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TEMPERATURE RECONSTRUCTION ERRORS CAUSED BY REPRESENTATION OF A PRIORI INFORMATION WITH USE EMPIRICAL ORTOGONAL FUNCTIONS

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The restoration of mean temperature of water layer is considered when the expansion of inhomogeneities of temperature field on empirical orthogonal functions (EOF) is used for description of a priori information. In practice only several first EOF giving the basic contribution into average square of a sound speed variation are taken into account. Even small modification of statistical model of inhomogeneities can noticeably change for the worse the quality of restoration of parameters of inhomogeneities. It is shown that the neglect by higher EOF, despite of their rather small contribution, can noticeably increase an error of the restoration of mean temperature of water layer.

INTRODUCTION

In classical approach of Munk and Wunsch [1] to the ocean acoustic tomography the temperature reconstruction reduces to solution of a set of linear equations. The number of equation in the set is determined by the number M of the measured ray travel times. Let's define N as a number of unknown parameters specifying the temperature field. As a rule, an inequality N>>M takes place and the set of equations is undetermined. There are two ways to solve this problem.

One way is reconstruction of average temperature in a water volume [2]. It is obvious that quality of detail reconstruction can not be satisfactory. But the ray travel time is determined by "integral" influence of all temperature inhomogeneities crossed by rays' trajectory. It means that temperature variation averaged over a large enough area may be reconstructed with good accuracy.

Second way is the use of parameterization. The expansion of temperature inhomogeneities field on orthogonal functions with rejection of its part [3,4] is frequently used in the ocean acoustic tomography. Empirical orthogonal functions (EOF) are often used for reconstruction. EOF are eigenfunctions of a correlation matrix of temperature inhomogeneities. In practice, only several firsts EOF giving the basic contribution in Average Square of a sound speed variation are taken into account. This rough model can cause the inaccuracies.

In this report we analyze those additional errors, which arise because of such rough model of inhomogeneities. The analysis was carried out for the western region of the Mediterranean Sea where a group of researchers of the Institute of Applied Physics, aboard a research vessel, participated in the international tomography experiment under the THETIS-2 project [5].

1.ANALYTICAL EVALUATIONS OF RELATIVE ERROR OF RESTORATION

We consider the simplest situation of a single point sound source and a single point receiver. In an unperturbed waveguide a source and a receiver are connected by M rays, which we will enumerate.

Let as usually there is an information about some basic sound speed profile and sound speed variation is small enough, that is possible to use the linear theory for account of ray arrival time.

The variation of arrival time δt, caused by difference of the real sound speed from the basic one is determined by expression

$$\delta t = -\int_{\eta} \frac{\delta c(\eta)}{c^2(\eta)} ds ,$$

where ds - the element of a ray trajectory length η.

As usual the field of temperature inhomogeneities we will present as expansion on EOF, eigenfunction of the correlation matrix. Let inhomogeneities field of sound speed (proportional temperature inhomogeneities) are presented as $\delta c(z) = \sum_n x_n \varphi_n(z)$, where $\varphi_n(z)$ - nth EOF.

Factors of expansion we will consider as the elements of a column vector $\mathbf{x}$. Ocean acoustic tomography assumes restoration of the given vector by a solution of a system of linear equations $\mathbf{y}= \mathbf{Ax}$. There $\mathbf{y}$ is column vector, which elements are the variations of times of ray arrivals δt_m, and the elements of a matrix $\mathbf{A}$ are equal $A_{mn} = \int \frac{ds\varphi_n(z_m(s))}{c^2}$. The size of $\mathbf{A}$ is equal $M \times N$, where M is number of solved rays, and N is number of empirical orthogonal functions. Here index m numbers rays hitting in a point observations, the integration is conducted along a trajectory mth ray $z = z_m(s)$, ds means the element of length of an arc of this ray, c is unperturbed sound speed in the point of a trajectory. A priori information about a statistic of inhomogeneities is presented by parameters EOF and diagonal matrix $\mathbf{K}$ with size $N \times N$ with the nonzero elements $K_{mm}=<x_m^2>$.

The mean temperature of water layer $z_1 < z < z_2$ is proportional to the scalar product $\gamma = \mathbf{g}^T \mathbf{x}$, where mth element of the weighting vector $\mathbf{g}$ is $g_m = \int_{z_1}^{z_2} dz \varphi_m(z)$.

Optimal linear estimate of quantity γ, which we designate through $\hat{\gamma}$, minimized the mean square $<(\gamma - \hat{\gamma})^2>$, is $\hat{\gamma} = \mathbf{s}^T\mathbf{y}$, where $\mathbf{s} = (\mathbf{AKA}^T)^{-1}\mathbf{AKg}$.

In this case, the relative accuracy of reconstruction of the quantity γ is

$$\varepsilon = <\left(\gamma - \hat{\gamma}\right)^2> / <\gamma>^2 = 1 - \frac{\mathbf{g}^T\mathbf{KA}^T(\mathbf{AKA}^T)^{-1}(\mathbf{AK})\mathbf{g}}{\mathbf{g}^T\mathbf{Kg}}.$$

This relation is obtained in the supposition that the correlation matrix of inhomogeneities is precisely known.

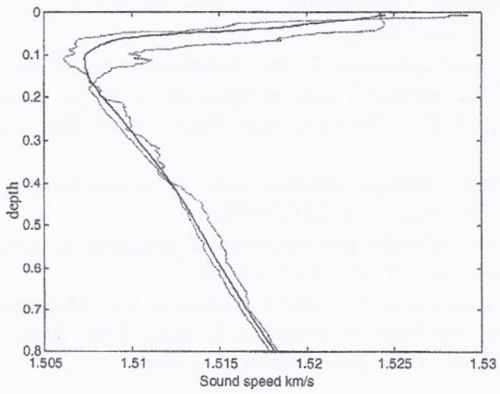
The rejection of a part EOF is equivalent to a replacement of true matrixes **K** and **A** to "short" matrixes $\mathbf{K_c}$ and $\mathbf{A_c}$ obtained by a rejection of the elements. These elements correspond to EOF not taken into account. The relative error of restoration of average temperature is determined by the relation

$$\varepsilon = 1 - \frac{\mathbf{g_c}^T\mathbf{K_c}\mathbf{A}^T(\mathbf{A_c}\mathbf{K_c}\mathbf{A_c}^T)^{-1}\left(2\mathbf{A_c}\mathbf{K_c} - \mathbf{AKA}^T\left(\mathbf{A_c}\mathbf{K_c}\mathbf{A_c}^T\right)^{-1}\mathbf{A_c}\mathbf{K_c}\right)\mathbf{g_c}}{\mathbf{g}^T\mathbf{Kg}}$$

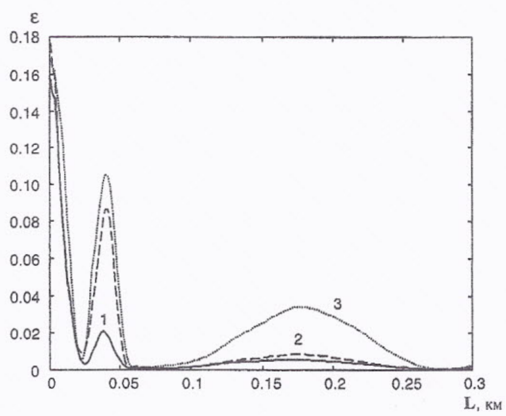
It is interesting to investigate a problem of sensitivity of restoration quality to a choice of an amount taken EOF. In other words what value it is necessary to pay for an inaccuracy of a used a priori information.

2.NUMERUCAL SIMULATION

In this report we analyze this problem for condition typical for the western region of the Mediterranean sea where the mentioned expedition was carried out. On the sound speed profiles, measured in expedition (see Figure) the evaluation of a correlation matrix of inhomogeneities of a medium was found and were calculated EOF.



Two typical profiles (dashed lines) and profile obtained by an average on all measurements, carried out in July, 1994 (solid line) are present on Figure. The basic inhomogeneities are concentrated in an interval of depths from a surface up to 200 m. That is why, EOF more than 200 m fast falls down on depths. Numerical simulation by ray code shows that pulses, propagated on separate rays, are not resolves in the accepted signal (it is supposed, that the duration pulse is 0.01 c). However, 5 group of four pulses (5 ray fours) are well resolved. It is 5 group of steep rays, which not concerning bottom. That is why restoration of layer mean temperature was produced on measurements "center of gravity" such group.



Relative error of mean temperature restoration in a horizontal layer of water from a surface up to depth L in dependence of L with the account all (curve 1), first ten (curve 2) and first seven (curve 3) empirical orthogonal functions is reduced on the following Figure.

The result of calculation according to exact formula coincides with a result obtained with restriction by 10 first EOF. However it is clear, that the further diminution of number taken into account EOF should noticeably increase the error of the of restoration. The result submitted in a figure presented that it happens already with restriction of number taken into account EOF up to seven. As shows the analysis of a correlation matrix of inhomogeneities, the summarized contribution all EOF with numbers n> 7 is small. So, the rejection of a component of a field formed these EOF will reduce in a diminution of average square of a sound speed variation only on 4%.

CONCLUSIONS

This example demonstrates the importance of using of an adequate a priori statistical model of inhomogeneities of a medium. Even small modification noticeably changes for the worse the quality of restoration of parameters of inhomogeneities. By the given report we want to attract attention in this possible source of errors.

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