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WAVE PROPAGATION IN MICROPOLAR MONOCLINIC THERMOELASTIC HALF SPACE

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Propagation of waves in a micropolar monoclinic medium possessing hermoelastic properties based on the Lord- Shulman (L-S),Green and Lindsay (G-L) and Coupled thermoelasticty (C-T) theories is discussed. The investigation is divided into two sections, viz., plane strain and anti-plane strain problem. After developing the solution, the phase velocities and attenuation quality factor have been derived and computed numerically. The numerical results have been plotted graphically.

Key words: micropolar, monoclinic, generalized thermoelastic.

1. Introduction

Any material is endowed with microstructure, like atoms and molecules at microscopic scale, grains and fibers or particulate at mesoscopic scale. Homogenization of a basically heterogeneous material depends on the scale of interest. When stress fluctuation is small enough compared to microstructure of the material, homogenization can be made without considering the detailed microstructure of the material. However, if it is not the case, the microstructure of the material must be considered properly in a homogenized formulation (Eringen, 1999; Hu *et al*., 2005). The concept of microcontinuum, proposed by Eringen (1999), can take into account the microstructure of the material while the theory itself remains still in a continuum formulation. The first grade microcontinuum consists of a number of theories, such as micropolar, microstretch and micromorphic, depending on how much micro-degrees of freedom are incorporated. These microcontinuum theories are believed to be potential tools to characterize the behavior of materials with complicated microstructures.

The most popular microcontinuum theory is the micropolar one, in this theory, a material point can still be considered as infinitely small, however, there are microstructures inside this point. So there are two sets of variables to describe the deformation of this material point, one characterizes the motion of the inertia center of this material point; the other describes the motion of the microstructure inside this point. In the micropolar theory, the motion of the microstructure is supposed to be an independently rigid rotation. Applications of this theory can be found in Eringen (1999); Hu *et al*. (2005), Lakes (1983).

The subject of generalized thermoelasticity has drawn the attention of researchers due to its relevance to many practical applications. Theories of generalized thermoelasticity have been developed mainly to overcome the shortcomings inherent in the classical coupled dynamical theory of thermoelasticity, such as the infinite speed of thermoelastic disturbances, unsatisfactory thermoelastic response of a solid body to short laser pulses and poor description of thermoelastic behaviour at low temperature. The generalized theories are characterized by the finite speed of thermal disturbance. A review of these theories is presented in the articles by Chandrasekharaiah (1998) and Hetnarski and Iganaczak (1999).

The theory of micropolar thermoelasticity has been a subject of intensive study. The linear theory of micropolar thermoelasticity was developed by Eringen (1970) and Nowacki (1966) to include thermal effects and is known as micropolar coupled thermoelasticity. A comprehensive review of works on the subject was given in Dhaliwal and Singh (1980). Boschi and Iesan (1973) extended a generalized theory of micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speed. Othman and Baljeet (2007), Othman *et al*. (2009) investigated various problems in the generalized theory of micropolar thermoelasticity. Verma (2002), Verma and Haseba (2001) discussed the propagation of waves in anisotropic media in generalized thermoelastcity.

Singh and Tomar (2007) derived the reflection and transmission coefficients when plane qP-wave is incident obliquely at acorrugated interface between two different monoclinic elastic half-spaces. They derived the expressions for reflection and transmission coefficients using Rayleigh's method and obtained the variation of the modulus of reflection and transmission coefficients with the angle of incidence, frequency and corrugation of the interface. Chattopadhyay *et al*. (2009) discussed the reflection and refraction when plane quasi-*P*-wave is incident at a corrugated interface between distinct triclinic elastic half spaces. Singh (2010) in his article studied the reflection of plane waves at the free surface of a monoclinic thermoelastic solid half-space. Sharma (2011) obtained energy flux characteristics of inhomogeneous waves in anisotropic thermoviscoelastic media.

The aim of the present paper is to discuss the propagation of waves in a micropolar monoclinic thermoelastic half space. The investigation is divided into two sections, viz., plane strain and antiplane strain problems. After developing the solution, the phase velocities and attenuation quality factor have been derived and computed numerically. The behavior of the components of displacement, microrotation and stresses for plane waves is obtained and plotted graphically. Some particular cases of interest are also deduced.

2. Basic equations

A homogeneous anisotropic thermally conducting micropolar elastic solid is considered at a uniform temperature T_o, in the undisturbed state. The balance equations for momentum, moment of momentum and energy in this medium, in the absence of body forces and heat sources, are given by

$$t_{ji,j} = \rho \ddot{u}_i, \tag{2.1}$$

$$m_{ik,i} + \in_{ijk} t_{ij} = \rho j \ddot{\varphi}_k, \qquad i,j,k = 1,2,3, \tag{2.2}$$

$$K^*_{ij} T_{,ij} - \rho C^* \left(\dot{T} + \tau_o \ddot{T}\right) = T_o \beta_{ij} \left(\dot{u}_{i,j} + n_o \tau_o \ddot{u}_{i,j}\right) \tag{2.3}$$

where, K^*_{ij} is (positive-definite) thermal conductivity tensor, ρ and C^* are density and specific heat at constant strain, respectively, u_i and φ_i are the components of the average displacements and microrotations of the particles. The thermal displacement T denotes variations in the temperature of the medium in undisturbed state. $\in_{ijk}$ is the permutation symbol, which is zero if any two of its suffixes are equal, takes the values 1 if the suffixes are in cyclic order and -1 if the suffixes are in cyclic order. The symmetric non-singular tensor β_{ij} represents thermal expansion of the medium. The components of the tensor define thermoelastic coupling, which is explained through constitutive relations. In terms of deformation, microrotation and temperature, the stress t_{ij} and couple stress m_{ij} components in the medium are expressed as

$$t_{ij} = A_{ijkl}\varepsilon_{kl} - \beta_{ij}\left(1+\tau_1\right)\dot{T}, \qquad m_{ij} = B_{ijkl}\psi_{kl} \tag{2.4}$$

where the fourth rank asymmetric tensors A_{ijkl} and B_{ijkl} represents the elastic constants of the medium. The two relaxation times are given by $\tau_1 \geq \tau_o > 0$. These time coefficients represent the thermal relaxation

mechanism and account for thermoelastic loss in the medium. Also, components of the micropolar strain tensor are given by $\varepsilon_{ij} = u_{j,i} + \in_{jik} \varphi_k, \psi_{ij} = \varphi_{i,j}$.
Also, we have
for Lord and Shulman(L-S)theory : $\tau_1 = 0, n_o = 1$,
for Green and Lindsay (G-L)theory: $n_o = 0, \tau_1 \geq \tau_o > 0$,
for Coupled Thermoelasticity (C-T) theory $n_o = 0, \tau_1 = \tau_o = 0$.

3. Formulation of the problem

Following Slaughter (2002), we have used appropriate transformations on the set of Eq.(2.4) to derive equations for a micropolar monoclinic medium. So, we have the following set of equations for the propagation of waves in the *x-z* plane

$$\begin{aligned} &A_{11}u_{1,11} + A_{81}u_{1,12} + A_{88}u_{1,22} + A_{99}u_{1,33} + (A_{77} + A_{18})u_{2,11} + (A_{12} + A_{87})u_{2,12} + \\ &+A_{82}u_{2,22} + A_{94}u_{2,33} + (A_{13} + A_{96})u_{3,13} + A_{59}u_{3,23} - (A_{77} + A_{18})\varphi_{3,1} + \\ &+A_{88}\varphi_{3,2} - A_{99}\varphi_{2,3} + A_{94}\varphi_{1,3} + A_{83}u_{3,22} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)(\beta_1 T_{,1} + \beta_8 T_2 + \beta_9 T_3) = \rho \ddot{u}_1, \end{aligned} \tag{3.1}$$

$$\begin{aligned} &(A_{11} + A_{21})u_{1,21} + A_{17}u_{2,11} + (A_{12} + A_{72})u_{2,12} + A_{28}u_{1,22} + A_{49}u_{1,33} + \\ &+A_{22}u_{2,22} + A_{44}u_{2,33} + (A_{13} + A_{46})u_{3,13} + (A_{23} + A_{45})u_{3,32} + (A_{18} - A_{17})\varphi_{3,1} + \\ &+(A_{28} - A_{27})\varphi_{3,2} + (A_{44} - A_{45})\varphi_{1,3} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)(\beta_7 T_{,1} + \beta_2 T_{,2} + \beta_4 T_{,3}) = \rho \ddot{u}_2, \end{aligned} \tag{3.2}$$

$$\begin{aligned} &(A_{69} + A_{31})u_{1,13} + A_{29}u_{1,32} + (A_{64} + A_{37} + A_{38})u_{2,13} + (A_{54} + A_{32})u_{2,23} + \\ &+A_{66}u_{3,11} + (A_{65} + A_{56})u_{3,12} + A_{55}u_{3,22}A_{33}u_{3,33} + (A_{37} - A_{38})\varphi_{3,3} + \\ &+(A_{66} - A_{69})\varphi_{2,1} + (A_{56} - A_{59})\varphi_{2,2} + (A_{64} - A_{65})\varphi_{1,1} + \\ &+(A_{54} - A_{55})\varphi_{1,2} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)(\beta_6 T_{,1} + \beta_5 T_{,2} + \beta_3 T_{,3}) = \rho \ddot{u}_3, \end{aligned} \tag{3.3}$$

$$\begin{aligned} &B_{11}\varphi_{1,11} + B_{17}\varphi_{1,21} + B_{18}\varphi_{2,11} + B_{12}\varphi_{2,21} + B_{13}\varphi_{3,13} + B_{71}\varphi_{1,12} + B_{77}\varphi_{1,22} + B_{78}\varphi_{2,12} + \\ &+B_{72}\varphi_{2,22} + B_{73}\varphi_{3,32} + B_{66}\varphi_{1,33} + B_{65}\varphi_{2,33} + B_{61}\varphi_{3,13} + B_{64}\varphi_{3,23} + A_{56}(u_{3,1} + \varphi_2) + \\ &+A_{55}(u_{3,2} - \varphi_1) + A_{59}(u_{1,3} - \varphi_2) + A_{54}(u_{2,3} + \varphi_1) - A_{46}(u_{3,1} + \varphi_2) - A_{45}(u_{3,2} - \varphi_1) + \\ &-A_{45}(u_{3,2} - \varphi_1) - A_{49}(u_{1,3} - \varphi_2) - A_{44}(u_{2,3} + \varphi_1) - (\beta_5 - \beta_3)\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T = \rho j \ddot{\varphi}_1, \end{aligned} \tag{3.4}$$

$$\begin{aligned} &B_{81}\varphi_{1,11} + B_{87}\varphi_{1,21} + B_{88}\varphi_{2,11} + B_{82}\varphi_{2,21} + B_{83}\varphi_{3,13} + B_{21}\varphi_{1,12} + B_{27}\varphi_{1,22} + B_{28}\varphi_{2,12} + \\ &+B_{22}\varphi_{2,22} + B_{56}\varphi_{1,33} + B_{55}\varphi_{2,33} - A_{66}(u_{3,1} + \varphi_2) - A_{65}(u_{3,2} - \varphi_1) - A_{69}(u_{1,3} - \varphi_2) + \\ &-A_{64}(u_{2,3} + \varphi_1) + A_{96}(u_{3,1} + \varphi_2) + A_{95}(u_{3,2} - \varphi_1) + A_{99}(u_{3,2} - \varphi_2) + \\ &+A_{94}(u_{1,3} + \varphi_1) - (\beta_9 - \beta_6)\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T = \rho j \ddot{\varphi}_2, \end{aligned} \tag{3.5}$$

$$B_{96}\varphi_{1,31}+B_{95}\varphi_{2,31}+B_{99}\varphi_{3,11}+B_{94}\varphi_{3,21}+B_{46}\varphi_{1,32}+B_{45}\varphi_{2,32}+B_{49}\varphi_{3,12}+B_{44}\varphi_{3,22}+$$
$$+B_{31}\varphi_{1,13}+B_{37}\varphi_{1,23}+B_{38}\varphi_{2,13}+B_{32}\varphi_{2,23}+B_{33}\varphi_{3,33}+A_{11}u_{1,1}+A_{77}\left(u_{2,1}-\varphi_3\right)+$$
$$+A_{78}\left(u_{1,2}+\varphi_3\right)-A_{82}u_{2,2}-A_{83}\left(u_{3,2}-\varphi_1\right)-\left(\beta_7-\beta_8\right)\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\rho j\ddot{\varphi}_3,$$

$$K_1^*T_{,11}+K_2^*T_{22}+K_3^*T_{,33}=\rho C^*\left(\frac{\partial T}{\partial t}+\tau_o\frac{\partial^2 T}{\partial t^2}\right)+$$
$$+T_o\left(\frac{\partial}{\partial t}+n_o\tau_o\frac{\partial^2}{\partial t^2}\right)\left(\beta_1u_{1,1}+\beta_2u_{2,2}+\beta_3u_{3,3}\right). \tag{3.6}$$

4. Solution of the problem

We consider a homogeneous, micropolar monoclinic generalized thermoelastic half space. When the displacement and rotation fields depend on two space variables x_1; x_3 and time 't', the field equations can be decomposed into two independent sets, (i) plane strain and (ii) anti-plane strain.

4.1. Plane-strain

In this case we assume the components of displacement and microrotation of the form

$$\boldsymbol{u}=\left(u_1,0,u_3\right),\qquad \varphi=(0,\varphi_2,0), \tag{4.1}$$

and we assume the solution of the form

$$\left(u_1,u_3,\varphi_2,T\right)=\left(\bar{u}_1,\bar{u}_3,\bar{\varphi}_2,\bar{T}\right)e^{i\xi\left(p_1x_1+p_3x_3-ct\right)} \tag{4.2}$$

where the components $(\bar{u}_1,\bar{u}_3,\bar{\phi}_2,\bar{T})$ define the amplitude and polarization of particle displacement, microrotation and temperature distribution in the medium.

To facilitate the solution, we use the following non-dimensional variables

$$\left(x_1',x'\right)_3=\frac{\omega^*}{c_1}\left(x_1,x_3\right),\qquad \left(u_1',u_3'\right)=\frac{\omega^*}{c_1}\left(u_1,u_3\right),\qquad t_{ij}'=\frac{t_{ij}}{A_{1111}},$$
$$m_{ij}'=\frac{m_{ij}c_1}{B_{2323}\omega^*},\qquad \varphi_2'=\frac{\varphi_2A_{1111}}{X},\qquad T'=\frac{T}{T_0},$$
$$t'=\omega^*t,\qquad \tau_0'=\omega^*\tau_0,\qquad \tau_1'=\omega^*\tau_1, \tag{4.3}$$
$$\omega^{*2}=\frac{X}{\rho j},\qquad c_1^2=\frac{A_{1111}}{\rho}$$

where ω is the characteristic frequency of the material and c_1 is the longitudinal wave velocity of the medium.

Using Eqs (4.3) and (4.4) on Eqs (3.2)-(4.1), and solving the resulting equations for the non-trivial solution, we obtain,

$$A\xi^4 + B\xi^3 + C\xi^2 + D\xi + E = 0 \tag{4.4}$$

where the list of symbols is given at the end of the paper (Appendix A).

The phase propagation velocity and attenuation quality factor of the quasi-longitudinal displacement (qLD) wave, quasi thermal wave (qT), quasi transverse microrotational (qTM) wave and quasi transverse displacement (qTD) wave, are obtained from the complex solutions ξ_i , $i = 1, 2, 3, 4$, of the equation as

$$\upsilon_i = \omega / \mathrm{Re}\,al(\xi_i), \tag{4.5}$$

$$Q_i^{-1} = 2\,\mathrm{Im}\,ag(\xi_i) / \mathrm{Re}\,al(\xi_i). \tag{4.6}$$

These waves are called quasi-waves because polarizations may not be along the dynamic axes. The waves with velocities *v1*, *v2*, *v3*, *v4* may be named as that are propagating with the descending phase velocities *vi*, (*i=1,2,3,4*), respectively.

4.2. Antiplane strain

In this case we assume the the components of displacement and microrotation of the form

$$\boldsymbol{u} = (0, u_2, 0), \qquad \varphi = (\varphi_1, 0, \varphi_3), \tag{4.7}$$

and we assume the solution of the form

$$(u_2, \varphi_1, \varphi_3) = (\overline{u}_2, \overline{\varphi}_1, \overline{\varphi}_3)\, e^{i\xi(p_1 x_1 + p_3 x_3 - ct)} \tag{4.8}$$

where the components $(\overline{u}_2, \overline{\varphi}_1, \overline{\varphi}_3)$ define the amplitude and polarization of particle displacement and microrotation in the medium.

Using Eqs (4.5) and (4.6) on Eqs (3.2)-(4.1), and solving the resulting equations for the non-trivial solution, we obtain,

$$\xi^3 + F\xi^2 + G\xi + H = 0 \tag{4.9}$$

where the list of symbols is given at the end of the paper. The phase propagation velocity and attenuation quality factor of the quasi-transverse displacement (qLD) wave, quasi-transverse microrotational (qTM1, qTM2) waves, are obtained from the complex solutions, ξ_i , $i = 1, 2, 3$, of the equation as

$$\upsilon_i = \omega / \mathrm{Re}\,al(\xi_i), \tag{4.10}$$

$$Q_i^{-1} = 2\,\mathrm{Im}\,ag(\xi_i) / \mathrm{Re}\,al(\xi_i). \tag{4.11}$$

These waves are called quasi-waves because polarizations may not be along the dynamic axes. The waves with velocities *v1*, *v2*, *v3* may be named as that are propagating with the descending phase velocities *vi*, (*i=1,2,3*), respectively.

5. Particular cases

5.1. Orthotropic material

Taking $A_{17} = A_{18} = A_{72} = A_{73} = A_{65} = A_{64} = A_{82} = A_{83} = A_{59} = A_{94} = 0$, in Eqs (4.5) and (4.10), we obtain the corresponding expressions for an orthotropic micropolar generalized thermoelastic medium.

5.2. Tetragonal material

Taking $A_{65} = A_{94} = 0$ and $A_{88} = A_{77}, A_{22} = A_{11}, A_{82} = -A_{17}$, $A_{72} = -A_{18}$, $A_{59} = -A_{64}$ $A_{82} = -A_{17}$, $A_{55} = A_{66}$, $A_{99} = A_{44}$, in the Eqs (4.5) and (4.4), we obtain the corresponding expressions for a tetragonal micropolar generalized thermoelastic medium.

5.3. Cubic material

Taking $A_{17} = A_{18} = A_{72} = A_{73} = A_{65} = A_{64} = 0$, $A_{82} = A_{73} = A_{59} = A_{94} = 0$, $A_{13} = A_{12} = A_{23}$, $A_{69} = A_{78} = A_{54}$ $A_{88} = A_{44} = A_{66}$, $A_{22} = A_{11} = A_{33}$, $A_{55} = A_{99} = A_{77}$, in the Eqs (4.5) and (4.10), we obtain the corresponding expressions for a tetragonal micropolar generalized thermoelastic medium.

6. Numerical results and discussion

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computation, we take the values for relevant parameters for a micropolar monoclinic thermoelastic solid as

$$A_{11} = 33.6 \times 10^{10}\, Nm^{-2}, \qquad A_{99} = 17.43 \times 10^{10}\, Nm^{-2}, \qquad A_{69} = 16 \times 10^{10}\, Nm^{-2},$$

$$A_{31} = 5.3 \times 10^{10}\, Nm^{-2}, \qquad A_{31} = 5.3 \times 10^{10}\, Nm^{-2}, \qquad A_{66} = 17.2 \times 10^{10}\, Nm^{-2},$$

$$A_{96} = 16.8 \times 10^{10}\, N, \qquad B_{88} = 2.91 \times 10^{9}\, N, \qquad B_{77} = 1.44 \times 10^{9}\, N,$$

$$\rho = 1.74\ Kg / m^3, \qquad j = 0.2 \times 10^{-15}\, m^2, \qquad K^* = 0.6 \times 10^{-2}\, J / m \sec {}^oC,$$

$$C^* = 0.23\, J / Kg\, {}^oC, \qquad T = 298\, K.$$

Figures 1 and 2 show the variations of phase velocities V_i, $i = 1, 2, 3, 4$, and attenuation quality factors Q_i, $i = 1, 2, 3, 4$. In these figures the solid curves represent the cases of a micropolar monoclinic generalized thermoelastic (MMT1) half space ($\xi = 0.25$), while dotted curves represent the cases of a micropolar monoclinic generalized thermoelastic (MMT2) half space ($\xi = 0.35$). The three theories of generalized thermoelasticity, viz, Lord Shulman (L-S) and Green Lindsay (G-L) are compared in all the graphs. The solid and dotted line without center symbol corresponds to L-S theory, the solid and dotted line with center symbol ($-o-o-$) corresponds to G-L theory.

It can be seen from Fig.1a that the value of phase velocity V_1 goes on decreasing with an increase in frequency for both the theories of thermoelasticity. The variation pattern of MMT1 and MMT2 is similar with the difference in their amplitudes. Figure 1b shows that the value of phase velocity V_2, attains a constant value. It is evident form Fig.1c that the value of phase velocity V_3 for the case of MMT1 and for both the theories of thermoelasticity, sharply decreases and then attains a constant value. However, a very small oscillation in its value is observed for the case of MMT2. The values for the case of MMT1 are less in magnitude as compared to those of MMT2. Figure 1d represents the variations in the value of phase velocity V_4 with frequency. It can be seen form this figure that the variation pattern is similar to the case of Fig.1c except for the difference in there amplitudes.

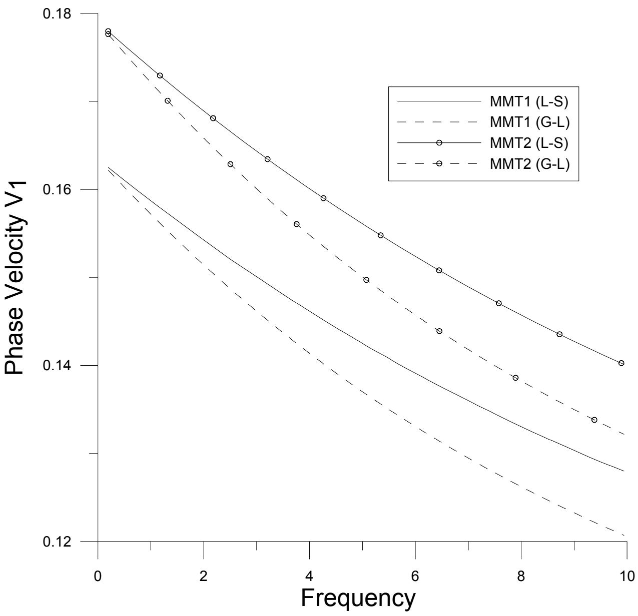

Fig.1a. Variation of phase velocity V_1 with frequency.

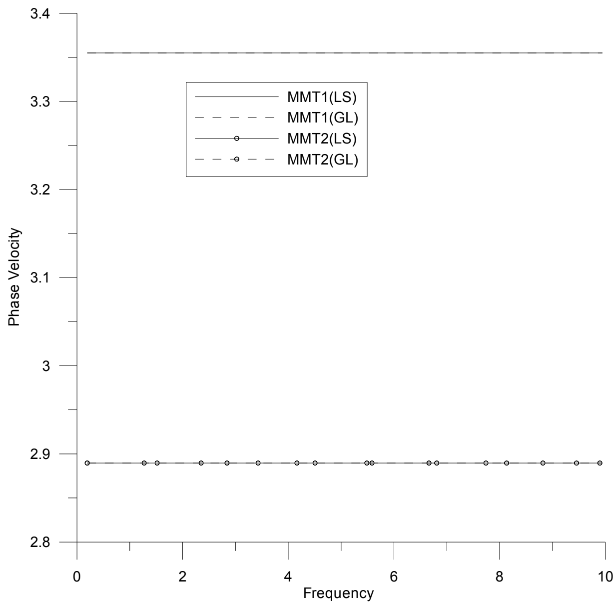

Fig.1b. Variation of phase velocity V_2 with frequency.

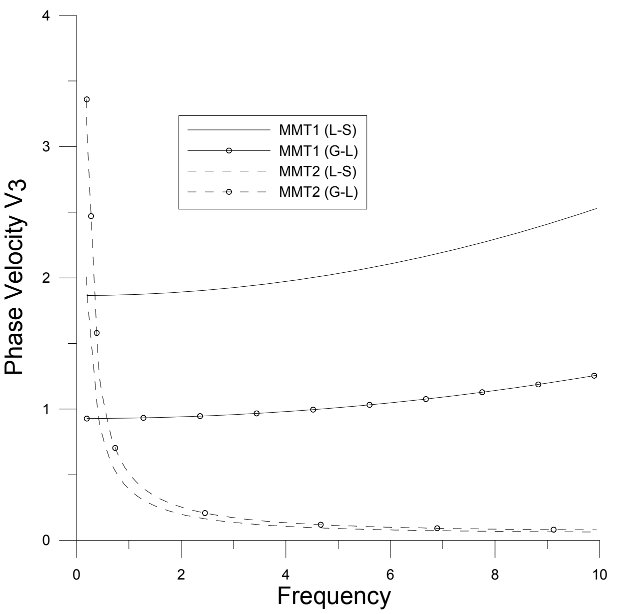

Fig.1c. Variation of phase velocity V_3 with frequency.

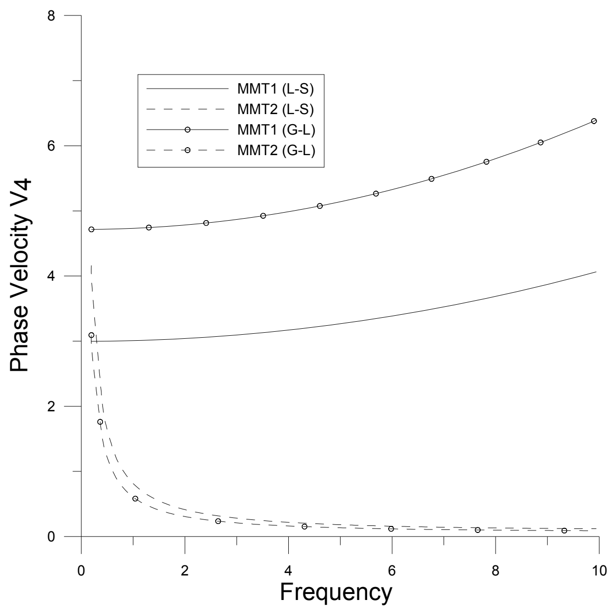

Fig.1d. Variation of phase velocity V_4 with frequency.

Figure 2 represent variations in the value of attenuation quality factors Qi, $i=1, 2, 3, 4$. It is shown in Fig.2a that the value of attenuation quality Q_1 for the case of MMT1 and MMT2 increases with an increase in frequency, for both the theories. Figures 2b and 2d represent the variations of attenuation quality factors

Q_2,Q_4 with frequency. It can be seen from these figures that, for the case of MTIS, the values of attenuation quality factor decrease and then attain a constant value for both the theories, while for Q_4, its value initially decreases, then attains a constant value with an increase in frequency. It can be seen from Fig.2c that the value of attenuation quality factor Q_3 increases, then decreases and then attains a constant value for all the cases.

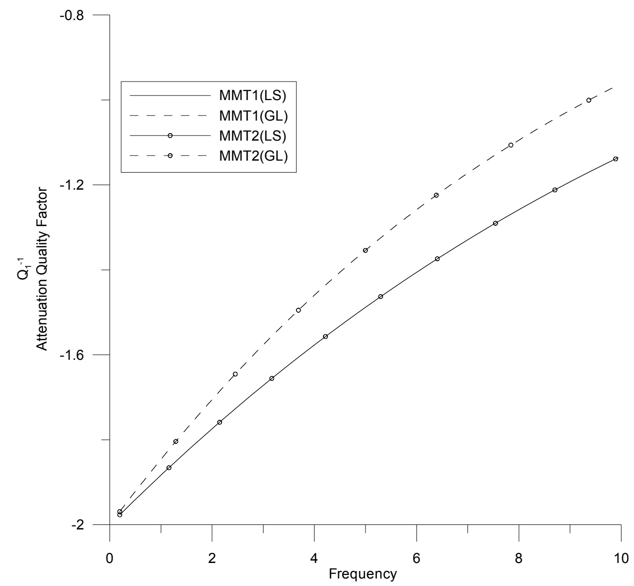

Fig.2a. Variation of attenuation quality factor Q_1^{-1} with frequency.

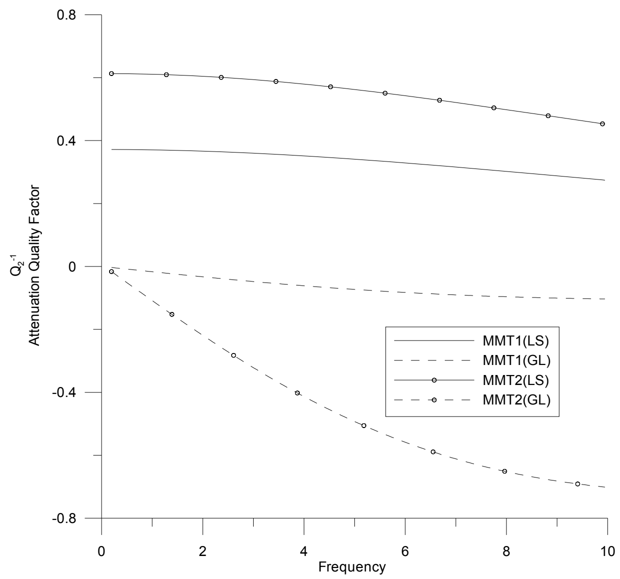

Fig.2b. Variation of attenuation quality factor Q_2^{-1} with frequency.

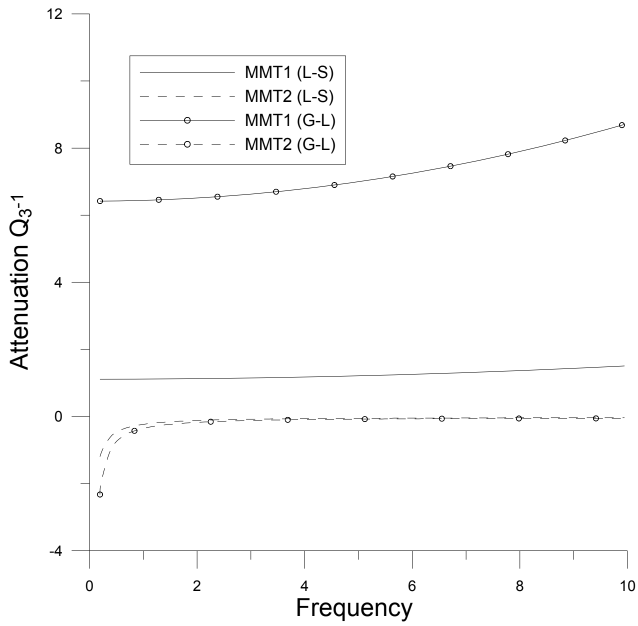

Fig.2c. Variation of attenuation quality factor Q_3^{-1} with frequency.

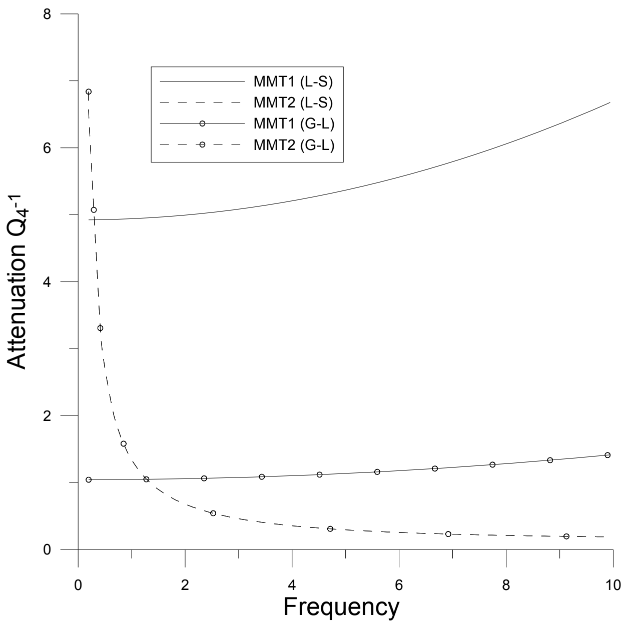

Fig.2d. Variation of attenuation quality factor Q_4^{-1} with frequency.

Figures 3 and 4 shows the variations of phase velocities and attenuation quality factor for the anti-plane problem. It can be seen in the figure that their behavior is similar.

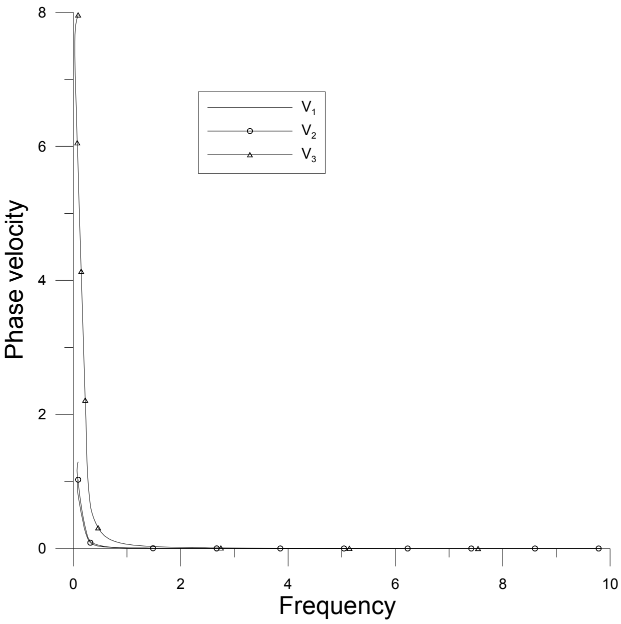

Fig.3. Variations of phase velocities with frequency.

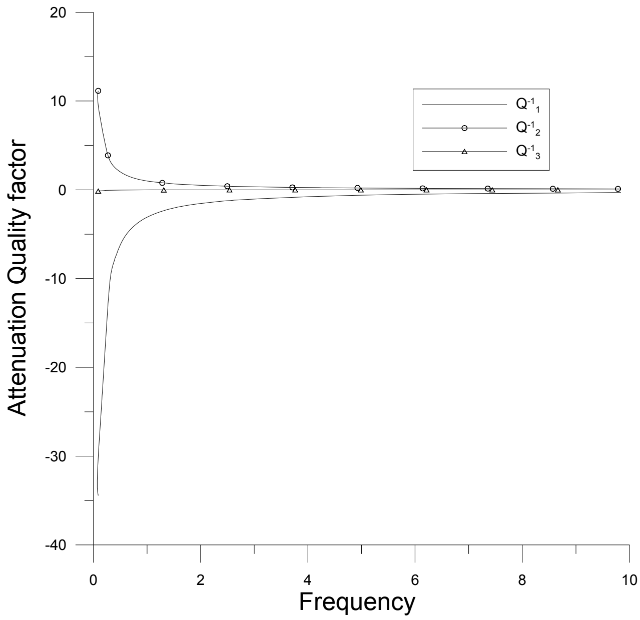

Fig.4. Variations of attenuation quality factor with frequency

7. Conclusion

The importance of thermal stresses in causing structural damages and changes in functioning of the structure is well recognized whenever thermal stress environments are involved. Wave propagation in a micropolar monoclinic thermoelastic half space has been discussed. The phase velocities and attenuation quality factor have been computed and plotted graphically. The numerically computed results are found to be in close agreement with the theoretical results.

Nomenclature

A_{ijkl} and B_{ijkl} – represents the elastic constants of the medium

C^* – specific heat at constant strain

K^*_{ij} – (positive-definite) thermal conductivity tensor

T – thermal displacement

u_i and φ_i – components of the average displacements and microrotations of the particles

β_{ij} – represents thermal expansion of the medium

$\in_{ijk}$ – permutation symbol

ρ – mass density

The two relaxation times are given by $\tau_1 \geq \tau_o \rangle 0$. These time coefficients represent the thermal relaxation mechanism and accounts for thermoelastic loss in the medium.

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Appendix

$$A = a_9 a_{13}\left(a_1 a_5 - a_2 a_9\right),\quad B = -ia_4 a_9 a_5 a_{10}\left(1 - i\omega\tau_1\right)\left(i\omega\tau_o n_o - 1\right) + ia_9\left(a_1 a_5 - a_2^2\right)\left(i\omega\tau_o - 1\right),$$

$$C = -\left[a_4 a_6\left(a_8 a_{11} - a_6 a_{10}\right) + ia_7 a_9\left(a_2 a_{10} - a_1 a_{11}\right)\right]\left(1 - i\omega\tau_1\right)\left(i\omega\tau_o n_o - 1\right) +$$
$$+ a_{13}\left(a_2 a_3 a_6 - a_1 + a_2 a_6 a_8\right),$$

$$D = ia_4 a_{12}\left(a_2 a_6 - a_5 a_8\right)\left(1 - i\omega\tau_1\right)\left(i\omega\tau_o n_o - 1\right) + i\left(i\omega\tau_o - 1\right)\left(a_2 a_3 a_6 - a_1 a_6 + a_2 a_6 a_8 - a_3 a_5 a_8\right),$$

$$E = -ia_7\left(1 - i\omega\tau_1\right)\left(i\omega\tau_o n_o - 1\right)\left(a_3 a_8 a_{11} - a_3 a_6 a_{10}\right),$$

$$F=-a_1^*-a^*{}_5-a^*{}_9,\qquad G=a^*{}_5a^*{}_9-a^*{}_6a^*{}_8+a^*{}_1\left(a^*{}_5+a_9{}^*\right)-a^*{}_2a^*{}_4-a^*{}_3a^*{}_7,$$

$$H=a^*{}_2\left(a^*{}_4a^*{}_9+a^*{}_6a^*{}_7\right)+a^*{}_3\left(a^*{}_4a^*{}_8+a^*{}_5a^*{}_7\right)-a^*{}_1\left(a^*{}_5a^*{}_9-a^*{}_6a^*{}_8\right),$$

$$a_1=p_1^2+b_1p_3^2-1,\quad a_2=p_1p_3b_2,\quad a_3=p_3b_7,\quad a_4=p_1b_4,\quad a_5=p_1^2b_5+p_3^2b_6-1,$$

$$a_6=p_1b_3^2,\qquad a_7=p_3b_{14},\qquad a_8=p_3b_1b_3,\qquad a_9=p_1^2b_9+p_3^2b_{10}-b_{11},\qquad a_{10}=p_1b_4,$$

$$a_{13}=p_1^2b_{12}+p_3^2b_{13},\qquad a_1^*=p_1^2b_{15}+b_{16}p_3^2,\qquad a_2^*=-p_3^2b_{18},\qquad a_3=-b_{17}p_1^2,$$

$$a_4^*=ip_3b_{23}\big/\xi,\qquad a_5^*=p_1^2b_{19}+p_3^2b_{21}-b_{22},\qquad a_6^*=-p_1p_3b_{20},$$

$$a_7^*=ip_1b_{28}\big/\xi,\qquad a_8^*=-p_3p_1b_{24},\qquad a_9^*=p_1^2b_{25}+p_3^2b_{26}-b_{27}\big/\xi^2.$$

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