GEOMETRY EDUCATION FOR DEVELOPING SPATIAL VISUALISATION ABILITIES OF ENGINEERING STUDENTS

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Abstract. The importance of descriptive geometry was pushed back in the last years in many curricula of engineering studies. Geometry Education was often substituted by training of CAD-systems and representation techniques. This development leads to a deficiency in spatial visualisation abilities of engineering students. Descriptive Geometry provides the foundations for creating and understanding 2-D drawings of 3-D objects and helps to develop spatial visualisation abilities. As a branch of mathematics geometry deals with properties, relationships, measurements of points, lines, planes and solids as well as transformations and projection methods. There is a need of fundamental knowledge in geometry combined with visualisation methods and an understanding of visual perception and communication. Studies and also our experiences showed that spatial visualisation abilities of the students are most improved by sketching and drawing 3-D objects and by creating and working at the same time with 3-D models, not by training CAD-Systems. Well developed spatial visualisation abilities are important predictors of success in engineering studies. How can the aim of well developed spatial visualisation abilities reached by geometry education? Which are the most important elements of such a geometry course? The experiences with students of architecture and civil engineering at our university along several years will be analysed. The conclusion of this analysis will be presented with the most relevant elements for developing spatial visualisation abilities. Examples of student projects and teaching materials will illustrate the content and way of such a geometry education which is able to support the development of spatial visualisation abilities.

Key Words: computer graphics, descriptive geometry, representation techniques, spatial visualization.

1. Historical Notes on the Role of Geometry Education

The old discipline of geometry has become its special importance for engineering education in the 18th century. The development of the 'École polytechnique' pushed forward a new basic discipline 'Descriptive Geometry' in engineering education, in particular supported by Gaspard Monge (1746-1818). But already before, much earlier, the basic knowledge about the geometrical projection methods were important above all for architecture. The 'École centrale des travaux publics' in Paris, was founded in 1794 by Gaspard Monge and Lazare Carnot to support mathematical and physical sciences in refer to their technical importance. There and also later in the 'École polytechnique' in France 'Descriptive Geometry' was one of the most important subjects. In Germany 'Descriptive Geometry' got also an important role in the newly founded technical universities in the 19th century. In these relationships the subject 'Descriptive Geometry' was given the role to deal with the representation of technical objects in the design process. In this new concept of 'Descriptive Geometry' Mathematics was combined with the practical demand of drawing 3-D objects. This new foundations of the technical universities arose out of the critique at the literal humanistic traditional education system. Diderot's and d'Alembert's 'Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers' (1751-1772) played an important role in this historical development. In the curriculum conception Gaspard Monge's 'Descriptive Geometry' was seen as a necessary and common language for engineers, craftsmen and workers. It is "le meilleure moyen que l'on

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puisse employer pour étudier et décrire les formes et les positions respectives des objectes" [6]. The problem of communication about spatial objects was paid more attention than before because the division of labour increased. Descriptive Geometry was seen in addition as the method of research. The education concept was a break with the old pedagogic ideal of reproducing things. Monge did not see descriptive geometry as a fixed topic, but it should always change according new developments of sciences. In geometry the teachers should pay attention to the development of spatial visualisation abilities. The task of 'Descriptive Geometry' is according Monge, given in the introduction of his textbook, published in 1800:
"This art (Descriptive Geometry) has two principal objects, the first to represent with exactness, from drawings which have only two dimensions, objects which have three, and which are susceptible of a strict definition; under this point of view it is a language necessary to the man of genius when he conceives a project, and to those who are to have the direction of it; and lastly, to the artists who are themselves to execute the different parts. The second object of descriptive geometry, is to deduce from the exact description of bodies all that necessarily follows of their forms and their respective positions; in this sense it is a means of seeking truth, as it offers perpetual examples of the passage from what is known to what is unknown, and as it is always applied to objects susceptible of the minutest evidence, it is necessary that it should form part of the plan of a national education." [10]

In the 'Écoles polytechniques' the amount of descriptive geometry with applications took 50% of the curriculum [6]. Experiments and exercises as well as the mutual coherence of the subjects rated high in the didactic concept. In 1804 the Écoles polytechniques' were subordinated to the ministry of war. The 'Écoles polytechniques' changed to cram institutions. Perhaps this development had an influence to an understanding of Descriptive Geometry as a fixed theory of teaching graphical techniques like learning recipes and not as an education in spatial visualisation and thinking.

2. New Developments of Geometry Education for Engineering Students

In the last decades the importance of descriptive geometry was pushed back more or less in many countries. The applied orientation of descriptive geometry to technical and architectural disciplines led to an understanding of this geometry education to learn drawing by ruler and compasses. So Descriptive Geometry was more or less equalised with drawing techniques. But geometry education is the necessary background for drawing of 3-D objects also today. Based on this misunderstanding the development of computer technologies and CAD-systems led to the opinion at several engineering faculties that there is no need for descriptive geometry any more. Descriptive geometry was substituted by training of CAD-systems. But the situation today shows that the knowledge of descriptive geometry is even more necessary than before. The determination of descriptive geometry by Monge hits also the requests of an actual geometry education for engineering: the spatial translation process between 3-D object and 2-D representation. The new requests of 3-D modelling and transformation processes especially by the new developments in architectural design demand in addition detailed geometrical knowledge about creation of forms and their transformations. In architecture and engineering education the importance of descriptive geometry in the aspect of communicating about spatial objects still remains valid. Descriptive geometry as a common language for engineers may be transformed today to descriptive geometry as a sign system for engineering. Therefore geometry education for engineering today should be included in communication theory, in an understanding of visual perception and communication about spatial objects.

3. Developing Spatial Visualisation Abilities

Well developed spatial visualisation abilities are important conditions for all engineering studies where 3-D reality should be created, designed and changed. The students mostly enter our universities without those well developed abilities. Nowadays a visual competence is more required than before to interpret the various digital representations of spatial objects and situations. We need a high competence in understanding computerised visualisations, animations and 'virtual realities'. We tested our students in the Mental Rotation Test (MRT) [9] and the Mental Cutting Test (MCT) [1], later often applied in research by Suzuki et al. in Japan [8] and in international co-operations [3], over several years at the very beginning of their studies in architecture and civil engineering. In most cases students entered university in 2003/2004 with less spatial visualisation abilities than in 1996/1997. A high Standard Deviation indicated the large differences in the levels of our entering students. This statistical result corresponds also with our experience while teaching that we have to find ways to arrange our teaching method with the large range in the spatial skills levels.

International Research in the topic of spatial visualisation abilities showed that it is possible to develop those abilities. Also in our studies in international co-operations we looked for significant predictors of success on the spatial visualisation tests. We found that playing with construction toys, previous drafting and working experience, the type of secondary education received had an influence to the test results [2]. We studied also the influence of descriptive geometry courses for developing spatial visualisation abilities by administering post-tests and found positive impacts. The study of S. Sorby and R. Gòrska about improvements in spatial abilities by various courses gives interesting answers. They found that those courses where sketching and hand drawing was a primary focus of activity enhanced most visualisation skills. Computer graphics courses had no significant impact in visualisation skills [7]. We evaluated our Descriptive Geometry program and asked the students in a questionnaire about their experiences and assessment of our teaching program, especially about the impact of drawing examples in the lectures and the labs, the models, the axonometric visualisations, the architecture examples and sketching [5]. The answers showed that the drawing examples by hand and the touchable models were most helpful for them. The students expressed the position that it is very important for them to work on the exercise material independently and then get supported in finding and working out the solutions. The experiences with the students in the last years and the research results about developing spatial visualisation abilities form the basis of our geometry education program for students of architecture and civil engineering.

4. Consequences for Geometry Education

After these considerations we conclude the most important elements of geometry education for architecture and civil engineering for developing the spatial visualisation abilities. The ideas can be also transferred to other engineering disciplines by choosing suitable examples.

4.1 Hand Drawing and Sketching Examples

There is a direct connection between our brain and our hand. Therefore it is important to use also this connection for spatial visualisation and thinking. As a first project in descriptive geometry the student got for example a painting of the artist Guenther Fruhtrunk to create spatial ideas out of the 2-D image in refer to the theme 'living'.

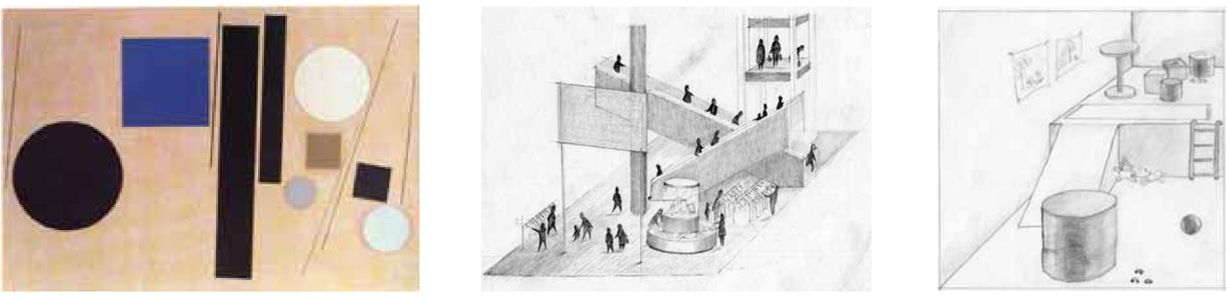

Figure 1: G. Fruhtrunk "Pour J. Arp et M. Hagenbach" – Derived Spatial Configurations

To get started ideas for spatial configurations out of the image it turned out to be very helpful to combine the abstract painting with well known living situations. Sketching was the first step in the creation process. In the second step the students had to draw their spatial configuration in three orthographic views, in the third step in two kinds of axonometries to get a visual representation. In this first project students have always to translate between 2-D and 3-D in their imagination. They learn in addition working with a co-ordinate system in the 3-D geometrical space and its representation in the 2-D drawing plane which is also the foundation for working with 3-D modelling software.

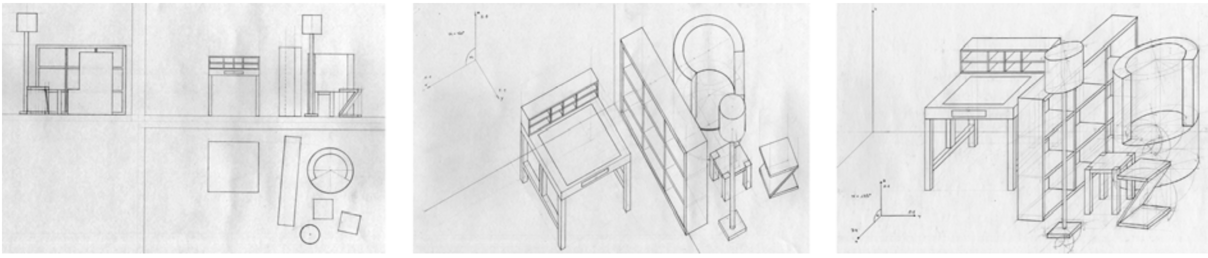

Figure 2: Multiview and Axonometric Drawings of the Spatial Configuration

4.2 Creating and Using Touchable Models

Those students starting with very low spatial abilities have problems by working directly with drawings and spatial configurations only in their imagination. Therefore we found it effective to start with the students in 3-D space and to ask them creating 3-D touchable models at the very beginning. To support the 2-D to 3-D thinking process the students created in the first step 2-D collages with given geometric figures, in the second step they derived 3-D models.

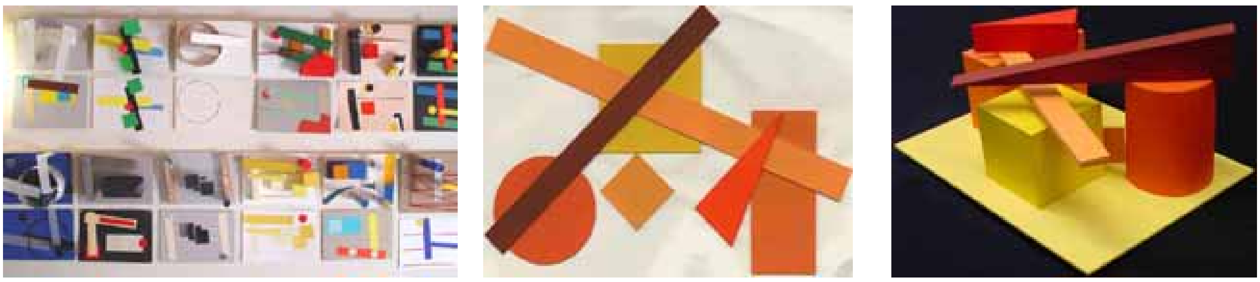

Figure 3: 2-D Collages and derived 3-D Models of Spatial Configurations

Compared with a drawing or sketching the 2-D collage has already a tendency of a three-dimensional interpretation of two-dimensional figures. After the students have done their multiview drawings and axonometries it turned out to be effective of the spatial thinking process to compare the drawings with the models and to take up the various viewpoints in refer to the model. Then it is very easy for the students to verify or falsify their drawings.

In all lectures and labs models are used for explaining the spatial thinking process when we solve a spatial problem by drawing on a two-dimensional plane. Figure 4 shows the geometric problem to find the cutting point of a line and a plane in space. The drawings are quoted from the textbook accompanying the lectures [4]. In the book the spatial imagination process will be supported by axonometric drawings.

Activities with the model help to protect the students from drawing just lines instead of transforming the spatial thinking process in drawing steps.

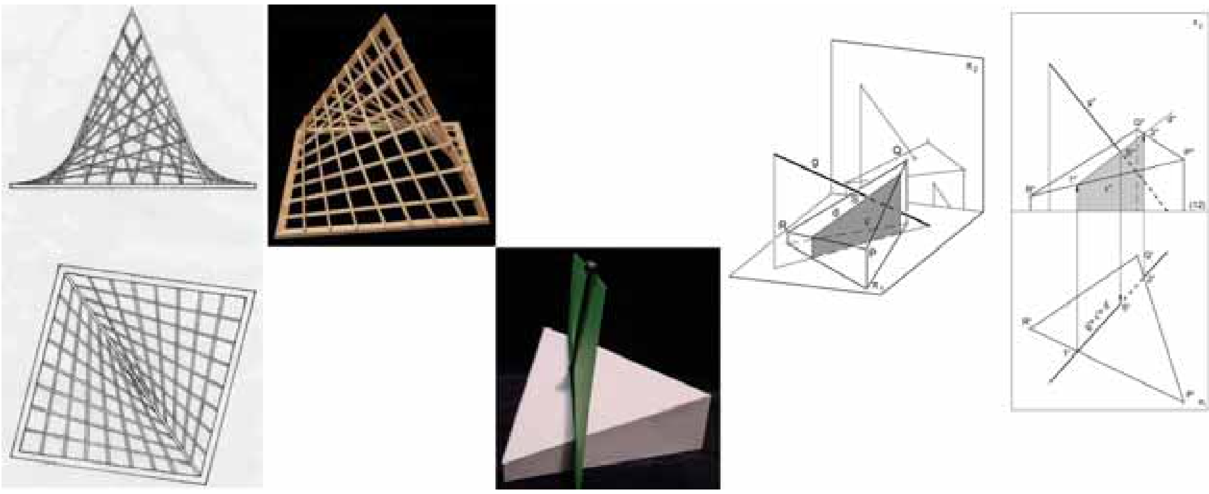

Figure 4: Drawing and Model of Hyperbolic Paraboloids and of the Solution to Find the Cutting Point of a Line and a Plane in Space

4.3 Supporting the Learning Process in Consecutive Steps

The geometry course consists of three parts: lectures, labs and homework projects. The role of the lectures is to explain the theoretical background, always accompanied by examples. The geometrical theory is illustrated by touchable and virtual models, axonometries and photos of built architecture. In Figure 5 the various elements are presented along the topic sections of a cone cutting. The most important point here is to understand the origin of the different cases of cone cuttings elliptic, parabolic and hyperbolic.

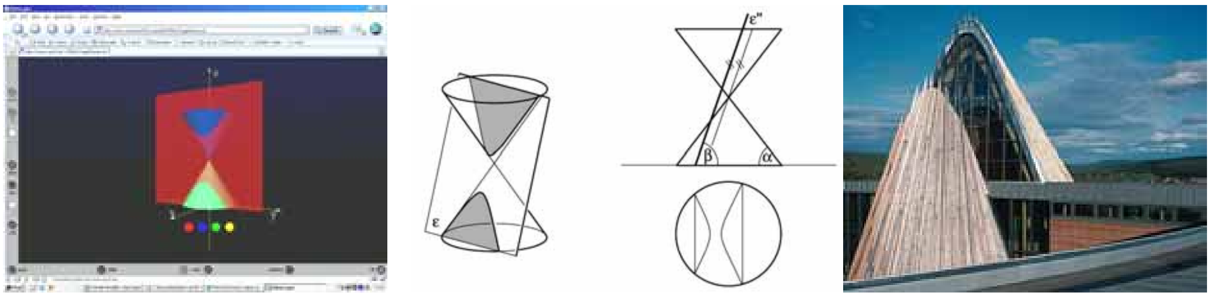

Figure 5: Virtual Model and Drawing of a Hyperbolic Cone Cutting, Photo of the Parliament of Laplander in Norway

In the labs and later in the homework projects students have to transfer the basic ideas to more complex problems as shown for example in Figure 6. The projects are worked out by the students independently for themselves with the possibility to ask for tutoring after they have started to work on it. In the shown example an elliptic cone cutting, an intersection of the cone with a cylinder and finally the development of the solids have to be constructed at the architectural example of the Ferry Terminal in Nagasaki, Japan, designed by Takamatsu and Lahyani in 1993-97. The projects are most worked out individually or sometimes arranged as group projects in order to learn team work.

Figure 6: Homework Projects – Ferry Terminal in Nagasaki – Photo, Task and Solution

4.4 Combining Geometry with 3-D modelling CAD-Programs and Digital Visualisations

Working by hand and working with the computer should not be separated. By transferring the geometric knowledge about space, representation of space, forms and combinations between forms to the digital working process with a 3-D modelling software the spatial visualisation ability can be enhanced by visualising different variants of the spatial objects from various viewpoints and by getting aware of the parameters of the used representation method.

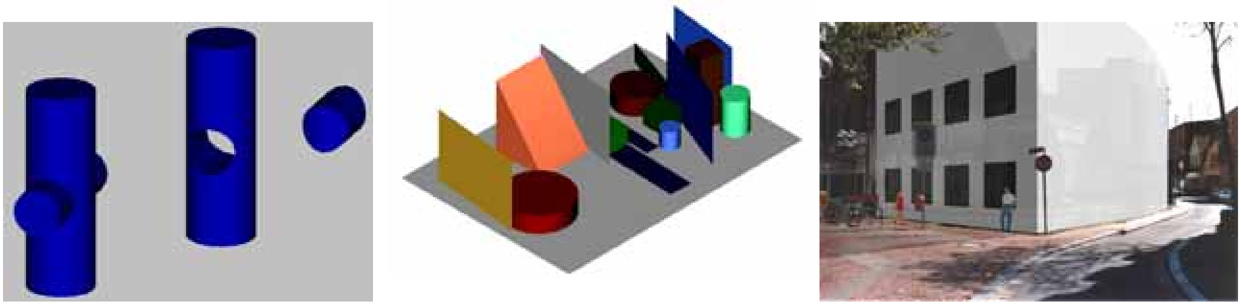

Figure 7: Boolean Operations, 3-D modelling of a Spatial Configuration (Fig. 3), Perspective Representation by Photomontage

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NAUCZANIE GEOMETRII W ROZWIJANIU WYOBRAŹNI PRZESTRZENNEJ STUDENTÓW KIERUNKÓW INŻYNIERSKICH

Badania naukowe w zakresie kształtowania oraz rozwijania wyobraźni przestrzennej oraz doświadczenia dydaktyków pokazują, iż najlepsze wyniki nauczania na kierunkach inżynierskich studiów technicznych uzyskuje się w sytuacji, kiedy podstawowa wiedza w zakresie geometrii wykreślnej jest uzupełniona przez wykonywanie szkiców odręcznych oraz rysunków dwuwymiarowych na podstawie modeli trójwymiarowych, które mogą być modelami rzeczywistymi lub wirtualnymi. Zadania uzupełniane są technikami prezentacji poprzez wykonanie komputerowej wizualizacji projektu. W pracy przedstawiono genezę rozwoju metod nauczania przedmiotu geometria wykreślna oraz podano konkretne przykłady zadań rozwiązywanych przez studentów architektury i inżynierii lądowej w Technicznym Uniwersytecie w Kaiserslautern (Niemcy). Długoletnie doświadczenie dydaktyczne prowadzi autorkę do konstrukcji optymalnego programu nauczania geometrii wykreślnej w takim kierunku, aby przedmiot spełniał warunek rozwijania technicznej wyobraźni przestrzennej studentów kierunków inżynierskich.