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Influence of time delay on fractional-order PI-controlled system for a second-order oscillatory plant model with time delay

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Abstract: The paper aims at presenting the influence of an open-loop time delay on the stability and tracking performance of a second-order open-loop system and continuous-time fractional-order PI controller. The tuning method of this controller is based on Hermite-Biehler and Pontryagin theorems, and the tracking performance is evaluated on the basis of two integral performance indices, namely IAE and ISE. The paper extends the results and methodology presented in previous work of the authors to analysis of the influence of time delay on the closed-loop system taking its destabilizing properties into account, as well as concerning possible application of the presented results and used models.

Key words: fractional-order control, FOPI controller, stability analysis, time delay, tracking

1. Introduction

The PID controllers are still one of the most popular devices applied to industrial automation being inexpensive and simple to install and to use. However, in recent years, there has been an increased interest in a fractional-order approach to describe dynamical systems or systems with higher complexity in a more precise manner, which is the main advantage of the fractional-order description over the integer-order one. The fractional-order system can be treated as a generalization of the integer-order system [3, 5], and can bring benefits in such areas of application as temperature control systems, mechanical systems, magnetic levitation systems [6] or in biological processes.

Among non-standard applications of fractional-order controllers, one can find in [16] the design procedure of the controller of mean arterial blood pressure with given gain/phase margins which is computationally simpler than the solution of a standard fractional-order design problem and gives a certain degree of robustness.

As stated above, in a standard approach, the problem is to solve non-integer order differential equations. Multiple papers present different possibilities of applying fractional-order systems in control theory [9, 10]. Should a system comprise a time delay, it is possible to analyze it by means of approximation of the delay based on Padé approximation [7], with the use of Smith Predictor [8] or as a first-order plant approximation [12]. Another possibility is to describe the system with a quasi-polynomial and fractional-order PI controller [3], as in this paper, resulting in ranges of admissible controller parameters that assure closed-loop stability of the considered system.

In comparison, in [1] one can find the approach dedicated to systems without time delay, and described by first-order transfer functions, based on an analytical tuning method, giving smaller overshoot, shorter settling time, improved noise reduction in comparison to the PI or PID controller, better robustness to plant's gain uncertainty in a low frequency range in comparison to the PI controller. The Matignon's stability theorem is used there to tune the gain of the controller. Whereas in [17] one can find a Ziegler-Nichols-like procedure based on the ultimate gain method giving tuning rules applicable to time-delayed systems, where authors obtain the optimal parameters for the considered system based on some specifications.

In this paper, the influence of time delay on the stability region and tracking performance of a closed-loop system with a second-order plant with time delay is analyzed, for the system with a fractional-order PI (FOPI) controller, as a natural extension of the results presented by the Authors in their previous research [14], and at the same time it is similar to [4], where a first-order plant model is considered. The main point of [14] was the analysis of the impact of a fractional order of integration on a stability area for a single delay, whereas this paper focuses on stability analysis changing both the delay time and order of the integral part. In addition, in [12, 15] it has been shown that the time delay usually leads to unstable closed-loop systems with an integer-order PID controller, on the contrary to the case of a non-integer order PI controller which ensures stable closed-loop response at some time delay value chosen. On the contrary to the papers cited in the previous paragraph, the main point in the paper is not about any analytical tuning method, but on presenting the impact of the time delay in the process on closed-loop stability for different fractional orders of integration.

The paper considers the second-order model of the plant, which is motivated by the previous research of the authors, dedicated to the motor-rotor model of a multirotor flying robot [11] in such a form. The final aim of this research is to implement fractional-order controllers in an unmanned aerial vehicle (UAV) to test tracking abilities in such systems, as well as stabilization quality.

The structure of the paper is as follows. Firstly, Section 2 includes the statement of the problem. The tuning method of the FOPI controller is presented in Section 3, whereas Section 4 includes information concerning selected performance indices, as well as conditions under which closed-loop stability has been assessed. The simulation results are presented in Section 5, with the last section providing conclusions and giving directions to further research.

2. Statement of the problem

The article is focused on the influence of time delay on both the tracking performance and the stabilizing range of controller parameters K_p and K_I that ensures stability of the closed-loop system when the plant is modeled by the second-order transfer function:

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_0} e^{-sL}, \tag{1}$$

where all coefficients are assumed to be known, and L is a time delay. A reason for using this model, and its more comprehensive description can be found in [12, 15].

The fractional-order PI^{λ} controller is described with the following transfer function:

$$C(s) = K_p + \frac{K_I}{s^{\lambda}}, \tag{2}$$

where K_p and K_I denote proportional and integral gains, respectively, and $0 < \lambda \leq 1$ is the order of integration. For this type of controller, the impact of its parameters on the closed-loop system is analyzed, taking selected performance measures into account.

The Simulink-based block diagram of the considered control system is shown in Figure 1, with the reference signal denoted as $r(t)$, tracking error as $e(t)$, control signal as $u(t)$ and, finally, the output signal as $y(t)$.

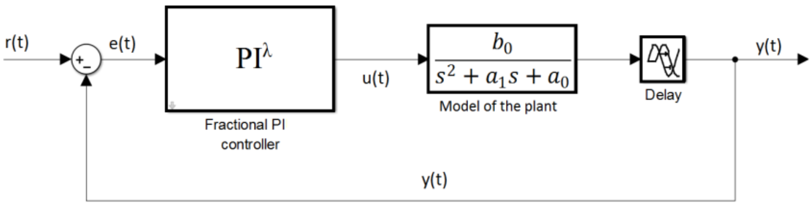

Fig. 1. Matlab-Simulink diagram of the considered control system

In order to perform performance and stability analyses, Pontryagin and Hermite-Biehler theorems will be used to derive theoretical bounds on controller parameters, defining stability areas that will be compared further to simulation-based results. By assuming the model to be of the second order, it is possible to obtain relatively simple formulas that define ranges of stabilizing parameters.

3. Tuning of a fractional-order PI controller for a second-order time delay system

The quasi-polynomial describing the closed-loop characteristic equation of the system presented in Figure 1 with $C(s)$ given by equation (2) is [3, 14]:

$$\delta^*(s) = b_0\left(K_p + \frac{K_I}{s^\lambda}\right)\mathrm{e}^{-sL} + s^2 + a_1 s + a_0, \tag{3}$$

and for this description Hermite-Biehler and Pontryagin theorems will be applied.

Theorem 1. Hermite-Biehler Theorem [3, 4]
Let δ^* be a complex function of ω and be described by the equation

$$\delta^*(j\omega) = \delta_r^*(\omega) + j\delta_i^*(\omega), \tag{4}$$

where $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ represent the real and imaginary parts of $\delta^*(j\omega)$. $\delta^*(j\omega)$ is stable if:

1) $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ have only simple real roots and these are interlaced;
2) $\delta_i^{'*}(\omega)\delta_r^*(\omega) - \delta_r^{'*}(\omega)\delta_i^*(\omega) > 0$ for some $\omega = \overline{\omega}$ in $(-\infty, +\infty)$,

where $\delta_i^{'*}(\omega)$ and $\delta_r^{'*}(\omega)$ are the derivatives of $\delta_i^*(\omega)$ and $\delta_r^*(\omega)$ with respect to ω.

It is important to ensure that $\delta_i^*(\omega)$ and $\delta_r^*(\omega)$ have only real roots which can be achieved by applying the Pontryagin Theorem.

Theorem 2. Pontryagin Theorem
Let $\delta^*(s)$ be described by (4), assuming that $s = j\omega$. To ensure that $\delta_r^*(\omega) = 0$ and $\delta_i^*(\omega) = 0$ have only real roots, in the following intervals:

$$-2l\pi + \eta \le \omega \le 2l\pi + \eta \qquad (l = 1, 2, 3, \ldots), \tag{5}$$

where η is an appropriate constant such that the coefficients of terms of the highest degrees in $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ do not vanish at $\omega = \eta$, $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ must have exactly $4lN + M$ roots, where N and M denote the orders of the integer part of the numerator and denominator polynomials, respectively. For the cases when the closed-loop characteristic equation is of fractional order, the polynomials $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ must have $4l([N] + 1)+[M] + 1$ roots, where [.] denotes the integer part.

Proofs of the theorems can be found in [2, 3].

Now, it is necessary to rewrite the quasi-polynomial $\delta^*(s)$ as:

$$\delta^*(s) = b_0(K_p + K_I) + (s^2 + a_1 s + a_0)s^\lambda \mathrm{e}^{sL}. \tag{6}$$

Assuming that $g = Ls$ and $\lambda = \frac{a}{b}$, we have

$$\delta^*(g) = b_0 K_p\left(\frac{g}{L}\right)^{\frac{a}{b}} + b_0 K_I + \left(\left(\frac{g}{L}\right)^2 + a_1\left(\frac{g}{L}\right) + a_0\right)\left(\frac{g}{L}\right)^{\frac{a}{b}}\mathrm{e}^g, \tag{7}$$

which for $g = (j\omega)$ becomes

$$\delta^*(j\omega) = b_0 K_p\left(\frac{j\omega}{L}\right)^{\frac{a}{b}} + b_0 K_I + \left(\left(\frac{j\omega}{L}\right)^2 + a_1\left(\frac{j\omega}{L}\right) + a_0\right)\left(\frac{j\omega}{L}\right)^{\frac{a}{b}}\mathrm{e}^{j\omega}. \tag{8}$$

By replacing the term $e^{j\omega}$ with $\cos(\omega) + j\sin(\omega)$ the real, $\delta_r^*(\omega)$, and imaginary part, $\delta_i^*(\omega)$, of $\delta^*(\omega)$ can be described by:

$$\delta_r^*(\omega) = \frac{\left[b_0 K_p + a_0\cos(\omega) - \frac{\omega^2}{L^2}\cos(\omega) - \frac{a_1\omega\sin(\omega)}{L}\right]}{L^{\frac{a}{b}}}\left|\mathrm{Re}\left(j^{\frac{a}{b}}\right)\right|\left|\omega^{\frac{a}{b}}\right| + b_0 K_I - \frac{\left[a_0\sin(\omega) + \frac{\omega^2}{L^2}\sin(\omega) + \frac{a_1\omega\cos(\omega)}{L}\right]}{L^{\frac{a}{b}}}\left|\mathrm{Im}\left(j^{\frac{a}{b}}\right)\right|\left|\omega^{\frac{a}{b}}\right|\mathrm{sign}(\omega), \tag{9}$$

$$\delta_i^*(\omega) = \frac{\left[b_0 K_p + a_0\cos(\omega) - \frac{\omega^2}{L^2}\cos(\omega) - \frac{a_1\omega\sin(\omega)}{L}\right]}{L^{\frac{a}{b}}}\left|\mathrm{Im}\left(j^{\frac{a}{b}}\right)\right|\left|\omega^{\frac{a}{b}}\right|\mathrm{sign}(\omega) + \frac{\left[a_0\sin(\omega) + \frac{\omega^2}{L^2}\sin(\omega) + \frac{a_1\omega\cos(\omega)}{L}\right]}{L^{\frac{a}{b}}}\left|\mathrm{Re}\left(j^{\frac{a}{b}}\right)\right|\left|\omega^{\frac{a}{b}}\right|. \tag{10}$$

Now, according to Pontryagin Theorem it must hold that $\delta_r^*(\omega) = 0$ and $\delta_i^*(\omega) = 0$, and the proportional gain K_p can be described by the following equation:

$$K_p = -\left[\frac{a_0}{b_0}\sin(\omega) + \frac{\omega^2}{b_0 L^2}\sin(\omega) + \frac{a_1\omega\cos(\omega)}{b_0 L}\right]\frac{\left|\mathrm{Re}\left(j^{\frac{a}{b}}\right)\right|}{\left|\mathrm{Im}\left(j^{\frac{a}{b}}\right)\right|}\mathrm{sign}(\omega) + -\left[\frac{a_0}{b_0}\cos(\omega) - \frac{\omega^2}{b_0 L^2}\cos(\omega) - \frac{a_1\omega\sin(\omega)}{b_0 L}\right]. \tag{11}$$

When $\delta_r^*(\omega)$ is rewritten as:

$$\delta_r^*(\omega) = b_0 K_I - m(\omega)K_p - b(\omega), \tag{12}$$

with:

$$m(\omega) = \frac{-\left|\mathrm{Re}\left(j^{\frac{a}{b}}\right)\right|\left|\omega^{\frac{a}{b}}\right|}{L^{\frac{a}{b}}},$$

$$b(\omega)=\left[-a_0\cos(\omega)+\frac{\omega^2}{L^2}\cos(\omega)+\frac{a_1\omega\sin(\omega)}{L}\right]\frac{\left|\mathrm{Re}(j^{\frac{a}{b}})\right|\left|\omega^{\frac{a}{b}}\right|}{L^{\frac{a}{b}}}-\frac{\left[a_0\sin(\omega)+\frac{\omega^2}{L^2}\sin(\omega)+\frac{a_1\omega\cos(\omega)}{L}\right]\left|\mathrm{Re}(j^{\frac{a}{b}})\right|\left|\omega^{\frac{a}{b}}\right|\mathrm{sign}(\omega)}{L^{\frac{a}{b}}},\tag{13}$$

then according to [4] the range of K_I to assure closed-loop stability must meet the conditions

$$\max\left(-m_j(\omega)K_p-b_j(\omega)\right)<K_I<\min\left(-m_j(\omega)K_p-b_j(\omega)\right)\tag{14}$$

with $j = 1, 2, 3\ldots$

Since $\delta_i^*(\omega)$ is an odd function, it has a root at $\omega = 0$. For $\omega = \omega_0 = 0$ it holds that

$$\delta_i^*(\omega)=b_0K_p+a_0.\tag{15}$$

To ensure the interlace property between $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$, the following condition must be imposed:

$$\delta_i^*(\omega)>0\Rightarrow b_0K_p+a_0>0\Rightarrow K_p>-\frac{a_0}{b_0}.\tag{16}$$

Based on the theoretical derivations presented in this section, the range of stabilizing values K_p and K_I to fulfill the stability of the closed-loop system requirement is given in Section 5.

4. Tracking performance evaluation and stability criteria

The tracking performance in the closed-loop system with controller parameters satisfying (11), (14), (16) is evaluated on the basis of two standard indices, namely, the integral of the absolute error (IAE) and integral of the squared error (ISE), as in [13-15].

Since the stability criteria for fractional-order systems with a time delay are complicated, the authors have decided to use the approach to utilize simulation results of the closed-loop systems.

In order to verify stability of the closed-loop system, the BIBO criterion is applied, and based on two conditions (i.e. stability is evaluated on the basis of the results from performed simulations). First one verifies if the duration of the simulation is the same as the desired simulation time, whereas the second condition verifies if the consecutive peaks of the output signal $y(t)$ are increasing or decreasing in the sense of their absolute values, but only for successfully conducted simulations. If the differences between consecutive peaks are decreasing in time, the closed-loop system is considered to be stable, and unstable in the other cases.

5. Simulation results

The stability analysis is performed for ω changing in the range [−π, π] with the step of 2π/200 The ranges of parameters K_p and K_i are calculated from (11) and (14), and, finally parameter λ varies from 0.1 to 1 with a step of 0.1. For all combinations of the parameters, the simulation time has been set to 200 s with the input signal $r(t) = 1(t)$.

The second-order plant parameters (1) have been chosen as: $b_0 = 1$, $a_0 = 1$, $a_1 = 2$, and the delay time L has been changed from 0.1 s to 2 s with a step of 0.1 s.

The surface plots presented in Figures 2-8 correspond to λ in the range [0.1, 1] for $L = 0.2$, 0.4, 0.6, 0.8, 1, 1.5 and 2 s. The results presented for $\lambda = 1$ are the same as in [12] but the tuning method defined by (11), (14) [3, 4] enables one to obtain a set of stabilizing parameters of a FOPI controller.

As can be seen from Figure 2, when the order λ in the FOPI controller increases, the stability region (stability area for the combination of controller gains) gets smaller, as in the case of the classical continuous-time PI-type controller ($\lambda = 1$). In Figure 2, for $\lambda > 0.7$ the stability region is not shown (equal axes limits are kept in all figures), but it extends to approximately $K_p = 5$. In addition, it is impossible to obtain a stable closed-loop system for $L > 0.2$ s which can be easily verified using, e.g. the Routh stability criterion with Padé approximation of the dead time.

With increasing L, as presented in Figures 3-8, the stability areas also get smaller, similarly to the case depicted in Figure 2, however, the span of admissible proportional gains is larger in comparison with integral gains, which can form a rule when tuning a FOPI controller by hand.

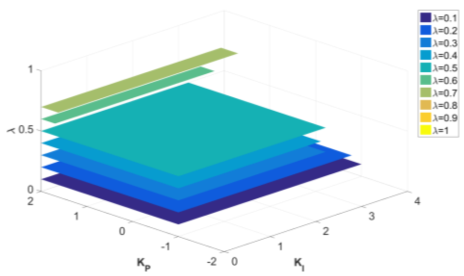

Fig. 2. Stability regions w.r.t. K_p and K_I for $\lambda \in [0.1, 1]$ and $L = 0.2$ s

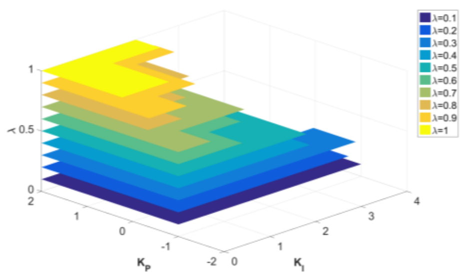

Fig. 3. Stability regions w.r.t. K_p and K_I for $\lambda \in [0.1, 1]$ and $L = 0.4$ s

For the combination $\lambda = 0.5$ and variable L it can be observed that when dead time is increased, similarly to the case of integer-order controllers, stability margins of the closed-loop systems decrease, and the stability area gets smaller. In such a situation, it is more difficult to find a set of stabilizing parameters (see Fig. 9).

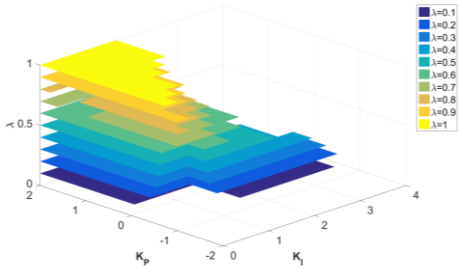

Fig. 4. Stability regions w.r.t. K_p and K_I for $\lambda \in [0.1, 1]$ and $L = 0.6$ s

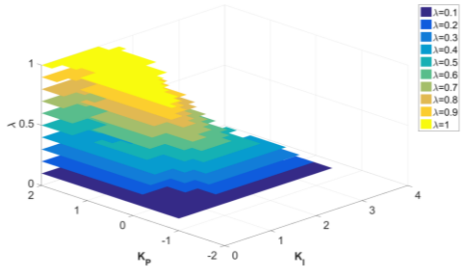

Fig. 5. Stability regions w.r.t. K_p and K_I for $\lambda \in [0.1, 1]$ and $L = 0.8$ s

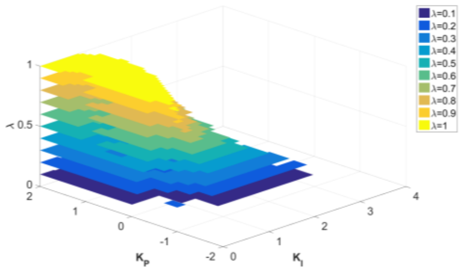

Fig. 6. Stability regions w.r.t. K_p and K_I for $\lambda \in [0.1, 1]$ and $L = 1$ s

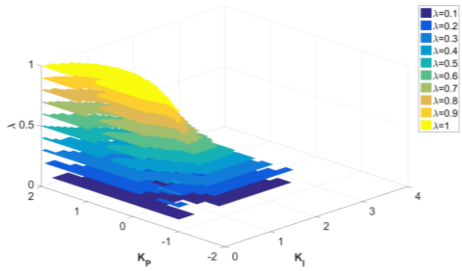

Fig. 7. Stability regions w.r.t. K_p and K_I for $\Lambda \in [0.1, 1]$ and $L = 1.5$ s

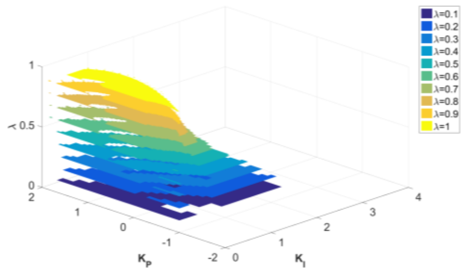

Fig. 8. Stability regions w.r.t. K_p and K_I for λ in [0.1, 1] and L = 2 s

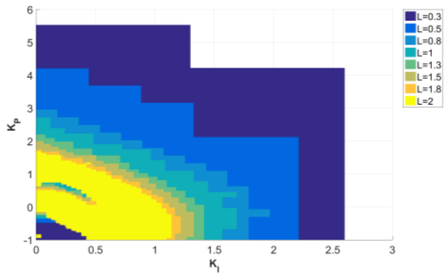

Fig. 9. Stability regions of parameters K_p and K_I for $\lambda = 0.5$ and increasing time-delay

The stability region is not the only point of interest of this paper and its analysis is accompanied below by the values of IAE and ISE performance indices presented when both λ and L are changed.

In Table 1, there are two rows per each combination, with the values placed in the upper row referring to minimal IAE and in the lower – to ISE performance indices obtained for controller parameters in the considered stability areas.

Table 1. Comparison of quality indices IAE and ISE for changing parameters λ and time-delay and *L*

	Time delay *L* [s]					
	0.4	0.6	0.8	1	1.5	2
λ = 0.1	28.43	32.19	37.85	42.17	48.32	53.13
	4.75	6.83	8.48	10.28	14.12	16.98
λ = 0.2	19.87	23.74	27.21	29.95	35.11	38.71
	3.15	4.27	5.31	6.21	8.23	9.85
λ = 0.3	15.67	18.63	21.01	23.08	27.02	29.96
	2.48	3.17	3.85	4.46	5.83	6.98
λ = 0.4	12.62	14.80	16.69	18.37	21.76	24.35
	1.85	2.40	2.90	3.35	4.37	5.25
λ = 0.5	9.31	11.39	12.88	14.30	17.08	19.31
	1.47	1.86	2.22	2.55	3.29	3.94
λ = 0.6	7.40	8.61	9.69	10.76	13.05	14.86
	1.20	1.51	1.74	1.97	2.50	2.97
λ = 0.7	5.13	6.17	7.10	7.93	9.67	11.10
	1.10	1.20	1.40	1.58	1.96	2.30
λ = 0.8	3.99	4.49	5.10	5.66	6.89	7.97
	0.88	1.06	1.18	1.31	1.60	1.84
λ = 0.9	**2.63**	3.11	3.49	3.85	4.69	5.42
	0.80	0.93	1.04	1.15	1.37	1.55
λ = 1	1.88	**1.68**	**1.84**	**1.99**	**2.33**	**2.62**
	0.84	**0.86**	**0.96**	**1.05**	**1.23**	**1.38**

As it is expected, the initial knowledge gathered from both stability and performance analysis of integer-order systems shows that the performance deteriorates with increasing dead time L. In addition, in the majority of cases, the integer-order PI controller gives superior performance indices with respect to the FOPI controller, and the problem here is a long tail in the response, corresponding to the integral action of the FOPI controller. The analysis results for $L = 0.2$ s are omitted here, since the closed-loop system has been likely to be unstable. It should be stressed that for $L = 0.4$ s, $\lambda = 0.9$ and $\lambda = 1$ the ISE index has the lower value for $\lambda = 0.9$ than in the case when $\lambda = 1$. To enable further analysis, Figure 10 presents closed-loop step response in these cases.

It is worth to note that in the case of the FOPI controller with $\lambda = 0.9$ the lower overshoot (maximum overshoot is 30%) is obtained, in comparison with the PI controller ($\lambda = 1$) case (maximum overshoot is 40%) but has two times longer settling-time (50 s in comparison with 25 s for the PI controller, due to a long tail in the step response).

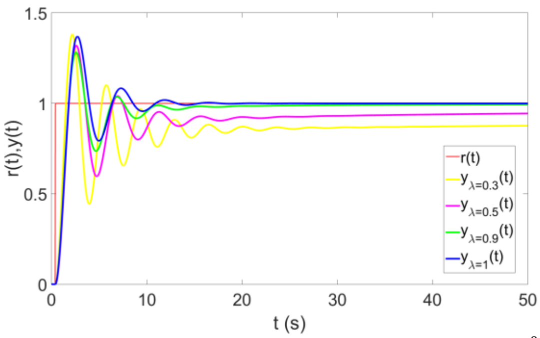

Fig. 10. Comparison of step responses of the closed-loop systems with $PI^{0.3}$, $PI^{0.5}$, $PI^{0.9}$ and PI^{1}

It is to be stressed that there are two contradictory aims – one to keep the stability area, presented in Figures 2-8 as large as possible (which is met for small values of λ parameter), but on the other hand, the time responses presented in Figure 11 show problems in static-like accuracy, related to a position error when initially visible transients decay to zero for this case.

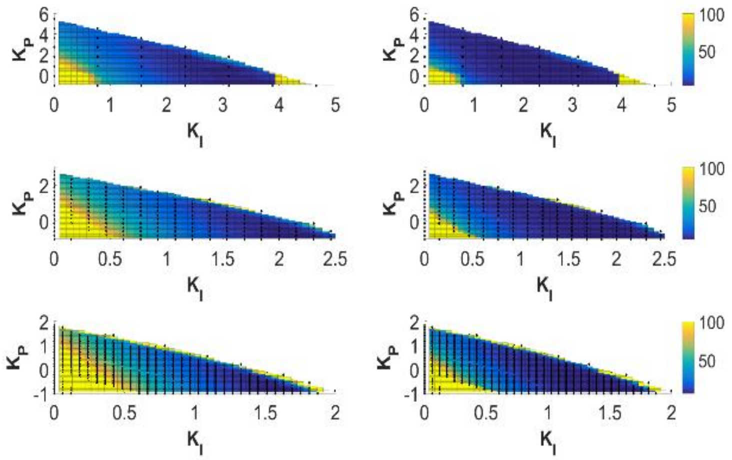

Fig. 11. Tracking performance (IAE – left column, ISE – right column) for $PI^{0.3}$ controller with L = 0.4 s (upper row), L = 1 s (middle row) and L = 2 s (lower row)

The fractional-order integrator for the considered system is therefore applicable for relatively big fractional orders, yielding at the same time, the closed-loop system with relatively small stability margins, expressed as the size of the area of stabilizing parameters. Using the fractional-order integrator must be, in this case, the result of other requirements imposed on the closed-loop system, as the need to non-standard slopes in log-magnitude or log-phase plots (other than resulting from integer-order analysis), or treating overshoot as the primary factor determining closed-loop performance, with the secondary factor as small tracking errors after transients have decayed.

To make our analysis complete, Figure 12 presents quality index surfaces for varying L and the $PI^{0.3}$ controller, where superscript denotes the FOPI controller with λ = 0.3.

The shape of performance surfaces presented in Figure 11 resembles the upper view on stability areas presented in Figures 3-8. Furthermore, the area covered by the surfaces also gets smaller with increasing L. These surfaces have been obtained by means of simulations for BIBO-stable closed-loop systems and are presented in the range of positive values of K_I to present behavior of closed-loop systems with stabilizing fractional-order PI controllers.

The decreasing stability margins that are visible in Figures 2-8 can also be estimated here when observing the rapid increase in both the IAE and ISE indices when combination of (K_p, K_I) is near the border of the presented surfaces.

As could be expected from the point of view of integer-order control systems, the best control performance can be achieved for relatively small dead times.

6. Conclusions

The results presented in the paper are representative examples of influence of time delay on tracking performance and stability of fractional-order systems. It was observed that with increasing time delay the range of stabilizing parameters K_p and K_I is decreasing, as well as that the non-integer order PI controller does not provide accurate tracking of the reference signal $r(t)$ in a short time horizon as shown in Figure 9. This tracking is possible in a long time horizon, in which a long tail in the response vanishes. However, the overshoot in many situations is lower in comparison with a control system with an integer-order PI controller. This is often more important from practical point of view, when small steady-state errors are acceptable, as well as slowly-decaying components in responses. This situation resembles long tails present in classical control systems when lag compensators attached in series are used.

As in the case of integer-order control systems, the presence of time delay has the destabilizing effect on the closed-loop, supporting the results presented in [2], where an approximation of time delay was used.

By using the fractional-order PI controller tuned according to Pontryagin and Hermite-Biehler theorems it is possible to compute the range of parameters K_p and K_I that guarantees the stability of the closed-loop system, giving rise to further, performance, analysis.

In comparison to approaches of the authors presented in their prior papers, see [2, 7], the method presented in this paper does not require making any approximations of the time delay.

Further research should be focused now on performing analysis of closed-loop systems with fractional-order controllers and anti-windup compensators, to verify the impact of possible saturations in a control signal on stability and performance for different anti-windup compensation schemes. As remarked in Introduction, the long-perspective gain is to use the results of this research to design and implement a fractional-order controller in an UAV.

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