SOME SIMULATION AND OPTIMIZATION EXPERIMENTS ON PREY-PREDATOR MODEL

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Abstract: The paper is an extension of the authors Kasperska E., Kasperski A., Bajon T. and Marjasz R. work (Kasperska et al., 2015) in the area of modeling and simulation of ecosystems on the base of "prey-predator" system dynamics model (presented on DSTA conference in 2015). The problem of connecting simulation with optimization is specially expanded to include both types of optimization - the policy optimization and the calibration. This new policy uses different set of parameters, model criteria and set of optimized parameters. The comparison between sensitivity analysis, optimization results and gaming outcome gives us a new view of learning process used by decision makers, model builders and students. We use the possibilities of visualization of simulation that are given by simulation language Vensim, and we try to apply calibration to detect the conditions, that stabilized that ecosystem and, what is quite new in the literature of this old, build the simulation game on base of that model to examine different scenarios of the human intervention in that ecosystem. At the end some conclusions are formulated.

Keywords: simulation, system dynamics, optimization, Vensim.

1. Introduction

Ecosystems are the complex, dynamical, nonlinear and multilevel systems that should be analyzed, modeled, and simulated by appropriate methods and tools. One of such method – System Dynamics (Akhar et al., 2013; Coyle, 1996; Coyle, 1999; Garcia, 2006; Kasperska, 2005; Kasperska, 2009; Kasperska et al., 2014; Kasperska et al., 2015; Kasperska and Mateja-Losa, 2005; Krupa, 2008; Marjasz, 2014) – was developed in the late 1950s and early 1960s at the Massachusetts Institute of Technology Sloan School of Management by Jay W. Forrester. The approach can be applied to dynamics problems arising in complex social, managerial, economic or ecological systems. The main purpose of System Dynamics is to try to discover the structure that conditions the observed behavior of the system over time.

System Dynamics tries to pose dynamic hypotheses that endogenously describe the observed behavior of system.

The problem of managing of ecosystems is in the center of interest for observers of contemporary changes in the surrounding world. The methods of analysis and modeling of changes should be interdisciplinary, connecting such disciplines like ecology, economy, mathematics and informatics. Achievement of sustained development or opposing the effects of climate changing in nature or the disturbed relationships type: prey-predator, all of this requires from the decision makers the ability of prognostic looking into the future. The effects of the human activities are long-wave in time and space and sometimes unintuitive.

In the complex systems, like ecosystems, there are many feedbacks, thus the dynamic behavior is a result of cooperation of those positive and negative loops. Like we already said, System Dynamics method is the appropriate tool for modeling and simulating such ecosystems. Many authors (Akhar et al., 2013; Fiddman, 2002; Garcia, 2006; 16. Ruth et al., 2012; Sterman, 2002) have undertaken this problem, but in the literature of this field we have lack of the papers connecting the simulation with the optimization (Kasperka et al., 2014; Kasperska et al., 2015; Kasperska and Mateja-Losa, 2005). Such connection gives new opportunities for the analysis of decision making problems in ecosystems, and because of this we have undertaken this problem in our paper.

The research work that had been done and described by authors in paper (Kasperska et al., 2015) is expanded by 12 new experiments consisting of policy optimization scope. The theoretical idea is visualized on figure 1 that illustrates the relationships between both types of Optimization (policy and calibration), Sensitivity Analysis and Gaming. In the authors opinion this idea gives a new look on simulation and optimization issues, helping in the process of organizing the research work and furthermore in applying the results of research in practice.

The object of experiments is described in the literature of this field (Coyle, 1996). The structure of Prey-Predator model in Vensim (Ventana) convention is presented on figure 2. A list of model variables and equations is presented below:

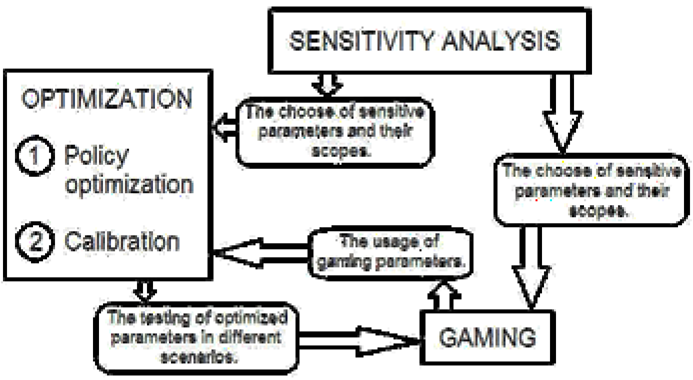

Figure 1. Relationships between OPTIMIZATION, SENSITIVITY ANALYZIS and GAMING. Source: own idea.

Food Requirement Per Fox – FRPF = 5 (1)

Fox Fecundity – FF = 0.05 (2)

Fox Initial Population – FIP = 20 (3)

Normal Fox Life – NFL = 120 (4)

Average Fox Food Intake – AFFI(t) = ARKR(t)/FP(t) (5)

Average Fox Lifetime – AFL(t) = NFL(t)*FLR(t) (6)

Fox Birth Rate – FBR(t) = DELAY3(FIR(t),7) (7)

Fox Death Rate – FDR(t) = FP(t)/(AFL(t) + 1) (8)

Fox Insemination Rate – FIR(t) = FF*FP(t) (9)

Fox Lifetime Reduction – FLR(t) = W ITH LOOKUP (FFSF(t)) (10)

Fox Lifetime Reduction – FLR(t) = W ITH LOOKUP (FFSF(t)) (11)

Fox Population – FP(t + dt) = FP(t) + dt *(FBR(t)- FDR(t)) (12)

Rabbit Initial Population – RIP = 300 (13)

Rabbit Insemination Rate – RIR(t) = RF*RP(t) (14)

Rabbit Birth Rate – RBR(t) = DELAY3(RIR(t), 4) (15)

Rabbit Population – RP(t + dt) = RP (t) + dt*(RBR(t)-RKR(t)) (16)

Rabbit Kill Rate – RKR(t) = FHE*RP(t)*FP(t) (17)

Average Rabbit Kill Rate – ARKR(t+dt) = SMOOTH(RKR(t), 2) (18)

Rabbit Fecundity – RF = 0.1 (19)

Fox Hunting Efficiency – FHE=0.005 (20)

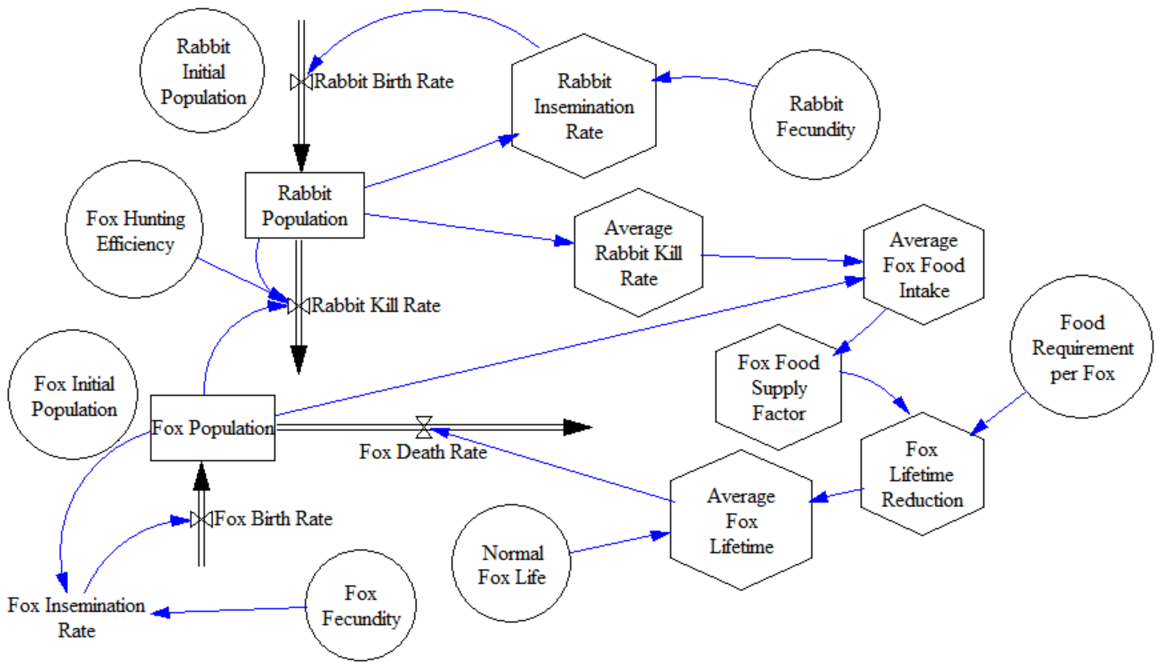

Figure 2. Structure of Predator Prey Model in simulation Language Vensim. Source: own idea on the base of Coyle, 1996.

Like professor Coyle said: the dynamics of the relationship between two populations of animals, one of which preys upon the other, is of great interest in ecological thinking and has been a fruitful area for analysis. One topic has been the academic need to understand why populations of animals undergo extreme fluctuations; another has been to suggest politics by which populations might be managed for economic reason or to preserve threatened species. In the paper of professor Coyle (Coyle, 1996) the mutual in influences between populations of rabbits and foxes were modeled. Using the model of rabbits - foxes, we have conducted many types of experiments including sensitivity analysis, calibration, and gaming. Those features (included in Vensim) weren't available for prof. Coyle in his times.

2. Experimental procedures

The SD models contain usually many parameters. It is interesting to examine the effect of their variation on simulation output. We select some parameters and assign maximum and minimum values along with a random distribution over which to vary them to see their impact on model behavior.

Vensim has a method of setting up such sensitivity simulation. Monte Carlo multivariate sensitivity works by sampling a set of numbers from within bounded domains. To perform one multivariate test, the distribution for each parameter specified is sampled, and the resulting values used in a simulation. When the number of simulation is set, for example, at 200, this process will be repeated 200 times. In order to do sensitivity simulation you need to de ne what kind of probability distribution values for each parameter will be drawn from. The simplest distribution is the Random Uniform Distribution, in which any number between the minimum and maximum values is equally likely to occur. The Random Uniform Distribution is suitable for most sensitivity testing and is selected by default. Another commonly used distribution is the Normal Distribution (or Bell Curve) in which value near the mean is more likely to occur that values far from the mean. Results of sensitivity testing can be displayed in different formats. Time graphs dis-play behavior of a variable over a period of time and are displayed either in terms of confidence bounds or a separate values which combine to form an individual simulation traces.

The second type of experiments we performed, was so called calibration, which is a type of optimization. Optimization can be used to validate and estimate parameters (calibration), or to select among alternative policies (policy optimization).

In order to use optimization, you will need to de ne what is good and what is bad it's called the payoff. The payoff is a measure, reported at the end of the simulation, stating numerically how good the simulation was. The payoff collapses your entire model, over the entire time it was simulated, into a single number. After defining the payoff, you need to select which constants will vary in order to maximize the payoff. Validation of a model relies in part on comparing the model behavior to time data string collected in the real world. When a model is structurally completed and simulated properly, calibration of the model can proceed. Calibration involves finding the values of model constants that make the model generate behavior curves that best fit to real world data. It is possible to manually alter model constants, to try to achieve a better fit between the real world data and simulation output. For a complex model with many constants to optimize and many variables of data sets to fit, the process is very time consuming. Using optimization, Vensim will automatically vary the constants of your choice and look for the best t between the simulation output and your real world data.

The third type of experiments we have performed was so called gaming. What are games in System Dynamics? Games are a way of actively engaging in process of a simulation. Games are examples of the flight simulator approach, where the user participate in the decisions that affect the simulation outcome for each step in time. A Vensim simulation model can be run as a game by stepping through time and making changes to gaming variables along the way. In contrast, a normal simulation model runs through the complete time span based on the initial setup of the model.

dt simulation step (0.125 [week]), (21)
t_0 simulation start time (0 [week]), (22)
t_H simulation end time (300 [week]), (23)
Rabbits Initial Population – 300 [Rabbits], (24)
F ox Initial Population – 20 [Fox], (25)

The last type of experiments we performed is a novelty in this paper. As mentioned in the explanation of calibration method the policy optimization uses the same mechanism utilizing the value of a payoff function as a method of finding an optimum value of chosen parameter. After defining the payoff, we selected which constants will vary in order to maximize or in the contrary to calibration to minimize the payoff. This approach gave us the possibility of conducting 12 new experiments, that are varying in object of optimization. Our objective changed from finding the equilibrium by utilizing calibration to finding the maximum or minimum of one objective function by utilizing policy optimization to some chosen parameter values. In the next section of this paper we present the results of some experiment types: sensitivity analysis, policy optimization, calibration and gaming.

3. Results

We performed several sensitivity analysis experiments and as a result we can see, that initial values of both population have great influences on the behavior of the model (see: figure 3, 4, 5, 6). The question is: how to stabilize the ecosystem? The model is closed. The dynamics arise entirely from the structure, initial conditions and parameter values (including sharp non-linarites). How the system might be managed by the human intervention? For instance, foxes might have to be culled, and wild rabbits are the delicious food. We have performed some calibration type experiments, to solve the problem of choosing the initial values of populations (see table 1).

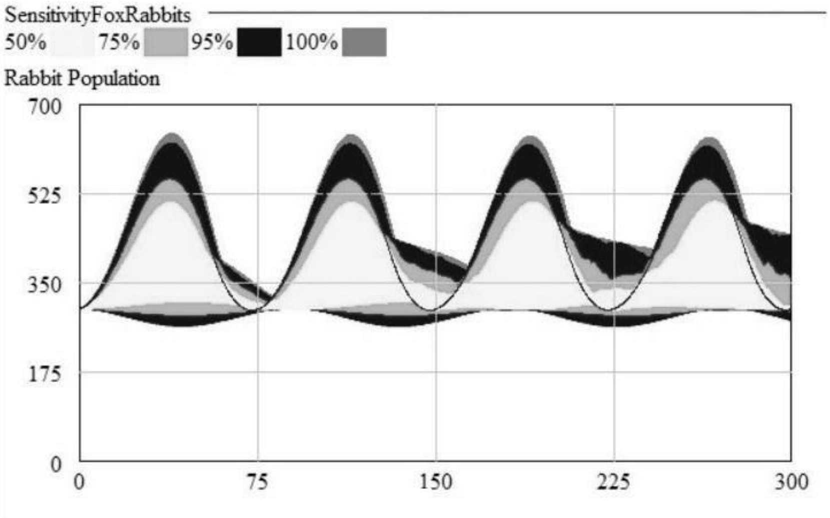

Figure 3. Confidence bounds of the variable Rabbit Population for Normal Fox Lifetime between 100 and 300 weeks. Source: own result.

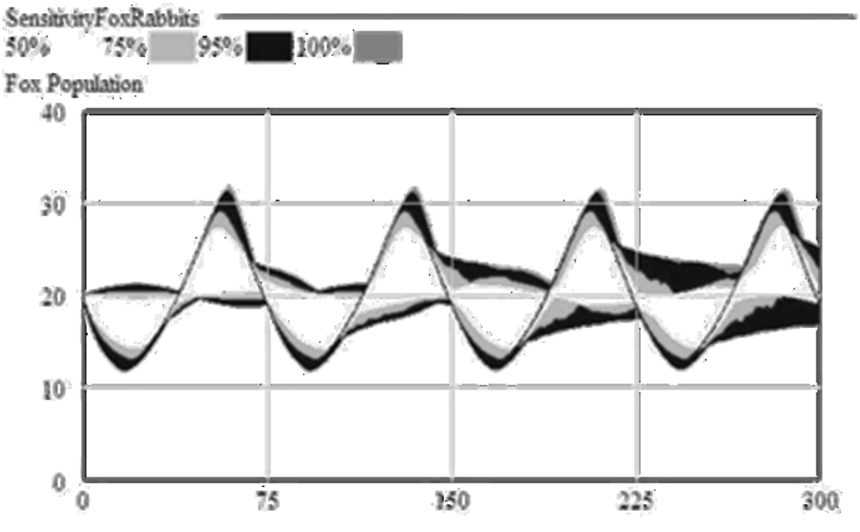

Figure 4. Confidence bounds of the variable Fox Population for Normal Fox Lifetime between 100 and 300 weeks. Source: own result.

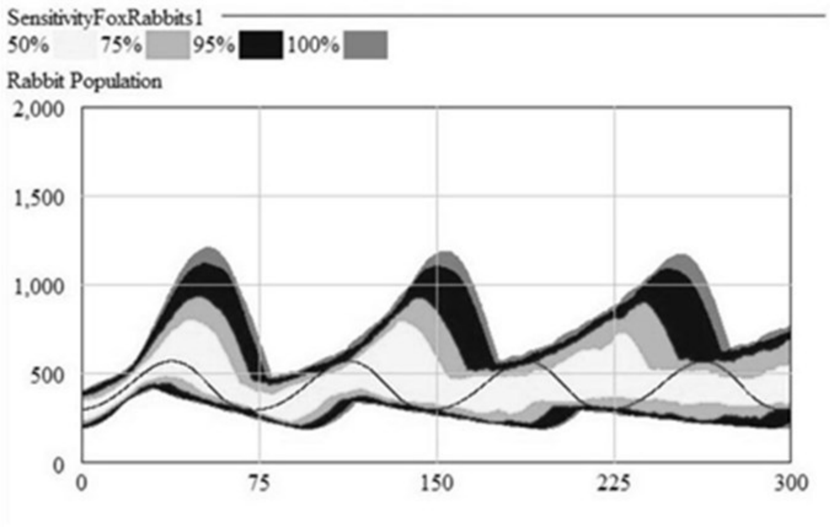

Figure 5. Confidence bounds of the variable Rabbit Population for: Rabbit Initial Population between 200 and 400 rabbits; Fox Initial Population between 15 and 25 foxes. Source: own result.

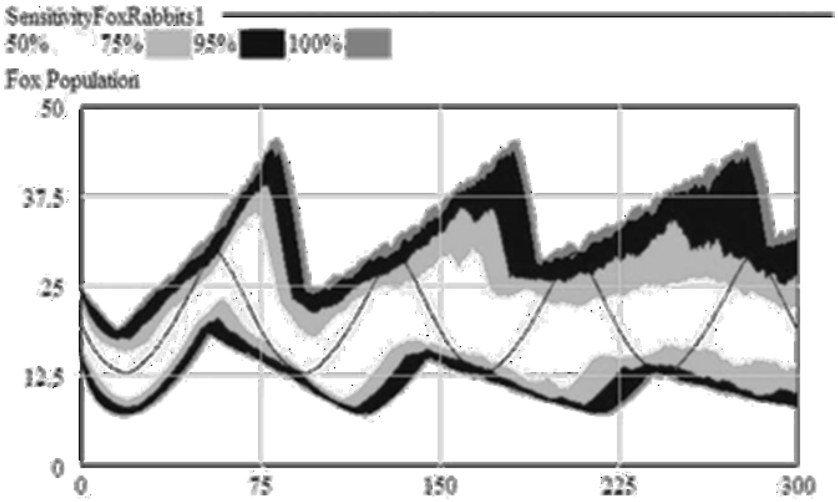

Figure 6. Confidence bounds of the variable Fox Population for: Rabbit Initial Population between 200 and 400 rabbits; Fox Initial Population between 15 and 25 foxes. Source: own result.

Table 1.

Results of Calibration on Prey-Predator model (searching the condition for equilibrium state of the system)

Assumptions for experiments							
initial values of levels		**conditions for calibration**		**initial values of levels**		**conditions for calibration**	
Rabbits	Foxes	What variable must be fit? (Data sets)	Results from calibration (round to total rabbits)	Rabbits	Foxes	What variable must be fit? (Data sets)	Results from calibration (round to total foxes)
300	20	Fox population = 20	Rabbit initial population ≈ 411 [figure 7, 8, 9]	300	20	Rabbit population = 300	Fox initial Population ≈ 15, Fox hunting efficiency ≈ 0.0068

cont. table 1

300	25	Fox population = 25	Rabbit initial population ≈ 514, Fox efficiency ≈ 0.004	350	20	Rabbit population = 350	Fox initial Population ≈ 17, Fox hunting efficiency ≈ 0.00587
300	30	Fox population = 30	Rabbit initial population ≈ 617, Fox efficiency ≈ 0.003333	400	20	Rabbit population = 400	Fox initial Population ≈ 19, Fox hunting efficiency ≈ 0.0051

Source: own idea.

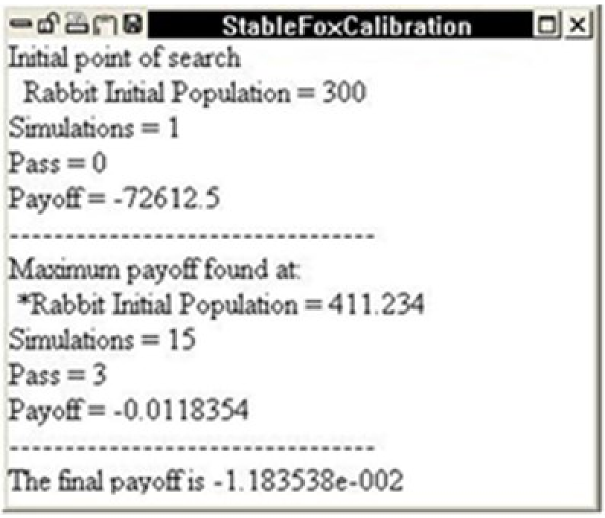

Figure 7. Example of calibration result shown in Vensim window. Source: own result.

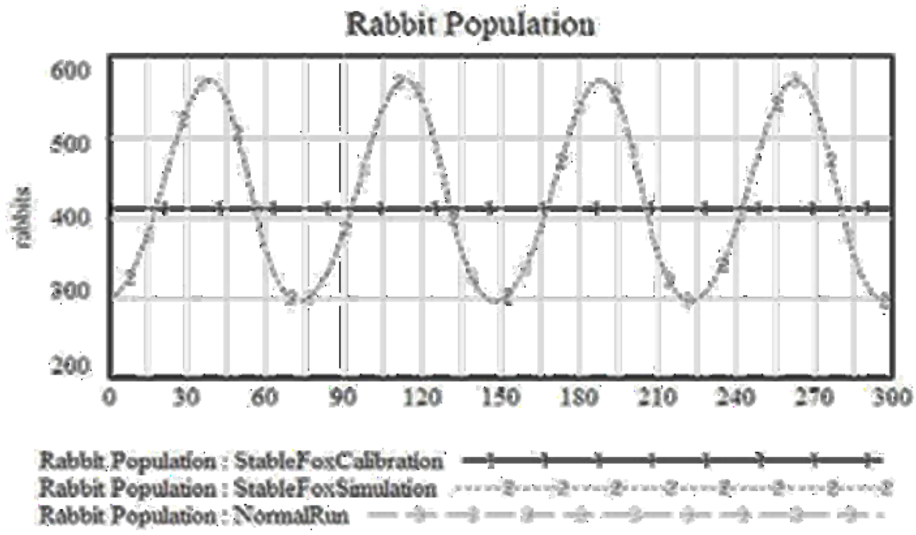

Figure 8. Graph illustrating the equilibrium for Rabbit Population achieved in calibration. Source: own result.

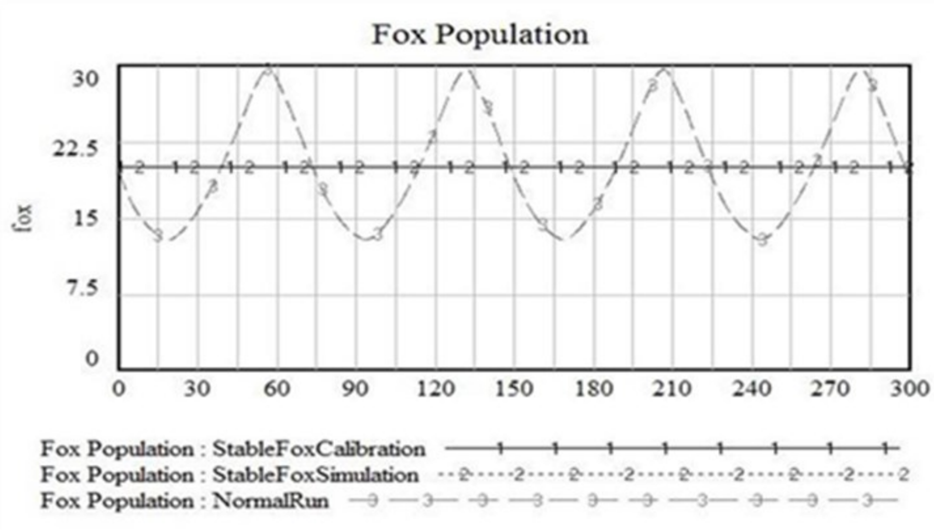

Figure 9. Graph illustrating the equilibrium for Fox Population achieved in calibration. Source: own result.

As an extension of our work we performed 12 new policy optimization type experiments. Their assumptions are presented in Table 2 and furthermore as an example we also presented two figures containing graphics taken from second experiment performed in Vensim program (see: figure 10 and figure 11). What must be explained is the difference in priorities for decision-makers, which indicates what type of optimization is performed: maximization or minimization. Technically speaking this is not a problem of simulation language Vensim, but the need of careful choosing of so called built-in payoff function, which is summarized during simulation process. For example, if our goal is the minimization of the number of killed rabbits, then the payoff function will be Rabbit Kill Rate.

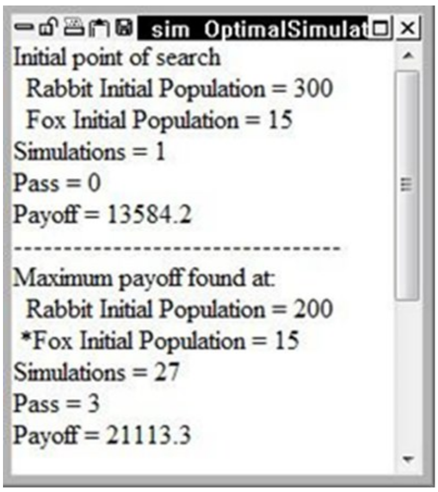

Figure 10. Example of policy optimization result shown in Vensim window. Source: own result.

Table 2.
Results of Policy optimization on Prey-Predator model (searching the optimized value of chosen parameters and payoff function)

Experiment no.	Type of optimization	Payoff function / Objective function	Optimized parameters	Scope for value of parameters	Optimal value for payoff function	Optimal value for parameters
1	MAX	Rabbit Birth Rate / Number of Birthed Rabbits	Rabbit Initial Population (RIP)	(200, 400)	18394.4	200
2	MAX	Rabbit Birth Rate / Number of Birthed Rabbits	Rabbit Initial Population (RIP) Fox Initial Population (FIP)	(200, 400) (15, 25)	21113.3 [figure 10] [figure 11]	200 15
3	MAX	Rabbit Birth Rate / Number of Birthed Rabbits	Rabbit Initial Population (RIP) Fox Initial Population (FIP) Normal Fox Life (NLF)	(200, 400) (15, 25) (100, 300)	269399	200 15 100
4	MIN	Rabbit Kill Rate / Number of Killed Rabbits	Rabbit Initial Population (RIP)	(200, 400)	12348.7	400
5	MIN	Rabbit Kill Rate / Number of Killed Rabbits	Rabbit Initial Population (RIP) Fox Initial Population (FIP)	(200, 400) (15, 25)	12741.3	352.18 23.15
6	MIN	Rabbit Kill Rate / Number of Killed Rabbits	Rabbit Initial Population (RIP) Fox Initial Population (FIP) Normal Fox Life (NLF)	(200, 400) (15, 25) (100, 300)	84340.3	265.77 20.26 300
7	MAX	Fox Birth Rate / Number of Birthed Foxes	Fox Initial Population (FIP)	(15, 25)	300.757	21.1327
8	MAX	Fox Birth Rate / Number of Birthed Foxes	Fox Initial Population (FIP) Rabbit Initial Population (RIP)	(15, 25) (200, 400)	303.3	25 341.69

cont. table 2

9	MAX	Fox Birth Rate Number of Birthed Foxes	Fox Initial Population (FIP) Rabbit Initial Population (RIP) Normal Fox Life (NLF)	(15, 25) (200, 400) (100, 300)	304.401	25 400 134.633
10	MIN	Fox Death Rate Number Dead Foxes	Fox Initial Population (FIP)	(15, 25)	289.039	15
11	MIN	Fox Death Rate Number Dead Foxes	Fox Initial Population (FIP) Rabbit Initial Population (RIP)	(15, 25) (200, 400)	272.514	15 286.361
12	MIN	Fox Death Rate Number Dead Foxes	Fox Initial Population (FIP) Rabbit Initial Population (RIP) Normal Fox Life (NLF)	(15, 25) (200, 400) (100, 300)	272.213	15 298.956 100.129

Source: own idea.

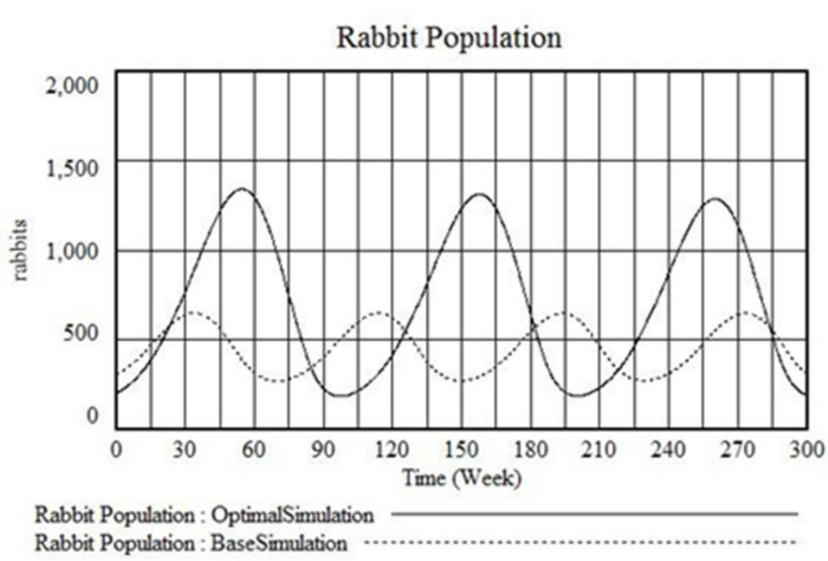

Figure 11. Graph illustrating the Rabbit Population for maximized Number of Birthed Rabbits with Rabbit initial population and Fox Initial Population optimum values. Source: own result.

4. Discussion of results and gaming experiment

On model Prey-Predator we have performed four types of experiments. First we used so called sensitivity analysis. That experiment shows the sensitivity of the system to the changes in parameter values. Large scale models with non-linarites are very sensitive to changes in several parameters and are non-sensitive for the rest of them. Searching this problem is very important, because it's an entrance for the process of optimization. In his book (Coyle, 1996) prof. Coyle said that: "the purpose of the model is to represent the dynamics of predator/prey behavior and to study how those dynamics change with uncertainties in the data. When the system has been well understood, the purpose might develop into understanding how the system might be managed by human intervention." Of course in prof. Coyle's times it wasn't possible to use automatic search of sensitivities in data. Now with the use of language Vensim, we have unlimited possibilities of experimentation, and we performed few of them. They show that populations of fox and rabbits are characterized by sustained oscillations. It must be emphasized that the oscillations arise entirely from within the model and not due to outside influence. Prof. Coyle achieved by "trial and error" method the result that with 300 rabbits, there is no value of fox population, which stabilize the system. In our paper we presented results of so called calibration, and they conclude that it is possible (see: table 1) to stabilize the system with small modification of data and the process of automatic calibration by Vensim.

The third type of experiment – the so called policy optimization (see: table 2) - gives us a new look on the priorities taken into account in decision-making part of optimization process. Until now we considered only the scenarios of finding an equilibrium which stabilizes the system. Now we change our goal to find a maximum or minimum value of one parameter without the requirement of system stability. As a result we obtain higher periodical system oscillations (kept in sensible boundaries) with an achievement of obtaining the maximum or minimum value by the chosen parameter from time to time. In real world such situations may be desirable – for example if we want to rise the population of rabbits (prey) before the declaration of hunting season organized by humans. The creation of conditions for high system oscillations can be a part of wider strategy to obtain an equilibrium that t our changing demands. This new approach shows the need for simulation of models with the possibility of human interventions.

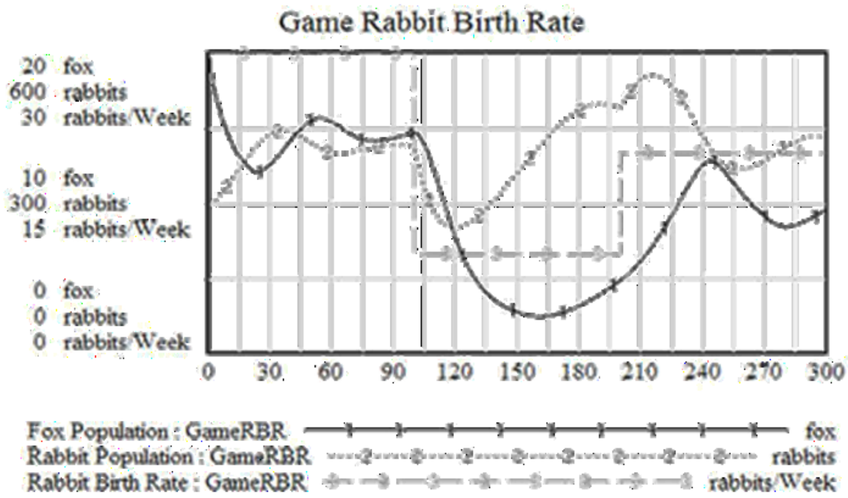

Figure 12. Graph for game 1 disease in rabbit birth population. Source: own result.

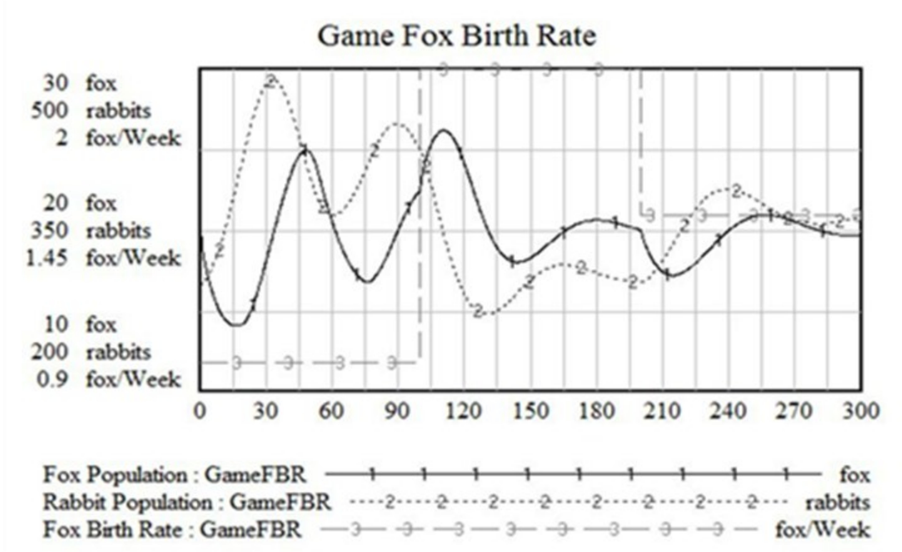

Figure 13. Graph for game 2 growth of fox population. Source: own result.

Table 3.
Results of Gaming on Prey-Predator Model (different human intervention and acting of nature)

Assumptions for gaming			
Game 1 – Disease in rabbit birth population			
Initial value of levels	Gaming variables		Dynamic behavior presented on figures
	Initial value	Changing the value during time horizon	
Rabbit population: 300 Fox population: 20	Rabbit Birth Rate: 30	In time t = 100 weeks Rabbit Birth Rate: 10 In time t = 200 weeks Rabbit Birth Rate: 20	[Figure 12] Rabbit Population, Fox Population, Rabbit Birth Rate

cont. table 3

Game 2 – Growth of fox population			
Initial value of levels	Gaming variables		Dynamic behavior presented on figures
	Initial value	Changing the value during time horizon	
Rabbit population: 300 Fox population: 20	Fox Birth Rate: 1	In time t = 100 weeks Fox Birth Rate: 2 In time t = 200 weeks Fox Birth Rate: 1.5	[Figure 13] Rabbit Population, Fox Population, Fox Birth Rate

Source: own idea.

In the context of the results of our experimentation: sensitivity analysis and both type of optimization, it was very interesting and innovative to perform the fourth type of experiments, so called – gaming. We examined different scenarios (see: game 1, game 2 in table 3) of the human intervention in that ecosystem or the intervention of nature. For example, we changed Rabbit birth rate (hypothetically) two times during the horizon of simulation, and we have the estimation how the system will behave after such scenario. It has practical implication, because in the real world the population of rabbits can die from disease such as myxomatosis, which is highly infectious and almost inevitably fatal for rabbits. Of course it has effects on the fox food supply. So these feedbacks can be examine by performing different games, with different scenarios. Problems similar to this arise in cases such as fishing policy. How many fish can safely be caught and what are the consequences for both fish and fisherman versus fishing industries subsides are obvious policy areas.

5. Final conclusion

Firstly, we would like to draw a number of theoretical conclusions:

- System Dynamics method is appropriate for modeling and simulation of the ecosystems, specially prey-predator systems,
- experimental procedures such as: sensitivity analysis, calibration, policy optimization and gaming; allow to search: sensitive parameters, conditions that stabilize the system, optimal value of parameters and payoff functions and examine different scenarios of the human intervention in that ecosystem (or "intervention" of nature).

Secondly, we would like to offer some practical conclusions:

- the simulation language Vensim is "friendly" and easily used tool for simulation and different type of research,
- many "old" and widely known models never were investigated from perspectives that Vensim offers, so this is a promising field for researchers and practitioners as well.

The comparison of possibilities for different types of research opens new perspectives for connecting them and applying results in the process of model building, the process of decision-making and learning (for practitioners as well for students who are learning new methods and discovering new tools).

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