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An integrated optimization model for procurement and production lot sizing and scheduling problems

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Abstract
Lot sizing is a prevalent issue within manufacturing companies, where determining the optimal procurement and production lot sizes is crucial for maximizing profits. This problem has become more complex, given that numerous suppliers can provide the same raw materials with different prices and quantity discount schemes. A company should also determine optimal carriers to deliver materials to the company's warehouse. In a manufacturing process, the company should determine the optimal production lot size and its schedules. In this paper, a model was developed to solve simultaneously procurement and production lot sizing, as well as production scheduling problems. The model encompasses multiple suppliers offering quantity discounts, aiming to maximize company profit by accounting for various costs, including procurement, production, inventory, and quality costs. A case study is taken from a company producing noodles and its related derivative products to illustrate the application of the model. Based on the optimization results, the company obtained a total profit of IDR. 14,656,550,000 or $950,921.30 (the exchange rate of $1 at IDR. 15,413). The sensitivity analysis results show that the objective function is sensitive to changes in the purchase cost, sale revenue, and discount rate parameters. The decision variables for accepted product demand, product quantity, and the starting and completion time of product family are only sensitive to changes in certain parameters. Meanwhile, the decision variables for product inventory, product backlog, raw material inventory, and purchased raw material quantity are sensitive to the changes in all the analyzed parameters.

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1. Introduction

Lot sizing and scheduling are inherent problems in a manufacturing company. In lot sizing, a company must make a decision regarding the production quantity of a particular product produced by a specific machine in a certain production process (Badri et al., 2020). Lot sizing will determine the production efficiency, which usually can be achieved by carefully determining the lot size based on the demand to be met and available stock (Almada-Lobo et al., 2010) and at the same time minimize the total production cost (Ramezanian et al., 2013). While lot sizing deals with the determination of economic production amount, scheduling deals with the determination of optimal production sequence along with the start and finish time of each production to minimize or maximize a certain objective function (Pinedo, 2009 and Liu et al., 2013). Lot sizing and scheduling are performed in two different planning levels, namely tactical planning and operational planning and hence treated as separate problems (Almeder et al., 2014 and Quadt, 2004). Another approach to solving such problems is by considering both lot-sizing and scheduling as one problem and solving both sequentially or simultaneously.

Lot sizing problems and scheduling have been solved as separate problems. The lot sizing problems can be differentiated between purchased lot sizing and production lot sizing. Ertogral et al. (2007) developed a model to determine the optimal procurement lot size under an equal-size shipment policy to minimize transportation cost. Another research in procurement lot sizing was done by Lee et al. (2013) and Choudhary et al. (2013) who developed a model to determine the optimal lot size under quantity discounts in multi-supplier and multi-period environments. The objectives of the models were to minimize total costs comprised of ordering cost, holding cost, purchase cost, and transportation cost. For production lot sizing, Zhou et al. (2018) developed an optimization model to determine the optimal production lot with a time-varying setup. The integrated model of purchasing and production lot-sizing has been developed by Cunha et al. (2018) and Su (2018), and Sutrisno et al. (2020). The solutions of the integrated models for both problems resulted in better performance compared to the separated ones.

For scheduling problems, Liu et al. (2013) study the scheduling problems of parallel machine configuration. In their study, two considerations were taken, namely past-sequence dependent delivery times and the effect of machine deterioration. Georgiadis (2019) developed a model to solve the scheduling problem in the diaries industry. Several constraints were imposed in the model such as inventory constraint and equipment capacity to minimize cost. The model also considered the existence of new orders including the order modifications or cancellations.

The alternative approach in lot sizing and scheduling problems are by solving them simultaneously. Quadt and Kuhn (2005) developed a model to simultaneously determine the optimal lot size and schedule on a flexible flow line with the existence of one or several parallel machine configurations in the line. Almada-Lobo (2010) and Ramezanian et al. (2013) developed an integrated model of lot-sizing and scheduling in an environment facing sequence dependent setup time sand costs. In the former research, the model was developed for a glass container industry where the production uses common resources. In the latter research, a more efficient model was proposed with fewer constraints in multiproduct multiperiod production systems. Almeder et al. (2015) developed a model to solve the problems of lead time assumptions which usually resulted in both infeasible production plans or costly needless inventory. They synchronized the problems of batch formation and lot streaming in scheduling

The more recent researches in model development of integrated lot-sizing and scheduling were done by Badri et al. (2020), Avadiappan et al. (2022), and Koch et al. (2022). In the research of Badri et al. (2020), an integrated model was proposed to minimize production costs and processing time. Avadiappan et al. (2022) developed a model to determine the optimal schedule for the continuous manufacturing process under the uncertainty of demand. The model took into consideration the feedback due to demand uncertainty and other uncertain aspects in which the scheduler has to reschedule the demand. The problem was solved using robust optimization and a stochastic programming approach. While in Koch et al. (2022), an integrated model was developed to solve production lot-sizing and scheduling in a tyre industry under uncertain demand with the objectives related to inventory and customer service level. Several researchers developed another decision in the integration of lot-sizing and scheduling, namely the procurement lot-sizing. This addition integrated the internal and external decision-making and resulted in a much better decision. One of the most recent models in this research stream was developed by Mohammadi et al. (2020). The model integrated the procurement decisions on the supplier side with the production lot sizing and scheduling on the internal side of the manufacturer. The model was fitted in the production situations where the manufacturer has several product families which have to be produced using certain capacitated production facilities.

In raw materials procurement, a manufacturer faces two main problems, namely supplier selection and order allocation or lot-sizing. In supplier selection, the manufacturer has to select one or several suppliers from an eligible set of suppliers. The selection is usually based on a set of criteria determined by the decision-makers. In the procurement lot-sizing, the suppliers offer quantity discounts to attract the manufacturer to purchase bigger amounts of raw materials. Many models have been developed to solve the lot-sizing problem under quantity discounts such as Hamdan and Cheaitou (2017), Ceraghalipour and Farsad (2018), and Shalke et al. (2018). Mirzaee et al. (2018) studied three quantity discount schemes, namely all-unit quantity discount, incremental quantity discount, and no discount. In an all-unit quantity discount, the discount was given to every unit of items purchased. The incremental quantity discount scheme was given when the manufacturer purchased several items above a predetermined threshold, while no discount is a scheme where no discount is applied.

This research aims to develop an optimization model in determining the optimal procurement and production lot sizes along with the starting and finishing time of each production lot size, as well as the carrier selection. The optimization model developed in this research is the extended version of the research of Permatasari et al. (2023). Compared to the above research, this research is more comprehensive in terms of the inclusion of the backlogs, defective products, and production time constraints in the model. This research also involves more products in the numerical example and provides sensitivity analysis which has not been done in the research of Permatasari et al. (2023). Hence, this research has several contributions. First, more detail of the schedule is added in terms of the starting and completion time of each product item and family. Second, this research considers the backlogs, product defects, and production time constraints in the model. In this research, the problem was modelled using Mixed Integer Linear Programming (MILP) and solved using LINGO 18.0 software.

2. System Description

The system description is shown in Fig.1. A manufacturing company faces a problem in determining the optimal lot-sizing and scheduling of its production. Two kinds of lot-size must be determined, namely production and purchasing lot sizes. The company produces several products in which the products can be grouped into several product families and each product family needs a certain production line. The company purchases raw materials for their production from several suppliers under a certain discount scheme. The raw materials purchased from suppliers contain some proportions of defects. Several choices of carriers are available to send the raw materials an upon the arrival of the materials, they will be stored in the warehouse for production uses according to the production schedules.

Production changeover between families or between products in a family needs setup time. Under the sequence-dependent setup assumption, then the setup time will depend on the resulting sequence. In each period, the planner must determine the optimal purchased amounts of each raw material, production line assignment and its quantity, as well as the production schedules.

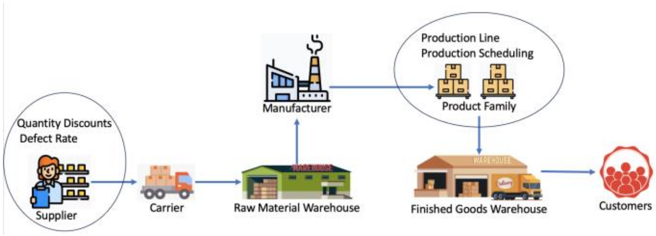

Fig. 1. System Description

3. Model Development

In this section, the basic assumption and notations of the model are listed as well as the model formulation

3.1. Assumptions and Notations

The assumptions used in this research are as follows:

(1) Each supplier has a limited capacity.
(2) The capacity of company warehouses is limited.
(3) No raw material inventory, product inventory, and product backlog at the first and last of the planning period.
(4) The completion time of a setup equals-the starting time of production.

The following notations are used in this paper:

Indices

t : period index
f : raw material index
s : supplier index
k : product index
d : discount level index
i : product family index
l : production line index
m : carrier index

Parameters

Ld_{kt} : lower bound of the demand of product k at period t
Ud_{kt} : upper bound of the demand of product k at period t
O_k^K : occupied space of product k in the warehouse
O_f^F : occupied space of raw material f in the warehouse
df_{fs} : defect rate of raw material f from supplier s
df_{fs} : penalty cost for defect rate of raw material f from supplier s
Cap_{ft}^F, : warehouse capacity of raw material f at period t
Cap_{kt}^K : warehouse capacity of product k at period t
Cap_{lt} : available capacity of line l at period t
DIS_{ds} : discount rate of level d from supplier s
u_{ds} : upper bound of discount level d from supplier s
h_f^F : holding cost of raw material f
h_k^K : holding cost of product k
hb_k^K : backlog cost of product k
$\emptyset$: service level of the manufacturer
oc_s : ordering cost of supplier s
bc_{fs} : purchasing cost of raw material f from supplier s
trc_{fsm} : transportation cost of raw material f from supplier s using carrier m
r_{kt} : unit sale revenue of product k at period t
e_{fkl}^F : consumption of raw material f for product k produced in line l
cp_{kl} : production cost of product k produced in line l
p_{kl} : production time of product k produced in line l
scp_{kl} : setup cost of product k produced in line l
stp_{kl} : setup time of product k produced in line l
scb_{ijl} : setup cost of family
stb_{ijl} : setup time of product family i to j produced in line l
LBL_{kl} : the amount of minimum production of product k in line l
M^{big} : a big number

Decision Variables

D_{kt} : demand for product k at period t
Q_{fsdtm}^F : purchased quantity of raw material f from supplier s at discount level d using carrier m in period t
Q_{klt} : production quantity of product k produced in line l at period t (unit)
I_{ft}^F, I_{kt}^K , B_{kt}^K : inventory of raw material f, product k, and backlog of product k at period t (unit)

ST_{ilt}, CT_{ilt} : start and completion time of family i, produced in line l, at period t (minute)

V_{jlt} : auxiliary variables for scheduling sub-cycle elimination of product j, produced in line l, at period t.

X_{ijlt} : 1, if family i precedes family j in production line l at period t (0, otherwise)

A_{jlt} : 1, if the setup of family j is done in line l start at period t (0, otherwise)

P'_{klt} : 1, if product k is produced in line l at period t (0, otherwise)

Z_{sdt} : 1, if an acquisition of supplier s is occurred occurs at discount level d at period t (0, otherwise)

Y_{st} : 1, if a raw material is supplied by supplier s in period t (0, otherwise)

3.2. Model Formulation

The objective function of the model is maximizing the company's profit. The product demand is uncertain but the company has the information about the demand range of each product. Eq. (1) expresses the objective function of the model. The revenue generated from the selling is expressed in Eq. (2). Several costs are involved in the model: cost of raw materials purchasing under discount scheme (Eq. 3); fixed order cost (Eq. 4); transportation cost (Eq. 5); total production costs (Eq. 6); holding costs of raw materials and final products (Eq. 7); product backlog cost (Eq. 8); penalty costs for defect raw materials (Eq. 9); and setup costs (Eq. 10).

$$\boldsymbol{Max}\ Z_1 - (Z_2 + Z_3 + Z_4 + Z_5 + Z_6 + Z_7 + Z_8 + Z_9) \quad (1)$$

where,

$$Z_1 = \sum_k \sum_t r_{kt} D_{kt} \quad (2)$$

$$Z_2 = \sum_s \sum_d \sum_f \left((1 - DIS_{sd}) \sum_f \sum_m bc_{fs}\, Q^F_{fsdtm} \right) \quad (3)$$

$$Z_3 = \sum_s \sum_t \sum_m oc_s\, Y_{st} \quad (4)$$

$$Z_4 = \sum_f \sum_s \sum_d \sum_t \sum_m trc_{fsm}\, Q^F_{fsdtm} \quad (5)$$

$$Z_5 = \sum_k \sum_l \sum_t cp_{kl}\, Q_{klt} \quad 6)$$

$$Z_6 = \sum_f \sum_t h^F_f I^F_{ft} + \sum_k \sum_t h^K_k\, I^K_{kt} \quad (7)$$

$$Z_7 = \sum_k \sum_t hb^K_k B^K_{kt} \quad (8)$$

$$Z_8 = \sum_f \sum_s \sum_d \sum_t \sum_m dfc_{fs}\, df_{fs} Q^F_{fsdtm} \quad (9)$$

$$Z_9 = \sum_k \sum_l \sum_t scp_{kl} P_{klt} + \sum_i \sum_j \sum_l \sum_t scb_{ijl}\, X_{ijlt} \quad (10)$$

s.t

$$u_{s,d-1} Z_{sdt} < \sum_f \sum_m Q^F_{fsdtm} \leq u_{s,d} Z_{sdt} \quad \forall s, d, t \quad (11)$$

$$\sum_d Z_{sdt} \leq Y_{st} \quad \forall s, t \quad (12)$$

$$\sum_F \sum_d \sum_m Q^F_{fsdtm} \leq M^{big}\, Y_{st} \quad \forall s, t, \quad (13)$$

$$I^F_{f.t-1} + \sum_s \sum_d \sum_m (1\text{-}df_{fs}) Q^F_{f,s,d,t,m} = \sum_{k \in K^F(f)} \sum_l e^F_{fkl} Q_{klt} + I^F_{ft} \quad \forall s, t \quad (14)$$

$$I^K_{kt} - B^K_{kt} = I^K_{k,t\text{-}1} - B^K_{k,t\text{-}1} + \sum_l Q_{klt} - D_{kt}, \forall k, t \quad (15)$$

$$B^K_{kt} \leq (1 - \emptyset) D_{kt} \quad \forall k, t \quad (16)$$

$$Ld_{kt} \leq D_{kt} \leq Ud_{kt} \quad \forall k, t \quad (17)$$

$$\sum_{k \in K_j} P'_{klt} \leq |K_j| \left(A_{jlt} + \sum_i X_{ijlt} \right), \forall j, l, t \quad (18)$$

$$Q_{klt} \leq \frac{Cap_{lt}}{p_{kl}} P'_{klt} \quad \forall k, l, t \quad (19)$$

$$\sum_j X_{ijlt} + A_{i,l,t+1} = \sum_j X_{jilt} + A_{ilt} \quad \forall i, l, t \quad (20)$$

$$\sum_j X_{ijlt} \leq \sum_j X_{jilt} + A_{ilt}, \forall i, l, t. \quad (21)$$

$$\sum_j A_{jlt} = 1, \forall l, t \quad (22)$$

$$V_{ilt} + N X_{ijlt} - (N - 1) \leq V_{jlt}, \forall i, j, l, \quad (23)$$

$$\sum_{k \in K_j} (p_{kl} Q_{klt} + stp_{kl} P'_{klt}) + \sum_i \sum_{j \neq i} stb_{ijl} X_{ijlt} \leq Cap_{lt} \quad (24)$$

$$Q_{klt} \geq LBL_{lk} P'_{klt} \quad \forall k, l, t \quad (25)$$

$$ST_{ilt} = CL_{ilt} - \sum_{k \in K_i} (p_{kl} Q_{klt} + stp_{kl} P'_{klt}), \forall i, l, t \quad (26)$$

$$CL_{ilt} + stb_{ijl} X_{ijlt} \leq ST_{jlt} + Cap_{lt} (1 - X_{ijlt}), \forall i, j, l, t \quad (27)$$

$$ST_{ilt} \geq \sum_{j \neq i} stb_{ijl} X_{ijlt} \quad \forall i, l, t \quad (28)$$

$$CL_{ilt} \leq Cap_{lt} (1 - X_{ijlt}) \quad \forall i, j, l, t \quad (29)$$

$$O^F_f I^F_{ft} \leq Cap^F_{ft} \forall f, t \quad (30)$$

$$O^K_k I^K_{kt} \leq Cap^K_{kt} \quad \forall k, t \quad (31)$$

$$D_{kt}, Q_{klt}, I^F_{ft}, I^K_{kt}, B^K_{kt}, Q^F_{fsdtm,}, V_{jlt}, ST_{ilt}, CL_{ilt} \geq 0 \quad (32)$$

$$X_{ijlt}, A_{jlt}, P'_{klt}, Z_{sdt}, Y_{stm} \in \{0,1\}, \ \forall k, t, l, m, s, d, i, j \quad (33)$$

Eq. (11) defines the discount level with its respective purchase amount range from each supplier at each period. Eq. (12) ensures only one discount level from a certain supplier for a certain raw material is selected at each period. Eq. (13) ensures the orders come from the selected suppliers. Eqs. (14) and (15) calculate respectively the inventory of raw materials and products. Eqs. (16) and (17) show respectively the upper limit of product backlog and the upper and lower limit of product demand. Eq. (18) indicates that the production of product k in family j can be started at period t after the completion of the setup for that family at the beginning of a certain period or a changeover occurs during the period. Eq. (19) shows that the amount of production quantity cannot exceed the production line capacity. Eq. (20) ensures that at each production line, the input flow equals to the output flow. Eq. (21) indicates that in a certain production line, the changeover of two consecutive families can be started after finishing the previous family, or the production line is assigned for a certain product family at the beginning of period t. Eq. (22) ensures that at the beginning of a period, one family experiences only one setup. Eq. (23) is needed to eliminate scheduling sub-cycles. Eq. (24) defines the production line capacity at each period. Eq. (25) shows the minimum production quantity on a certain production line. Eq. (26) determines the production starting time of family i. Eq. (27) is needed to avoid overlapping schedules. Eqs. (28) and (29) respectively show the lower bounds and upper bounds of the family starting and completion time. Eqs. (30) and (31) ensure inventory of raw materials and products must not exceed the warehouse capacity, respectively. Eq. (32) shows the non-negativity of the continuous decision variable, while Eq. (33) defines the binary variables in the proposed model.

4. Results and discussion

4.1. Numerical Example

The production lines operate for six working days with 24 working hours from Monday through Friday and 12 hours on Saturday. This numerical example covers only four periods with a total working time of 7920 minutes for each period. Currently, the company produces three product families (F_1: MKAP, MKHC, MKJB, MKMG; F_2: SHORR, MYG, MYCBQ, MIFGH, MISE; F_3: MISCC). are processed by two production lines (Line A and Line B). Four raw materials are needed in the production, namely Wheat Flour (TT), Tapioca Flour (TP), Cassava Flour (TG), and Salt (G). Two suppliers are available to provide those raw materials (CK and MKJ). Each supplier offers three discount levels, namely d_1, d_2, and d_3. To deliver the raw materials from suppliers to the company, three carriers are available, namely Box Truck (TA) and Wing Box Truck (TB). The service level of the manufacturer is assumed to be 90%. The model parameters are provided in the Appendix as shown in Table 1-6. Table 1 lists the upper and lower demand for each product in each period. Table 2 provides the following data for each product: occupied space, warehouse capacity, product price, holding cost, and backlog cost. Table 3 lists the production cost, processing time, setup cost, setup time, minimum production amount, and raw material consumption of each product. The following data for each raw material are shown in Table 4: occupied space, warehouse capacity, holding cost, purchasing cost, defect penalty cost, and defect rate. Table 5 shows the supplier order cost, discount level the limit order for each level, and transportation cost for both transportation alternatives on each raw material. Table 6 shows the inter-family setup time and cost.

Using Lingo 18.0 software, the optimal solution results in the profit of IDR. 14,656,550,000 or \$950,921.30. The optimal values of the decision variables are provided in Table 7, Table 8, Table 9, Table 10, Table 11 and Table 12 as shown in the Appendix. The Gantt chart of the optimal production schedule is shown in Fig. 2. Table 7 shows the optimal solution for accepted product demand and production quantity. Except for MISCC, the optimal accepted product demands for nine products have different values at each period. There are two periods in which MISCC has the same optimal accepted product demands. All products have different optimal production quantities in the four planning periods. From period two until period four, MISCC product was not produced. Meanwhile, the MIFGH product was not produced only in period three.

The optimal product inventory and product backlog are shown in Table 8. From the table, only two products have inventory, namely MISCC from period one to period three and MYCBQ in period three. Hence, the rest of the products do not have inventory in the four periods. In addition, MKAP, MKHC, MKJB, MKMG, SHORR, MYG, MYCBQ, MIFGH, and MISE have product backlogs with different values in several periods, and MISCC has no backlog in all periods.

The optimal purchased raw materials and their inventory are shown in Table 9. There are inventories for TP and TG with different values in Periods 1-3. The company purchases most of TT's raw materials from MKJ suppliers with more than 90% of the total company needs. For the rest of the raw materials, the company purchases 100% of TP and TG raw materials from the CK supplier and 100% of G raw materials from the MKJ supplier.

Fig. 2. shows the starting and completion times of all product families in both production lines. The figure also shows the changeover setup between families. In periods 2 and 4, both production lines have the same schedule. On Line A in both periods F2 precedes F1, while Line B produces only F2. In period 1, the available lines have different family assignments. On-Line A, F1 precedes F2, while on Line B F3 precedes F2. In period 3, Line A was used to produce F1 and F2, while Line B was fully assigned for F2.

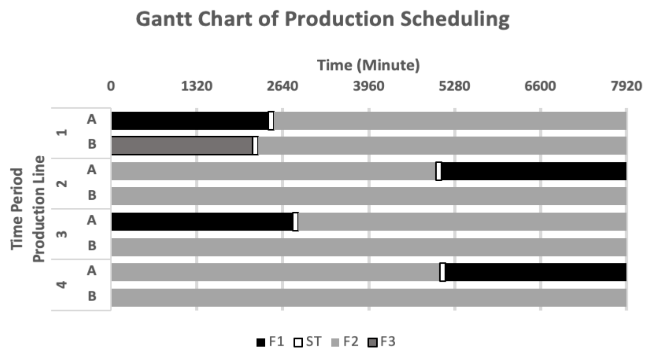

Fig. 2. Gantt Chart of Production Scheduling

4.2. Sensitivity Analysis

This section discusses the effect of several parameters on the objective function and decision variables. Three important parameters are considered in this analysis, namely raw material purchasing cost, unit sale product revenue, and the discount level. For the first two parameters, the values are changed from 25%-75% with an increment of 25%. For the last parameter, the discount level was changed to 4%-12% higher than the baseline value and 4% lower until no discount was applied. The sensitivity analysis results are presented in Table 10.

From the results shown in Table 10, the objective function is sensitive to the changes in all parameters. The company will gain lower profit as the purchase cost increases, while it will increase as unit sale revenue and discount rate increase gradually. Meanwhile, the decision variables for accepted product demand and produced product quantity are sensitive to changes in the purchase cost and sale revenue parameters. However, they are not sensitive to changes in the discount rate parameter. In addition, the decision variables for product inventory, product backlog, raw material inventory, and purchased raw material quantity are sensitive to changes in all parameters.

The effect of parameter change on the production scheduling is shown in Figure 3-5. Based on those figures, the starting and completion times of the family are sensitive to changes in the purchase cost and sale revenue parameters. From the figures, one can see that there is a change in the value of the family starting and completion time when the purchase cost parameter value is increased by 50% and 70% and when the sale revenue parameter value is decreased by 50% and 70%. However, the starting and completion time of the family are not sensitive to changes in the discount rate parameter because there is no change in the value that occurs at the family's starting time, completion time, and production sequence.

The model has several practical implications. First, by using the developed model the company can determine optimal procurement and production lot sizes, production schedules as well as optimal carriers simultaneously. Hence, there will be a significant increase in efficiency in the decision-making process. Second, by using the results of sensitivity analysis, the company has guidance on the change of important parameters.

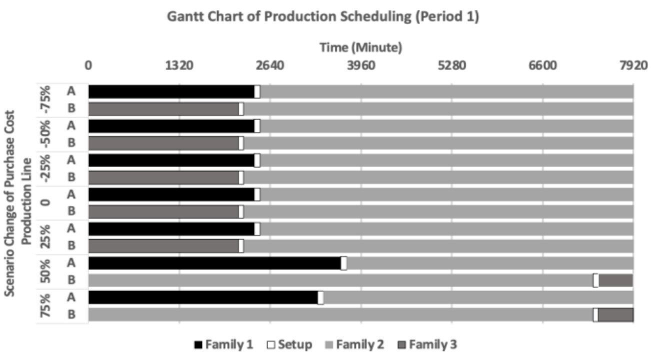

Fig. 3. Gantt Chart Effect of Changes in Purchase Cost Parameters on Family Beginning and Completion Time

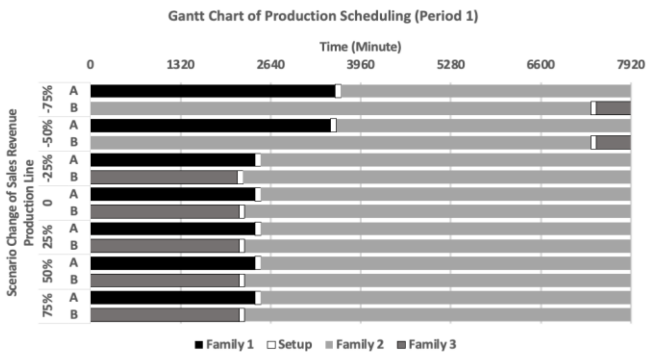

Fig. 4. Gantt Chart Effect of Changes in Sale Revenue Parameters on Family Beginning and Completion Time

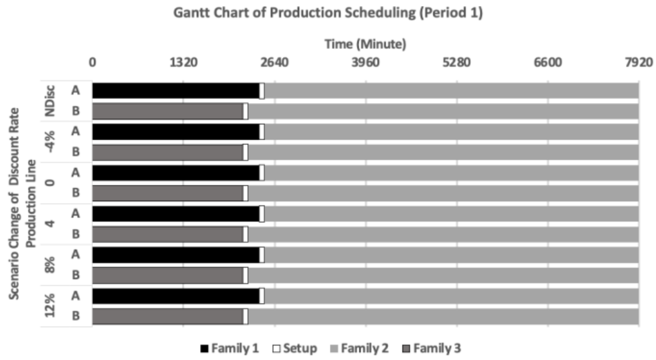

Fig. 5. Gantt Chart Effect of Changes in Discount Rate Parameters on Family Beginning and Completion Time

Some parameter changes will significantly make the decision variables and objective function depart from the baseline. Hence, new optimal values should be found using the current value of such parameters. Third, the dynamic aspects of the environment have not been considered in this model. When a new variable exists and has a significant influence on the system, then the model should be updated to include the new variable.There are also some limitations of the model, mainly due to the third and fourth assumptions. According to the third assumption, the model assumed the amount of beginning and ending inventory for raw materials and end products are zero as well as product backlogs. There will be many companies that have to stock the raw materials and also their end products as an anticipation of sudden demand or order to avoid any lost sales or backlogs. In this case, the model should be slightly modified to include those aspects. The fourth assumption was needed to ensure that the production could be started once the setup was finished. It represents the ideal condition of the production lines. Sometimes, the production should be delayed due to certain conditions in the production line such as process parameter setting and production ramp-up. This will make the production line produce some defective products in the early production time. Hence, the defective products should be included and calculated to get more accurate results.

5. Conclusions

In this research, a model was developed to simultaneously solve procurement and production lot sizing and scheduling problems to maximize profit. The model considers several aspects, namely quantity discounts, backlogs, and carriers. The decision variables of the model included accepted product demand, produced product quantity, purchased raw material quantity, product inventory, product backlog, raw material inventory, and the product family's beginning and completion time. Based on the sensitivity analysis results, the objective function is sensitive to the changes in the purchase cost, sale revenue, and discount rate parameters. The decision variables of accepted product demand, production quantity, and product family schedules are sensitive to certain parameters. The other decision variables are sensitive to the changes in all analyzed parameters. For further research, several considerations can be added to the model, such as defect rate and uncertainty in certain model parameters. Another development can be directed to involve a metaheuristic approach to improve the efficiency of computation time, especially for the more complex and larger problems.

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Appendix

Table 1. Product Demand Bounds

Product	Upper Limit				Lower Limit			
	Period				Period			
	1	2	3	4	1	2	3	4
MKAP	6150	6200	6300	6350	3884	3884	3884	3884
MKHC	5400	5450	5500	5650	3884	3884	3884	3884
MKJB	4150	4200	4250	4400	3890	3890	3890	3890
MKMG	4400	4450	4500	4650	3861	3861	3861	3861
SHORR	62000	62500	63000	64500	6644	6644	6644	6644
MYG	2150	2200	2250	2400	1931	1931	1931	1931
MYCBQ	1900	1950	2050	2100	1874	1874	1874	1874
MIFGH	3900	3950	4000	4150	2436	2436	2436	2436
MISE	2400	2450	2550	2600	1585	1585	1585	1585
MISCC	12650	12700	12750	12900	4969	4969	4969	4969

Table 2. Product Space, Product Warehouse Capacity, Product Sale Price, Product Holding Cost, and Product Backlog Cost

Product	Occupied Space	Warehouse Capacity	Product Price	Holding Cost	Backlog Cost
MKAP	0.0089	95	42000	40	64
MKHC	0.0089	95	42000	40	64
MKJB	0.0089	75	42000	40	64
MKMG	0.0089	80	42000	40	64
SHORR	0.0032	240	50500	48	77
MYG	0.0208	70	114285	110	175
MYCBQ	0.0208	70	114285	110	175
MIFGH	0.0100	80	66300	64	102
MISE	0.0182	80	74600	72	114
MISCC	0.0069	122	17500	17	27

Table 3. Production Cost, Processing Time, Setup Cost, Setup Time, Production Lower Bound, and Raw Material Consumption.

Product	Production Cost; Processing Time; Setup Cost; Setup Time;Minimum Production Amount		Raw Material Consumption (Wheat Flour (TT); Tapioca Flour (TP); Cassava Flour (TG); Salt (G))
	Line A	Line B	
MKAP	829;0.13;3193;10;3884	882;0.14;3513;11;3651	0.02644;0.0044;0;0.00026
MKHC	829;0.13;3193;10;3884	882;0.14;3513;11;3651	0.02644;0.0044;0;0.00026
MKJB	828;0.13;3193;10;3890	881;0.14;3513;11;3656	0.0264;0.0044;0;0.00026
MKMG	834;0.13;3193;10;3861	887;0.14;3513;11;3629	0.0266;0.0044;0;0.00026
SHORR	196;0.16;3513;11;6245	185;0.15;3193;10;6644	0.0402;0.00252;0.00252;0.00036
MYG	3970;0.27;4471;14;1815	3732;0.25;4151;13;1931	0.12432;0.00622;0;0.00086
MYCBQ	4092;0.28;4471;14;1761	3846;0.26;4151;13;1874	0.12812;0.0064;0;0.00088
MIFGH	1172;0.21;3832; 2; 2290	1101;0.2;3513;11;2436	0.16728;0.00112;0;0.00084
MISE	1801;0.33;4151;13;490	1693;0.31;3832;12;585	0.15052;0.00126;0;0.00098
MISCC	509;0.11;5110;16;4671	478; 0.1;4790;15;4969	0.03896;0.00086;0;0.0002

Table 4. Raw Material Space, Raw Material Warehouse Capacity, Raw Material Holding Cost, Purchase Cost, Defect Penalty Cost, and Defect Rate.

Raw Material	Occupied Space	Warehouse Capacity	Holding Cost	Purchase Cost; Defect Cost; Defect Rate	
				Supplier CK	Supplier MKJ
TT	25	200000	239	256000;25600;4	242000;24200;6
TP	50	40000	269	268000;26800;5	293000;29300;3
TG	50	12000	607	628000;62800;8	637000;63700;7
G	50	20000	99	110000;1100;6	97000;9700;8

Table 5. Supplier Order Cost, Discount Levels, and Transportation Cost

Supplier	Ordering Cost	Disc. Level; Limit Order			Transportation Cost	
		d_1	d_2	d_3	Box Truck (TT; TP; TG; G)	Wing Box Truck (TT; TP; TG; G)
CK	7000	0;499	8;2499	10;6000	2990;5980;5980;5980	3115;6230;6230;6230
MKJ	6000	0;499	6;2499	8;6000	2743;5485;5485;5485	2286;4571;4571;4571

Table 6. Family Setup Time and Cost

Family	Setup Time; Setup Cost		
	F1	F2	F3
F1	-	85;40717	80;38321
F2	90;43112	-	75;35926
F3	85;40717	80;38321	-

Table 7. Optimal Solution for Accepted Product Demand and Produced Product Quantity

Product	D (unit)				Qk (unit)			
	Period				Period			
	1	2	3	4	1	2	3	4
MKAP	6150	6200	6300	6350	5535	6197	6917	6351
MKHC	5400	5450	5500	5650	4860	5990	5500	5650
MKJB	4150	4200	4250	4400	3891	4458	4251	4400
MKMG	4400	4450	4500	4650	3960	4890	4496	4654
SHORR	62000	62500	63000	64500	55800	62450	69250	64500
MYG	2150	2200	2250	2400	1935	2415	2250	2400
MYCBQ	1900	1950	2050	2100	1761	3934	0	2305
MIFGH	3900	3950	4000	4150	3510	3946	4340	4204
MISE	2400	2450	2550	2600	2160	2445	2540	2855
MISCC	6720	4969	4969	4969	21627	0	0	0

Table 8. Optimal Solution for Product Inventory and Product Backlog

Product	Ik (unit)				Bk (unit)			
	Period				Period			
	1	2	3	4	1	2	3	4
MKAP	0	0	0	0	615	618	1	0
MKHC	0	0	0	0	540	0	0	0
MKJB	0	0	0	0	259	1	0	0
MKMG	0	0	0	0	440	0	4	0
SHORR	0	0	0	0	6200	6250	0	0
MYG	0	0	0	0	215	0	0	0
MYCBQ	0	1845	0	0	139	0	205	0
MIFGH	0	0	0	0	390	394	54	0
MISE	0	0	0	0	240	245	255	0
MISCC	14907	9938	4969	0	0	0	0	0

Table 9. Optimal Solution for Raw Material Inventory and Purchased Raw Material Quantity

Raw Material	Raw Material Inventory (unit)				Purchased Raw Material Quantity (unit)	
	Period				Supplier	
	1	2	3	4	CK	MKJ
TT	0	0	0	0	1095	19593
TP	671	372	293	0	1213	0
TG	240	83	163	0	690	0
G	0	0	0	0	0	168

Table 10. Computational Results of Sensitivity Analysis

Parameter	Changes Scenario	Objective Function and Decision Variables						
		OF	D	Ik	Bk	Qk	If	Qf
bc_{fs}	-75%	18,644,750,000	398627	31659	17082	398627	1833	22757
	-50%	17,315,320,000	398627	31661	17083	398627	1833	22757
	-25%	15,985,850,000	398624	31660	17081	398624	1833	22757
	0	14,656,550,000	398627	31659	17065	398627	1821	22759
	25%	14,124,870,000	398627	31661	17065	398627	1821	22759
	50%	12,002,600,000	403893	28234	13995	403893	1936	22123
	75%	10,710,050,000	402726	12931	241	402726	1899	22074
r_{kt}	-75%	-309,693,000	385180	7165	0	385180	1944	20713
	-50%	4,572,276,000	386960	6660	0	386960	1932	20964
	-25%	9,582,424,000	398380	29814	13482	398380	1814	22749
	0	14,656,550,000	398627	31659	17065	398627	1821	22759
	25%	19,730,990,000	398627	31660	17066	398627	1871	22758
	50%	24,805,480,000	398638	31659	17064	398638	1821	22759
	75%	29,880,200,000	398639	31659	17066	398639	1821	22760
	No Disc	14,187,250,000	398627	31685	17106	398627	0	22782
DIS_{sd}	-4%	14,424,990,000	398624	31662	17084	398624	1833	22759
	0	14,656,550,000	398627	31659	17065	398627	1821	22759
	4%	14,888,410,000	398626	31659	17062	398626	1821	22759
	8%	15,120,160,000	398626	31839	17285	398626	1821	22759
	12%	15,352,760,000	398625	31748	17516	398625	910	22716