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MACHINE TOOL CONTROL WITH ADDITIONAL MEASUREMENT FOR INCREASING THE CONTROL SYSTEM DYNAMICS

As a typical type of controller in the area of machine tools the classical cascade controller is used. It consists of several PI control loops and allows the position control of the machine tool. This type of controller is easy to be implemented and gives satisfactory results but only in the case that the sufficiently stiff machine tool is being controlled. If adverse is true the performance of the controller deteriorates. This is due to the fact, that the controller is limited by the structural properties of the machine tool. The bandwidth of the controller is restricted by the position of the first anti-resonant frequency of the machine tool. The control techniques overcoming this limitation have been extensively researched. As a result the control technique employing the additional measurement of TCP, the model-based predictive control and the Kalman filter is used and delivers the increased control system dynamics. The paper deals with the description of the proposed control concept and the practical methods for additional measurement together with the Kalman filter tuning are described. The evaluation of the proposed control concept is based on the experimentally measured data on the machine tool axis with significant flexibility.

1. INTRODUCTION

The cascade control concept is the ordinary used type of control in the area of machine tools. It is simple and reliable control concept that gives satisfactory results, but the achievable dynamics of the controlled system is restricted [12], especially if the mechanical structure is flexible. It is limited by the structural properties of the system. Namely by the first anti-resonant frequency which is connected with the machine tool stiffness. The lower the stiffness is, the lower the value of the first anti-resonant frequency is which leads to the narrow passband of the controller and to the weak dynamic properties of the controlled system. To overcome this limitation and in general to increase the dynamic properties of the controlled system the model-based control techniques has been successfully tested in combination with the additional measurement of the Tool Centre Point (TCP). For example

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in [13] the state-feedback controller has been employed to deliver the increased system dynamics. The drawback of the technique presented in [13] is the difficulty of the controller tuning to meet the user's demands, especially when the state vector of the employed model has no physical meaning. For that reason another model-based control technique, the model-based predictive control technique with the Kalman filter has been used that delivers similar control results but is much easier and straightforward to be tuned.

The motivation of the investigation of the control of machine tools with flexible mechanical structure is to build much lighter, however much more flexible machine tools. Such machine tools would fit into the eco-design as they would consume significantly less energy during their manufacture as well as during their operation.

2. CONCEPT OF THE ADDITIONAL MEASUREMENT

The concept of the additional measurement of TCP which is utilized in the further presented control technique can be suitably explained on the simple two-mass model of an arbitrary machine tool, Fig. 1. It can be stated that the first mass in the model represents the motor itself and the second mass represents the tool tip itself. The spring element between the masses represents the machine tool stiffness (parameter k_t) and the damper element stands for the machine tool damping ratio (parameter b_t). The measuring devices can be placed on the structure to deliver the information about the position of both masses. Based on them their velocities can also be computed. This description of the machine tool includes the classical cascade control concept where only the position and velocity of the first mass (motor) are measured as depicted in the left part of Fig. 1, although the subject of the control is the position of the second mass (tool tip). This can lead to the satisfactory control results only if sufficiently stiff machine tools are being controlled. When adverse is true the concept of additional measurement is necessary, i.e. the position measurement of the TCP as depicted in the right part of Fig. 1 where the state-feedback controller is being employed as an example.

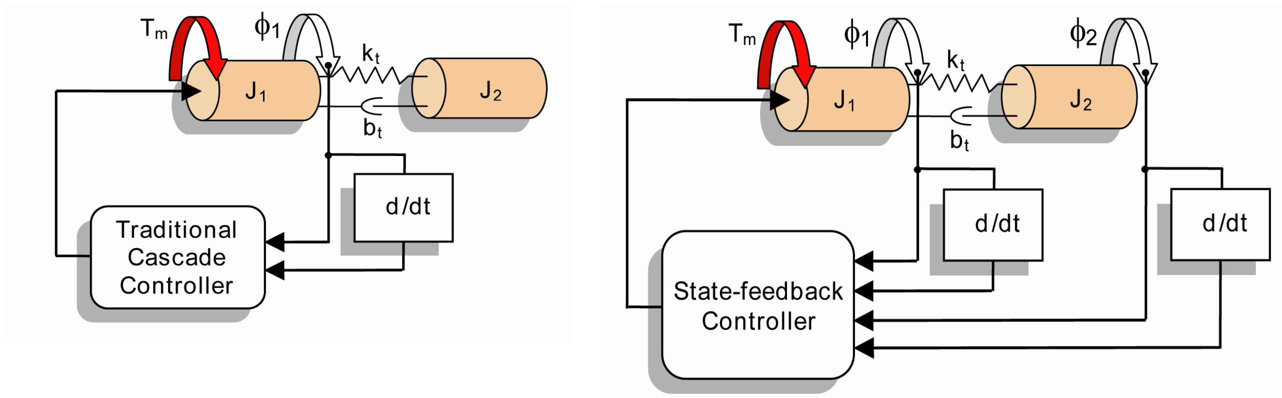

Fig. 1. Traditional cascade controller and the full state-feedback controller with additional measurement on simple two-mass system

3. PREDICTIVE CONTROL WITH KALMAN FILTERING

The model-based predictive control in the combination with the Kalman filter as a state observer can be used as a model-based control technique similar to the state-feedback control technique that can improve the control system dynamics. The experimental control results concerning the state-feedback control technique can be found in [13]. Here the description of the model-based predictive control technique is presented.

3.1 SYSTEM MODEL

As a system model the linear, time invariant, state-space model in discrete form is considered (1). Within the model also the feed-through relation between the input and output is reflected via the nonzero elements of matrix $\mathbf{D_d}$. Based on this fact the general form of the predictive controller has to be derived. This is not very common in the literature [2], [4], [10,11]. The system model is written as

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A_d}\mathbf{x}_k + \mathbf{B_d}\mathbf{u}_k \\ \mathbf{y}_k &= \mathbf{C_d}\mathbf{x}_k + \mathbf{D_d}\mathbf{u}_k \end{aligned}, \tag{1}$$

where $\mathbf{x}_k$ is the state vector of the dimension $n\times 1$, $\mathbf{u}_k$ is the vector of the inputs with the dimension $m\times 1$, $\mathbf{y}_k$ is the output vector with the dimension $l\times 1$ and the matrices $\mathbf{A_d}$, $\mathbf{B_d}$, $\mathbf{C_d}$, $\mathbf{D_d}$ are the state-space matrices with the dimensions $n\times n$, $n\times m$, $l\times n$, $l\times m$ respectively. The index k marks the sample in the time instant $k.T$, where T is the sampling period.

3.2. MODEL MODIFICATION FOR THE OFFSET FREE CONTROL

The offset free control is an important issue in the theory of predictive control. There are two main reasons causing the output offset. These are the presence of disturbances and the mismatch between the model and the real system. In order to solve the output offset several techniques have been developed. They can be found for example in [5], [8, 9]. If the

$$\begin{aligned} \underbrace{\begin{bmatrix} \Delta\mathbf{x}_{k+1} \\ \mathbf{y}_k \end{bmatrix}}_{\mathbf{x}_{\mathbf{p}k+1}} &= \underbrace{\begin{bmatrix} \mathbf{A_d} & \mathbf{0} \\ \mathbf{C_d} & \mathbf{I} \end{bmatrix}}_{\mathbf{A_p}} \underbrace{\begin{bmatrix} \Delta\mathbf{x}_k \\ \mathbf{y}_{k-1} \end{bmatrix}}_{\mathbf{x}_{\mathbf{p}k}} + \underbrace{\begin{bmatrix} \mathbf{B_d} \\ \mathbf{D_d} \end{bmatrix}}_{\mathbf{B_p}} \Delta\mathbf{u}_k \\ \mathbf{y}_k &= \underbrace{\begin{bmatrix} \mathbf{C_d} & \mathbf{I} \end{bmatrix}}_{\mathbf{C_p}} \begin{bmatrix} \Delta\mathbf{x}_k \\ \mathbf{y}_{k-1} \end{bmatrix} + \underbrace{\mathbf{D_d}}_{\mathbf{D_p}} \Delta\mathbf{u}_k \end{aligned}, \tag{2}$$

system model has no feed-through relation ($\mathbf{D_d}$ is zero matrix) then the suitable technique, that has been tested, can be found in [8]. The considered model (1) has the non-zero matrix $\mathbf{D_d}$ therefore the following model modification has been applied that solves the offset free control

where $\Delta = 1 - z^{-1} \Rightarrow \Delta\mathbf{x}_{k+1} = \mathbf{x}_{k+1} - \mathbf{x}_k$, $\Delta\mathbf{x}_k = \mathbf{x}_k - \mathbf{x}_{k-1}$, $\Delta\mathbf{u}_k = \mathbf{u}_k - \mathbf{u}_{k-1}$ and $\mathbf{0}$ and $\mathbf{I}$ are the zero and identity matrices of respective dimensions, $\mathbf{x}_{\mathbf{p}k}$ is the state vector of the new state-space model (2), $\Delta\mathbf{u}_k$ is the vector of input differences, $\mathbf{y}_k$ is the output vector and the matrices $\mathbf{A_p}$, $\mathbf{B_p}$, $\mathbf{C_p}$, $\mathbf{D_p}$ are the new state-space model matrices. The index k marks the sample in the time instant $k.T$ again, where T is the sampling period.

3.3. PREDICTION OF THE SYSTEM BEHAVIOUR

The prediction of the future system behaviour, which is the main principle of the predictive control, is created in the following manner

- Based on (2) the prediction of the state and the output in the k+1 time sample is created

$$\mathbf{x}_{\mathbf{p}k+1} = \mathbf{A_p}\mathbf{x}_{\mathbf{p}k} + \mathbf{B_p}\Delta\mathbf{u}_k, \quad \mathbf{y}_{k+1} = \mathbf{C_p}\mathbf{x}_{\mathbf{p}k+1} + \mathbf{D_p}\Delta\mathbf{u}_{k+1} \tag{3}$$

- The prediction is also created in the k+2 time sample

$$\mathbf{x}_{\mathbf{p}k+2} = \mathbf{A_p}\mathbf{x}_{\mathbf{p}k+1} + \mathbf{B_p}\Delta\mathbf{u}_{k+1}, \quad \mathbf{y}_{k+2} = \mathbf{C_p}\mathbf{x}_{\mathbf{p}k+2} + \mathbf{D_p}\Delta\mathbf{u}_{k+2} \tag{4}$$

- Substituting (3) in (4) in order to eliminate $\mathbf{x}_{\mathbf{p}k+1}$, it is derived

$$\begin{aligned} \mathbf{x}_{\mathbf{p}k+2} &= \mathbf{A}_\mathbf{p}^2\mathbf{x}_{\mathbf{p}k} + \mathbf{A_p}\mathbf{B_p}\Delta\mathbf{u}_k + \mathbf{B_p}\Delta\mathbf{u}_{k+1} \\ \mathbf{y}_{k+2} &= \mathbf{C_p}\mathbf{A}_\mathbf{p}^2\mathbf{x}_{\mathbf{p}k} + \mathbf{C_p}\mathbf{A_p}\mathbf{B_p}\Delta\mathbf{u}_k + \mathbf{C_p}\mathbf{B_p}\Delta\mathbf{u}_{k+1} + \mathbf{D_p}\Delta\mathbf{u}_{k+2} \end{aligned} \tag{5}$$

- Starting with (5) and creating the prediction in the k+3 time sample, it is obtained

$$\begin{aligned} \mathbf{x}_{\mathbf{p}k+3} &= \mathbf{A}_\mathbf{p}^2\mathbf{x}_{\mathbf{p}k+1} + \mathbf{A_p}\mathbf{B_p}\Delta\mathbf{u}_{k+1} + \mathbf{B_p}\Delta\mathbf{u}_{k+2} \\ \mathbf{y}_{k+3} &= \mathbf{C_p}\mathbf{A}_\mathbf{p}^2\mathbf{x}_{\mathbf{p}k+1} + \mathbf{C_p}\mathbf{A_p}\mathbf{B_p}\Delta\mathbf{u}_{k+1} + \mathbf{C_p}\mathbf{B_p}\Delta\mathbf{u}_{k+2} + \mathbf{D_p}\Delta\mathbf{u}_{k+3} \end{aligned}, \tag{6}$$

  where $\mathbf{x}_{\mathbf{p}k+1}$ is eliminated using (3) and it is obtained

$$\begin{aligned} \mathbf{x}_{\mathbf{p}k+3} &= \mathbf{A}_\mathbf{p}^3\mathbf{x}_{\mathbf{p}k} + \mathbf{A}_\mathbf{p}^2\mathbf{B_p}\Delta\mathbf{u}_k + \mathbf{A_p}\mathbf{B_p}\Delta\mathbf{u}_{k+1} + \mathbf{B_p}\Delta\mathbf{u}_{k+2} \\ \mathbf{y}_{k+3} &= \mathbf{C_p}\mathbf{A}_\mathbf{p}^3\mathbf{x}_{\mathbf{p}k} + \mathbf{C_p}\mathbf{A}_\mathbf{p}^2\mathbf{B_p}\Delta\mathbf{u}_k + \mathbf{C_p}\mathbf{A_p}\mathbf{B_p}\Delta\mathbf{u}_{k+1} + \mathbf{C_p}\mathbf{B_p}\Delta\mathbf{u}_{k+2} + \mathbf{D_p}\Delta\mathbf{u}_{k+3} \end{aligned} \tag{7}$$

- Continuing in such recursion till the $k+i$ time sample it is derived

$$\begin{aligned}\mathbf{x}_{\mathbf{p}k+i} &= \mathbf{A}_\mathbf{p}^i \mathbf{x}_{\mathbf{p}k} + \mathbf{A}_\mathbf{p}^{i-1}\mathbf{B}_\mathbf{p}\Delta\mathbf{u}_k + \mathbf{A}_\mathbf{p}^{i-2}\mathbf{B}_\mathbf{p}\Delta\mathbf{u}_{k+1} + \ldots + \mathbf{B}_\mathbf{p}\Delta\mathbf{u}_{k+i-1} \\ \mathbf{y}_{k+i} &= \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^i \mathbf{x}_{\mathbf{p}k} + \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^{i-1}\mathbf{B}_\mathbf{p}\Delta\mathbf{u}_k + \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^{i-2}\mathbf{B}_\mathbf{p}\Delta\mathbf{u}_{k+1} + \ldots + \mathbf{C}_\mathbf{p}\mathbf{B}_\mathbf{p}\Delta\mathbf{u}_{k+i-1} + \mathbf{D}_\mathbf{p}\Delta\mathbf{u}_{k+i}\end{aligned} \tag{8}$$

- Based on the previous equations one matrix equation of the future predictions up to the horizon n_y can be written as

$$\underbrace{\begin{bmatrix}\mathbf{y}_{k+1}\\ \mathbf{y}_{k+2}\\ \vdots \\ \mathbf{y}_{k+n_y}\end{bmatrix}}_{\underset{\rightarrow}{\mathbf{y}}} = \underbrace{\begin{bmatrix}\mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}\\ \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^2\\ \vdots \\ \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^{n_y}\end{bmatrix}}_{\mathbf{P}} \underbrace{\mathbf{x}_{\mathbf{p}k}}_{\underset{\leftarrow}{\mathbf{x}}} + \underbrace{\begin{bmatrix}\mathbf{C}_\mathbf{p}\mathbf{B}_\mathbf{p} & \mathbf{D}_\mathbf{p} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}\mathbf{B}_\mathbf{p} & \mathbf{C}_\mathbf{p}\mathbf{B}_\mathbf{p} & \mathbf{D}_\mathbf{p} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^{n_y-1}\mathbf{B}_\mathbf{p} & \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^{n_y-2}\mathbf{B}_\mathbf{p} & \mathbf{C}_\mathbf{p}\mathbf{A}_\mathbf{p}^{n_y-3}\mathbf{B}_\mathbf{p} & \cdots & \mathbf{D}_\mathbf{p}\end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix}\Delta\mathbf{u}_k\\ \Delta\mathbf{u}_{k+1}\\ \Delta\mathbf{u}_{k+2}\\ \vdots \\ \Delta\mathbf{u}_{k+n_y}\end{bmatrix}}_{\Delta\underset{\rightarrow}{\mathbf{u}}} \tag{9}$$

- The equation of the prediction can be written in the compact form as

$$\underset{\rightarrow}{\mathbf{y}} = \mathbf{P}.\underset{\leftarrow}{\mathbf{x}} + \mathbf{H}.\Delta\underset{\rightarrow}{\mathbf{u}} \tag{10}$$

3.4. PREDICTIVE CONTROL LAW

For the derivation of the predictive control law the quadratic cost function is being used. Here the following quadratic cost function has been used

$$J = Q_y \sum_{i=1}^{n_y} \left\| \mathbf{r}_{k+i} - \mathbf{y}_{k+i} \right\|_2^2 + dQ_y \sum_{i=1}^{n_y} \left\| \mathbf{y}_{k+i} - \mathbf{y}_{k+i-1} \right\|_2^2 + dQ_u \sum_{i=0}^{n_u} \left\| \Delta\mathbf{u}_{k+i} \right\|_2^2, \tag{11}$$

where the vectors $\mathbf{r}_{k+i}$, $\mathbf{y}_{k+i}$, $\Delta\mathbf{u}_{k+i}$ stands for the vector of the desired output values, the vector of the predicted outputs and the vector of the predicted input increments. The coefficients n_y a n_u are the horizons of the predicted outputs and the input increments, where $1 \le n_y < \infty$ and $0 \le n_u \le n_y - 1$. The coefficients Q_y, dQ_y a dQ_u are the weighting factors for each part of the quadratic cost function by which the resulting control action is tuned. For example the coefficient Q_y influences the amount of the position deviation during the control, the coefficient dQ_y influences the rate of the change in the controlled output and the coefficient dQ_u influences the dynamics and the amplitude of the control input. The cost function (11) can be written in the compact form as

$$J = Q_y \left\| \underset{\rightarrow}{\mathbf{r}} - \underset{\rightarrow}{\mathbf{y}} \right\|_2^2 + dQ_y \left\| \underset{\rightarrow}{\mathbf{y}} - \underset{\rightarrow}{\mathbf{y}}_{old} \right\|_2^2 + dQ_u \left\| \Delta\underset{\rightarrow}{\mathbf{u}} \right\|_2^2, \tag{12}$$

where $\underset{\rightarrow}{\mathbf{y}}_{old} = z^{-1}\underset{\rightarrow}{\mathbf{y}}$. The control law is derived by the minimisation of the cost function (12) using the condition

$$\frac{dJ}{d\Delta\underset{\rightarrow}{\mathbf{u}}} = 0 \tag{13}$$

This leads to the expression

$$\Delta\mathbf{u}_k = \mathbf{e_1}.\left[\left(Q_y + dQ_y\right).\mathbf{H}^T.\mathbf{H} + dQ_u.\mathbf{I}\right]^{-1}.\mathbf{H}^T.\left[Q_y.\underset{\rightarrow}{\mathbf{r}} + dQ_y.\underset{\rightarrow}{\mathbf{y}}_{old} - \left(Q_y + dQ_y\right).\mathbf{P}.\underset{\leftarrow}{\mathbf{x}}\right], \tag{14}$$

which is the first element of the vector $\Delta\underset{\rightarrow}{\mathbf{u}}$ that is implemented in the time sample $k.T$ as the control action. In (14) the matrix $\mathbf{e_1}$ consists of

$$\mathbf{e_1} = \left[\mathbf{I} \quad \mathbf{0} \quad \mathbf{0} \quad \dots \quad \mathbf{0}\right], \tag{15}$$

where **I** and **0** are the identity and zero matrices of the dimensions equal to $\Delta\mathbf{u}_k$.

3.5. KALMAN FILTER AS A STATE OBSERVER FOR THE PREDICTIVE CONTROLLER

Since the Kalman filter is the optimal observer for the systems affected by the measurement and process noise a slightly different model than (1) is used for its derivation

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A_d}.\mathbf{x}_k + \mathbf{B_d}.\mathbf{u}_k + \mathbf{G_d}.\mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C_d}.\mathbf{x}_k + \mathbf{D_d}.\mathbf{u}_k + \mathbf{v}_k \end{aligned}, \tag{16}$$

where $\mathbf{w}_k$ is the process noise, $\mathbf{v}_k$ is the output or measurement noise and the matrix $\mathbf{G_d}$ couples the effect of the process noise to the model states. Here and further it is assumed $\mathbf{G_d}$=$\mathbf{B_d}$. The process noise then becomes the input noise. The noises $\mathbf{w}_k$, $\mathbf{v}_k$ are assumed the white, uncorrelated, with zero mean value and covariances **Q**, **R**

$$\mathrm{E}\left(\mathbf{w}_k\right) = \mathrm{E}\left(\mathbf{v}_k\right) = \mathbf{0},\ \mathrm{E}\left(\mathbf{w}_k\mathbf{w}_k^T\right) = \mathbf{Q},\ \mathrm{E}\left(\mathbf{v}_k\mathbf{v}_k^T\right) = \mathbf{R},\ \mathrm{E}\left(\mathbf{w}_k\mathbf{v}_k^T\right) = \mathbf{0} \tag{17}$$

Having such knowledge about the system, the state vector in the time sample $k.\,T$ can be estimated using the recursion as [3]

- The correction

$$\begin{aligned} \mathbf{K}_{k-1} &= \mathbf{P}'_{k-1}.\mathbf{C}_\mathbf{d}^T.\left(\mathbf{C_d}.\mathbf{P}'_{k-1}.\mathbf{C}_\mathbf{d}^T + \mathbf{R}\right)^{-1} \\ \hat{\mathbf{x}}_{k-1} &= \hat{\mathbf{x}}'_{k-1} + \mathbf{K}_{k-1}.\left(\mathbf{y}_{k-1} - \left(\mathbf{C_d}.\hat{\mathbf{x}}'_{k-1} + \mathbf{D_d}.\mathbf{u}_{k-1}\right)\right) \\ \mathbf{P}_{k-1} &= \mathbf{P}'_{k-1} - \mathbf{K}_{k-1}.\mathbf{C_d}.\mathbf{P}'_{k-1} \end{aligned} \tag{18}$$

- The prediction

$$\begin{aligned} \hat{\mathbf{x}}'_k &= \mathbf{A_d}.\hat{\mathbf{x}}_{k-1} + \mathbf{B_d}.\mathbf{u}_{k-1} \\ \mathbf{P}'_k &= \mathbf{A_d}.\mathbf{P}_{k-1}.\mathbf{A}_\mathbf{d}^T + \mathbf{G_d}.\mathbf{Q}.\mathbf{G}_\mathbf{d}^T \end{aligned} \tag{19}$$

The matrix $\mathbf{K}_k$ is the Kalman gain, $\mathbf{P}_k^{'}$ is the predicted value of the estimation error covariance matrix, $\mathbf{P}_k$ is the corrected value of the estimation error covariance matrix, $\hat{\mathbf{x}}_k^{'}$ is the predicted value of the estimated state vector and $\hat{\mathbf{x}}_k$ is the corrected value of the estimated state vector. The state vector estimation is started by setting the initial values $\hat{\mathbf{x}}_0^{'}$ and $\mathbf{P}_0^{'}$. Because the so far presented technique of Kalman filtering delivers the estimation of the true state $\mathbf{x}_k$, meanwhile within the predictive controller vector of the state the increment $\Delta\mathbf{x}_k$ is used, it is necessary to store also the previous value of the state vector $\mathbf{x}_{k-1}$ since $\Delta\mathbf{x}_k = \mathbf{x}_k - \mathbf{x}_{k-1}$. This is done in shift registers where also $\mathbf{y}_{k-1}$ and $\mathbf{u}_{k-1}$ are stored.

For the correct state estimation not only the system model (16) has to be in good concordance with the real system but also the covariance matrices $\mathbf{Q}$, $\mathbf{R}$ have to truly describe the affecting noise. There exist several techniques for estimating the covariance matrices based on the real measurement. They can be found in [1], [3], [6,7]. For example in [7] the autocovariance matrices are expressed and used together with least-square method to estimate the system covariance matrices. Unfortunately the technique is sensitive to any non-linearity present in the real system, for example like Coulomb friction. In the area of machine tools the Coulomb friction is a very common phenomenon. Due to this fact a new technique for estimating the covariance matrices which can handle the effect of Coulomb friction has been developed as described in the following chapter.

4. PRACTICAL METHOD FOR ESTIMATING THE TRUE SYSTEM COVARIANCES

The proposed method is based on the known equations for the calculation of the signal covariances (20). The negative effect of the Coulomb friction is eliminated by using the measured closed loop data from the system. As a regulator the classical feedback controller is used with all feedback loops active (position, velocity and current control loop). Also the velocity feedforward is being activated during the measurement. For the position setpoint a position ramp is used meaning that after the start the system settles on the constant speed. Then also the Coulomb friction force becomes constant and the position deviation settles at zero. Based on all these conditions it is possible by using the simple arithmetic to filter out the time behaviour of the process and the measurement noises. The obtained noises are then processed through (20) to get the true covariance matrices $\mathbf{Q}$, $\mathbf{R}$

$$\mathbf{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix},\ \mathbf{Y} = \begin{bmatrix} Y_1 \\ \vdots \\ Y_m \end{bmatrix},\ \overline{X}_i = \mathrm{E}(X_i),\ \overline{Y}_i = \mathrm{E}(Y_i) \quad , \qquad (20)$$

$$\mathrm{cov}(\mathbf{X},\mathbf{Y}) = \begin{bmatrix} \mathrm{E}[(X_1-\overline{X}_1)(Y_1-\overline{Y}_1)] & \mathrm{E}[(X_1-\overline{X}_1)(Y_2-\overline{Y}_2)] & \cdots & \mathrm{E}[(X_1-\overline{X}_1)(Y_m-\overline{Y}_m)] \\ \mathrm{E}[(X_2-\overline{X}_2)(Y_1-\overline{Y}_1)] & \mathrm{E}[(X_2-\overline{X}_2)(Y_2-\overline{Y}_2)] & \cdots & \mathrm{E}[(X_2-\overline{X}_2)(Y_m-\overline{Y}_m)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{E}[(X_n-\overline{X}_n)(Y_1-\overline{Y}_1)] & \mathrm{E}[(X_n-\overline{X}_n)(Y_2-\overline{Y}_2)] & \cdots & \mathrm{E}[(X_n-\overline{X}_n)(Y_m-\overline{Y}_m)] \end{bmatrix}$$

As an example the mechanical system consisting of a linear motor and a table is used. The motor force is considered as the system input and the position of the motor primary part and the table position are considered as the system outputs. The signal arithmetic is well described from Fig. 2 and Fig. 3. Fig. 2 (left) shows the time behaviour of the requested and the true value of the motor force. The data are truncated. The transient effect, here from 0s to 1s, is missed out. Each signal is then centred (mean value is subtracted) and then the resulting signals are subtracted from each other. As a result the record of the process noise is obtained Fig. 2 (right).

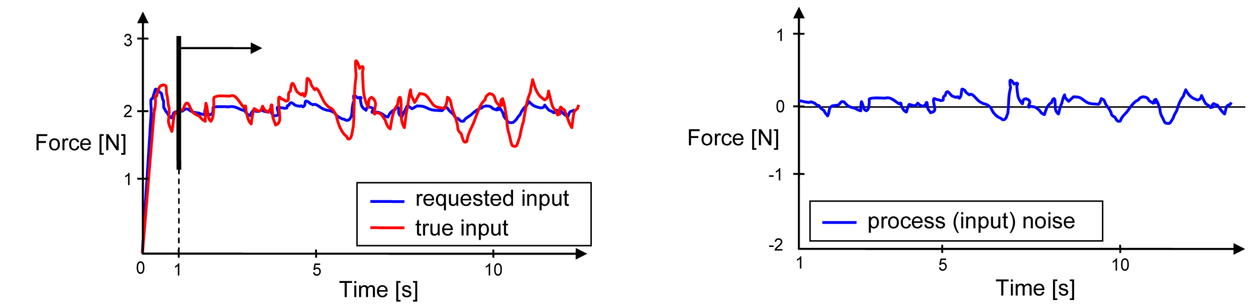

Fig. 2. Time behaviour of requested and true value of motor force (left); time behaviour of process (input) noise (right)

In the case of the measurement noises the procedure is similar as in the case of the motor force, Fig. 3. From the position setpoint the remaining measured positions are subtracted. Then the resulting signals are truncated. Again the transient effect from 0s to 1s is missed out. The corresponding measurement noises can be seen in Fig. 3 (right).

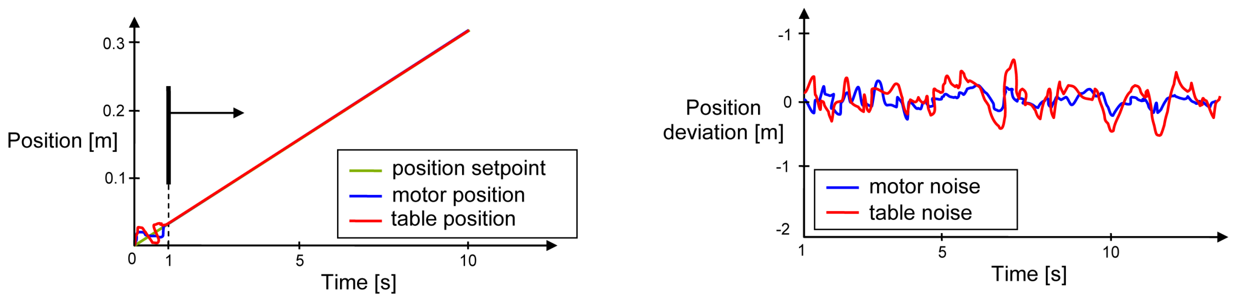

Fig. 3. Time behaviour of measured positions (left); time behaviour of measurement noise (right)

The noise sequences are then processed through (20) and the requested true covariance matrices are calculated.

5. EXPERIMENTAL RESULTS

The real experiment of the presented control technique has been done on the experimental mechanical system consisting of the bed, motor, ball screw feed drive, table and the flexible arm as seen in Fig. 4. The flexibility of the arm mounted on the table is significant. As the measured outputs the motor shaft angular position, the table position and the arm tip position have been used with the aim to precisely control the arm tip position. The model of the mechanical system has been assembled using FEM method. The four most important eigenmodes have been included in the model.

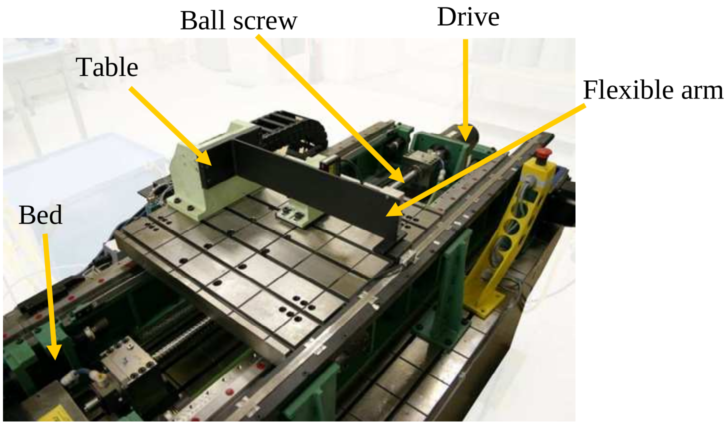

Fig. 4. Experimental mechanical system ETB-1

For the test of the described control technique, the positioning and dynamical stiffness tests have been carried out and compared with the results of the traditional cascade controller where the additional measurement of the flexible arm tip position (TCP) has been also used. But before that the covariance matrices **Q**, **R** for the Kalman filter have been evaluated. They have been determined as

$$
\begin{gathered}
Q = 3.960e^{-2} \\
\mathbf{R} = \begin{bmatrix} 4.550e^{-9} & 0 & 0 \\ 0 & 4.210e^{-9} & 0 \\ 0 & 0 & 1.210e^{-8} \end{bmatrix}
\end{gathered}
\tag{21}
$$

The matrix Q is scalar due to only one measured input and the matrix $\mathbf{G_d}$=$\mathbf{B_d}$. The matrix **R** is 3×3 because of three measured outputs.

Setting of the predictive controller within the tests allowing the best achievable dynamics has been as follows:

- T = 0.001s
- n_y = 300, n_u = 200
- Q_y = 1500, dQ_y = 50000, dQ_u = 0.05

The positioning test has been done using the position ramp as a setpoint. The setpoint changed from 0 to 0.15m. Comparison of the control techniques can be seen in Fig. 5.

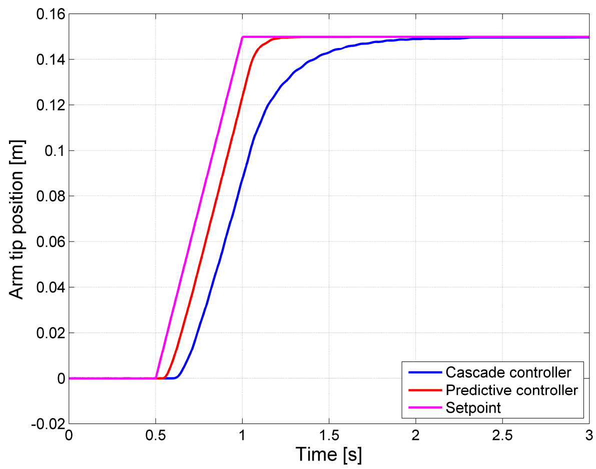

Fig. 5. Time behaviour of the flexible arm tip position during the positioning test

From the measured data also the frequency response plots have been evaluated and the controller bandwidths have been examined. In Fig. 6 the frequency response plots of the predictive controller and cascade controller are shown. Also the system frequency response between the flexible arm tip position (x_{p2}) and the motor torque (M_k) is depicted. The bandwidth of the proposed control compared to the cascade controller however with the TCP measurement has been increased by 80%. The bandwidth of the cascade controller without the TCP measurement is more than 10 times lower.

The dynamical stiffness test has been performed as the reaction of the flexible arm tip position to the step disturbance in the motor torque. The amount of the step disturbance has been 7.45 Nm. The results can be seen in Fig. 7. The increase is 5 times. The amplitude of the position deviation caused by the torque disturbance was also attenuated faster.

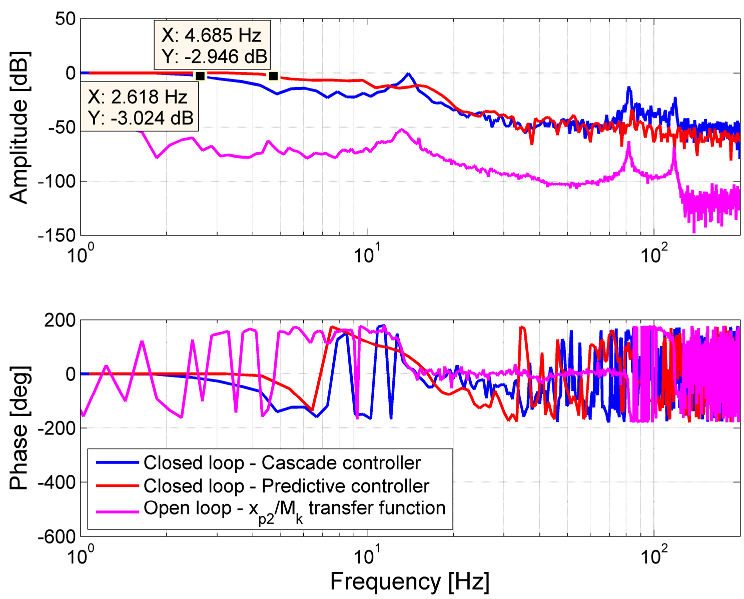

Fig. 6. System frequency response plots with examined controllers

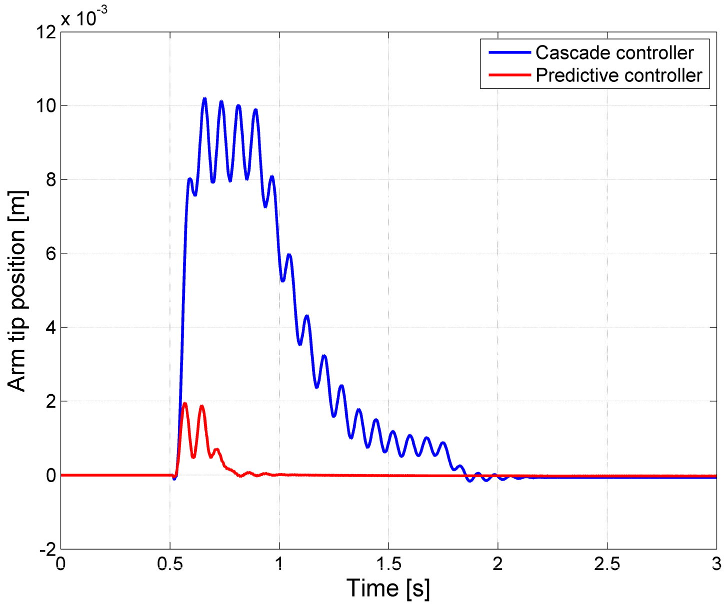

Fig. 7. Dynamical stiffness test, compensation of the position deviation due to the step disturbance

6. CONCLUSION

The paper deals with the position control of flexible mechanical system using the additional TCP measurement and it is aimed mainly to the area of machine tools. The motivation is to build much lighter, however much more flexible machine tools.

Within the paper the modified model-based predictive control technique which includes also the feed-through term $\mathbf{D_d}$ has been derived. The presented control technique works with the Kalman filter that reconstructs the vector of the states. For the correct state estimation the good knowledge of the process and the measurement noises is necessary. For that purpose the practical method to estimate the true system covariances has been introduced. All of the presented techniques have been tested within the real experiments on the position control of the flexible mechanical structure. The results showed much better capabilities of the proposed predictive controller than the so far used cascade controller. The control system dynamics has been increased as proved from the positioning test in Fig. 5 and also the frequency response test in Fig. 6. The passband of the system just using the additional TCP measurement was increased more than 10 times [13], the passband of the predictive controller was almost twice as high as that of the system with the cascade controller, both with the additional TCP measurement. Also the dynamical stiffness test in Fig. 7 proved the better dynamical response in the case of the predictive controller.

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