} u}{u^H \hat{R}_{xx}^{-1} u} ,$$

where u is the mth column vector of the identity matrix I_{NxN} such that the index m represents the mth element of the ULA. No optimized criterion is proposed for the choice of this element.

The power spectrum can be computed as:

$$P(\theta) = P_{LP}(\theta) = \frac{u^H \hat{R}_{xx}^{-1} u}{|u^H \hat{R}_{xx}^{-1} a(\theta)|^2} . \qquad (12)$$

The choice of the mth element affects the resolution capability of this method which is dependent on the SNR, and the minimum angle separating the sources.

2.5. Maximum Entropy

Maximum entropy technique [9] is an improvement of the beamforming approach, based on extrapolation the covariance matrix. The extrapolation should be selected with maximized signal entropy where its maximum is achieved by searching for the coefficients of an auto-regressive (AR) model that minimize the expected prediction error:

$$a = \arg\min \left\{ a^H \hat{R}_{xx} \right\} ,$$

subject to the constraint that the first AR coefficient satisfies $a^H e_1 = 1$ where $a = [a_1, a_2, \ldots, a_N]^T$ and e_1 is the first column of the identity matrix I_N. Applying the Lagrange multiplier technique yields to

$$a = \frac{\hat{R}_{xx}^{-1} e_1}{e_1^T \hat{R}_{xx}^{-1} e_1} .$$

Next the spatial spectrum can be computed as

$$P(\theta) = P_{ME}(\theta) = \frac{1}{|a(\theta)^H C_j|^2} , \qquad (13)$$

where C_j represents the jth column of the inverse cross correlation matrix $\hat{R}_{xx}^{-1}$.

The quality of the resolution of the maximum entropy method depends on the choice of column C_j.

2.6. Pisarenko Harmonic Decomposition

Pisarenko harmonic decomposition method [9] minimizes the Mean Square Error (MSE) of the array output under the constraint that the norm weight vector to be equal to unity. The eigenvector that minimizes the MSE corresponds to the smallest eigenvalue of the cross-correlation of array output data, the output power is given by:

$$P(\theta) = P_{PHD}(\theta) = \frac{1}{|a(\theta)^H \bar{e}_1|^2} , \qquad (14)$$

where $\bar{e}_1$ is the eigenvector associated with the smallest eigenvalue σ_1.

2.7. Minimum Norm

The minimum norm technique [1], [9] is generally considered to be a high-resolution method which assumes a ULA structure.

The algorithm is described as the following. After estimating the cross correlation matrix $\hat{R}_{xx}$, a Singular Value Decomposition (SVD) is performed to extract the matrices U, S and V such that $\hat{R}_{xx} = USV'$. Next, a noise subspace is constructed by selecting the set of vectors $E_N = U(:, P+1:N)$ where P and N denotes the number of radiating sources and the number of elements in the ULA respectively. Constructing the spectrum is based on minimum norm vector lying in the noise subspace whose first element equals 1 and having minimum norm, this condition is satisfied by using the first column of the identity matrix $u = [1 \ 0 \ 0 \ \ldots \ 0]^T$ to compute the following spatial spectrum:

$$P_{MN}(\theta) = \frac{1}{|a(\theta) E_N E_N^H u|^2} , \qquad (15)$$

where $a(\theta)$ is the array steering vector and E_N is the noise subspace with columns representing the eigenvectors $[e_1, e_2, \ldots, e_{N-P}]$.

2.8. MUSIC Algorithm

Multiple Signal Classification (MUSIC) method [10] is widely used in signal processing applications for estimating and tracking the frequency and emitter location.

This method is considered as a generalization of the Pisarenko's one [9]. It is based on spectral estimation which exploits the orthogonality of the noise subspace with the signal subspace.

Assume that $\hat{R}_{xx}$ is *NxN* matrix with rank *P*, therefore it has $N - P$ eigenvectors corresponding to the zeros/smallest eigenvalues in the absence/presence of noise. The eigende-composition of $\hat{R}_{xx}$ is given by:

$$\hat{R}_{xx} = \sum_{i=1}^{N} \lambda_i q_i q_i^H = Q_s \Delta_s Q_s^H + Q_n \Delta_n Q_n^H, \qquad (16)$$

where

$$\Delta_s = diag[\lambda_1, \lambda_2, \ldots, \lambda_P],$$

$$\Delta_n = diag[\lambda_{P+1}, \lambda_{P+2}, \ldots, \lambda_N],$$

$$\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_P > \lambda_{P+1} = \lambda_{P+2} = \ldots = \sigma_N^2,$$

$Q_s = [q_1, q_2, \ldots, q_P,]$ is the signal subspace corresponding to Δ_s and $Q_n = [q_{P+1}, q_{P+2}, \ldots, q_N]$ is the noise subspace corresponding to Δ_n.

The MUSIC spectrum is given by:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) Q_n Q_n^H a(\theta)}. \qquad (17)$$

When scanning the angles in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, if θ is DOA of one of signals, so $a(\theta) \perp Q_n$ the denominator is identically zero and the spectrum identifies the angle as a peak.

2.9. Propagator Method

Unlike the MUSIC algorithm, the propagator method [12]–[14] is computationally low complex because it does not need eigendecomposition of the covariance matrix, but it uses the whole of it, to obtain the propagation operator. Therefore, this algorithm is only suitable to the presence of white Gaussian noise and its performance will be degraded in spatial non-uniform colored noise. The propagator is constructed as the following. The covariance matrix can defined as:

$$\hat{R}_{xx} = \begin{bmatrix} R_1 & R_2 \end{bmatrix}^T,$$

where R_1 and R_2 are *PxN*, $(N-P)xN$ matrices respectively.

In noiseless system:

$$R_2 = P^H R_1. \qquad (18)$$

In noisy environment the least mean squares technique (LMS) is used to estimate *P* that minimizes the Frobenius norm $||R_2 - P^H R_1||$:

$$P^H = R_2 (R_1^H R_1)^{-1} R_1^H. \qquad (19)$$

Next, the matrix *Q* is constructed, such that:

$$Q^H = \begin{bmatrix} P^H & -I_{N-P} \end{bmatrix}. \qquad (20)$$

The spectrum is given by:

$$P(\theta) = P_{propag}(\theta) = \frac{1}{||Q^H a(\theta)||^2}. \qquad (21)$$

2.10. Partial Covariance Matrix

Partial covariance matrix technique [15] is an enhanced version of the propagator method, where no eigendecom-position is needed. The different approach for computing the propagation operator is based on using three submatri-ces of the estimated cross-correlation matrix $\hat{R}_{xx}$. The array manifold matrix can be portioned as:

$$A = \begin{bmatrix} A_1^T, A_2^T, A_3^T \end{bmatrix}, \qquad (22)$$

where A_i, $i = 1, 2, 3$ is matrix with dimensions $P \times P$, $P \times P$ and $(N - 2P) \times P$ respectively.

The following partial cross-correlation matrices of the array output are defined as :

$$R_{12} = E\left[X(1:P,:) X(P+1:2P,:)^H \right] = A_1 R_{ss} A_2^H, \qquad (23)$$

$$R_{31} = E\left[X(2P+1:N,:) X(1:P,:)^H \right] = A_3 R_{ss} A_1^H, \qquad (24)$$

$$R_{32} = E\left[X(2P+1:N,:) X(P+1:2P,:)^H \right] = A_3 R_{ss} A_2^H. \qquad (25)$$

Based on these sub-matrices, the matrix Q is:

$$Q^H = \begin{bmatrix} R_{32} R_{12}^{-1} & R_{31} R_{21}^{-1} & -2I_{N-2P} \end{bmatrix}$$

Multiplying *Q* with the steering matrix yields to:

$$Q^H A = 0, Q^H a(\theta_k) = 0 \quad (k = 1, 2 \ldots, p). \qquad (26)$$

The spectrum is then, similarly to the propagator method, given by:

$$P(\theta) = P_{partial}(\theta) = \frac{1}{||Q^H a(\theta)||^2}. \qquad (27)$$

3. Simulation Results

A comparative study [17] has been made between 7 algo-rithms for DOA, using 4 elements and 2 sources with fixed SNR = 10 dB and the 2 sources were separated by $d = 80°$. This study focused on the performance of the algorithms based on the number of snapshots by simulating the first time with L1 = 10 then with L2 = 100 snapshots.

In this paper, real life scenario is simulated by studying the performance of each method based on the noise environ-ment by testing with SNR1 = 1 dB (high noise level) and SNR2 = 20 dB (low noise level). To evaluate the Rayleigh angle resolution limit, for example the second and the third radiating sources were chosen to be separated by 6° while the number of snapshots was fixed.

The authors consider Uniform Linear Array (ULA) com-posed of $N = 10$ identical sensors with half wavelength inter-element spacing and $P = 4$ almost equally powered emitting sources with carrier frequency $fc = 1$ GHz. The distance between two sensors is $d = 15$ cm so the total distance of the array is 135 cm and $K = 200$ snapshots. For simulation on evaluating each method the Monte-Carlo method was used such as each result is an average of $L = 100$ runs.

The sources are non-coherent as given by the normalized cross-correlation matrix R_{ss}:

$$R_{ss} = \begin{pmatrix} 1.00 & 0.00 & 0.00 & 0.04 \\ 0.00 & 1.00 & 0.05 & -0.05 \\ 0.00 & 0.05 & 1.00 & 0.08 \\ -0.04 & -0.05 & 0.08 & 1.00 \end{pmatrix}.$$

In Table 1 the configuration of the described sources is presented.

Table 1
Sources characteristics

Sources	S1	S2	S3	S4
DOAS [°]	−24	15	21	70
Power [W]	1.20	1.30	1.44	1.50

Figure 2 shows the results of the Bartlett spectrum, apparently the maximum resolution for this method is more than 6°, which makes inappropriate for this case. In the previous studies [17], the authors show that ideal resolution of this algorithm is 20°.
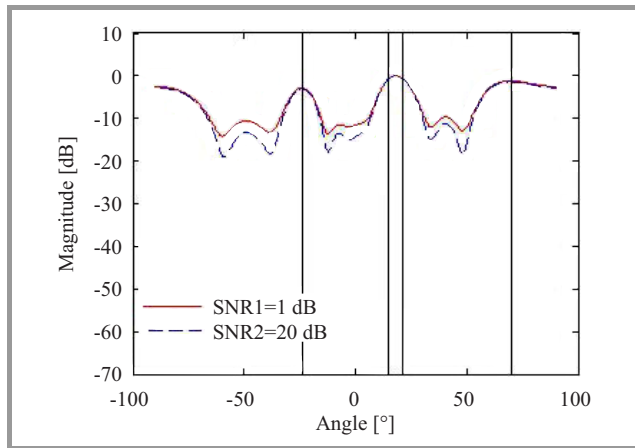
Fig. 2. Bartlett spectrum.

Figure 3 represents the Capon beamformer spectrum which is better performing than the Bartlett method, at SNR = 20 dB the algorithm detects well the sources, but
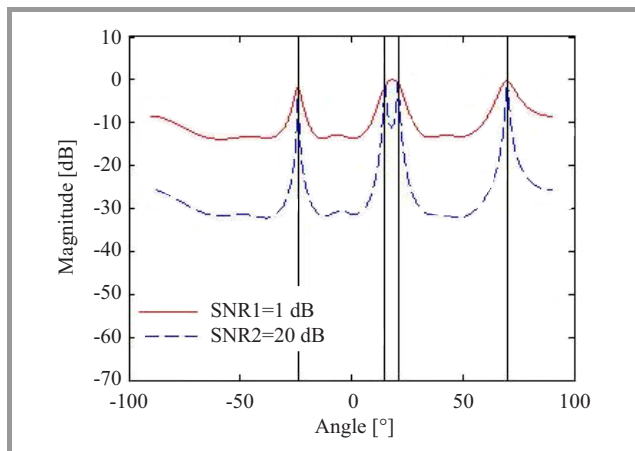
Fig. 3. Capon beamformer spectrum.

in high-level noise it fails to separate the second and the third sources located at (15°, 21°). The numerical tests at SNR = 1 dB showed that the algorithm can separate the sources with minimal difference of 9°.

Figure 4 shows the result of Linear Prediction algorithm, by choosing the fifth element as the vector u, in the Eq. (12), from the identity matrix $I_{10\times10}$

$$u = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

This algorithms performs better than the two previous techniques, it separates well the closed sources at low SNR.
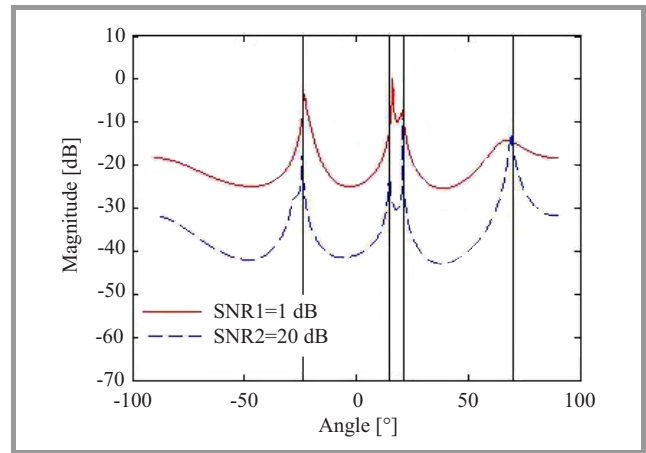

Fig. 4. Linear prediction spectrum.

Figure 5 shows the result of the maximum entropy DOA estimate, by choosing the vector $\bar{C}_j$ as the first column of the inverse cross-correlation matrix $\hat{R}_{xx}^{-1}$ in the Eq. (13).
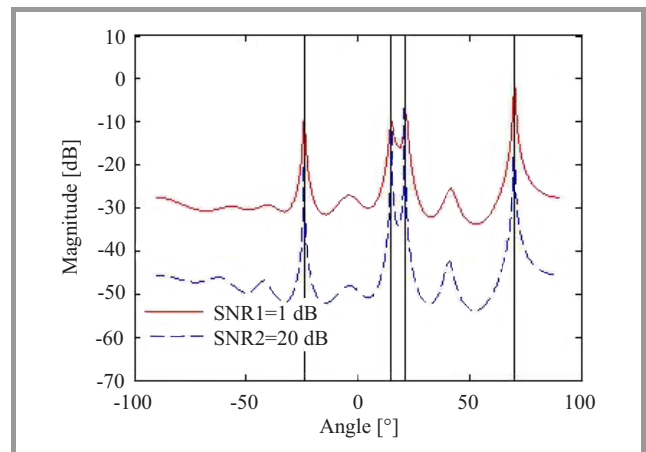

Fig. 5. Maximum entropy spectrum.

This technique performs well by separating the sources at both noise levels which makes it better than Bartlett, Capon and linear prediction methods, however the choice of the column $\bar{C}_j$ influences the performance. As in [17], the jth column was chosen to be in the center of the cross correlation matrix, but in this study the first column was chosen which gives also good results.

In Fig. 6, the application of the Pisarenko harmonic decomposition, at SNR = 20 dB, gives almost the same spectrum

of the maximum entropy method, while at SNR = 1 dB, the spectrum detected well the first source at $-20°$, could not separate the second and the third angles while the last source is detected at $67°$, which makes this technique non convenient in low SNR condition.
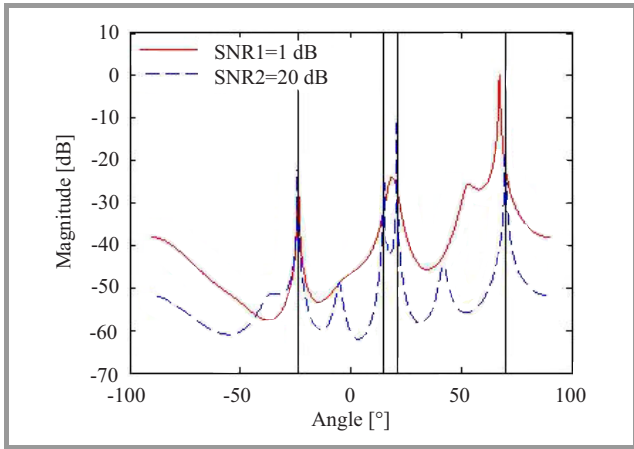

Fig. 6. Pisarenko harmonic decomposition spectrum.

Figure 7 illustrates the minimum norm spectrum which is almost identical with maximum entropy method but with higher number of floating point operations.
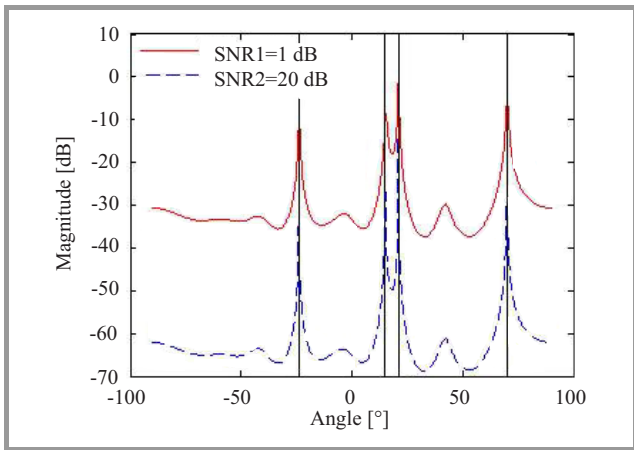

Fig. 7. Minimum norm spectrum.

It should be noted that all the methods are computed using MATLAB and the results are plotted in decibel using the formula

$$P(dB) = 10\log_{10}\left(\frac{spectrum}{Max[spectrum]}\right),$$

to produce a unique frame for comparison [18].

The MUSIC algorithm gives the best result compared to the previous algorithms, as illustrated in Fig. 8, because it detects well all the sources in any noise level and its spectrum does not contain side lobes unlike other techniques.

Note that in high level noise, the spectrum has minimum magnitude of –50 dB while the minimum norm presents a minimum at –60 dB.
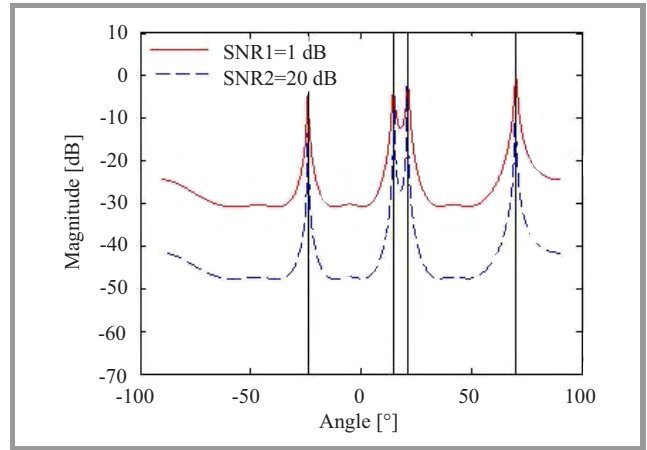

Fig. 8. MUSIC spectrum.

Although, the MUSIC algorithm may fail to resolve the high correlated sources which makes preprocessing techniques like the forward backward averaging or spatial smoothing mandatory to decorrelate the sources.

The propagator method, shown in Fig. 9, has identical performance in both noise levels with minimum apparition of side lobes.

The main advantage of the propagator method is that the constructed matrix Q in Eq. (20) does not need any eigendecomposition, hence the complexity is reduced to $NPK + O(P^3)$ [9].
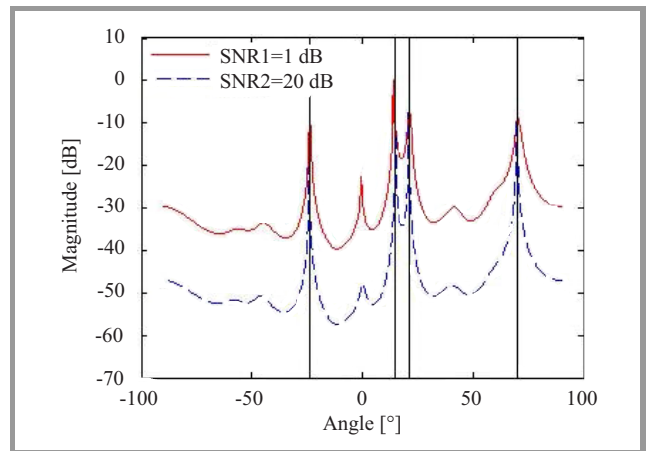

Fig. 9. Propagator spectrum.

Finally, the partial covariance matrix algorithm (without eigendecomposition) is shown in Fig. 10. The results are almost identical with the propagator method, except a noticeable increase in the two side lobes. What makes this technique better than that of the PM method is that the complexity [15] is reduced to $(N-P)PK + O(P^3)$ and takes only partial cross correlation matrices to compute the spectrum. Therefore it is effective in the case of nonuniform colored noise.

The second simulation is based on the average Root Mean Square Error (RMSE) over K = 100 runs between the true DOAs and the nine normalized spectrums, with chrono-
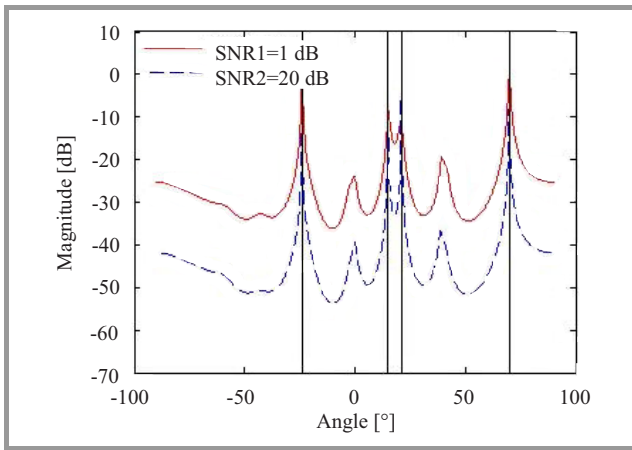
Fig. 10. Partial covariance spectrum.

logical order as described in this paper, computed for two values of SNR:

$$RMSE(\hat{P}(\theta), P(\theta)) = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (\hat{P}(\theta_n) - P(\theta_n))^2}.$$

Figures 11–12 represent the RMSE between each method and the true spectrum for SNR = 1 dB and SNR = 20 dB respectively.
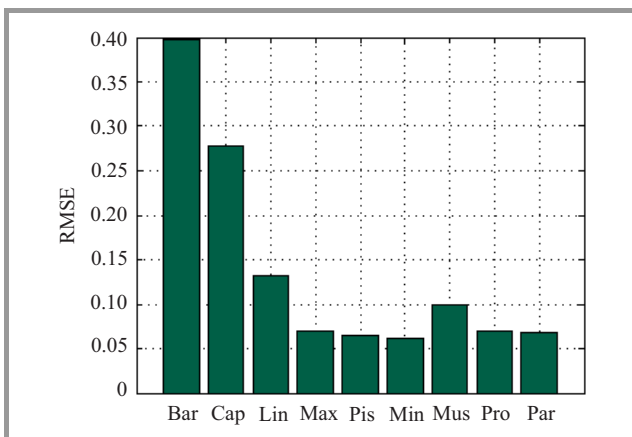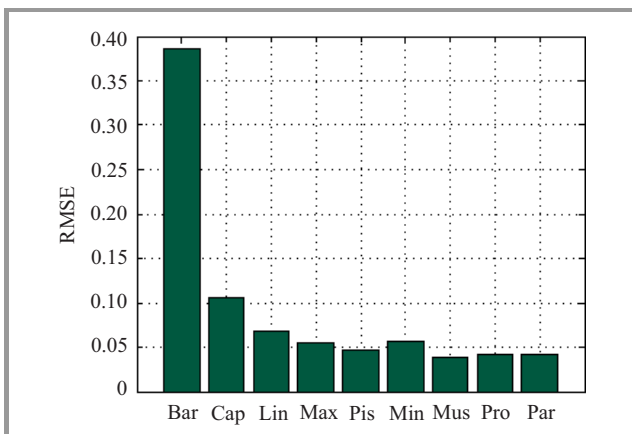

Fig. 11. RMSE, SNR = 1 dB.

Fig. 12. RMSE SNR = 20 dB.

4. Conclusions

In this paper, some algorithms for one dimensional narrow-band direction of arrival (DOA) estimation in stationary case for smart antennas, and for spatially uniform AWGN was compared, starting with the Bartlett method to the recent algorithm which is the partial covariance. In order to evaluate its performance four non-correlated and almost equally powered emitting sources was considered such that two of the sources are separated of 6°, the SNR of 1 dB and 20 dB was the key factor for evaluation. The results showed that in high-level noise, the minimum norm algorithm performs well while in the low-level noise the MUSIC, propagator and partial covariance matrix methods are almost the same and give good results.

In the perspective study, the authors will try to evaluate the partial covariance matrix algorithm in the case of two dimensional wideband sources.

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