Algorithms of Sustainable Estimation of Unknown Input Signals in Control Systems

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Abstract:
The problem of estimating unknown input effects in control systems based on the methods of the theory of optimal dynamic filtering and the principle of expansion of mathematical models is considered. Equations of dynamics and observations of an extended dynamical system are obtained. Algorithms for estimating input signals based on regularization and singular expansion methods are given. The above estimation algorithms provide a certain roughness of the filter parameters to various violations of the conditions of model problems, i.e. are not very sensitive to changes in the a priori data.

Keywords: *control system, state, signal reconstruction, regularization, regularization parameter*

1. Introduction

When constructing and implementing automatic control systems, the problem of statistical signal reconstruction is extremely important, since at these stages of system development reliable statistical characteristics of disturbances are usually absent. The main requirement for signal restoration is the requirement to obtain qualitative estimates of uncontrolled signals in accordance with the chosen optimality criterion. The processing of measurement results of experimental data under real operating conditions of automatic systems can be performed on the basis of statistical signal reconstruction, and signal recovery algorithms can conveniently be synthesized on the basis of optimal filtering methods [1–7].

In the theory of optimal Kalman filtration, a Gaussian Markov random process is generated, as is well known, at the output of a dynamical system whose input receives an independent Gaussian random process of white noise type. In this case, the measurement interference is also a Gaussian random process of white noise type, the influence of which is taken into account in the mathematical model of the measuring system [1, 2].

Descriptions of object noise and interference measurements by time-correlated random Gaussian Markov processes violate the conditions for the general formulation of the classical optimal Kalman filtering problem. However, in these cases, the Kalman filtration method can also be used, since there are methods for generalizing the filtration method to the cases under consideration [1–3]. This is achieved by using the principle of expanding the mathematical models of the original dynamical system [3]. In [7], a general formalized scheme for constructing regularized algorithms for restoring input effects in control systems based on the principle of model expansion is presented.

2. Formulation of the Problem

Consider a linear dynamical system

$$x_{i+1} = Ax_i + f_i,\ x_{i_0} = x_0,\ i = 0, 1, ..., \tag{1}$$

the input effect f_i of which is the output of the shaping filter, the input of which receives white noise w_i

$$f_i = D_1 \xi_i , \tag{2}$$

$$\xi_{i+1} = D_2 \xi_i + D_3 w_i ,\ \xi_{i_0} = \xi_0 , \tag{3}$$

where the x_i – n_1-dimensional state vector of the initial system, the f_i – q-dimensional vector of statistical input actions, A – the matrix of the corresponding dimension, the ξ_i – n_2-dimensional state vector of the additional linear dynamical system; w_i – μ-dimensional noise vector of a white noise type with characteristics $E\left[w_i\right]=0$, $E\left[w_i w_k\right] \quad Q_i \quad {}_{ik}$; D_1, D_2, D_3 – matrices of corresponding dimensions.

We assume that observations of the state of system (1) are carried out in accordance with equations

$$z_{1,i} = H_1 x_i + v_{1,i} , \tag{4}$$

$$\xi_{i+1} = D_2 \xi_i + D_3 w_i ,\ \xi_{i_0} = \xi_0 ,$$

where $z_{1,i}$ and $z_{2,i}$ – are respectively m_1- and m_2-dimensional observation vectors characterizing the behavior of the initial dynamical system, $v_{1,i}$ and $v_{2,i}$ – are the m_1- and m_2-dimensional noise interference vectors of Gaussian white noise type with the characteristics $E[v_{1,i}] = 0$, $E[v_{1,i} v_{1,k}^T] = R_{1,i}\delta_{ik}$, $E[v_{1,i}] = 0$, $E[v_{2,i} v_{2,k}^T] = R_{2,i}\delta_{ik}$, $k = 0, 1, ...,$ δ_{ik} – symbol of Kronecker, H_1 and H_2 – matrix of measurements corresponding to dimensions.

We assume that the covariance matrices Q_i, $R_{1,i}$, $R_{2,i}$ are unknown, but they are functions of time. In the conditions formulated above, the equations of dynamics and observation of the extended dynamical system can be written in the following form:

$$\left[\frac{x_{i+1}}{\xi_{i+1}}\right] = \left[\begin{array}{c|c} A & D_1 \\ \hline 0 & D_2 \end{array}\right]\left[\frac{x_i}{\xi_i}\right] + \left[\frac{0}{D_3}\right] w_i ,$$

$$\left[\frac{z_{1,i}}{z_{2,i}}\right]=\left[\begin{array}{c|c} H_1 & 0 \\ \hline 0 & H_2 \end{array}\right]\left[\frac{x_i}{\xi_i}\right]+\left[\frac{v_{1,i}}{v_{2,i}}\right],$$

or in a more compact form:

$$x^*_{i+1}=A^*x^*_i+D^*w^*_i, \tag{5}$$

$$z^*_i=H^*x^*_i+v^*_i, \tag{6}$$

where:

$$A^*=\left[\begin{array}{c|c} A & D_1 \\ \hline 0 & D_2 \end{array}\right],\ H^*=\left[\begin{array}{c|c} H_1 & 0 \\ \hline 0 & H_2 \end{array}\right],\ D^*=\left[\frac{0}{D_3}\right],$$

$$x^*_{i+1}=\left[\frac{x_{i+1}}{\xi_{i+1}}\right],\ v^*_i=\left[\frac{v_{1,i}}{v_{2,i}}\right],\ z^*_i=\left[\frac{z_{1,i}}{z_{2,i}}\right],\ w^*_i=w_i.$$

To estimate the extended state vector on the basis of equations (5), (6), one can use the traditional Kalman filter [1,4]:

$$x^*_{i+1|i}=A^*x^*_{i|i}, \tag{7}$$

$$P^*_{i+1|i}=A^*P^*_{i|i}A^{*T}+D^*\hat{Q}_iD^{*T}, \tag{8}$$

$$x^*_{i+1|i+1}=x^*_{i+1|i}+K^*_{i+1}y^*_{i+1}, \tag{9}$$

$$y^*_{i+1|i}=z^*_{i+1}-H^*x^*_{i+1|i}, \tag{10}$$

$$K^*_{i+1}=P^*_{i+1|i}H^{*T}[C^*_{i+1}]^{-1}, \tag{11}$$

$$C^*_{i+1}=H^*P^*_{i+1|i}H^{*T}+R_{i+1}, \tag{12}$$

$$P^*_{i+1|i+1}=(I-K^*_{i+1}H^*)P^*_{i+1|i}. \tag{13}$$

On the basis of the values of ξ_{i+1} computed at each step, expression (2) gives the value of the unknown input action f_i. Based on the Kalman filter form (7)–(13), it is easy to see that the amplification factor of the Kalman filter algorithm K^*_{i+1} depends on the matrices Q_i and R_{i+1}. Thus, in the case where the noise covariance matrices Q_i and R_{i+1} are unknown, the coefficient K^*_{i+1} , which must be found to determine the state vector estimate, is also unknown.

Analyzing the expressions (7)–(13), we can conclude that the relations (8), (11)–(13) should be excluded from the extrapolation and filtering algorithms, because the estimates Q_i and R_{i+1} are used in their calculation. In [1, 2] we show that in this case the gain K^*_{i+1} can be calculated from a sample of measurements Z^{i+l}_{i+1} based on the following two-stage computational procedure:

$$SL = M, \tag{14}$$

and

$$K^*_{i+1}\Phi^*=L, \tag{15}$$

where:

$$S=\left[\begin{array}{c} H^* \\ \hline H^*A^* \\ \hline \cdots\cdots \\ \hline H^*A^{*l-2} \end{array}\right]A^*,\quad M=\left[\begin{array}{c} y^*_{i+2|i}y^{*T}_{i+1|i} \\ \hline y^*_{i+3|i}y^{*T}_{i+1|i} \\ \hline \cdots\cdots\cdots \\ \hline y^*_{i+l|i}y^{*T}_{i+1|i} \end{array}\right],$$

$$L=P^*_{i+1|i}H^{*T},\quad \hat{O}^*=[y^*_{i+1|i}y^{*T}_{i+1|i}].$$

Such an approach allows the use of real measurement information, which is of significant importance when a priori information is given inaccurately. In other words, the above algorithm is robust to changing a priori data.

The system of equations (14) for determining the matrix L can be poorly conditioned, which contributes to a decrease in the accuracy of the calculation of the sought solution. This circumstance in the solution of this equation leads to the necessity of applying regularization methods. Among them we should mention a group of methods based on the general regularization principle of A.N. Tikhonov [8–13], and methods of effective pseudoinversion, based on the singular matrix decomposition [14–17].

3. Solution of the Task

In the real situation, the initial data of system (14) are known only approximately. Almost in place of system (14) another system is used

$$S_hL=M_\delta, \tag{16}$$

such that $\|S_h-S\|\le h,\ \|M_\delta-M\|\le\delta$. Thus, approximate data are characterized by a set of S_h, M_δ, η, where $\eta=\{\delta,h\}$ – is the error vector.

We write down the expression for the smoothing functional of A.N. Tikhonov

$$G^\alpha[L^\alpha_\eta]=\left\|S_hL^\alpha_\eta-M^\delta\right\|+\alpha\left\|L^\alpha_\eta\right\|,$$

where $\alpha > 0$ – regularization parameter.

We introduce the following functions [10,11]:

$$\gamma_\eta(\alpha)=\left\|L^\alpha_\eta\right\|,$$

$$\beta_\eta(\alpha)=\left\|S_hL^\alpha_\eta-M_\delta\right\|,$$

$$\rho_\eta(\alpha)=\beta_\eta(\alpha)-\left(\delta+h\sqrt{\gamma_\eta(\alpha)}\right)^2-\mu^2_\eta.$$

where: L^α_η – is the extremal of the functional A.N. Tikhonov $G^\alpha[L]$ for fixed $\alpha > 0$, the functions $\gamma_\eta(\alpha)$, $\beta_\eta(\alpha)$, $\rho_\eta(\alpha)$ are monotonic and continuous as functions of α in the domain $\alpha > 0$, $\mu_\eta=\inf_{L\in D}\left\|S_hL-M_\delta\right\|$ – a measure of incompatibility of equation (16) with approximate data on the set $D\in\Theta$.

The solution of equation (16) on the basis of the regularization method of A.N. Tikhonov is given by the formula [9, 10]

$$L_\alpha=(\alpha I+S^T_hS_h)^{-1}S^T_hM_\delta=g_\alpha(S^T_hS_h)S^T_hM_\delta,$$

where: $g_\alpha(\lambda) = (\alpha + \lambda)^{-1}$, $\alpha > 0$, $0 \le \lambda > \infty$ – generating system of functions for the method of A.N. Tikhonov.

We assume that the natural condition

$$\left\|M_\delta\right\|^2>\delta^2+\mu^2_\eta. \tag{17}$$

The function of the generalized residual $\rho_\eta(\alpha)$ has the following limit values at the ends of the segment [11]

$$\lim_{\alpha\to+\infty}\rho_\eta(\alpha)=\left\|M_\delta\right\|-\delta^2-\mu^2_\eta,\ \overline{\lim_{\alpha\to0+0}}\rho_\eta(\alpha)=-\delta^2.$$

Thus, if condition (17) is satisfied, equation $\rho_\eta(\alpha) = 0$ has in root $\alpha > 0$ a root $\alpha'(\eta)$, and element $L^{\alpha'(\eta)}_\eta$ is uniquely defined.

If, however, the numbers h and d are unknown or their computation is associated with considerable difficulties, then the regularization parameter a is expedient to be determined on the basis of the quasioptimality or relations [13]

$$\left\| L^{\alpha_{i+1}} - L^{\alpha_i} \right\| = \min, \quad \alpha_{i+1} = \zeta \alpha_i ,$$

$$i = 0,1,2,..., \quad 0 < \zeta <,$$

$$r_{\text{rel}}(\alpha) = r_1(\alpha)/r(\alpha),$$

where:

$$r_1(\alpha) = \left\| S_h \gamma_\alpha - \left(S_h L_\alpha - M_\delta \right) \right\|, \ \gamma_\alpha = \alpha \left(dL_\alpha / d\alpha \right).$$

When solving the system of equations (16), it is expedient to use regularization on the basis of extended systems [15, 17, 18]. It is known [17] that the normal pseudosolution $\tilde{L}_* = S_h^+ M_\delta$ is a normal solution of the normal system of equations

$$S_h^T S_h L = S_h^T M_\delta \tag{18}$$

or $S_h^T \tilde{\varsigma} = 0$, where $\tilde{\varsigma} = M_\delta - S_h L$.

Thus, the normal system of equations (18) is equivalent to an extended system of equations

$$R_h q = b_\delta, \tag{19}$$

where:

$$R_h = \begin{bmatrix} I_{l(m-1)} & S_h \\ S_h^T & 0 \end{bmatrix}; \quad z = \begin{bmatrix} \tilde{\varsigma} \\ L \end{bmatrix}; \quad b_\delta = \begin{bmatrix} M_\delta \\ 0 \end{bmatrix},$$

and $q = \left(\tilde{\varsigma}^T, L^T \right)^T \in R^{l(m-1)+n}$.

Since the normal system of equations (18) is always consistent [15, 17], it follows immediately from this that the extended system (19) for any initial data $\tilde{d} = \{S_h, M_\delta\}$ is also consistent, and the normal solution of the extended system is defined as $\tilde{q}_* = R_h^+ b_\delta = \left(\tilde{\varsigma}_*^T, \tilde{L}_*^T \right)^T$, where $\tilde{L}_* = S_h^+ M_\delta$ and $\tilde{\varsigma}_* = M_\delta - S_h \tilde{L}_*$.

Using the regularization method of A.N. Tikhonov, regularized solution $\tilde{q}_\alpha$ of the system (19) is defined as the unique solution of the Euler equation

$$(R_h^2 + \alpha I_{l(m-1)+n}) q = R_h b_\delta. \tag{20}$$

where: $R_h^2 = R_h^T R_h \quad (R_h = R_h^T), \quad \|R_h - R\| = \|S_h - S\| \le h$, $\|b_\delta - b\| = \|M_\delta - M\| \le \delta$.

It was shown in [18] that if we put $\alpha = h$ in equation (20), then the deviation error has the form

$$\|\tilde{q}_\alpha - q_*\| = O(h + \delta),$$

where $\tilde{q}_\alpha$ – decision (20), $q_* = R^+ b = \begin{bmatrix} M - SL \\ L \end{bmatrix}$, $L = S^+M$.

The last estimate shows that in the case under consideration it is sufficient to match the regularization parameter a only with the value h, i.e. with a measure of the error of the matrix S, in other words, in essence, the problem of choosing the regularization parameter is removed.

The method of effective pseudoinversion, as is known [14, 16], is based on the singular expansion of the matrix S_h, i.e. on its presentation in the form

$$S_h = UTV^T,$$

where U – orthogonal $((l-1)m \times 2p)$–matrix; V – orthogonal $(2p \times n)$–matrix; T – diagonal $(2p \times 2p)$–matrix.

The columns u_i and v_i of the matrices U and V are the eigenvectors of the matrices $S_h S_h^T$ and $S_h^T S_h$, and the diagonal elements μ_i of the matrix T – are the positive roots of the eigenvalues λ_i of the matrix $S_h^T S_h$ (or $S_h S_h^T$).

The pseudoinverse Moore-Penrose matrix S_h^+ makes it possible to obtain the estimate [14, 16]

$$L^* = VT^+U^T M_\delta = \sum_{i=1}^{r} \frac{1}{\mu_i} v_i u_i^T, \tag{21}$$

where $T^+ = diag(t_1^+, ..., t_i^+)$ – pseudo-inverse matrix for the matrix T; n – rank of the matrix S_h, i.e. the number of non-zero singular numbers $\mu_i (i = 1,...,p)$; $t_i^+ = 1/\mu_i$, if $\mu_i \ne 0$, and $t_i^+ = 0$, if $\mu_i = 0$.

In the case where the rank of the matrix S_h $n = p$, the pseudoinverse estimate (21) coincides with the estimate (16) for the least squares and, correspondingly, is characterized by low accuracy. In connection with this, the so-called effective pseudoinverse matrices and the estimates

$$L_\tau = VT_\tau U^T M_\delta = \sum \text{—} v_i \cdot u_i^T,$$

where T_τ^+ – effective pseudo-matrix $T = diag(t_{1\tau}^+, ..., t_{n\tau}^+)$; $n' < n$, $t_{i\tau}^+ = 1/\mu_i$, if $\mu_i > \tau$, and $t_{i\tau}^+ = 0$, if $\mu_i = 0$.

Taking into account the symmetry and the positive definiteness of the matrix $\hat{O}^*$ in (15) for the stable calculation of the matrix gain factor K_{i+1}^*, it is expedient to use the M.M. Lavrentev computational scheme

$$K_{i+1}^* = L^* (\Phi^* + \alpha I)^{-1},$$

where the regularization parameter a is expedient to be determined on the basis of the quasioptimality method.

4. Conclusion

The above algorithms of stable recovery of unknown input signals in control systems allow to raise the level of a priori information about the control object, the reliability of statistical characteristics of external influences, and thus the quality of control processes in statistically indeterminate situations.

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