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Research paper

Experimental justification of implementation of the composite projection in Azerbaijan

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Abstract: Transnational communication lines differ from other objects of the economy both in form and size, and in the task of their geodetic support, which requires the development of special methods for its solution. One of the ways of geodetic support with high accuracy is the choice of optimal (with minimal distortion) geodetic projection for the territories of transnational communication lines. Composite projection is one of the most appropriate methods to choose the optimal projection. This article presents the argumentation and results of experimental calculations for the implementation of optimal geodetic projection for transnational communication lines in Azerbaijan.

Keywords: transnational communications, geodetic support, composition, cartographic distortion, geodetic projection

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1. Introduction

After Azerbaijan has gained its independence in order to develop its economy it has implemented a lot of large scale domestic and international communication projects such as oil and gas pipelines, transportation and other projects. One of the important practical aspects while arranging any kind of activities on the physical surface of the Earth (including the implementation of transnational communication projects) is ensuring geodetic support for the project works in compliance with applicable norms and guidelines. Generally geodetic support can be done in a different way depending on the type, parameters (length, area and etc.) accuracy requirements of the project to be implemented and also location of the object (Piskunov and Krylov, 1989; Gojamanov, 2011; Levchuk et al., 1983). Transnational communication lines differ from other civil works not only in regards to their geometric parameters, but also in terms of the geodetic support systems that they require. Therefore geodetic support of transnational communication projects requires the formulation of specific approaches and solutions. Selection of optimal geodetic projection for work area is very important in terms of increasing the accuracy of the geodetic support. In special cases the selection of optimal geodetic projection can be beneficial for establishing appropriate coordinate system for the projects implemented in Azerbaijan Republic. Considering this some researchers propose to use composite projection method (Lisichansky, 1970; Podshivalov, 2000; Gojamanov and Imbabi, 2010). The elements of composite projection are identified by using experimental calculations. This article covers such calculations carried out for the territory of Azerbaijan Republic and optimal geodetic projection selected to meet the actual demand.

2. The importance of establishing common coordinate system framework for the transnational communication lines

Transnational communication lines cross the territories of several countries and in order to better manage them from single center, it is required that they are located in the same coordinate framework. It is possible to establish such a coordinate framework using geodetic coordinates system (Boyko, 2003; Morozov, 1979). However, geodetic coordinates (B, L) are curvilinear coordinates which are indicated as angle degrees. Even though this aproach is efficient in solving several scientific and practical issues, it creates difficulties in the different sectors of economy including implementation of transnational communication projects. Indeed, values of geodetic coordinates expressed as linear units varies depending on geographical latitude and since the geographical longitudes are not parallel to each other, azimuths calculated based on them also vary. Whilst simple formulas can be used to solve the direct and inverse geodesy exercises on the plane, it is difficult to do the same on the ellipsoid. With this in mind conclusion can be made that it is more efficient to provide geodetic support to the transnational communication projects by using plane coordinates. This means that there has to be transition from ellipsoid to the plane. There might be infinite number of geodetic projections that can serve for this purpose. Therefore the type of projection has to be chosen based on the functionality use of the end picture on the plane.

The technical requirements applied to the geodetic projections are as follows (Bugayevskiy, 1998; Ogorodova, 2006):

– The selected geodetic projection should ensure the establishment of unified coordinate system in the whole area of the country and should create opportunity for the calculation of the topographic-geodetic works in a uniform way.
– The values of linear and angular distortions should be kept to the minimum.

In order to meet above-mentioned conditions, the symmetrical composite projections are used for geodetic support purposes. The theoretical foundations for the selection of optimal projections and resolution of related problems have been studied by scientists such as Bugayevskiy (1998), Lisichansky(1970) and Podshivalov (2000). It should also be mentioned that the optimal geodetic projection is selected among the composite projections. In such cases the composite projection is considered as one of the best suitable methods.

3. Theoretical-methodological base of composite projection

Let us search for the optimal geodetic projection which meets the Chebyshev–Grave conditions (principles) by taking the benefit of Lisichansky's method which he used in cartographic projections (Lisichansky, 1970). It is also important to note that Podshivalov used this methodology as well in his research (Patsiya, 2008; Podshivalov, 2000).

The essence of this method is that in order to create optimal, accurate coordinate framework (for the specific area or object) not one, but several conformal projections are being used based on common algorithms, which differs from traditional approach (using only one projection for geodetic purposes) and this is called "derivative" or "composite" projection according to Podshivalov (2000). The coordinates of composite projection are identified in the form of linear function of two or more conformal projection that are part of it. The conformity of projection is identified by meeting the Koshi–Riman condition (Boyko, 2003; Bugayevskiy, 1998; Evglevsky, 2005):

$$\begin{aligned} \frac{\partial x}{\partial B} &= \frac{M}{N \cdot \cos B} \cdot \frac{\partial y}{\partial L}; \\ \frac{\partial y}{\partial B} &= -\frac{M}{N \cdot \cos B} \cdot \frac{\partial x}{\partial L} \end{aligned} \tag{1}$$

where:

B, L – the geodetic coordinates of point on the ellipsoid;

x, y – the rectangular (decart) coordinates on the plane of the same point;

M – the curvature radius of the meridian;

N – the curvature radius of the first vertical.

Since the composite projection is identified as a linear function of two or more conformal projections it also meets the Koshi–Riman condition and that is why it is conformal. On the other hand, if the scales of the central points of the projections that are used for the creation of the composite projection are selected among the groups of modelled (planable) projections, then it is possible to make zero distortion lines to have required shape and to be at required location and direction within the work area. This meets the second case of the Chebyshev–Grave condition (Bugayevskiy, 1998; Podshivalov, 2000) which defines as optimal projection. In other words, in this case there is an opportunity that lines connecting the points with the same distortion values (isoclines) are similar or pass closer to the area's contour line, i.e. there is an opportunity to manage the distortions by meeting the Chebyshev–Grave condition.

The composite projection is written in a mathematical language as below:

$$\begin{aligned} X &= k_1 \cdot x_1 + k_2 \cdot x_2 + \ldots + k_n \cdot x_n; \\ Y &= k_1 \cdot y_1 + k_2 \cdot y_2 + \ldots + k_n \cdot y_n; \end{aligned} \tag{2}$$

where:

x_i, y_i – the rectangular coordinates of the point on the plane in the various conformal projections;

X, Y – the rectangular coordinates of the same point on the plane in the composite projection;

k_i – composite coefficient.

The essence of the k_i coefficients is that with their help the isoclines are passed very close to the contour lines of the depicted area i.e. the most optimal (accurate) projection for the area is selected.

These coefficients are selected based on the following condition:

$$k_1 + k_2 + \ldots + k_n = 1 \tag{3}$$

Equation (3) makes easier to determine the values for the conditional coefficients. Podshivalov (2000) proposes that the value of the main scale of the composite projection to be the same with the value of the m_0 accepted in the projections which are part of composite projection. According to Podshivalov (2000) if condition (3) is met, the selected projection will have all the characteristics that are specific for the composite projections: symmetry; slight difference in the coordinates of points forming composite projection and projections which are part of it; the opportunity to arrange the optimal distribution of the linear distortions within the area by changing (modeling) the value of the m_0 scale.

4. Experimental calculations for selection of composite projection in Azerbaijan Republic

Based on the theoretical-methodological knowledge given in the previous chapter we will try to justify with the experimental calculations the importance and applicability of implementing the composite projection for the creation of optimal (accurate) coordinate framework for the geodetic support during the realization of transnational communication projects in Azerbaijan Republic. We should also note that the implementation of the composite projection in terms of values of linear distortions (at least $1,5 \div 2$ times less) is considerably efficient in all cases and is more efficient than the other projections which are being used and till now considered the efficient at the work sites currently (Ismayilov, 2018; Gojamanov, 2011; Patsiya, 2008). Moreover there is an opportunity to depict the area in a common coordinate framework in other words, to provide the geodetic support in a common coordinate framework. The territory of Azerbaijan Republic is located in the two 6° zones according to the Gauss–Kruger coordinate system however, it is possible to use common zone coordinate system in a composite projection. Let's show it with experimental calculations.

It should be mentioned that it is expedient to use cylindrical and conic projections in the territory of Azerbaijan Republic for the creation of the composite projection. Therefore these projections have been used in experimental calculations. The main condition in this case is that the sum of the participating coefficients (k_1 and k_2) is

$$k_1 + k_2 = 1 \tag{4}$$

The values of coefficients are calculated with the following formulas (Patsiya, 2008; Podshivalov, 2000):

$$k_1 \approx \frac{\Delta y_{12}^2}{\Delta y_{12}^2 + \Delta x_{12}^2}; \qquad k_2 \approx \frac{\Delta x_{12}^2}{\Delta y_{12}^2 + \Delta x_{12}^2}, \tag{5}$$

where: Δx and Δy coordinate differences of the two typical points of the area.

It is known that, distortion characteristics (accuracy) of any given projection are characterized with the value of maximum special scale at the edge points of the projection area zone (Boyko, 2003; Bugayevskiy, 1998; Morozov, 1979). In this case the coordinates of the central (characteristic) point for the projection area must be known. In turn, finding the central point of the area requires determination of the coordinates of the typical points along the border line of area (Gojamanov et al., 2007). Therefore, 101 typical points along the land border of the Azerbaijan Republic's territory (uneven and bending points of the border line) have been selected and B_i, L_i coordinates have been determined graphically on the general geographical map of the Azerbaijan Republic in 1:500 000 scale (2017 issue).

As mentioned above, the values for the k_1 and k_2 coefficients are calculated using Equation (5) based on the coordinates of the two typical points of the area. Transnational communication lines usually pass through the Azerbaijan territory in the direction of Alat-Gazakh region and then cross into the Georgian territory. Considering this aspect as a typical points of transnational communication lines in the territory of the Azerbaijan Republic the broder points of No. 1 (Gazakh) and No. 63 (Alat town) have been selected. The geodetic coordinates of these point are following (determined in a 1:500 000 scale map with a graphic method):

$$\text{No. 1} \begin{cases} B_1 = 41.20°; \\ L_2 = 45.11°; \end{cases} \qquad \text{No. 63} \begin{cases} B_{63} = 40.00°; \\ L_{63} = 49.30°. \end{cases}$$

After that based on geographical coordinates the rectangular coordinates of the same points on the plane have been calculated in accordance to the transverse cylindrical Gauss–Kruger projection. The rectanguar coordinates in relation to the newly determined geographical central point ($B_0 = 40.5653°$; $L_o = 47.6112°$) of the country in the single zone are calculated using the following formulas (Boyko, 2003; Bugayevskiy, 1998; Morozov, 1979):

$$\begin{aligned} x &= \left[(a_4 \cdot l^2 + 0,5) \cdot l^2 \cdot N - a_0\right] \sin B \cdot \cos B + 6367558.5 \cdot B \\ y &= \left[(a_5 \cdot l^2 + a_3) \cdot l^2 + 1\right] l \cdot N \cos B \end{aligned} \tag{6}$$

where:

$$a_0 = (0.7 \cos^2 B - 135.3) \cos^2 B + 32140.4;$$
$$a_3 = (0.0011 \cos^2 B + 0.3333) \cos^2 B - 0.1667;$$
$$a_4 = 0.25 \cos^2 B - 0.012;$$
$$a_5 = (0.2 \cos^2 B - 0.17) \cos^2 B.$$

In Equation (6) B, L coordinates are identified in radians and x, y plane coordinates are identified in metres with 0.1 m accuracy.

The formulas for the coordinate calculations (6) are written in the Excel (Table 1).

Table 1. The algorithms for the calculation of x, y coordinates in accordance to the B, L coordinates

Operation No.	Letter symbol of algorithms in Excel	Calculation formula
1	A	B_i
2	B	L_i
3	C	L_0
4	D	$l = L_i - L_o = B_1 - C_1$
5	E	$N_i = a(1 - e^2 \cdot \sin^2 B_0)^{-\frac{1}{2}} = \dfrac{6378245}{\sqrt{(1 - 0.0066934216 \cdot \sin^2(A_1))}}$
6	F	$a_0 = (0.7 \cdot \cos^2 B - 135.3) \cdot \cos^2 B + 32140.4 =$ $= (0.7 \cdot \cos^2(A_1) - 135.3) \cdot \cos^2(A_1) + 32140.4$
7	G	$a_3 = (0.0011 \cdot \cos^2 B + 0.3333) \cdot \cos^2 B - 0.1667 =$ $= (0.0011 \cdot \cos^2(A_1) + 0.3333) \cdot \cos^2(A_1) - 0.1667$
8	H	$a_4 = 0.25 \cdot \cos^2 B - 0.042 = 0.25 \cdot \cos^2(A_1) - 0.042$
9	I	$a_5 = (0.2 \cdot \cos^2 B - 0.17) \cdot \cos^2 B = (0.2 \cdot \cos^2(A_1) - 0.17) \cdot \cos^2(A_1)$
10	J	$X = \left[\left(a_4 \cdot l^2 + 0.5\right) l^2 \cdot N - a_0\right] \cdot \sin B \cdot \cos B + 6367558.5 \cdot \dfrac{B^\circ}{\text{rad}} =$ $= \left[\left(H_1 \cdot D_1^2 + 0.5\right) \cdot D_1^2 \cdot E_1 - F_1\right] \cdot \sin(A_1) \cdot \cos(A_1) =$ $= 6367558.5 \cdot B^\circ \pi 1/180^\circ$
11	K	$Y = \left[\left(a_{54} \cdot l^2 + a_3\right) l^2 + 1\right] \cdot lN \cdot \cos B =$ $= \left[\left(I_1 \cdot D_1^2 + G_1\right) \cdot D_1^2 + 1\right] \cdot sD_1 \cdot E_1 \cdot \cos(A_1)$

As a result of calculation the following coordinates for typical No. 1 and No. 65 points have been identified:

$$x_1 = 4580518 \text{ m}; \qquad x_{63} = 4431317 \text{ m};$$
$$y_1 = -203249 \text{ m}; \qquad y_{63} = 161297.9 \text{ m}.$$

And as follows from this, $\Delta x_{1.63} = -149201$ m; $\Delta y_{1.63} = +364546.9$ m.

If put these figures into Equation (5) then we get the following: $k_1 = 0.86$; $k_2 = 0.14$.

In other words in order to create the composite projection first the participation coefficients of cylindrical and conic projections have been identified and then by multiplying these respective coefficients to the algorithm for the calculation of the special linear scale the general algorithms for the composite calculation are identified. After that the values of the special linear scale for the specific points selected along the state border of the Azerbaijan Republic are calculated using the starting points of composite projection $B_0, L_0, m_0 = 1$. Since the calculation results were big, only the distribution scheme of distortions and the general results according to the calculation versions are indicated in table 2 given below.

The maximum special linear scale No. 92 in the composite projection with the parametres of $k_1 = 0.86$; $k_2 = 0.14$ overlaps with the border point and its value is $m_{\max} = 1.000624068$ and the relative linear distortion is 1:1 600 (Figure 1).

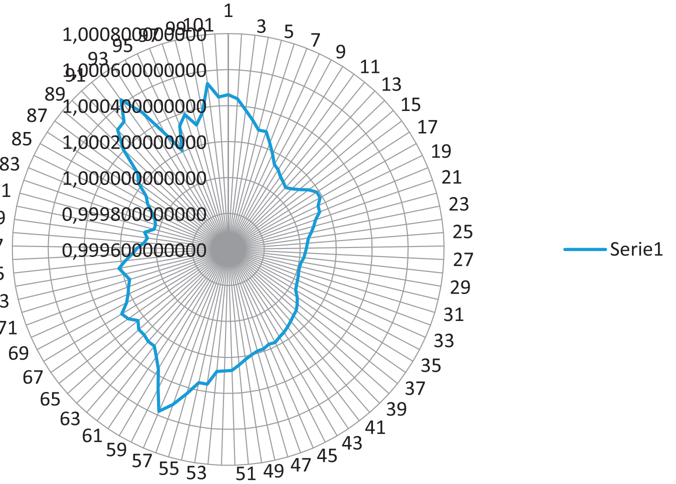

Fig. 1. Scheme of linear distortion in composute projection (transverse cylindrical Gauss–Kruger projection ($k_1 = 0.86$) + Lambert conic projection ($k_2 = 0.14$)

In the next stage the optimal scale for the area's central point was calculated in order to decrease the distortion figure of the projection. The formula of this calculation is given below (Gojamanov, 2011; Patsiya, 2008):

$$m_0 = \frac{2}{1 + m'_{\max}}; \qquad m_0 = \frac{2}{1 + 1.0005624068} = 0.99968806 \tag{7}$$

On the other hand it should be noted that the maximum values of the distortion in composite projection coincidence with the edge points of the experimental area and value of the distortion is considerably small in the most parts of the area i.e. the image has high accuracy (relative accuracy of 1:2 000–1:5 000). This means that, it is possible to provide the geodetic support by using the composite projection with more higher accuracy compared to the traditional approach (based on the Gauss–Kruger projection).

Besides the option stated above, additional 14 options for the composite projection with the different input parametres have been tested as an experimental model in order to search for the more optimal geodetic projection for the territory of Azerbaijan Republic. The combination of the cylindrical and conic projections have been reviewed in these additional options as well. In the options No. (1–12) the central geographical point of Azerbiajan (B_0, L_0) was chosen as a central point of projection and in the options No. 13 and No. 14 the central point (coordinates) of projection was changed.

In the options No. (1–12) the values of coefficients for the k_1 and k_2 were changed with the 0.05 step for each new option between the interval of k_1 $(0.86 \div 0.46)$, $k_2 (0.14 \div 0.60)$ and in the options No. 10 and No. 11 the values of coefficients for the k_1 and k_2 were opposite each other and in the option No. 12 values accepted as respectively $k_1 = -0.50$, $k_2 = 0.50$.

As stated above since the results of the calculations are quite big they are not presented here, however, summary results for the all options are given in the Table 2.

Table 2. Calculation of special linear scales of composite projection

Version No	Coordinates of central point		Composite projection coefficients		No of border point and rate of special linear scale		Relative linear distortion	
	B_0	L_0	K_1 (cylindric)	K_2 (conic)	m_{max}	m_{min}	ΔS/S, max	ΔS/S, min
1	40°,563	47°,611	0.80	0.20	2; 1.0005890410	30; 1.0000124022	1:1 700	1:80 600
2	40°,563	47°,611	0.75	0.25	92; 1.000559805	30; 1.0000154590	1:1 800	1:64 700
3	40°,563	47°,611	0.70	0.30	92; 1.000530660	30; 1.0000185158	1:1 900	1:54 000
4	40°,563	47°,611	0.65	0.35	92; 1.000501470	30; 1.000021573	1:2 000	1:46 000
5	40°,563	47°,611	0.60	0.40	92; 1.000472279	30; 1.000024629	1:2 100	1:40 000
6	40°,563	47°,611	0.55	0.45	92; 1.000443088	30; 1.000027686	1:2 250	1:36 000
7	40°,563	47°,611	0.50	0.50	92; 1.00041000	30; 1.000030743	1:2 450	1:32 500
8	40°,563	47°,611	0.45	0.55	74; 1.000433970	30; 1.000033800	1:2 300	1:29 500
9	40°,563	47°,611	0.40	0.60	74; 1.000460440	30; 1.000036856	1:2 170	1:27 100
10	40°,563	47°,611	1.14	−0.14	58; 1.000761801	81; 0.999999366	1:1 300	1:1 600
11	40°,563	47°,611	−0.14	1.14	75; 1.000790303	45; 1.000005367	1:1 260	1:190 000
12	40°,563	47°,611	−0.50	1.50	75; 1.001011702	11; 1.000002862	1:990	1:350 000
13	41°,0	47°,0	0.86	0.14	58; 1.000861059	28; 1.000007045	1:1 160	1:142 000
14	41°,0	47°,0	0.50	0.50	74; 1.0006419125	12; 1.000007726	1:1 550	1:130 000

By reviewing Table 2 it is obvious that the projection selected based on the composite principle in all cases is more suitable rather than traditional projections in terms of accuracy, however the smallest distortion value is in the option No. 7 – 1:2 450. In terms of the distortion indicators the value of the maximum special scale in this option is two times smaller than analogical value in the Gauss–Kruger 6° zones projection which is currently in use in the Azerbaijan Republic and 6–8 times smaller than in the case where the image transferred from one 6° zone to the other (i.e. calculation from 9^{th} zone to the 8^{th} zone's central meridian or vice versa). One of the advantages of the composite projection in comparison to the traditionally used projection is that it is created in the common coordinate (projection) framework. All the assumptions prove that utilising the composite projection for the geodetic support of the transnational communication projects is both suitable and efficient.

5. Conclusion

The development of modern geodetic measurement geo-technologies demands the preparation of unified coordinate systems and geodetic reference projections with high precision which meet the modern requirements.

1. The composite projection ensures the unified coordinate zone framework in the territory of Azerbaijan Republic and also results in improving of accuracy $1.5 \div 2$ times compared to the traditional geodetic support approach – just utilizing only one projection (1:2 500). Thus it is more appropriate to use composite projection for providing geodetic support to transnational communication projects.
2. One of the advantages of composite projection is that the percentage shares of included projection in composite projection might be selected in a such way that the shape of equal distortion lines are similar to the shape of outline lines of area. This means that the optimal projection can be created while meeting the Chebyshev–Grave conditions most.
3. Among the 14 composite projections which are involved in experimental calculation the 7^{th} one had high accuracy. In this composite projection cylindrical and conic projections with the respective ratios of $k_1 = 0.5$ and $k_2 = 0.5$ were used and the central point of composite projection was overlapping with the geographical center of Azerbaijan Republic that was determined by us ($B_0 = 40°,5653 = 40°33'55''$; $L_0 = 47°,6112 = 47°36'40''$). In this case the maximum relative linear distortion is 1:2 500. However, relative linear distortion in most parts of area varies between $\dfrac{\Delta S}{S} = \dfrac{1}{3000} \div \dfrac{1}{5000}$.

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References

Boyko, Y.Q. (2003). *High Geodesy. Part 2. Sferoidal geodesy*. Moscow: Cartgeosenter-Geodezizdat.

Bugayevskiy, L.M. (1998). *Mathematical cartography*. Moscow: Zlatoust.

Evglevsky, I.V. (2005). Selection of the optimal geodesic projections for areas of arbitrary sizes and configurations. *Computer Science*, 3, 126–134.

Gojamanov, M.H. (2011). Studies on the choice of the optimal projection. Materials of the International scientific-practical conference. Tashkent Institute of land reclamation and irrigation. Tashkent. 3–8.

Gojamanov, M.H. and Imbabi, A.R. (2010). Selection of the optimal projection for the territory of Arab Republic of Egypt. Proceedings of the BSU branch of the Geographical Society of the Republic of Azerbaijan. Baku: Volume III, 62–72.

Gojamanov, M.H., Jafarova, D.B. and Mustaphayeva, E.V. (2007). About geographical center of Azerbaijan. Present and tomorrow of geography. Materials of republican scientific and practical conference, BSU, 181–185 (in Azerbaijani).

Ismayılov, A.I. (2018). Selection of optimal geodetic projection for transnational communication projects. Materials of XII Republican scientific conference of PhDs and young researchers. SPUAR, Baku, 36–40.

Levchuk, G.P., Novak, V.E. and Lebedev, N.N. (1983). *Applied geodesy. Geodetic works in surveys and construction of engineering structures*. Moscow: Nedra.

Lisichansky, A.S. (1970). Theory of the generalized class of conformal azimuthal-conical projections. *Geodesy, Cartography and Aerial Photography*, 11, 83–87.

Morozov, V.P. (1979). *Course sfeoroidal geodesy*. Moscow: Nedra.

Ogorodova, L.V. (2006). *Higher geodesy. Part III*. Theoretical geodesy. Moscow: Geodesicartisdat.

Patsiya, A.M. (2008). The choice of projection for the territory of the subject of the Russian Federation. *Geodesy and Cartography*, 7, 45–48.

Piskunov, M.E. and Krylov, V.N. (1989). *Geodesy in the construction of gas, water and sewer networks and structures*. Moscow: Stroyizdat.

Podshivalov, V.P. (2000). Compositional geodetic projections. *Geodesy and Cartography*, 8, 39–43.