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ANALYSIS OF CONTACT STRESSES AND SLIP IN POWER SHIFT GEARINGS USING MULTI-CRITERION OPTIMISATION

ANALIZA NAPRĘŻEŃ KONTAKTOWYCH I POŚLIZGÓW MIĘDZYZĘBNYCH W PRZEKŁADNIACH ZĘBATYCH POWER SHIFT Z WYKORZYSTANIEM OPTYMALIZACJI WIELOKRYTERIALNEJ

Key words: multi-criterion optimisation, gear meshing, contact stress, slippage.

Abstract: The work analysed contact stresses and slippage occurring in specific toothed pairs of two power shift gearings with eight ratios. The analysed gearings have ratios of identical values. Differences in contact stresses and slippage values within individual toothed pairs of both gearings (at same values of input torque and engine speed) result from the internal configuration of the kinematic chains created by the toothed gears within individual gear ratios. The analysis included 5 characteristic contact points within the tooth engagement area. They were selected analytically depending on geometrical parameters of gears that constitute the toothed pair. Using computer-assisted design works that employ multi-criterion optimisation, it is possible to minimize slippage and take reasonable advantage of the fatigue contact durability of the material that was used for producing the toothed gears.

Słowa kluczowe: optymalizacja wielokryterialna, zazębienie kół zębatych, naprężenia kontaktowe, poślizg międzyzębny.

Streszczenie: W pracy analizowano naprężenia kontaktowe i poślizgi międzyzębne występujące w poszczególnych parach zębatych dwóch przekładni typu power shift o ośmiu stopniach przełożenia. Analizowane przekładnie zębate posiadają jednakową liczbę przełożeń oraz jednakowe ich wartości. Różnica w wartościach naprężeń kontaktowych i poślizgów międzyzębnych w poszczególnych parach zębatych obydwu przekładni, przy tych samych wartościach wejściowego momentu obrotowego i prędkości obrotowej, wynika z konfiguracji wewnętrznej łańcuchów kinematycznych utworzonych przez koła zębate na poszczególnych stopniach przełożenia. Do analizy wybrano 5 charakterystycznych punktów przyporu w strefie zazębienia, które określono analitycznie w zależności od parametrów geometrycznych kół tworzących parę zębatą. W pracach projektowych wspomaganych komputerowo przy stosowaniu optymalizacji wielokryterialnej istnieje możliwość racjonalnego wykorzystania zmęczeniowej wytrzymałości kontaktowej materiału, z którego wykonane są koła zębate oraz minimalizacja poślizgów międzyzębnych.

INTRODUCTION

Power shift gearings **[L. 1–4]** are used in transmissions of engineering machines, because they allow changing the gear ratio at full load. Such functionality is provided by toothed gears which remain in constant engagement as well as by multidisc clutches integrated with appropriate toothed gears. When a gear is changed, the load exerted on a toothed gear and clutch disks increases and slippage in the first stages of friction coupling within the clutch disks is inevitable.

Research on contact stresses and slippage that employ multi-criterion optimisation and cover each toothed pair in a gearing make it possible to choose the geometrical parameters of the toothed gears which allow using the fatigue contact durability σ_{Hlim} of the material that served for producing the toothed gears. The multi-criterion optimisation strives for minimizing contact stress and slippage at specific geometrical parameters of toothed gears.

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CHARACTERISTICS OF THE RESEARCH SUBJECT

Research was conducted on two power shift gearings **[L. 1]** marked as PZA and PZB. Each of those gearings has eight gear ratios accomplished through specific wet multidisc clutches integrated with toothed gears. Kinematic schemes in axial alignment for PZA and PZB gearings are illustrated in **Fig. 1**.

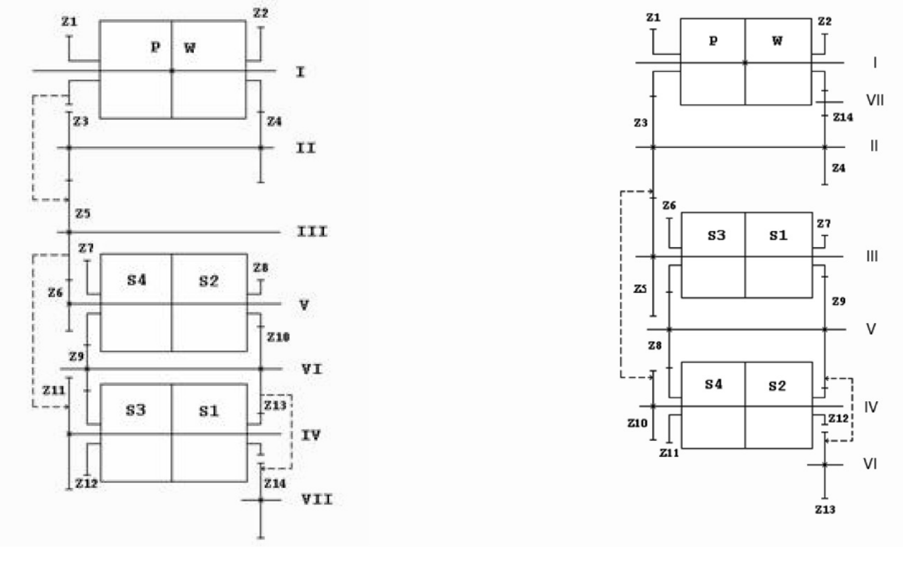

Fig. 1. Kinematic scheme in axial alignment of PZA gearing and PZB gearing
Rys. 1. Schemat kinematyczny w układzie osiowym przekładni zębatej PZA i PZB

The PZA and PZB gearings feature 14 toothed gears located on 7 shafts. By engaging, the gears create 10 toothed pairs (kinematic pairs) which are positioned within an appropriate kinematic chain from input Shaft I to output Shaft VII and denote gear ratios. Shafts I, IV, and V (III for PZB gearing) feature the following clutches: P, W, S_1, S_2, S_3, and S_4. Clutch P allows forward motion, while Clutch W allows backward motion. Clutches S_1, S_2, S_3, and S_4 serve for maintaining ratios at stages 1–8.

The positioning of tooth gears inside a gearing affects tooth engagement between the given toothed gear and the nearest gears. A good indication of the tooth engagement cycles for any toothed gear is a kinematic scheme of the gearing in radial alignment, which is presented in **Fig. 2**.

Using **Figs 1** and **2**, it is possible to record the routes of kinematic chain within individual ratios in PZA and PZB gearings.

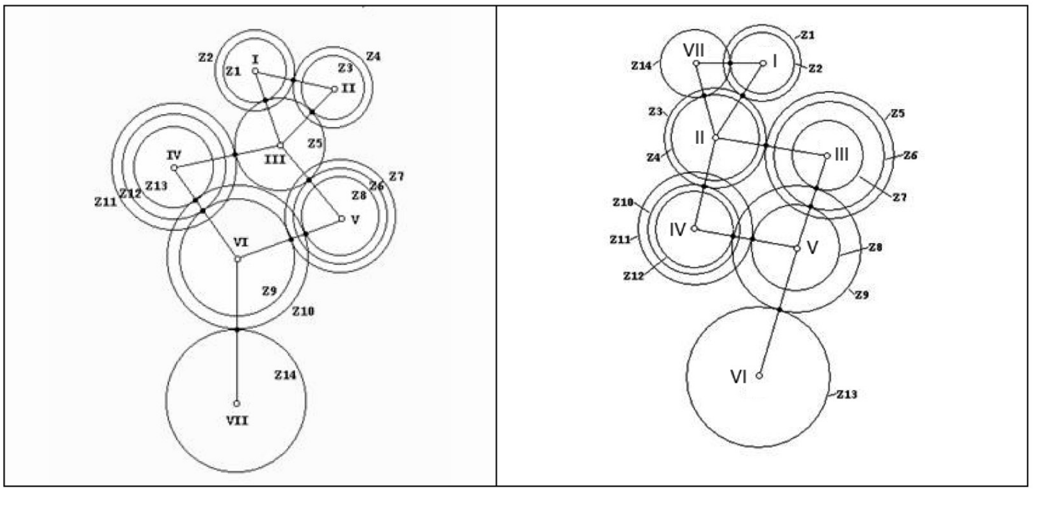

Fig. 2. Kinematic scheme in radial alignment of PZA gearing and PZB gearing
Rys. 2. Schemat kinematyczny w układzie promieniowym przekładni zębatej PZA i PZB

PZA gearing:

$$i_1 = \frac{z_5}{z_1} \cdot \frac{z_{11}}{z_5} \cdot \frac{z_{10}}{z_{13}} \cdot \frac{z_{14}}{z_{10}} \qquad i_5 = \frac{z_4}{z_2} \cdot \frac{z_5}{z_3} \cdot \frac{z_{11}}{z_5} \cdot \frac{z_{10}}{z_{13}} \cdot \frac{z_{14}}{z_{10}}$$

$$i_2 = \frac{z_5}{z_1} \cdot \frac{z_6}{z_5} \cdot \frac{z_{10}}{z_8} \cdot \frac{z_{14}}{z_{10}} \qquad i_6 = \frac{z_4}{z_2} \cdot \frac{z_5}{z_3} \cdot \frac{z_6}{z_5} \cdot \frac{z_{10}}{z_8} \cdot \frac{z_{14}}{z_{10}}$$

$$i_3 = \frac{z_5}{z_1} \cdot \frac{z_{11}}{z_5} \cdot \frac{z_9}{z_{12}} \cdot \frac{z_{14}}{z_{10}} \qquad i_7 = \frac{z_4}{z_2} \cdot \frac{z_5}{z_3} \cdot \frac{z_{11}}{z_5} \cdot \frac{z_9}{z_{12}} \cdot \frac{z_{14}}{z_{10}}$$

$$i_4 = \frac{z_5}{z_1} \cdot \frac{z_6}{z_5} \cdot \frac{z_9}{z_7} \cdot \frac{z_{14}}{z_{10}} \qquad i_8 = \frac{z_4}{z_2} \cdot \frac{z_5}{z_3} \cdot \frac{z_6}{z_5} \cdot \frac{z_9}{z_7} \cdot \frac{z_{14}}{z_{10}}$$

PZB gearing:

$$i_1 = \frac{z_3}{z_1} \cdot \frac{z_5}{z_3} \cdot \frac{z_9}{z_7} \cdot \frac{z_{13}}{z_9} \qquad i_5 = \frac{z_{14}}{z_2} \cdot \frac{z_4}{z_{14}} \cdot \frac{z_5}{z_3} \cdot \frac{z_9}{z_7} \cdot \frac{z_{13}}{z_9}$$

$$i_2 = \frac{z_3}{z_1} \cdot \frac{z_{10}}{z_3} \cdot \frac{z_9}{z_{12}} \cdot \frac{z_{13}}{z_9} \qquad i_6 = \frac{z_{14}}{z_2} \cdot \frac{z_4}{z_{14}} \cdot \frac{z_{10}}{z_3} \cdot \frac{z_9}{z_{12}} \cdot \frac{z_{13}}{z_9}$$

$$i_3 = \frac{z_3}{z_1} \cdot \frac{z_5}{z_3} \cdot \frac{z_8}{z_6} \cdot \frac{z_{13}}{z_9} \qquad i_7 = \frac{z_{14}}{z_2} \cdot \frac{z_4}{z_{14}} \cdot \frac{z_5}{z_3} \cdot \frac{z_8}{z_6} \cdot \frac{z_{13}}{z_9}$$

$$i_4 = \frac{z_3}{z_1} \cdot \frac{z_{10}}{z_3} \cdot \frac{z_8}{z_{11}} \cdot \frac{z_{13}}{z_9} \qquad i_8 = \frac{z_{14}}{z_2} \cdot \frac{z_4}{z_{14}} \cdot \frac{z_{10}}{z_3} \cdot \frac{z_8}{z_{11}} \cdot \frac{z_{13}}{z_9}$$

Ratios i_1 to i_4 allow forward motion of the machine, while ratios i_5 to i_8 provide the backward motion. The gear which is characterized by the most tooth engagement cycles has been selected from within the internal structure of the considered gearings. In case of the PZA gearing, it is the z_5 gear of tooth engagement cycles equalling 4, while the z_3 gear in the PZB gearing has 3 tooth engagement cycles. The teeth engagement cycles are illustrated in **Fig. 3**.

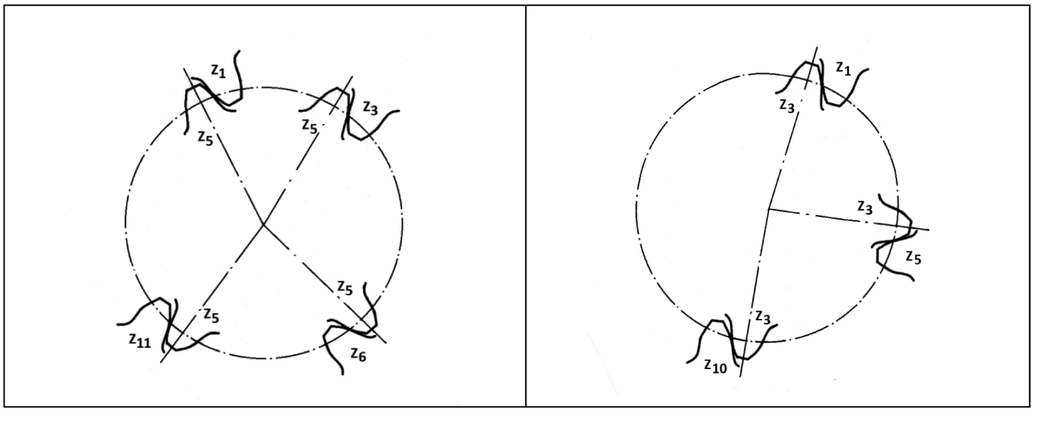

Fig. 3. Teeth engagement cycles in individual gearings: PZA gearing and PZB gearing
Rys. 3. Krotność cykli zazębienia poszczególnych przekładni: przekładnia PZA i PZB

In case of the PZA gearing, the tooth engagement cycles of z_5 gear are indicated by toothed gears z_1, z_3, z_6, and z_{11} while tooth engagement cycles of z_3 gear are indicated by toothed gears z_1, z_5, and z_{10}. During the time of operation assumed for the PZA gearing, the highest number of load cycles will be accomplished by the gear z_5. This indicates that pitting will probably be the initial cause of fatigue wear. In the PZB gearing, toothed gear z_3 suffers the highest number of load cycles at the given time. Hence, this gear will be the first to suffer undesired effects of fatigue wear.

NUMERICAL TESTS ON CONTACT STRESSES AND SLIPPAGE

The contact stresses and slippage have been calculated within the characteristic contact points at the active surface of tooth engagement. Characteristic contact points located on the sides of teeth **[L. 5–8, 10]** are presented in **Fig. 4**.

During tooth engagement, the characteristic points of teeth in the driver (active) gear and the follower (passive) gear overlap each other, which can be recorded as follows: $E_{2cz} = E_{2bier}$, $B_{1cz} = B_{1bier}$, $C_{cz} = C_{bier}$, $B_{2cz} = B_{2bier}$, $E_{1cz} = E_{1bier}$. Considering the engagement of the driver (active) gear and the follower (passive) gear, it can be stated that part of the involute profile of tooth E_2B_1 at the tooth root as well as part of the involute profile of tooth E_2B_1 at the tooth tip are forming an area of double-tooth engagement. Part of the involute profile B_2E_1 is also located within the double-tooth engagement at the tooth tip of the driver (active) gear. It is positioned together with the involute profile B_2E_1 at the tooth root of the follower (passive) gear.

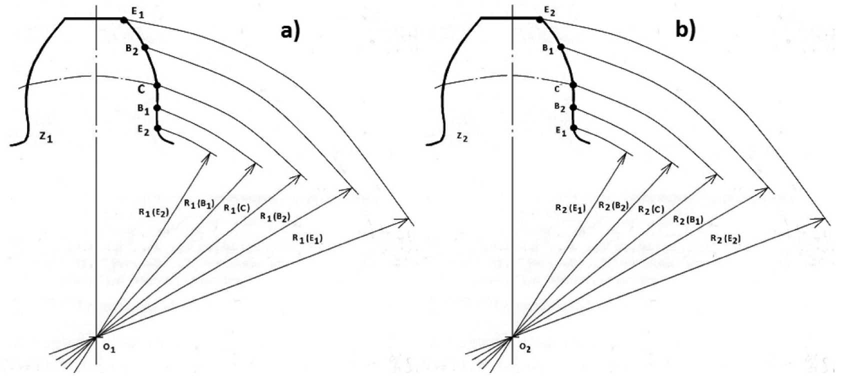

Fig. 4. Characteristic contact points: a) on the tooth of the driver gear; b) on the tooth of the follower gear
Rys. 4. Charakterystyczne punkty zazębienia: a) koło napędzające, b) koło napędzane

Contact stresses and slippage have been measured with an original computer software **[L. 9]** at each of the five characteristic contact points, at an input load moment of 1500 [Nm] and an input engine speed of 1600 [min^{-1}]. Due to their multiplicity, however, **Tables 1–4** present stress values only for the extreme points E_1 and E_2 obtained after the first calculation stage (before optimisation) and after the optimisation for PZA

Table 1. Contact stress values [MPa] for E_1 and E_2 points in the PZA gearing before optimization
Tabela 1. Naprężenia kontaktowe [MPa] w punkcie E_1 i E_2 przekładni PZA przed optymalizacją

ratio	toothed pair										contact point
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	632.9	849.6	934.5	1161.2							E_1
i_2	632.9			1038.8	1037.3	586.2					
i_3	632.9	849.6		899.5			948.5				
i_4	632.9			734.9	1037.3			673.6			
i_5		850.0	935.0	1161.9					681.1	617.3	
i_6				1039.3	1037.9	586.6			681.1	617.3	
i_7		850.0		900.0			949.0		681.1	617.3	
i_8				735.3	1037.9			674.0	681.1	617.3	
i_1	681.0	914.2	1005.5	1249.5							E_2
i_2	681.0			1117.7	1116.1	630.8					
i_3	681.0	914.2		967.8			1020.6				
i_4	681.0			790.7	1116.1			724.8			
i_5		914.6	1006.1	1250.1					732.9	664.2	
i_6				1118.3	1116.7	631.1			732.9	664.2	
i_7		914.6		968.4			1021.1		732.9	664.2	
i_8				791.1	1116.7			725.2	732.9	664.2	

Table 2. Contact stress values [MPa] for E_1 and E_2 points in the PZB gearing before optimization
Tabela 2. Naprężenia kontaktowe [MPa] w punkcie E_1 i E_2 przekładni PZB przed optymalizacją

ratio	toothed pair										contact point
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	**712.7**	**556.7**	**956.0**	**1112.3**							E_1
i_2	**712.7**			**994.9**	**603.9**	**726.8**					
i_3	**712.7**	**556.7**		**824.9**			**1175.2**				
i_4	**712.7**			**710.0**	**603.9**			**984.1**			
i_5		**556.7**	**956.0**	**1112.3**					**702.7**	**648.0**	
i_6				**994.9**	**603.9**	**726.8**			**702.7**	**648.0**	
i_7		**556.7**		**824.9**			**1175.2**		**702.7**	**648.0**	
i_8				**710.0**	**603.9**			**984.1**	**702.7**	**648.0**	
i_1	**777.1**	**607.0**	**1042.3**	**1212.7**							E_2
i_2	**777.1**			**1084.7**	**658.4**	**792.5**					
i_3	**777.1**	**607.0**		**899.4**			**1281.3**				
i_4	**777.1**			**774.1**	**658.4**			**1073.0**			
i_5		**607.0**	**1042.3**	**1212.7**					**766.2**	**706.5**	
i_6				**1084.7**	**658.4**	**792.5**			**766.2**	**706.5**	
i_7		**607.0**		**899.4**			**1281.3**		**766.2**	**706.5**	
i_8				**774.1**	**658.4**			**1073.0**	**766.2**	**706.5**	

Table 3. Contact stress values [MPa] for E_1 and E_2 points in the PZA gearing after optimization
Tabela 3. Naprężenia kontaktowe [MPa] w punkcie E_1 i E_2 przekładni PZA po optymalizacji

ratio	toothed pair										contact point
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	**892.7**	**1273.2**	**1032.5**	**1271.2**							E_1
i_2	**892.7**			**1137.1**	**1410.3**	**669.3**					
i_3	**892.7**	**1273.2**		**984.7**			**1376.1**				
i_4	**892.7**			**804.5**	**1410.3**			**927.8**			
i_5		**1273.9**	**1033.1**	**1271.9**					**1192.5**	**846.8**	
i_6				**1137.8**	**1411.1**	**669.7**			**1192.5**	**846.8**	
i_7		**1273.9**		**985.2**			**1376.9**		**1192.5**	**846.8**	
i_8				**804.9**	**1411.1**			**928.3**	**1192.5**	**846.8**	
i_1	**969.0**	**1382.0**	**1120.7**	**1379.8**							E_2
i_2	**969.0**			**1234.3**	**1530.8**	**726.5**					
i_3	**969.0**	**1382.0**		**1068.8**			**1493.7**				
i_4	**969.0**			**873.2**	**1530.8**			**1007.1**			
i_5		**1382.7**	**1121.3**	**1380.6**					**1294.4**	**919.2**	
i_6				**1235.0**	**1531.7**	**726.9**			**1294.4**	**919.2**	
i_7		**1382.7**		**1069.4**			**1494.5**		**1294.4**	**919.2**	
i_8				**873.7**	**1531.7**			**1007.6**	**1294.4**	**919.2**	

and PZB gearings, respectively. **Table 1** presents contact stress values after the first stage of calculations within E_1 and E_2 points in the PZA gearing.

Table 2 presents results from the first calculation stage of PZB gearing at the same load M=1500 [Nm], n = 1600 [min^{-1}].

Results of contact stresses and slippage **[L. 9]** have been obtained during multi-criterion optimisation with 11 criteria that include the following: the maximum number of contact points, minimal tooth shape coefficient, minimal thickness at the tooth tip, the total weight of toothed gears, the total mass inertial moment

Table 4. Contact stress values [MPa] for E_1 and E_2 points in the PZB gearing after optimization
Tabela 4. Naprężenia kontaktowe [MPa] w punkcie E_1 i E_2 przekładni PZB po optymalizacji

ratio	toothed pair										contact point
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	**866.9**	**695.3**	**1106.6**	**1294.5**							E_1
i_2	**866.9**			**1157.9**	**702.9**	**864.3**					
i_3	**866.9**	**695.3**		**960.1**			**1474.9**				
i_4	**866.9**			**826.3**	**702.9**			**1197.4**			
i_5		**695.3**	**1106.6**	**1294.5**					**852.7**	**871.5**	
i_6				**1157.9**	**702.9**	**864.3**			**852.7**	**871.5**	
i_7		**695.3**		**960.1**			**1474.9**		**852.7**	**871.5**	
i_8				**826.3**	**702.9**			**1197.4**	**852.7**	**871.5**	
i_1	**936.5**	**751.2**	**1195.5**	**1398.6**							E_2
i_2	**936.5**			**1250.9**	**759.4**	**933.7**					
i_3	**936.5**	**751.2**		**1037.2**			**1593.4**				
i_4	**936.5**			**892.7**	**759.4**			**1293.6**			
i_5		**751.2**	**1195.5**	**1398.6**					**921.2**	**941.5**	
i_6				**1250.9**	**759.4**	**933.7**			**921.2**	**941.5**	
i_7		**751.2**		**1037.2**			**1593.4**		**921.2**	**941.5**	
i_8				**892.7**	**759.4**			**1293.6**	**921.2**	**941.5**	

Table 5. Slippage values ($m \times s^{-1}$) for E_1 and E_2 points in the PZA gearing before optimization
Tabela 5. Poślizgi międzyzębne ($m \times s^{-1}$) w punkcie E_1 i E_2 przekładni PZA przed optymalizacją

ratio	toothed pair										contact point
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	**2.639**	**2.737**	**1.117**	**1.099**							E_1
i_2	**2.639**			**1.374**	**2.727**	**3.257**					
i_3	**2.639**	**2.737**		**1.832**			**2.283**				
i_4	**2.639**			**2.746**	**2.727**			**2.160**			
i_5		**2.732**	**1.116**	**1.097**					**7.277**	**2.420**	
i_6				**1.372**	**2.722**	**3.252**			**7.277**	**2.420**	
i_7		**2.732**		**1.830**			**2.281**		**7.277**	**2.420**	
i_8				**2.740**	**2.722**			**2.156**	**7.277**	**2.420**	
i_1	**2.880**	**1.368**	**2.006**	**1.361**							E_2
i_2	**2.880**			**1.701**	**2.364**	**1.727**					
i_3	**2.880**	**1.368**		**2.269**			**2.575**				
i_4	**2.880**			**3.401**	**2.364**			**5.696**			
i_5		**1.365**	**2.003**	**1.359**					**2.918**	**2.681**	
i_6				**1.699**	**2.359**	**1.724**			**2.918**	**2.681**	
i_7		**1.365**		**2.266**			**2.572**		**2.918**	**2.681**	
i_8				**3.394**	**2.359**			**5.686**	**2.918**	**2.681**	

of toothed gears, the maximal durability of tooth root and tooth edge, material effort uniformity within toothed gears, minimal relative thickness of the oil film within the area between teeth, gearing efficiency, and minimal slippage value.

Table 3 presents contact stresses values which were obtained during the PZA gearing optimisation that included 11 criteria.

Results of optimization calculations for the PZB gearing, which were obtained accordingly to the PZA gearing, are presented in **Table 4**.

The maximum slippage values occur in the extreme points E_1 and E_2. The numerical values for PZA gearing obtained in the first calculation stage are presented in **Table 5**.

Table 6 presents slippage values after the first calculation cycle for the PZB gearing and its extreme points E_1 and E_2.

Apart from the contact stresses and due to the 11 criteria mentioned above, optimisation calculations in the PZA gearing also included the slippage in points E_1 and E_2. The slippage values are presented in **Table 7**.

Table 6. Slippage values ($m \times s^{-1}$) for E_1 and E_2 points in the PZB gearing before optimization
Tabela 6. Poślizgi międzyzębne ($m \times s^{-1}$) w punkcie E_1 i E_2 przekładni PZB przed optymalizacją

ratio	**toothed pair**										**contact point**
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	**3.293**	**3.319**	**1.942**	**1.212**							
i_2	**3.293**			**1.514**	**3.837**	**3.223**					
i_3	**3.293**	**3.319**		**2.203**			**3.015**				
i_4	**3.293**			**2.976**	**3.837**			**2.851**			E_1
i_5		**3.317**	**1.942**	**1.212**					**4.583**	**4.684**	
i_6				**1.514**	**3.835**	**3.221**			**4.583**	**4.684**	
i_7		**3.317**		**2.203**			**3.015**		**4.583**	**4.684**	
i_8				**2.974**	**3.835**			**2.849**	**4.583**	**4.684**	
i_1	**3.512**	**2.250**	**1.597**	**1.766**							
i_2	**3.512**			**2.207**	**2.427**	**1.980**					
i_3	**3.512**	**2.250**		**3.210**			**2.603**				
i_4	**3.512**			**4.337**	**2.427**			**5.108**			
i_5		**2.249**	**1.597**	**1.766**					**3.890**	**4.248**	E_2
i_6				**2.207**	**2.425**	**1.978**			**3.890**	**4.248**	
i_7		**2.249**		**3.210**			**2.603**		**3.890**	**4.248**	
i_8				**4.334**	**2.425**			**5.104**	**3.890**	**4.248**	

Table 7. Slippage values ($m \times s^{-1}$) for E_1 and E_2 points in the PZA gearing after optimization
Tabela 7. Poślizgi międzyzębne ($m \times s^{-1}$) w punkcie E1 i E_2 przekładni PZA po optymalizacji

ratio	**toothed pair**										**contact point**
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	**2.871**	**2.554**	**1.315**	**1.143**							
i_2	**2.871**			**1.429**	**2.863**	**2.979**					
i_3	**2.871**	**2.554**		**1.905**			**2.433**				
i_4	**2.871**			**2.856**	**2.863**			**3.248**			E_1
i_5		**2.551**	**1.314**	**1.141**					**2.962**	**3.097**	
i_6				**1.427**	**2.860**	**2.976**			**2.962**	**3.097**	
i_7		**2.551**		**1.903**			**2.430**		**2.962**	**3.097**	
i_8				**2.852**	**2.860**			**3.244**	**2.962**	**3.097**	
i_1	**3.079**	**1.642**	**2.085**	**1.518**							
i_2	**3.079**			**1.898**	**3.013**	**2.134**					
i_3	**3.079**	**1.642**		**2.530**			**1.655**				
i_4	**3.079**			**3.793**	**3.013**			**4.412**			
i_5		**1.641**	**2.082**	**1.515**					**3.989**	**2.936**	E_2
i_6				**1.895**	**3.010**	**2.132**			**3.989**	**2.936**	
i_7		**1.641**		**2.528**			**1.653**		**3.989**	**2.936**	
i_8				**3.788**	**3.010**			**4.407**	**3.989**	**2.936**	

The optimization calculations of slippage in contact points E_1 and E_2 in the PZB gearing are illustrated in **Table 8**.

Optimization of the PZA and PZB gearings has been conducted in relation to 11 criteria referenced by the global criterion. This work focuses on contact stresses and slippage. It is believed that multi-criterion optimisation is an optimal solution when global criterion reaches the minimum. **Figure 5** presents the contact stress chart in relation to optimisation steps for the PZA gearing.

Figure 6 presents the contact stress chart in relation to optimisation steps for the PZB gearing.

Table 8. Slippage values (m×s^{-1}) for E_1 and E_2 points in the PZB gearing after optimization
Tabela 8. Poślizgi międzyzębne (m×s^{-1}) w punkcie E_1 i E_2 przekładni PZB po optymalizacji

ratio	**toothed pair**										**contact point**
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	**3.920**	**3.504**	**2.048**	**1.295**							E_1
i_2	**3.920**			**1.618**	**3.758**	**2.987**					
i_3	**3.920**	**3.504**		**2.354**			**2.850**				
i_4	**3.920**			**3.180**	**3.758**			**2.991**			
i_5		**3.504**	**2.048**	**1.295**					**3.537**	**3.908**	
i_6				**1.618**	**3.758**	**2.987**			**3.537**	**3.908**	
i_7		**3.504**		**2.354**			**2.850**		**3.537**	**3.908**	
i_8				**3.180**	**3.758**			**2.991**	**3.537**	**3.908**	
i_1	**3.796**	**3.093**	**1.544**	**1.744**							E_2
i_2	**3.796**			**2.180**	**3.644**	**2.073**					
i_3	**3.796**	**3.093**		**3.171**			**2.251**				
i_4	**3.796**			**4.284**	**3.644**			**4.765**			
i_5		**3.093**	**1.544**	**1.744**					**3.791**	**3.848**	
i_6				**2.180**	**3.644**	**2.073**			**3.791**	**3.848**	
i_7		**3.093**		**3.171**			**2.251**		**3.791**	**3.848**	
i_8				**4.284**	**3.644**			**4.765**	**3.791**	**3.848**	

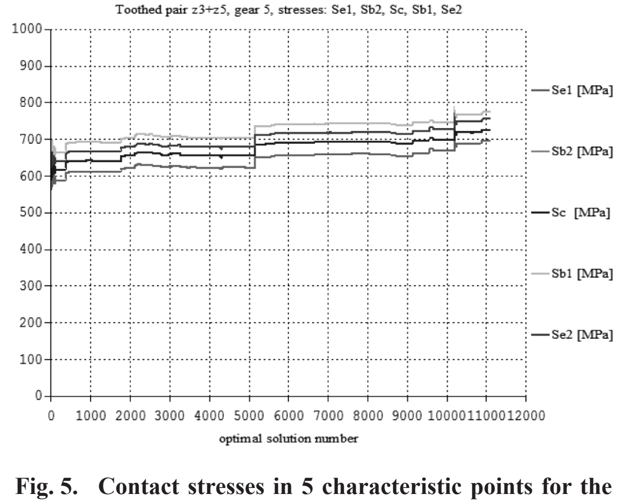

Fig. 5. Contact stresses in 5 characteristic points for the PZA gearing
Rys. 5. Naprężenia kontaktowe w 5 charakterystycznych punktach przekładni PZA

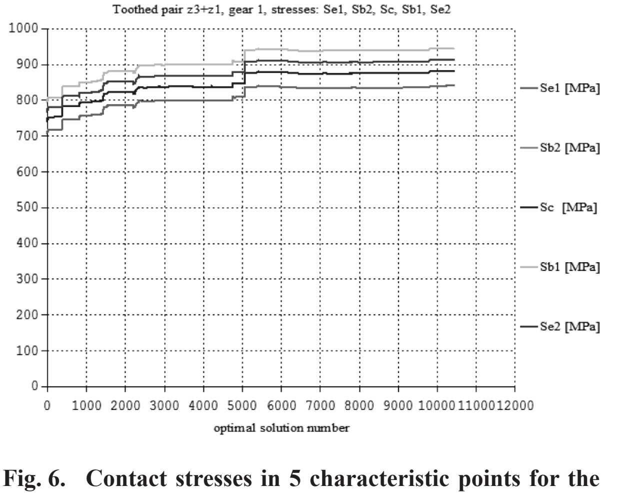

Fig. 6. Contact stresses in 5 characteristic points for the PZB gearing
Rys. 6. Naprężenia kontaktowe w 5 charakterystycznych punktach przekładni PZB

ANALYSIS OF THE RESULTS

For each ratio from i_1 to i_8 for contact points E_1 and E_2, contact stresses in toothed pair z_{14}/z_{10} in the PZA gearing presented in **Table 1** are higher than contact stresses in toothed pair z_{13}/z_9 in the PZB gearing illustrated in **Table 2**. The purpose for comparing stress values between toothed pairs z_{14}/z_{10} and z_{13}/z_9 is justified by the fact that they constitute the last pair in the kinematic chain of ratios from i_1 to i_8.

According to **Tables 1** and **3**, a comparison of contact stresses within toothed pairs z_5/z_1, z_{11}/z_5, z_6/z_5, and z_5/z_3 with 4 engagement cycles (being a part of the gear z_5 in the PZA gearing) between contact stresses within toothed pairs z_3/z_1, z_5/z_3, and z_{10}/z_3 with 3 engagement cycles (being a part of the gear z_3 in the PZB gearing) indicates that contact stresses in toothed gear z_5 in the PZA gearing are higher. Higher stress values and four tooth engagement cycles in toothed gear z_5 indicate that, after a given time of operation, this gear will suffer the highest number of load cycles. From this reason, it can be expected that first occurrences of fatigue contact durability take the form of pitting.

According to **Table 5**, after a single calculation stage, the slippage in toothed pairs z_5/z_1, z_{11}/z_5, z_6/z_5, and z_5/z_3 in the PZA gearing does not significantly differ from slippage in the same toothed pairs following optimisation according to **Table 7**. According to **Tables 5** and **7**, the highest difference between slippage values after a single calculation stage and the values obtained after optimisation was displayed by toothed pair z_4/z_2 in the PZA gearing at ratios from i_5 to i_8. Following optimisation, slippage in this toothed pair has decreased from 7.277 $m{\times}s^{-1}$ to 2.962 $m{\times}s^{-1}$.

The PZB gearing is characterized by higher slippage values within all ratios in each toothed pair. The toothed pairs z_{14}/z_2 and z_4/z_{14} in the PZB gearing described in **Table 6** display the highest slippage values after the first calculation stage within ratios i_5 to i_8 and point E_1 as well as within ratios i_5 to i_8 and point E_2. By comparing slippage values presented in Tables 6 and 8, it can be noticed that toothed pairs: z_3/z_4, z_5/z_3, and z_{10}/z_3 display an increase in slippage after optimization calculations.

SUMMARY

The analysis of calculations with multi-criterion optimisation conducted on the PZA and PZB gearings showed that toothed pairs of the both gearings do not display significant differences in slippage values after optimisation.

The criterion of contact stresses indicates that the configuration of the PZB gearing is more favourable than the configuration of the PZA gearing. The toothed gear z_3, being a part of its structure is characterized by 3 engagement cycles, while the toothed gear z_5 in the PZA gearing has four engagement cycles. Thus, the expected durability of the PZB gearing will be greater due to fatigue contact durability of the toothed gear edges.

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