TRANSNAV
http://www.transnav.eu

the International Journal
on Marine Navigation
and Safety of Sea Transportation

Volume 11
Number 3
September 2017

DOI: 10.12716/1001.11.03.19

Approach Parameters in Marine Navigation – Graphical Interpretations

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ABSTRACT: In this paper approach parameters widely used collision avoidance systems such as the distance at closest point of approach and time to the closest point of approach and less known and used as the distance on course, the distance abeam and any distance and the times intervals to their occurrences are derived, analyzed and graphically interpreted in the combined coordinate system for position and motion. They can be used in collision avoidance systems and for reversed purposes - manoeuvring to required approach parameters, intentional approaches and naval tactical manoeuvres.

1 INTRODUCTION

After the introduction of marine navigation radars for collision avoidance purposes, approach parameters of tracked objects were determined in a graphical manner by manual radar plots. At the beginning, analytical formulae for determination of motion and approach parameters and collision avoidance manoeuvres were derived in a polar coordinate system, natural for radar plots, with input values such as distances, bearings, velocities, courses and their changes.

The introduction of computer controlled Automatic Radar Plotting Aids (ARPAs) has created the need for algorithms for determination of motion and approach parameters but calculations in such systems are system-specific because they use mainly a Cartesian coordinate system. This is caused by:

- simple equations of motion in a system of Cartesian coordinates,
- simple estimation algorithms for motion parameters in digital tracking filters (because for objects travelling with constant velocities and courses their polar position changes – radial and angular velocities - are not constant and in Cartesian coordinates are constant),
- reduction of number of trigonometric and circular functions which, when used in numerical calculations, are connected with longer and less accurate calculations.

Publication of such algorithms is very rare - Jakševič (1967) and Lord (1968) are two of the very few that have been published. Only the second has some derivations and all of them are limited to the predicted object CPA (Closest Point of Approach) distance and the time interval to its occurrence. These parameters are well-established approach parameters used in collision avoidance systems featuring ARPAs as well as in manual radar plots.

In this paper other approach parameters such as:

- the predicted object distance on course and the time interval to its occurrence,
- the predicted object distance abeam and the time interval to its occurrence,
- the predicted object distance and the time interval to its occurrence

are presented in analytical and graphical form.

This paper is mainly a combined and shortened version of Lenart 1999a, 1999b, 2000a, 2000b and 2010 with emphasis to graphical interpretations.

2 ASSUMPTIONS AND INPUT PARAMETERS

For the purposes of this analysis, own vessel and extraneous objects of interest are regarded as if the mass of each object was concentrated at a point. It will be assumed that all moving external objects are travelling at constant velocity and course. In the movable plane tangential to the Earth's surface Cartesian coordinates system Ox, Oy (Fig. 1), with Oy pointing North, O is the present position of own vessel. It is also be assumed that manual plots or the radar processing and tracking (ARPA) or AIS (Automatic Identification System) has yielded:

- the present relative position of each object of interest X, Y,
- the components of its true velocity Vtx, Vty and/or
- the components of its relative velocity Vrx, Vry.

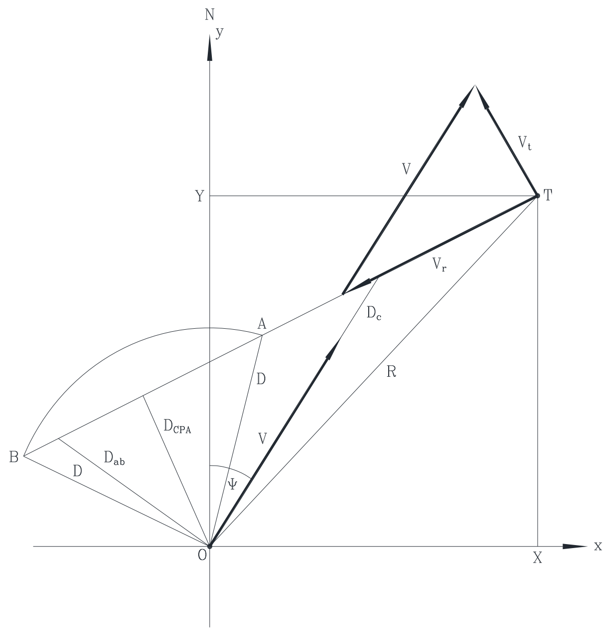

Figure 1. Input parameters

The relationship of own and an object's velocities can be described by equations

$$V_{tx} = V_{rx} + V_x \quad (1)$$

$$V_{ty} = V_{ry} + V_y \quad (2)$$

$$V_t = \sqrt{V_{tx}^2 + V_{ty}^2} \quad (3)$$

$$V_r = \sqrt{V_{rx}^2 + V_{ry}^2} \quad (4)$$

where V_x, V_y = own velocity components,

$$V_x = V \sin \psi \quad (5)$$

$$V_y = V \cos \psi \quad (6)$$

$$V = \sqrt{V_x^2 + V_y^2} \quad (7)$$

where ψ = own course (the angle measured clockwise from Oy to **V**).

From the above

$$V_{tx} = V_{rx} + V \sin \psi \quad (8)$$

$$V_{ty} = V_{ry} + V \cos \psi \quad (9)$$

and

$$V_{rx} = V_{tx} - V \sin \psi \quad (10)$$

$$V_{ry} = V_{ty} - V \cos \psi \quad (11)$$

Own and an object's motion parameters should be either ground or sea referenced and a drift angle is assumed to be zero.

3 COMBINED COORDINATES SYSTEM FOR POSITION AND MOTION

A conventional PPI displays the position of each object by plotting them in polar (r, β - distance, bearing) or Cartesian (x, y) coordinates. If we apply a scaling factor τ to the velocity coordinates (V, ψ) or (V_x, V_y) such that

$$r = V \tau \quad (12)$$

$$\beta = \psi \quad (13)$$

$$x = V_x \tau \quad (14)$$

$$y = V_y \tau \quad (15)$$

then the position and velocity coordinates coupled by time τ can be plotted on a common display. On such a display, besides own velocity vector (V, ψ) and positions of objects (X, Y), vectors of theirs true (V_{tx}, V_{ty}) or relative motion (V_{rx}, V_{ry}) can be plotted in a coordinate system parallel shifted to the point (X, Y). This corresponds to equations

$$x = X + V_{tx} \tau \quad (16)$$

$$y = Y + V_{ty} \tau \quad (17)$$

or

$$x = X + V_{rx} \tau \quad (18)$$

$$y = Y + V_{rx}\,\tau \qquad (19)$$

In a graphical interpretation the above equations mean that vectors of velocity are plotted in this coordinates system of position as τ – minutes vectors of predicted motion drawn from the present positions of own vessel and objects. The full area of (V_x, V_y) or (V, ψ) is the area our manoeuvres which can be limited by our maximum velocity V_{max} – the circle centred at (0, 0) and having radius V_{max}.

4 EQUATIONS OF RELATIVE MOTION

The relative position of an object, at time t, is given by

$$X(t) = X + V_{rx}\,t \qquad (20)$$

$$Y(t) = Y + V_{ry}\,t \qquad (21)$$

If D(t) is the distance to an object at time t, then

$$D(t) = \sqrt{X^2(t) + Y^2(t)} = \sqrt{R^2 + V_r^2 t^2 + 2(XV_{rx} + YV_{ry})t} \qquad (22)$$

or after squaring both sides and rearrangements

$$V_r^2 t^2 + 2(XV_{rx} + YV_{ry})t + R^2 - D^2(t) = 0 \qquad (23)$$

where

$$R = \sqrt{X^2 + Y^2} \qquad (24)$$

5 EQUATION OF TRUE MOTION

A substitution of Equations (10) and (11) to equation of relative motion (23) and rearranging yields equation of true motion

$$(V^2+V_t^2)T_D^2-2[(X+V_{tx}T_D)\sin\psi+(Y+V_{ty}T_D)\cos\psi]VT_D +2(XV_{tx}+YV_{ty})T_D+R^2-D^2 = 0 \qquad (25)$$

6 CPA DISTANCE AND TIME

6.1 *Equations for D_{CPA} and T_{CPA}*

In equation of relative motion (23) the distance reaches a minimum D_{CPA} (Lenart 1983)

$$D_{CPA} = \left| \frac{XV_{ry} - YV_{rx}}{V_r} \right| \qquad (26)$$

and time to achieve CPA - T_{CPA}

$$T_{CPA} = -\frac{XV_{rx} + YV_{ry}}{V_r^2} \qquad (27)$$

6.2 *Derivation of Equation $V = f(\psi, D_{CPA})$*

From Equations (26) and (4), by squaring both sides and rearranging terms, we obtain a quadratic equation in V_{ry}

$$(X^2-D^2_{CPA})V^2_{ry}-2XYV_{rx}V_{ry}+(Y^2-D^2_{CPA})V^2_{rx} = 0 \qquad (28)$$

whose solution is

$$V_{ry} = A_{DCPA}\,V_{rx} \qquad (29)$$

where

$$A_{DCPA} = \frac{XY \pm D_{CPA}\sqrt{R^2 - D^2_{CPA}}}{X^2 - D^2_{CPA}} \qquad (30)$$

A substitution of Equations (10, 11) to Equation (29) and rearranging yields

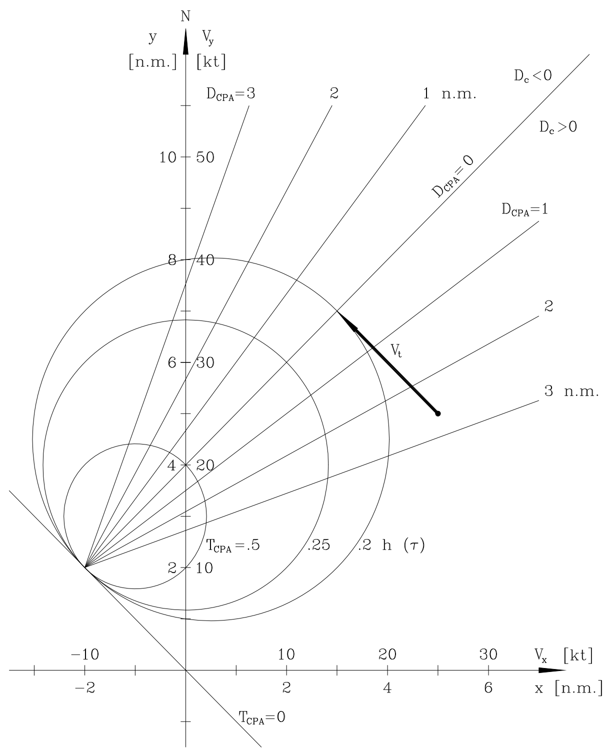

Figure 2. Lines D_{CPA}=const. and circles T_{CPA}=const. τ=0.2 h, X=Y=5 n.m., V_{tx}=-10 kt, V_{ty}=10 kt

$$V = \frac{B_{DCPA}}{A_{DCPA}\sin\psi - \cos\psi} \qquad (31)$$

where

$$B_{DCPA} = A_{DCPA}\,V_{tx} - V_{ty} \qquad (32)$$

and real solutions exist if (Equation (30))

$$R \geq D_{CPA} \tag{33}$$

Equation (31) gives the velocity V which own vessel must adopt to achieve the required CPA distance D_{CPA} (in respect to the selected object) for various assumed own courses ψ, but we should search for solution

$$V \geq 0 \tag{34}$$

and V, ψ for which

$$T_{CPA} \geq 0 \tag{35}$$

6.3 *Graphical interpretation*

A graphical interpretation of A_{DCPA} and B_{DCPA} can be obtained on a plotting in Cartesian coordinates of own velocity (V_x, V_y) by substituting Equations (5) and (6) to Equation (31)

$$V_y = A_{DCPA} V_x - B_{DCPA} \tag{36}$$

In these coordinates all points corresponding to a given value of D_{CPA} will lie on two straight lines having slopes A_{DCPA+} and A_{DCPA-} (values A_{DCPA} with + or – in the numerator of Equation (30) respectively), intersecting in the point (V_{tx}, V_{ty}) and cutting the V_y axis at $-B_{DCPA+}$ and $-B_{DCPA-}$ (the values of B_{DCPA} obtained on putting respective values of A_{DCPA} in Equation (32)).

In the combined coordinate system (Equations (12-15)) Equations (32) and (36) transform respectively to

$$r = \frac{B_{DCPA}\tau}{A_{DCPA}\sin\psi - \cos\psi} \tag{37}$$

and

$$y = A_{DCPA}\, x - B_{DCPA}\, \tau \tag{38}$$

Figure 2 illustrates a family of lines (36) or (37) and (38) for various D_{CPA} for an exemplary object.

6.4 *Collision Threat Parameters Area*

We can use Equations (36) or (38) and (30), (32) with a substitution of

$$D_{CPA} = D_S \tag{39}$$

where D_S = assumed safe value of D_{CPA} (thresholds set by the system's operator), to draw lines V(D_{CPA}=D_S). For that part of the area bounded by these lines and within which T_{CPA}>0, own vessel's motion parameters are leading to a threat of collision. This region is named the Collision Threat Parameters Area – CTPA (Lenart 1983) a new radar display and plot technique. The size and the position of CTPA are independent of own vessel's motion parameters. If we also plot own vessel's velocity vector (actual or simulated) and this terminates inside the CTPA then there is a collision threat. Any manoeuvre, by change of course and/or velocity, which deflects the end of this vector out of the CTPA is a possible means of avoiding the given threat.

6.5 *Derivation of equation V = f(ψ, T_{CPA})*

A substitution of Equations (4), (10) and (11) to Equation (27) gives a quadratic equation in V

$$T_{CPA}V^2-[(X+2V_{tx}T_{CPA})\sin\psi+(Y+2V_{ty}T_{CPA})\cos\psi]V$$

$$+(V_t^2T_{CPA}+XV_{tx}+YV_{ty}) = 0 \tag{40}$$

whose solution is

$$V = A_{TCPA}\sin\psi + B_{TCPA}\cos\psi \pm \sqrt{(A_{TCPA}\sin\psi + B_{TCPA}\cos\psi)^2 - C_{TCPA}} \tag{41}$$

where

$$A_{TCPA} = V_{tx} + \frac{X}{2T_{CPA}} \tag{42}$$

$$B_{TCPA} = V_{ty} + \frac{Y}{2T_{CPA}} \tag{43}$$

$$C_{TCPA} = V_t^2 + \frac{XV_{tx} + YV_{ty}}{T_{CPA}} \tag{44}$$

Real solutions exist if

$$(A_{TCPA}\sin\psi + B_{TCPA}\cos\psi)^2 \geq C_{TCPA} \tag{45}$$

Equation (41) can yield up to two velocities $V \geq 0$ which own vessel must adopt to achieve the required time to CPA - T_{CPA} (in respect to the selected object) for various assumed own courses ψ.

6.6 *Graphical interpretation*

A graphical interpretation of solutions given by Equation (41) can be obtained in Cartesian coordinates of own velocity (V_x, V_y) substituting Equations (5–7) to Equation (41)

$$(V_x - A_{TCPA})^2 + (V_y - B_{TCPA})^2 = \left(\frac{R}{2T_{CPA}}\right)^2 \tag{46}$$

The above equation reveals that the locus of points, for which T_{CPA} is a constant, is a circle centred at (A_{TCPA}, B_{TCPA}) and having radius $\left|R/(2T_{CPA})\right|$.

Figure 2 illustrates a family of circles for various values of T_{CPA}≥0 for an exemplary object.

Transformation of Equation (46) to (x, y) coordinates (Equations (14-15)) yields

$$(x - A_{TCPA}\tau)^2 + (y - B_{TCPA}\tau)^2 = \left(\frac{R\tau}{2T_{CPA}}\right)^2 \tag{47}$$

6.7 *Derivation of equation $\psi = g(V, D_{CPA})$*

If we search for own course ψ which will lead to the required CPA distance D_{CPA} at an assumed own speed V then we can get an inverse function $\psi=g(V, D_{CPA})$ to the function $V=f(\psi, D_{CPA})$ by a substitution to Equation (31) the trigonometric identities

$$\sin\psi = \frac{2\tan\frac{\psi}{2}}{1+\tan^2\frac{\psi}{2}} \tag{48}$$

$$\cos\psi = \frac{1-\tan^2\frac{\psi}{2}}{1+\tan^2\frac{\psi}{2}} \tag{49}$$

which will result in equation

$$(V - B_{DCPA})\tan^2\frac{\psi}{2} + 2A_{DCPA}V\tan\frac{\psi}{2} - (V + B_{DCPA}) = 0 \tag{50}$$

and its solution

$$\tan\frac{\psi}{2} = \frac{A_{DCPA}V \pm \sqrt{(A_{DCPA}^2+1)V^2 - B_{DCPA}^2}}{B_{DCPA} - V} \tag{51}$$

Real solutions exist if

$$V^2 \geq \frac{B_{DCPA}^2}{A_{DCPA}^2+1} \quad \text{and} \quad R \geq D_{CPA} \tag{52}$$

and Equation (51) can give up to four own courses ψ, which will lead to the required CPA distance D_{CPA} at an assumed own speed V if they additionally fulfil Condition (35). Graphically these solutions are the intersection points of lines $V(D_{CPA}=\text{const.})$ with a circle V=const. (a circle centred at (0, 0) and having radius V).

7 DISTANCE AND TIME ON COURSE

7.1 *Equations for D_c and T_{Dc}*

The predicted object distance on course D_c (Figure 1) and the time interval to its occurrence T_{Dc} are sometimes used as additional criteria for collision threat. They are used in some ARPAs for calculation of BCR - the bow crossing range and BCT - the bow crossing time. These parameters are given by equations (Lenart 1999b)

$$D_c = \frac{XV_{ry} - YV_{rx}}{V_{ry}\sin\psi - V_{rx}\cos\psi} \tag{53}$$

$$T_{Dc} = \frac{X\cos\psi - Y\sin\psi}{V_{ry}\sin\psi - V_{rx}\cos\psi} \tag{54}$$

$D_c>0$ means that an object will cross the course of own vessel ahead and $D_c<0$ that an object will cross the course astern. Interpretation of the sign of T_{Dc} is similar to T_{DCPA} – for $T_{Dc}<0$ D_c has taken place in the past.

7.2 *Derivation of equation $V = f(\psi, D_c)$*

A substitution of Equations (10) and (11) to Equation (53) and rearranging yields

$$V = \frac{A_{Dc}V_{tx} - V_{ty}}{A_{Dc}\sin\psi - \cos\psi} \tag{55}$$

where

$$A_{Dc} = \frac{Y - D_c\cos\psi}{X - D_c\sin\psi} \tag{56}$$

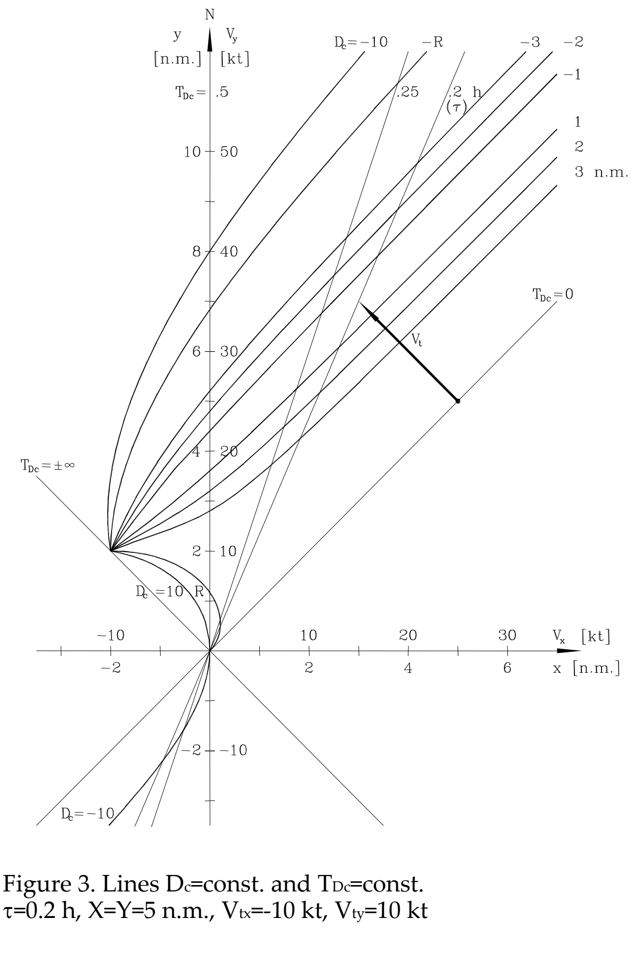

Figure 3. Lines D_c=const. and T_{Dc}=const. τ=0.2 h, X=Y=5 n.m., V_{tx}=-10 kt, V_{ty}=10 kt

Equation (57) is similar to Equation (31) with (32) but A_{Dc} is dependent on ψ. Equation (57) gives the speed V, which own vessel must adopt to achieve the required distance on course D_c (in respect to the selected object) for various assumed own courses ψ, but we should search for solution

$$V \geq 0 \tag{57}$$

and ψ for which

$$T_{Dc} \geq 0 \tag{58}$$

Condition (58) means that the approach on course is at present or will be in the future, not in the past.

7.3 *Derivation of Equation ψ = g(T_{Dc})*

Substituting Equations (10) and (11) to Equation (54) results in Equation

$$T_{Dc} = \frac{-Y\sin\psi + X\cos\psi}{V_{ty}\sin\psi - V_{tx}\cos\psi} \tag{59}$$

This equation reveals that the time to distance on course T_{Dc} is independent of own velocity V. Therefore from the above

$$\tan\psi = \frac{X + V_{tx}T_{Dc}}{Y + V_{ty}T_{Dc}} \tag{60}$$

and Equation (60) gives own course ψ, which will lead to the required time to distance on course T_{Dc}.

7.4 *Graphical interpretation*

A graphical interpretation of solutions given by Equation (60) can be obtained in Cartesian coordinates of own velocity (V_x, V_y)

$$V_y = \frac{X + V_{tx}T_{Dc}}{Y + V_{ty}T_{Dc}} V_x \tag{61}$$

and the locus of points, for which T_{Dc} is a constant, is a straight line crossing the origin of coordinates. Figure 3 illustrates a family of lines (53) for various required D_c and a family of straight lines (61) for various values of $T_{Dc}\geq 0$ for an exemplary object.

7.5 *Sign of D_c*

It can proved (Lenart 2010) that if for a given own course ψ exists own velocity $V(D_{CPA}=0)>0$ with $T_{DCPA}(D_{CPA}=0)>0$ then for $V>V(D_{CPA}=0)$ an object will pass astern ($D_c<0$), and for $V<V(D_{CPA}=0)$ an object will pass ahead ($D_c>0$). This sign of D_c is illustrated in Figure 2.

8 DISTANCE AND TIME ABEAM

8.1 *Equations for D_{ab} and T_{Dab}*

The predicted object distance abeam D_{ab} and the time interval to its occurrence T_{Dab} are sometimes used additional criteria for collision threat. These parameters are given by equations (Lenart 2000a)

$$D_{ab} = \frac{XV_{ry} - YV_{rx}}{V_{rx}\sin\psi + V_{ry}\cos\psi} \tag{62}$$

$$T_{Dab} = -\frac{X\sin\psi + Y\cos\psi}{V_{rx}\sin\psi + V_{ry}\cos\psi} \tag{63}$$

$D_{ab}>0$ means that an object will be abeam on the starboard side of own vessel, and $D_{ab}<0$ that an object will be abeam on the port side. Interpretation of the sign T_{Dab} is similar to T_{DCPA} – for $T_{Dab}<0$ D_{ab} has taken place in the past.

8.2 *Derivation of equation V = f(ψ, D_{ab})*

Substituting Equations (10) and (11) to Equation (62) and rearranging yields

$$V = \frac{A_{Dab}V_{tx} - V_{ty}}{A_{Dab}\sin\psi - \cos\psi} \tag{64}$$

where

$$A_{Dab} = \frac{Y + D_{ab}\sin\psi}{X - D_{ab}\cos\psi} \tag{65}$$

Equation (64) is similar to Equation (31) with (32) but A_{Dab} is dependent on ψ. Equation (64) gives the velocity V, which own vessel must adopt to achieve the required distance abeam D_{ab} (in respect to the selected object) for various assumed own courses ψ, but we should search for solution

$$V \geq 0 \tag{66}$$

and V, ψ for which

$$T_{ab} \geq 0 \tag{67}$$

Condition (67) means that the approach abeam is at present or will be in the future, not in the past.

8.3 *Derivation of equation V = f(ψ, T_{Dab})*

Substituting Equations (10) and (11) to Equation (63) results in Equation

$$T_{Dab} = -\frac{X\sin\psi + Y\cos\psi}{V_{tx}\sin\psi + V_{ty}\cos\psi - V} \tag{68}$$

hence

$$V = A_{TDab} \sin\psi + B_{TDab} \cos\psi \quad (69)$$

where

$$A_{TDab} = V_{tx} + \frac{X}{T_{Dab}} \quad (70)$$

$$B_{TDab} = V_{ty} + \frac{Y}{T_{Dab}} \quad (71)$$

Equation (69) can yield the velocity $V \geq 0$, which own vessel must adopt to achieve the required time to the distance abeam T_{Dab} (in respect to the selected object) for various assumed own courses ψ.

8.4 *Graphical interpretation*

A graphical interpretation of solutions given by Equation (69) can be obtained in Cartesian coordinates of own velocity (V_x, V_y) substituting Equations (5) through (7) to Equation (69)

$$(V_x - \tfrac{1}{2} A_{TDab})^2 + (V_y - \tfrac{1}{2} B_{TDab})^2 = \left(\tfrac{1}{2} A_{TDab}\right)^2 + \left(\tfrac{1}{2} B_{TDab}\right)^2 \quad (72)$$

The above equation reveals that the locus of points, for which T_{Dab} is a constant, is a circle centred at $(\tfrac{1}{2} A_{TDab}, \tfrac{1}{2} B_{TDab})$ and crossing the origin of a coordinates system.

Figure 4 illustrates a family of lines (64) for various required D_{ab} and a family of circles (72) for various values of $T_{Dab} \geq 0$ for an exemplary object.

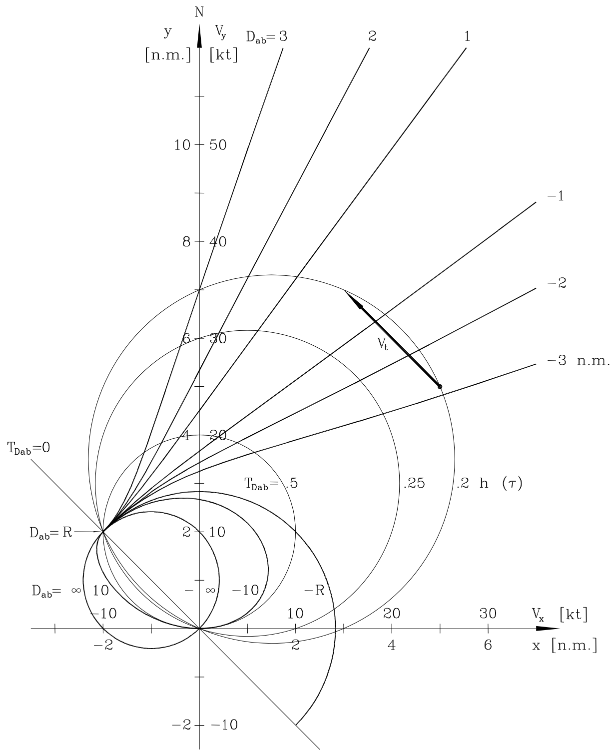

Figure 4. Lines D_{ab}=const. and circles T_{Dab}=const. τ=0.2 h, X=Y=5 n.m., V_{tx}=-10 kt, V_{ty}=10 kt

8.5 *Sign of D_{ab}*

It can be proved (Lenart 2010) that the sign of formula under the modulus in Equation (26) is the sign opposite to the sign of the distance abeam D_{ab}, if own course is equal to bearing to an object (when T_{Dab}>0 and T_{CPA}>0).

9 DISTANCE AND TIME

9.1 *Derivation of equation $T_D = f(V_r, D)$*

Solving a quadratic equation of relative motion in T_D (23) we obtain (for $t=T_D$)

$$T_D = \frac{-(XV_{rx} + YV_{ry}) \pm \sqrt{(DV_r)^2 - (XV_{ry} - YV_{rx})^2}}{V_r^2} \quad (73)$$

or (Equations (26) and (27))

$$T_D = T_{CPA} \pm \frac{\sqrt{D^2 - D_{CPA}^2}}{V_r} \quad (74)$$

and real solutions exist if

$$D \geq D_{CPA} \quad (75)$$

Equation (73) or (74) gives time T_D to achieve the distance D to the selected object.

9.2 *Time to safe distance*

Since in Equation (73) or (74) D can be any distance, we can substitute $D=D_S$ (as in Section 6.5) and this time can be named the time to safe distance and have been proposed analyzed and applied to detection of dangerous objects and to display the possible evasive manoeuvres (accurate Predicted Areas of Danger instead of their geometrical approximations) in Lenart (2015).

9.3 *Derivation of equation $V = f(\psi, D, T_D)$*

Solving a quadratic equation in V (25) we obtain

$$V = A_{VTd} \sin\psi + B_{VTd} \cos\psi \pm \sqrt{(A_{VTd} \sin\psi + B_{VTd} \cos\psi)^2 - C_{VTd}} \quad (76)$$

where

$$A_{VTd} = V_{tx} + \frac{X}{T_D} \quad (77)$$

$$B_{VTd} = V_{ty} + \frac{Y}{T_D} \quad (78)$$

$$C_{VTd} = V_t^2 + \frac{2(XV_{tx} + YV_{ty})}{T_D} + \frac{R^2 - D^2}{T_D^2} \qquad (79)$$

Real solutions exist if

$$(A_{VTd} \sin\psi + B_{VTd} \cos\psi)^2 \geq C_{VTd} \qquad (80)$$

Equation (76) can yield up to two own velocities $V \geq 0$, which own vessel must adopt to achieve the required distance D at the required time TD (in respect to the selected object) for various assumed own courses ψ.

9.4 *Graphical interpretation*

A graphical interpretation of solutions given by Equation (76) can be obtained in Cartesian coordinates of own velocity (V_x, V_y) substituting Equations (5–7, 24) to Equation (25)

$$(V_x - A_{VTd})^2 + (V_y - B_{VTd})^2 = \left(\frac{D}{T_D}\right)^2 \qquad (81)$$

The above equation reveals that the locus of points for which D and T_D are constants is a circle centred at (A_{VTd}, B_{VTd}) and having radius $|D/T_D|$.

Figure 5 illustrates a family of circles (81) for various required D and T_D for an exemplary object as well as, for comparison, circles $V(T_{CPA}$=const.) (Equation (47)) and the line $V(D_{CPA}=0)$ (Equation (36)).

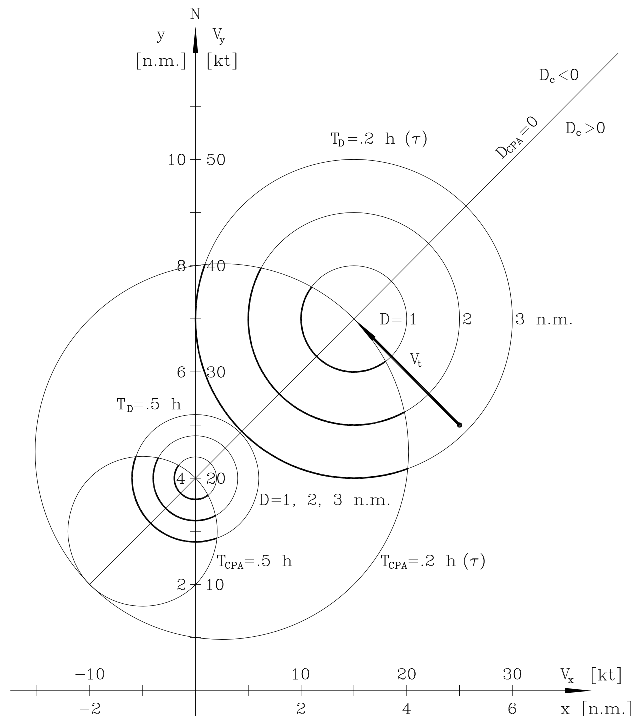

Figure 5. Circles D, T_D=const.
τ=0.2 h, X=Y=5 n.m., V_{tx}= -10 kt, V_{ty}=10 kt

It should be noted from Equations (73) and (74) that there can exist two times of approach at distance D: shorter - approach at the point A (Figure. 1) and longer - approach at the point B. If only the earlier (the first) approach is interesting for us, then, for this time condition $T_{CPA}>T_D$ for selected own motion parameters V, ψ should be fulfilled. This criterion fulfil points of circle V(D, T_D), which lie inside a circle $V(T_{CPA}$=const.) for the same time (marked in Figure 5 by the thicker line).

Graphically solutions of Equation (76) are the intersection points of the circle V=f(D, T_D) with a line of an assumed own course ψ.

10 TIME TO MANOEUVRE

It has to be emphasized that the manoeuvres calculated in the previous Sections are kinematic and should be undertaken immediately. If we require to have the time lapse Δt for calculations, for the decision to initiate a manoeuvre and for the execution of the calculated manoeuvre then (X, Y) in the previous equations should be replaced by ($X_{\Delta t}$, $Y_{\Delta t}$) respectively, given by equations

$$X_{\Delta t} = X + V_{rx}\,\Delta t \qquad (82)$$

$$Y_{\Delta t} = Y + V_{ry}\,\Delta t \qquad (83)$$

11 CONCLUSIONS

Formulae for such approach parameters as the predicted object CPA distance, the distance on course, the distance abeam, any distance and the times intervals to their occurrences in a Cartesian coordinates system have been derived, analyzed and graphically interpreted in the combined coordinate system for position and motion.

More than 80 directly applicable formulae for collision avoidance and quite reversed purposes - manoeuvring to required approach parameters, intentional approaches and naval tactical manoeuvres have been provided – almost all of them are derived from one basic equation of relative motion.

The introduction such auxiliary parameters as the distance on course and the distance abeam, apart from the main approach parameter - the distance to CPA, makes possible:

– a resignation from assumption (Section 2) that the mass of each object was concentrated at a point which can have significance when distances are comparable to objects' dimensions,
– more complete analysis of the main parameters (e.g. conclusions in Sections 7.5, 8.5 and 9.2).

Interpretation and plotting of derived formulae in the combined coordinate system of position and motion enable their applications as well in computer controlled radar systems as in manual radar plots – some of them are very simple in manual plots as it has been shown in Lenart (1983).

It must be emphasized that owing to the fact that in the derived formulae trigonometric and inverse trigonometric functions of extraneous objects' parameters are not used, computer calculations can be faster and more accurate.

REFERENCES

Jakševič, E.V. 1967. Algoritm posledovatel'noj obrabotki radiolokacionnych izmerenij. *Trudy CNIMF – Sudovoždenie i svjaz'*, 83: 58-63.

Lenart, A.S. 1983. Collision Threat Parameters for a New Radar Display and Plot Technique. *Journal of Navigation*, 36: 404-410.

Lenart, A.S. 1999a. Manoeuvring to Required Approach Parameters - CPA Distance and Time. *Annual of Navigation*, 1: 99-108.

Lenart, A.S. 1999b. Manoeuvring to Required Approach Parameters - Distance and Time on Course. *Annual of Navigation*. 1: 109-115.

Lenart, A.S. 2000a. Manoeuvring to Required Approach Parameters - Distance and Time Abeam. *Annual of Navigation*. 2: 81-88.

Lenart, A.S. 2000b. Manoeuvring to Required Approach Parameters - Distance, Time and Bearings. *Annual of Navigation*. 2: 89-97.

Lenart, A.S. 2010. *Approach Parameters in Marine Navigation – Theory and Applications*. Gdynia: Gdynia Maritime University Publishing.

Lenart, A.S. 2015. Analysis of Collision Threat Parameters and Criteria. *Journal of Navigation*, 68: 887-896.

Lord, R.N. 1968. A Controlled Azimuth Avoidance Manoeuvre. *Journal of the Institute of Navigation*, 21: 171-181.