ARCHIVES OF ELECTRICAL ENGINEERING VOL. 66(1), pp. 165-178 (2017)

DOI 10.1515/aee-2017-0012

Influence of output conductance on characteristic frequencies of switch mode BUCK and BOOST converter

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(Received: 13.09.2016, revised: 08.11.2016)

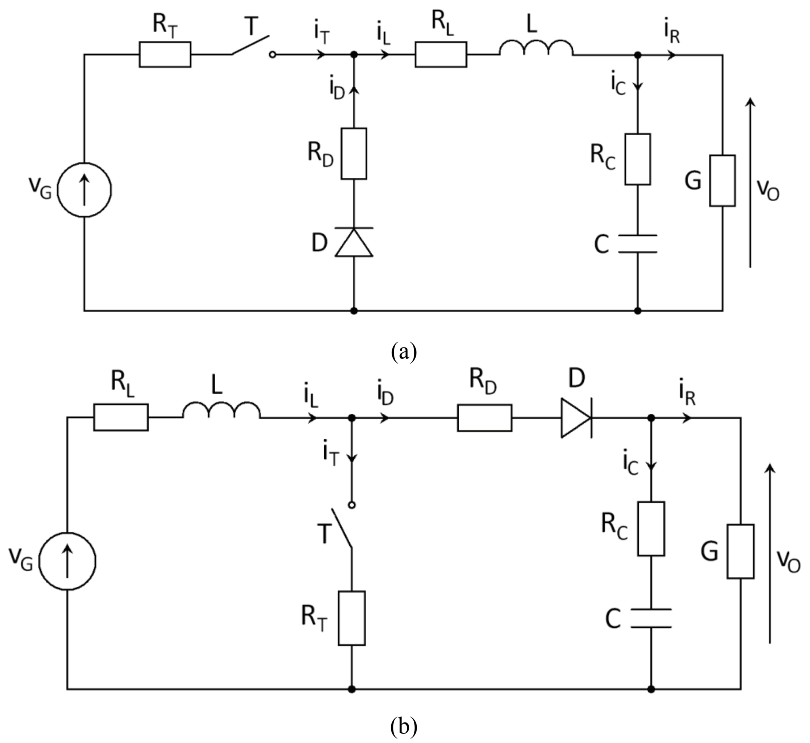
Abstract: Characteristic frequencies corresponding to poles and zeros of small-signal control-to-output transfer functions of popular DC-DC converters (BUCK and BOOST) are analyzed. The main attention is paid to influence of load conductance on the characteristic frequencies for converters working in continuous conduction mode (CCM) as well as in discontinuous conduction mode (DCM). Parasitic resistances of all converter components are included in calculations. In addition the improved description of CCM-DCM boundary is presented. The calculations are verified experimentally and good consistency of the results is observed.

Key words: pulse converters, BUCK, BOOST, small-signal models, control-to-output characteristics, DCM boundary, characteristic frequencies

1. Introduction

Switch-mode DC-DC power converters find great amount of applications. Their circuit solutions and control methods as well as component parameters are steadily improved [1, 2]. The most popular DC-DC converters – step-down (BUCK) and step-up (BOOST) are the objects of this paper. Typical converter contains a power stage and control subcircuit and its operation is based on a PWM principle. The precise knowledge of dynamic characteristics of the power stage is necessary for proper design of control circuit. The description of dynamic properties of the power stage has usually the form of small-signal transmittances in s-domain. These transmittances are obtained after linearization of an averaged model of the power stage [1-5, 9]. The most important small-signal transmittances of DC-DC converters are control-to-output and input-to-output transmittances. The characteristic frequencies corresponding to poles and zeros of transmittances are crucial parameters describing dynamic properties of converter power stage.

The converters may operate in continuous conduction mode (CCM) or discontinuous conduction mode (DCM). Formulas for small-signal transmittances and characteristic frequencies depend on the conduction mode, therefore the conditions for DCM-CCM boundary should be known. Characteristic frequencies in given operation mode depend mainly on the parameters of an induction coil and capacitor used in the power stage. In typical description of these frequencies, their dependence on load current (or load conductance) is not taken into account [1-6] but it is shown in [7] that in some situations, this dependence may be important. The considerations in [7] are only theoretical and restricted to a step-down converter (BUCK). In the present paper, formulas describing the influence of load conductance on characteristic frequencies of BUCK and BOOST converters are presented and theoretical results are verified by measurements. Section 2 of the paper is devoted to a general description of characteristic frequencies of DC-DC converters. In Section 3, new expressions defining a CCM-DCM boundary for nonideal converters are given. Theoretical description of characteristic frequencies of nonideal BUCK and BOOST is presented in Section 4. Examples of numerical calculation and measurement results are shown and compared in Section 5. Some concluding remarks are given in Section 6. The electrical schemes of the power stages of the converter under consideration are shown in Fig. 1. Apart from ideal components of converters – *T* (controlled switch), *D* (diode), *L* (inductor coil), *C* (capacitor), the parasitic resistances of each component are included.



(b)

Fig. 1. Power stage of BUCK (a) and BOOST (b) converters including parasitic resistances

2. General description of small-signal transmittances and characteristic angular frequencies

Small-signal control-to-output and input-to-output transmittances of basic DC-DC converters (such as BUCK and BOOST) may be expressed in the form of second-order or first-order function of *s* variable [1-3]. For continuous conduction mode of operation one usually obtains [1, 2, 7, 8, 10]:

$$H_t = H_0 \frac{1 + s/\omega_Z}{1 + s/(Q \cdot \omega_0) + s^2/\omega_0^2}. \tag{1}$$

In the case of the BOOST converter, a transfer function has additional zero. For the DCM mode, based on separation of variable technique, it is obtained [4, 7]:

$$H_s = H_{s0} \frac{1 + s/\omega_{ZS}}{1 + s/\omega_P}. \tag{2}$$

Transmittance (1), for typical component values of the BUCK or BOOST converter has a pair of complex poles:

$$s_{P1,2} = \sigma \pm j \cdot \omega_R. \tag{3}$$

The resonance frequency f_R correspond to relation:

$$\omega_R = 2\pi \cdot f_R = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}. \tag{4}$$

The maximum value of $|H_s|$ is obtained for frequency f_M, where:

$$\omega_M = 2\pi \cdot f_M = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}. \tag{5}$$

The numerical values of ω_R and ω_M are usually very close to ω_o.

Apart from characteristic frequencies f_o, f_R and f_M involved with the poles of type (1) transmittances, there are frequencies $f_Z = \omega_Z/2\pi$, $f_{ZS} = \omega_{ZS}/2\pi$ corresponding to zeros of transmittances (1) and (2).

For some specific set of converter parameters, one obtains $Q < 1/2$ and corresponding values of poles of expression (1) are real numbers. The characteristic angular frequency in this case is:

$$\omega_{1,2} = \frac{\omega_0}{2Q} \cdot \left(1 \pm \sqrt{\frac{1}{4} - Q^2}\right) \quad \text{for } Q < 0.5. \tag{6}$$

Dependencies of characteristic frequencies of BUCK and BOOST on the parameters of converter components are presented in the further part of the paper.

3. CCM and DCM boundary

Characteristic frequencies of converters working in CCM and in DCM are different. The CCM-DCM boundary are usually defined by threshold value G_C of load conductance. G_C depends on switching period T_S, duty ratio D_A of switching waveform and the inductance L of coil. The expressions for G_C given in the literature (e.g. [1, 2]) refer to ideal converters only. The equations given below describe threshold conductance G_C obtained with parasitic resistances accounted for.

For BUCK converter:

$$G_{C(\mathrm{BUCK})} = \frac{(1-D_A)T_S}{2L-(R_L+R_D)(1-D_A)T_S}. \tag{7}$$

For BOOST converter:

$$G_{C(\mathrm{BOOST})} = \frac{(1-D_A)^2 D_A T_S}{2L-(R_D-R_T)(1-D_A)D_A T_S}. \tag{8}$$

Converter works in DCM for load conductance $G < G_C$. A converter is always designed to work in a specific mode of operation. Due to work of the converters in a wide range of parameters, the worst case scenario should be considered, regarding Eqs. (7) and (8), to ensure that the converter stays in the CCM or DCM mode. Examples of modifications that include the worst case have been presented in [2].

4. Small signal transmittances and characteristic frequencies of BUCK and BOOST converters

In this section, the expressions for control-to-output transmittances H_d and corresponding characteristic angular frequencies derived for BUCK and BOOST converters are presented. Control-to-output transmittance is usually used in the procedure of control subcircuit design [1-3]. In addition, it is known [1-3] that the denominators of expressions describing control-to-output and input-to-output transmittances, as well as output impedance of a given converter type are the same, therefore the same angular frequencies of poles refer to various transmittances.

Control-to-output transmittance H_d is defined as follows:

$$H_d(s) = \left.\frac{V_o}{\theta}\right|_{V_g=0}, \tag{9}$$

where θ, V_g and V_o are small-signal values of duty ratio and input and output voltage.

Control to output transmittances of BUCK and BOOST converters working in CCM with parasitic resistances (see Fig. 1) may be expressed as [7]:

$$H_{d(\text{BUCK})} = \frac{(V_G - I_L(R_T - R_L))(1 + sCR_C)}{s^2 LC_Z + s(GL + R_Z C_Z + CR_C) + 1 + GR_Z}\,, \tag{10}$$

$$H_{d(\text{BOOST})} = \frac{-s^2 LCR_C I_L + s(V_A CR_C - LI_L - CR_C R_Z I_L) + V_A - I_L R_Z}{s^2 LC_Z + s(GL + R_Z C_Z + (1 - D_A)^2 + CR_C) + (1 - D_A)^2 + GR_Z}\,. \tag{11}$$

The example of transfer function derivation is given in Appendix A.

By comparison of (10) and (11) with general description in Eq. (1), one obtains:

$$\omega_{Z(\text{BUCK})} = \frac{1}{C \cdot R_C}\,, \tag{12}$$

$$\omega_{0(\text{BUCK})} = \frac{1}{\sqrt{LC}} \sqrt{\frac{GR_Z + 1}{GR_C + 1}}\,, \tag{13}$$

$$Q_{(\text{BUCK})} = \frac{\sqrt{LC_Z (G \cdot R_Z + 1)}}{GL + C_Z R_Z + CR_C}\,, \tag{14}$$

$$\omega_{0(\text{BOOST})} = \frac{1}{\sqrt{LC}} \sqrt{\frac{GR_Z + (1 - D_A)^2}{GR_C + 1}}\,, \tag{15}$$

$$Q_{(\text{BOOST})} = \frac{\sqrt{LC_Z \left(GR_Z + (1 - D_A)^2\right)}}{GL + C_Z R_Z + (1 - D_A)^2 CR_C}\,. \tag{16}$$

The notation in Eqs. (10) – (16) corresponds to following dependencies:

$$R_Z = R_L + D_A \cdot R_T + (1 - D_A) \cdot R_D\,, \tag{17}$$

$$C_Z = C \cdot (1 + G \cdot R_C)\,, \tag{18}$$

$$V_X = V_O - I_L (R_T - R_D)\,, \tag{19}$$

$$V_A = (1 - D_A) V_X\,, \tag{20}$$

$$I_{L(\text{BUCK})} = GV_O\,, \tag{21}$$

$$I_{L(\text{BOOST})} = \frac{GV_O}{1 - D_A}\,, \tag{22}$$

$$V_{O(\text{BUCK})} = \frac{D_A V_G}{1 + R_Z G}\,, \tag{23}$$

$$V_{O(\text{BOOST})} = \frac{(1 - D_A) V_G}{(1 - D_A)^2 + R_Z G}\,. \tag{24}$$

From Eqs. (4) – (6) and (10) – (24), the following expressions for characteristic angular frequencies ω_R, ω_M, $\omega_{1,2}$ are obtained:

$$\omega_{R\,(\mathrm{BUCK})} = \frac{1}{2LC_Z}\sqrt{4LC_Z(1+GR_Z)-(GL+R_ZC_Z+CR_C)^2}\ , \tag{25}$$

$$\omega_{M\,(\mathrm{BUCK})} = \frac{1}{2LC_Z}\sqrt{4LC_Z(1+GR_Z)-2(GL+R_ZC_Z+CR_C)^2}\ , \tag{26}$$

$$\omega_{1,2\,(\mathrm{BUCK})} = \frac{1}{4LC_Z}\left(2(GL+R_ZC_Z+CR_C)\pm\sqrt{(GL+R_ZC_Z+CR_C)^2-4LC_Z(1+GR_Z)}\right), \tag{27}$$

$$\omega_{R(\mathrm{BOOST})} = \frac{1}{2LC_Z}\sqrt{4LC_Z\left((1-D_A)^2+GR_Z\right)-\left(GL+R_ZC_Z+(1-D_A)^2CR_C\right)^2}\ , \tag{28}$$

$$\omega_{M(\mathrm{BOOST})} = \frac{1}{2LC_Z}\sqrt{4LC_Z\left((1-D_A)^2+GR_Z\right)-2\left(GL+R_ZC_Z+(1-D_A)^2CR_C\right)^2}\ , \tag{29}$$

$$\begin{aligned}\omega_{1,2\,(\mathrm{BOOST})} &= \frac{1}{4LC_Z}2\left(GL+R_ZC_Z+(1-D_A)^2CR_C\right)\pm\\ &+\frac{1}{4LC_Z}\sqrt{\left(GL+R_ZC_Z+(1-D_A)^2CR_C\right)^2-4LC_Z\left((1-D_A)^2+GR_Z\right)}\end{aligned}\ . \tag{30}$$

Control-to-output conductances of converters working in DCM are following:

$$H_{dD(\mathrm{BUCK})} = \frac{2G_ZV_GD_A(M_{ID}-1)}{sC+G_ZD_A{}^2M_{ID}{}^2+G}\ , \tag{31}$$

$$H_{dD(\mathrm{BOOST})} = \frac{2D_AG_ZV_G(M_{VD}-1)}{sC(M_{VD}-1)^2+G_ZD_A{}^2+(M_{VD}-1)^2G}\ , \tag{32}$$

where:

$$M_{VD(\mathrm{BUCK})} = \frac{G_ZD_A}{2G}\left(\sqrt{D_A^2+4\frac{G}{G_Z}}-D_A\right), \tag{33}$$

$$M_{VD(\mathrm{BOOST})} = \frac{1}{2}\left(1+\sqrt{1+D_A^2G_ZR}\right), \tag{34}$$

$$M_{ID} = \frac{1}{M_{VD}}\ , \tag{35}$$

$$G_Z = \frac{T_S}{2L}\ . \tag{36}$$

The formulas for small-signal transmittances may be derived for averaged models in the form of equivalent circuits shown in Appendix B. The resulting characteristic angular frequencies for BUCK and BOOST in DCM may be expressed as:

$$\omega_{PD(\mathrm{BUCK})} = \frac{1}{C}\left(G_Z D^2 M_{ID}{}^2 + G\right), \tag{37}$$

$$\omega_{PD(\mathrm{BOOST})} = \frac{1}{C}\left(\frac{G_Z D^2}{M_{VD} - 1} + G\right). \tag{38}$$

It may be shown that the influence of parasitic resistances on control-to-output transmittances and related characteristic frequencies may be neglected for DCM.

The influence of load conductance on characteristic frequencies of converters in DCM seems to be relatively strong, as it is seen from Eqs. (37), (38). It may be quantitatively evaluated by numerical experiments.

5. Examples of calculations and measurements

The measurements and numerical calculations have been performed for a laboratory model of a BUCK converter with parameters established by auxiliary measurements: $L = 32$ μH; $C = 345$ μF; $R_L = 53$ mΩ; $R_C = 91$ mΩ; $R_T = 20$ mΩ; $R_D = 281$ mΩ. Switching frequency of 100 kHz and 250 kHz was used for the converter working in DCM and CCM respectively. An indirect method of measuring frequency characteristic was used. First a response of output voltage to a step change of a duty cycle was measured with an MSO5104 digital oscilloscope. Next, the digital data were processed and an FFT function was used to find the spectrum of the measured function. Fluctuations appear at higher frequencies due to limited resolution of the oscilloscope. The exemplary frequency characteristics of the control-to-output transmittance magnitude calculated and measured for the BUCK converter are presents in Figs. 2 and 3.

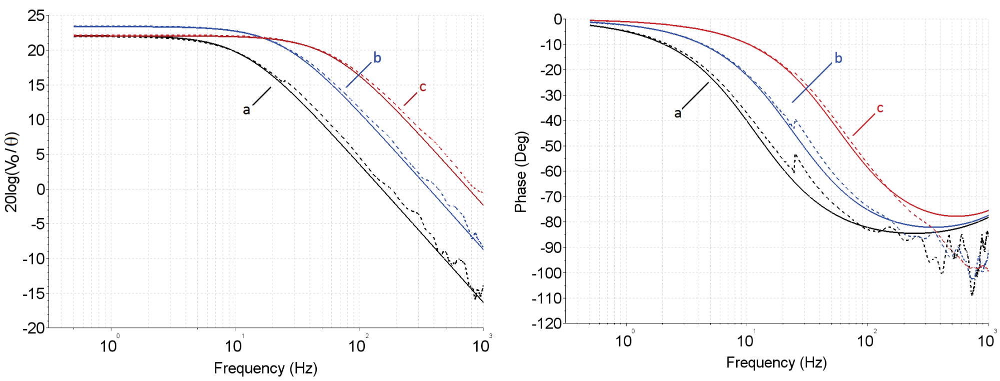


Fig. 2. Magnitude and phase of control-to-output transmittance calculated (solid lines) and measured (dotted lines) for BUCK converter for load resistance values corresponding to operation in DCM

Load resistance values assumed in Fig. 2 are: 198 Ω (curve a), 61.5 Ω (curve b) and 20 Ω (curve c), that corresponds to DCM. Load resistance values in Fig. 3: 4.7 Ω (curve a), 1 Ω (curve b), 0.5 Ω (curve c) and 0.2 Ω (curve d) correspond to the CCM mode. Values of $|H_d|$ in Figs. 2 and 3 are normalized to 1 V. The exemplary frequency characteristics of the control-to-output transmittance magnitude calculated and measured for the BOOST converter are presented in Figs. 4 and 5. Load resistance values assumed in Fig. 2 are: 198 Ω (curve a), 50 Ω (curve b) and 20 Ω (curve c), that corresponds to DCM. Load resistance values in Fig. 3: 4.7 Ω (curve a), 1 Ω (curve b), 0.5 Ω (curve c) and 0.2 Ω (curve d) correspond to the CCM mode. Values of $|H_d|$ in Figs. 4 and 5 are normalized to 1 V.

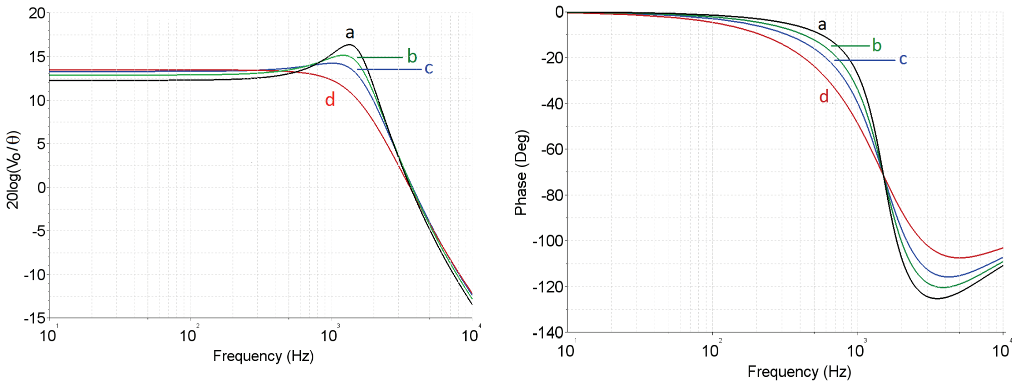


Fig. 3. Magnitude and phase of control-to-output transmittance calculated for BUCK converter for load resistance values corresponding to operation in CCM

Values of characteristic frequencies f_P corresponding to pole frequency ω_P in DCM (Fig. 2) are equal to: 12 Hz (curve a); 25 Hz (curve b) and 60.3 Hz (curve c). The influence of zero (ω_Z) of transmittance for DCM is not visible in Fig. 2 because $\omega_Z \gg \omega_P$.

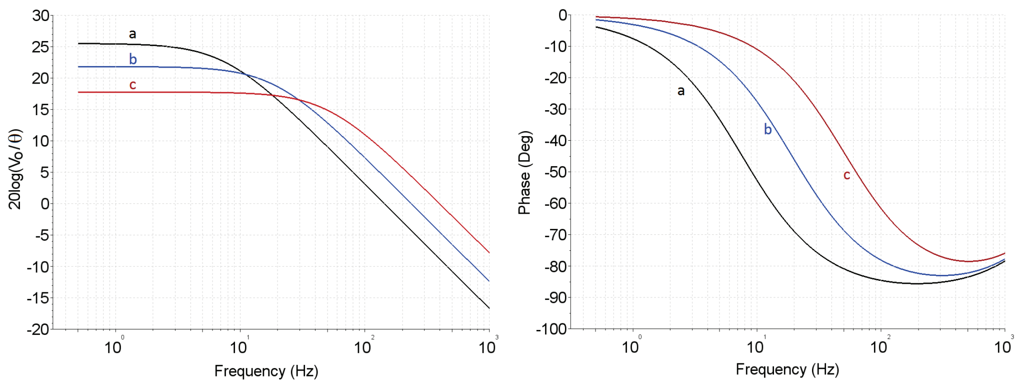


Fig. 4. Magnitude and phase of control-to-output transmittance calculated for BOOST converter for load resistance values corresponding to operation in DCM

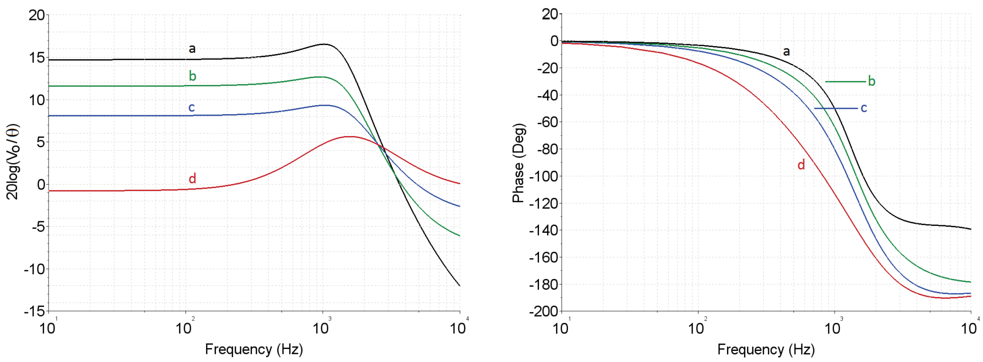


Fig. 5. Magnitude and phase of control-to-output transmittance calculated for BOOST converter for load resistance values corresponding to operation in CCM

6. Conclusions

Characteristic frequencies corresponding to poles and zero of control-to-output transmittance are important parameters, used in the procedure of control subcircuit design of switch-mode power converters. These frequencies are usually assumed to be independent of the load conductance. In some cases, the load conductance of a converter changes considerably and its influence on characteristic frequencies cannot be neglected. Such situations, for popular DC-DC converters BUCK and BOOST are considered in the paper. The expressions describing characteristic frequencies of converters, are derived with the parasitic resistances of converter components included. Calculations based on theoretical formulas are compared with the measurement results and good consistency is obtained. It is observed that the influence of load conductance on characteristic frequencies of converters working in discontinuous conduction mode (DCM) are much stronger than in CCM. Therefore, the precise description of the boundary between CCM and DCM is necessary and such description for converters with parasitic resistances is presented in Section 3.

Appendix A. A derivation of ideal BUCK converter transmittance for DCM

This derivation, presented in [5], is depicted here in brief for readers' convenience. A schematic used for the following derivation has been presented in Fig. A1.

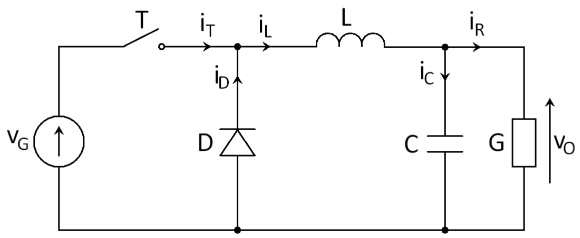


Fig. A.1. Ideal BUCK converter

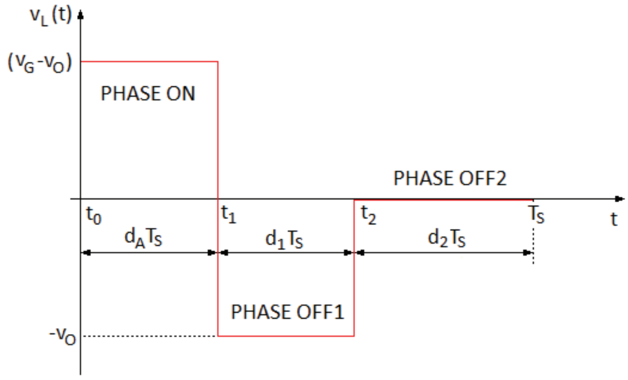


Fig. A.2. Inductor voltage of BUCK converter working in DCM

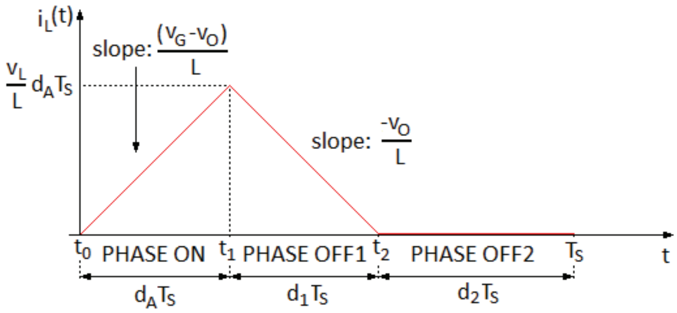


Fig. A.3. Inductor current of BUCK converter working in DCM

The waveform of inductor current in single switching period is shown in Fig. A3. The value of d_1 is:

$$d_1 = \frac{(v_G - v_O)}{v_O} d_A, \tag{A.1}$$

where d_A is the externally forced duty ratio.

Contrary to the situation in CCM the values of the inductor current is identical (and equal to zero) at the beginning and at the end of each period. The average value of inductor voltage v_{LS} is zero in DCM, not only for DC condition but also in transients:

$$v_{LS} = 0. \tag{A.2}$$

As a consequence, the inductor doesn't exist in the large-signal averaged model of BUCK and BOOST for DCM. This observation agrees with result of deriving the models for DCM based on state-space averaging.

The local average values of inductor current i_L(ON) for the ON phase, i_L(OFF1) for the OFF1 phase (see Fig. A.2) and average over the whole period, i_{LS} obtained from the figure regarding the expression for d_1 are obtained in the form:

$$i_{L(\mathrm{ON})} = \frac{d_A T_S}{2}\frac{v_G - v_O}{L}, \tag{A.3}$$

$$i_{L(\mathrm{OFF1})} = \frac{d_1 T_S}{2}\frac{-v_O}{L}, \tag{A.4}$$

with (42) and (43) One can calculate the average value of inductor current in the whole phase:

$$i_{LS} = \frac{v_G - v_O}{2L} d_A T_S \left(d_A + d_1\right). \tag{A.5}$$

From (A.1), (36) and (A.5):

$$i_{LS} = d_A^2 G_Z \frac{v_G}{v_O}(v_G - v_O) . \tag{A.6}$$

Small-signal relation based on (A.6) may be presented as:

$$I_l = \alpha_1 V_g + \alpha_2 \theta + \alpha_3 V_o , \tag{A.7}$$

where:

$$\alpha_1 = \frac{\partial f(v_G, d_A, v_O)}{\partial v_G} = D_A^2 G_Z (2\frac{V_G}{V_O} - 1) , \tag{A.8}$$

$$\alpha_2 = \frac{\partial f(v_G, d_A, v_O)}{\partial d_A} = 2 D_A G_Z V_G (\frac{V_G}{V_O} - 1) , \tag{A.9}$$

$$\alpha_3 = \frac{\partial f(v_G, d_A, v_O)}{\partial v_O} = -D_A^2 G_Z \frac{{V_G}^2}{V_O^2} , \tag{A.10}$$

substituting (A.8)-(A.10) into (A.7):

$$I_l(s) = D_A^2 G_Z (2\frac{V_G}{V_O} - 1) V_g(s) + 2 D_A G_Z V_G (\frac{V_G}{V_O} - 1)\theta(s) - D_A^2 G_Z \frac{{V_G}^2}{V_O^2} V_o(s) . \tag{A.11}$$

Small-signal dependence of I_L on V_O resulting from Fig. A.1 is:

$$I_l(s) = V_o(s)(sC + G) , \tag{A.12}$$

which leads to:

$$H_d(s) = \frac{2 D_A G_Z V_G (M_{ID} - 1)}{sC + G + D_A^2 G_Z M_{ID}^2}\theta(s) , \tag{A.13}$$

where static current transmittance is:

$$M_{ID} = \frac{1}{M_{VD}} \tag{A.14}$$

and

$$M_{VD} = \frac{V_O}{V_G} = \frac{G_Z D_A}{2G}\left(\sqrt{\left(D_A^2 + 4\frac{G}{G_Z}\right)} - D_A\right). \quad \text{(A.15)}$$

Appendix B. Equivalent circuit, representing small-signal models of BUCK and BOOST in CCM and DCM

The models have been previously presented in [5] and are shown here for reader's convenience.

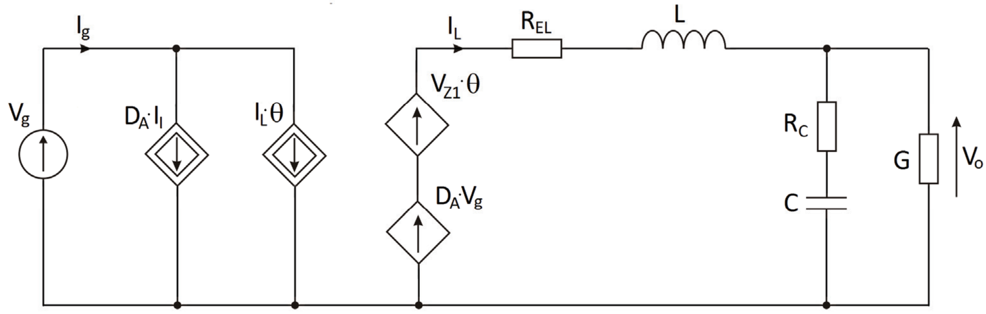


Fig. B.1. Small-signal model of BUCK converter working in CCM

$$R_{EL} = R_E + R_L, \quad \text{(B.1)}$$

$$R_E = R_T D_A + R_D(1 - D_A), \quad \text{(B.2)}$$

$$V_{Z1} = V_G + (R_D - R_T) I_L. \quad \text{(B.3)}$$

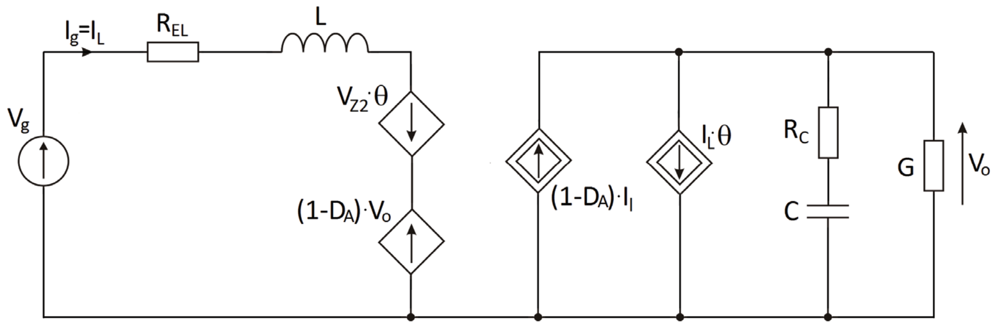


Fig. B.2. Small-signal model of BOOST converter working in CCM

$$V_{Z2} = V_O + (R_D - R_T) I_L. \quad \text{(B.4)}$$

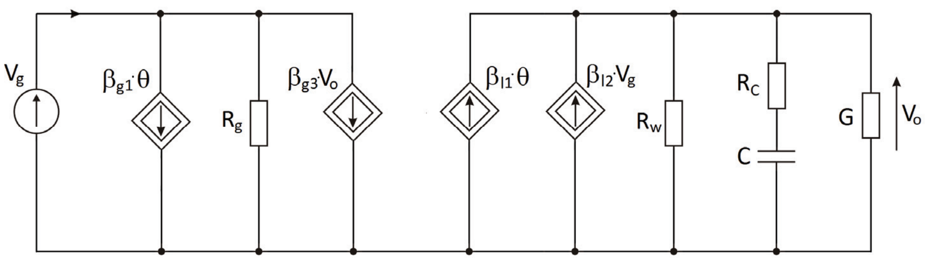


Fig. B.3. Small-signal model of BUCK converter working in DCM

$$\beta_{g1} = 2G_Z D_A (V_G - V_O)\,, \tag{B.5}$$

$$R_g = \frac{1}{G_Z D_A^2}\,, \tag{B.6}$$

$$\beta_{g3} = -G_Z D_A^2\,, \tag{B.7}$$

$$\beta_{l1} = 2G_Z V_G D_A\left(\frac{V_G}{V_O} - 1\right), \tag{B.8}$$

$$\beta_{l2} = G_Z D_A^2\left(\frac{2V_G}{V_O} - 1\right), \tag{B.9}$$

$$R_w = \left(G_Z D_A^2 \frac{V_G^2}{V_O^2}\right)^{-1}. \tag{B.10}$$

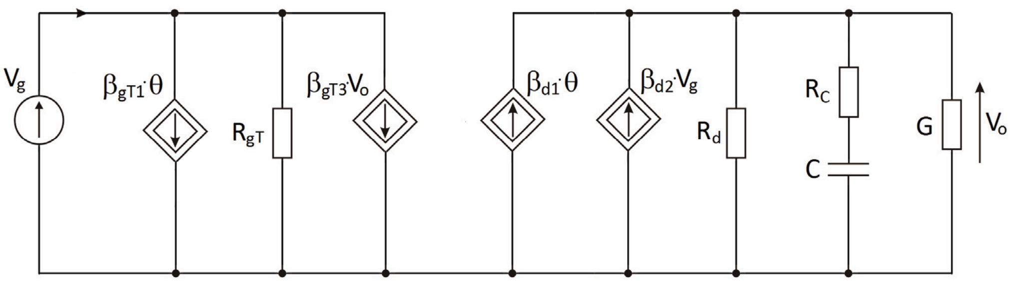


Fig. B.4. Small-signal model of BOOST converter working in DCM

$$\beta_{gT1} = 2G_Z D_A \frac{V_G V_O}{V_O - V_G}\,, \tag{B.11}$$

$$R_{gT} = \frac{(V_O - V_G)^2}{G_Z D_A^2 V_O^2}, \tag{B.12}$$

$$\beta_{gT3} = -\frac{G_Z D_A^2 V_G^2}{(V_O - V_G)^2}, \tag{B.13}$$

$$\beta_{d1} = 2G_Z D_A \frac{V_G^2}{V_O - V_G}, \tag{B.14}$$

$$\beta_{d2} = G_Z D_A^2 \frac{V_G(2V_O - V_G)}{(V_O - V_G)^2}, \tag{B.15}$$

$$R_w = \left(G_Z D_A^2 \frac{V_G^2}{V_O^2}\right)^{-1}. \tag{B.17}$$

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