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# Small-signal input characteristics of step-down and step-up converters in various conduction modes

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**Abstract.** Small-signal input characteristics of BUCK and BOOST DC-DC power converters in continuous conduction and discontinuous conduction mode have been presented. Special attention is paid to characteristics in discontinuous conduction mode. The input characteristics are derived from the general form of averaged models of converters. The frequency dependence of input admittance and other input characteristics has been observed in a relatively low-frequency range. The analytical formulas derived in the paper are illustrated by numerical calculations and verified by experiments with a laboratory model of BOOST converter. A satisfying level of conformity of calculations and measurements has been obtained.

**Key words:** pulse converters, BUCK, BOOST, small-signal models, input characteristics.

## List of symbols:

- C – capacitance,
- D – diode,
- $d_A$, $D_A$ – duty ratio and its steady-state value,
- f, $f_S$ – frequency, switching frequency,
- G – load conductance,
- $H_s$ – transmittance of control subcircuit,
- i – with subscripts in capital letters: instantaneous values of currents,
- I – with subscripts in capital letters: D.C. values of currents,
- I – with subscripts in small letters: small signal representations of currents in s domain,
- K – ideal controlled switch (transistor),
- L – inductance,
- $M_{VT}$, $M_I$ – voltage and current static transmittance,
- R – load resistance,
- $T_S$ – switching period,
- v – with subscripts in capital letters: instantaneous values of voltages,
- V – with subscripts in capital letters: D.C. values of voltages,
- V – with subscripts in small letters: small signal representations of voltages in s domain,
- Y – small-signal input admittance,
- Γ – quantity in dependence of input current on control signal,
- ω – angular frequency,
- θ – small-signal representations of duty ratio in s domain.

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## 1. Introduction

A typical switch-mode DC–DC power converter contains power stage and control subcircuit working together as a feedback dynamic system. The procedure of designing control subcircuit is based on the knowledge of power stage dynamics, usually described in the form of a set of small-signal transmittances. These transmittances are derived by linearization of large-signal averaged models of power stage [1–7]. The transmittances describing the influence of input voltage and control signal on the output voltage of converters are most frequently used, but in some applications the input small-signal characteristics are important [8, 9]. The examples are converters working in power-factor correctors (PFC) [10, 11] and in maximum power point tracing (MPPT) circuits used in photo-voltaic systems [12].

Small-signal averaged models of the power stage of converter have the form of equation systems in s-domain or equivalent circuits in which the small-signal representation of duty ratio θ, currents and voltages averaged over single switching period are used as variables [1–7]. The dependence of input current $I_g$ of converter on input voltage $V_g$ and duty-ratio representation θ may be obtained in the form:

$$I_g = Y \cdot V_g + \Gamma \cdot \theta \,. \tag{1}$$

Y is small-signal input admittance and Γ describes the dependence of input current on control quantity θ. The control subcircuit having transmittance $H_s$ delivers the control signal θ (after comparing the current sample with the proper reference signal):

$$\theta = H_s \cdot I_g \,. \tag{2}$$

From (1) and (2) one obtains:

$$I_g = \frac{Y}{1 - \Gamma \cdot H_s} \cdot V_g \,. \tag{3}$$

The proper design of $H_s$ transmittance of control subcircuit is possible if frequency dependencies of Y and Γ are known.

Small-signal input characteristics of the most popular DC-DC converters: step-down (BUCK) and step-up (BOOST) are considered in this paper. The analysis is performed separately for converters working in continuous conduction mode (CCM) and discontinuous conduction mode (DCM). The schemes of circuits under consideration are shown in Figs. 1 and 2, where K and D denote active and passive switch, respectively.

Special attention is paid to the analysis of converters working in DCM because some of the input characteristics of converters in CCM were discussed in the previous paper [9].

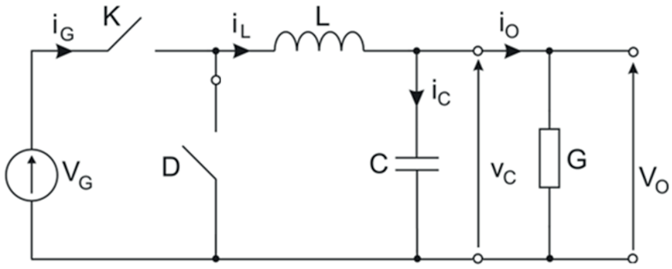


Fig. 1. Power stage of a BUCK converter

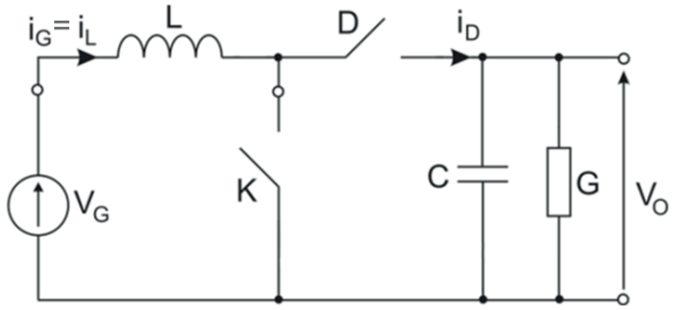


Fig. 2. Power stage of a BOOST converter

Small letters with capital subscripts denote instantaneous values of currents and voltages, capital letters with capital subscripts – quiescent values (D.C. values), and capital letters with small subscripts correspond to small-signal representations of currents and voltages in s-domain.

The analysis of input characteristics in s-domain for BUCK and BOOST converters is presented in Sec. 2 and 3, respectively. The selected dependencies in time domain and numerical examples are given in Sec. 4 and some concluding remarks in Sec. 5.

## 2. Step-down (BUCK) converter

**2.1. Continuous conduction mode (CCM).** Small-signal form of averaged model of BUCK converter power stage in CCM [5, 6] is expressed by following equations:

$$sL \cdot I_l = D_A \cdot V_g + V_G \cdot \theta - V_o, \tag{4}$$

$$\mathrm{I}_1 = (\mathrm{G} + \mathrm{s} \cdot \mathrm{C}) \cdot \mathrm{V}_\mathrm{o}, \tag{5}$$

$$I_g = I_l \cdot D_A + I_L \cdot \theta. \tag{6}$$

The dependence of small-signal input current on input voltage and duty ratio θ is obtained from equations (4–6) in the form:

$$\begin{aligned} I_g = {} & \left( D_A^2 \cdot G \frac{sCR+1}{s^2 LC + sLG + 1} \right) \cdot V_g + \\ & + I_O \cdot \left( \frac{sCR+1}{s^2 LC + sLG + 1} + 1 \right) \cdot \theta \end{aligned}. \tag{7}$$

Equation (7) corresponds to general dependence given by (1) with:

$$Y(BUCK) = D_A^2 \cdot G \frac{sCR+1}{s^2 LC + sLG + 1}, \tag{8}$$

$$\Gamma(BUCK) = I_O \cdot \left( \frac{sCR+1}{s^2 LC + sLG + 1} + 1 \right). \tag{9}$$

Low frequency values of Y (BUCK) and Γ (BUCK) (for s→0) are:

$$Y_0(BUCK) = D_A^2 \cdot G, \tag{10}$$

$$\Gamma_0(BUCK) = 2 \cdot I_O. \tag{11}$$

$D_A$ and $I_O$ are D.C. values of duty ratio and load current.

**2.2. Discontinuous conduction mode (DCM).** The starting point for derivation of input characteristics of BUCK converter in DCM is small-signal model presented in Fig. 3a obtained from [6]. In this case, quantities Y (BUCK) and Γ (BUCK), according to definition in (1), are obtained for θ = 0 or $V_g = 0$, respectively, which corresponds to Figs. 3b and 3c. For θ = 0 (Fig. 3b), one obtains:

$$I_g = G_g \cdot V_g + \alpha_{g3} \cdot V_o = G_A (V_g - V_o), \tag{12}$$

$$\alpha_{12} V_g = (G_w + G + sC) \cdot V_o, \tag{13}$$

where $G_A$ and $G_g$ are:

$$G_A = D_A^2 \cdot \frac{T_S}{2L}, \tag{14}$$

$$G_g = \frac{1}{R_g} = G_A, \tag{15}$$

After excluding quantity $V_o$ (output voltage) from (12) and (13), one obtains the dependence of $I_g$ on $V_g$ (for θ = 0) and, as a result, the admittance $Y_d$ (BUCK) for this case:

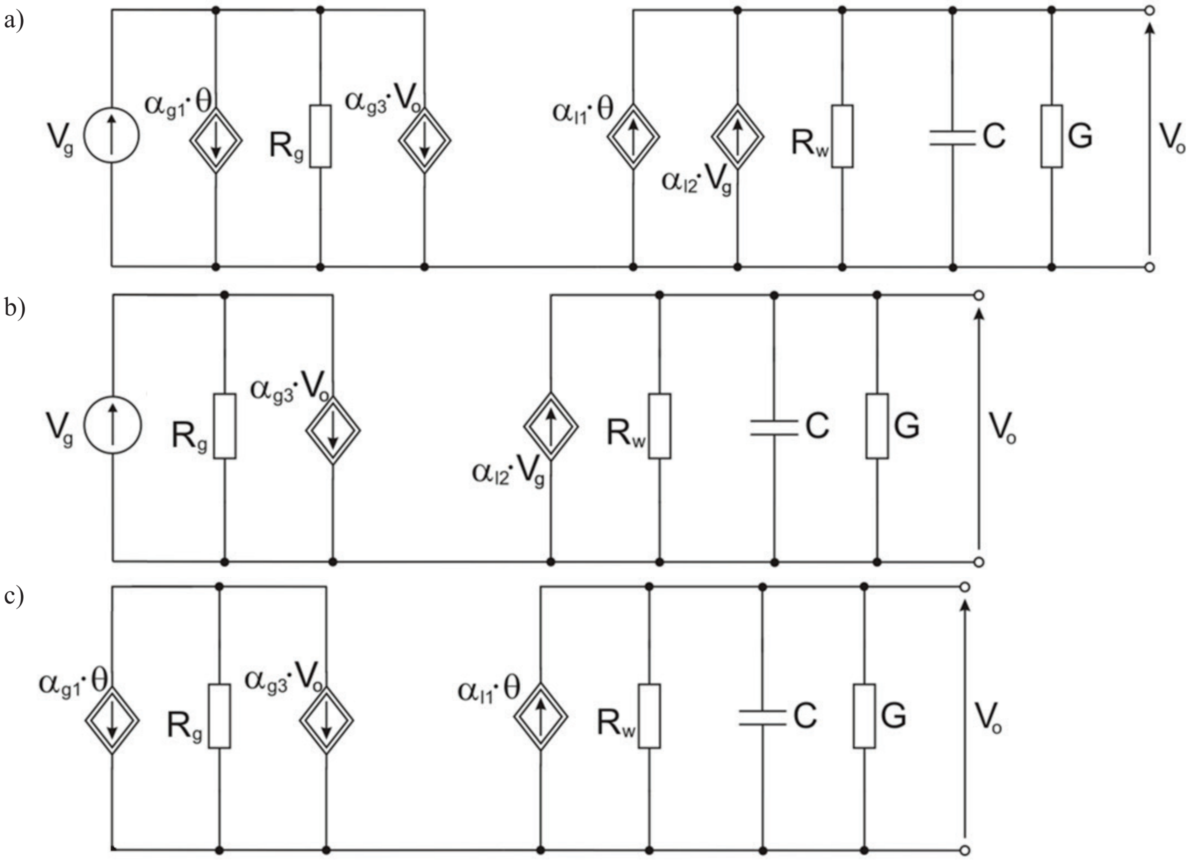


Fig. 3. Averaged small-signal model of BUCK converter working in DCM: (a) full; (b) θ = 0; (c) Vg = 0

$$Y_d(BUCK) = G_A \frac{G_w + G + sC - \alpha_{l2}}{G_w + G + sC}. \tag{16}$$

The additional subscript "d" in $Y_d$ (BUCK) refers to DCM. Quantities $G_w$ and $\alpha_{l2}$, according to [6] are:

$$G_w = G_A \frac{V_G^2}{V_O^2}, \tag{17}$$

$$\alpha_{l2} = G_A \cdot \left( \frac{2V_G}{V_O} - 1 \right). \tag{18}$$

After introducing (17) and (18) into (16), we obtain the following expression for $Y_d$ (BUCK):

$$Y_d(BUCK) = Y_{d0}(BUCK) \frac{s/\omega_{zd} + 1}{s/\omega_{pd} + 1}. \tag{19}$$

Low-frequency value $Y_{d0}$ (BUCK) in (19) is:

$$Y_{d0}(BUCK) = Y_d(BUCK)|_{s=0} = \\ = G_A \frac{G_A \cdot (M_I - 1)^2 + G}{G_A \cdot M_I^2 + G}, \tag{20}$$

where $M_I$ is D.C. current transmittance of BUCK in DCM:

$$M_I = \frac{I_O}{I_G} = \frac{1}{2}\left( \sqrt{1 + \frac{4G}{G_A}} + 1 \right). \tag{21}$$

$\omega_{zd}$ and $\omega_{pd}$ – characteristic angular frequencies in (19) are:

$$\omega_{zd} = 2\pi f_{zd} = \frac{1}{C}\left[ G_A (M_I - 1)^2 + G \right], \tag{22}$$

$$\omega_{pd} = 2\pi f_{pd} = \frac{1}{C}\left[ G_A M_I^{\,2} + G \right]. \tag{23}$$

Figure 3c corresponds to the condition $V_g$=0, and in this case the following equations are obtained:

$$I_g = \alpha_{g3} V_o + \alpha_{g1}\theta, \tag{24}$$

$$V_o = \alpha_{l1} \cdot \theta \cdot \frac{1}{G_w + G + sC}. \tag{25}$$

After excluding $V_o$, it is obtained:

$$\Gamma_d(BUCK) = \alpha_{g1} + \frac{\alpha_{g3} \cdot \alpha_{l1}}{G_w + G + sC}. \tag{26}$$

The expressions for $\alpha_{l1}$, $\alpha_{g1}$, $\alpha_{g3}$ [6] are:

$$\alpha_{l1} = 2 \cdot V_G \cdot \frac{G_A}{D_A} \cdot \left( \frac{V_G}{V_O} - 1 \right), \tag{27}$$

$$\alpha_{g1} = 2 \frac{G_A}{D_A} \cdot (V_G - V_O), \tag{28}$$

$$\alpha_{g3} = -G_A. \tag{29}$$

From (26–29), the expression for $\Gamma_d$ (BUCK) quantity is obtained in the form:

$$\Gamma_d(BUCK) = \Gamma_{d0}(BUCK) \cdot \frac{s/\omega_{z1} + 1}{s/\omega_{pd} + 1}, \tag{30}$$

where:

$$\Gamma_{d0}(BUCK) = \frac{2G_A}{D_A} \cdot V_G \cdot (1 - M_I^{-1}) \cdot \frac{G_A \cdot M_I \cdot (M_I - 1) + G}{G_A \cdot M_I^2 + G} \tag{31}$$

and:

$$\omega_{z1} = 2\pi f_{z1} = \frac{1}{C}\left[ G_A \cdot M_I (M_I - 1) + G \right]. \tag{32}$$

The pole of $\Gamma_d$ (BUCK) is the same as of $Y_d$ (BUCK).

## 3. Step-up (BOOST) converter

**3.1. Continuous Conduction Mode (CCM).** Small-signal form of averaged model of BOOST converter power stage in CCM can be expressed by equations (33–35) [6]:

$$sL \cdot I_g = V_g - V_o \cdot (1 - D_A) + V_O \cdot \theta, \tag{33}$$

$$I_d = (sC + G) \cdot V_o, \tag{34}$$

$$I_d = I_g \cdot (1 - D_A) - I_L \cdot \theta, \tag{35}$$

where $I_g$ is the input current and the inductor current as well, $I_d$ is the current of a diode. After excluding $I_d$ and the output voltage $V_o$ one obtains:

$$Y(BOOST) = \frac{G}{(1-D_A)^2} \cdot \frac{1 + sCR}{s^2 \frac{LC}{(1-D_A)^2} + s \frac{LG}{(1-D_A)^2} + 1} \tag{36}$$

$$\Gamma(BOOST) = \frac{2I_O}{(1-D_A)^2} \frac{1 + sCR/2}{s^2 \frac{LC}{(1-D_A)^2} + s \frac{LG}{(1-D_A)^2} + 1}. \tag{37}$$

For small frequencies (when $s \to 0$) quantities Y (BOOST) and Γ (BOOST) are equal to:

$$Y_0(BOOST) = \frac{G}{(1-D_A)^2} \tag{38}$$

$$\Gamma_0(BOOST) = \frac{2I_O}{(1-D_A)^2}. \tag{39}$$

**3.2. Discontinuous Conduction Mode (DCM).** The general small-signal scheme of a BOOST converter in DCM [6] shown in Fig. 4a can be used to derive dynamic input characteristic of the converter. Quantities $Y_d$ (BOOST) and $\Gamma_d$ (BOOST) for this case are obtained for $\theta = 0$ or $V_g = 0$, respectively, which corresponds to Figs. 4b and 4c.

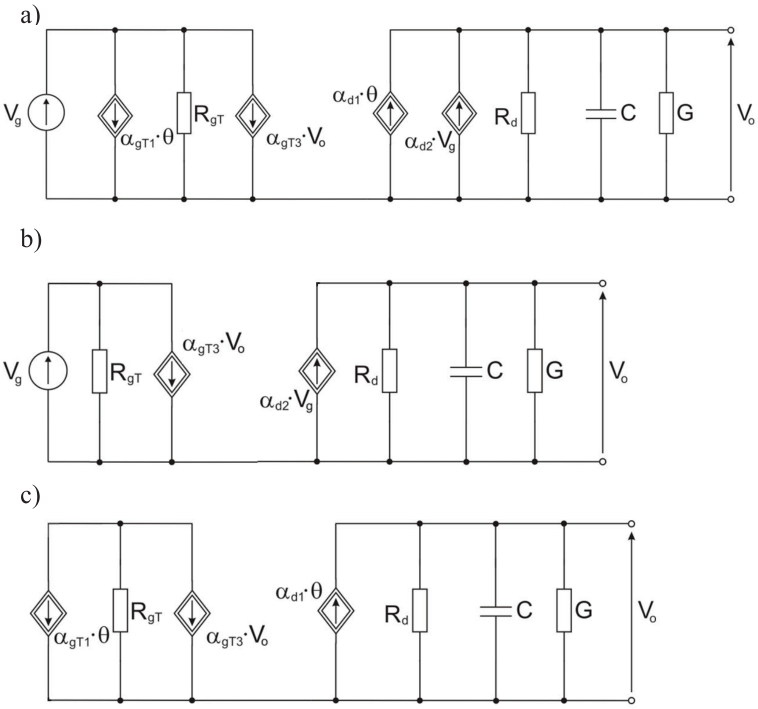


Fig. 4. Averaged small-signal model of a BOOST converter working in DCM: (a) full; (b) θ = 0; (c) $V_g = 0$

For $\theta = 0$ (Fig. 4b), one obtains:

$$I_g = G_{gT} \cdot V_g + \alpha_{gT3} \cdot V_o, \tag{40}$$

$$V_o = \alpha_{d2} \cdot V_g \cdot \frac{1}{G_d + G + sC}. \tag{41}$$

After excluding output voltage $V_o$, one obtains the dependence of $I_g$ on $V_g$ (for $\theta = 0$) and, as a result, the admittance $Y_d$ (BOOST) for this case:

$$Y_d(BOOST) = \frac{G_{gT} \cdot (G_d + G + sC) + \alpha_{gT3} \cdot \alpha_{d2}}{G_d + G + sC}. \tag{42}$$

Quantities $\alpha_{gT3}$, $\alpha_{d2}$, $G_{gT}$ and $G_d$ are [6]:

$$\alpha_{gT3} = -\frac{G_A \cdot V_G^2}{(V_O - V_G)^2}, \tag{43}$$

$$\alpha_{d2} = G_A \cdot \frac{V_G \cdot (2V_O - V_G)}{(V_O - V_G)^2}, \tag{44}$$

$$G_{gT} = \frac{1}{R_{gT}} = \frac{G_A \cdot V_O^2}{(V_O - V_G)^2}, \tag{45}$$

$$G_d = \frac{1}{R_d} = \frac{G_A}{M_{VT} - 1}. \tag{46}$$

After substituting expressions for coefficients α, $G_{gT}$, $G_d$ from (43–46) according to [6] into (42) we obtain the equation for $Y_d$ (BOOST):

$$Y_d(BOOST) = Y_{d0}(BOOST) \cdot \frac{1 + s/\omega_{zt}}{1 + s/\omega_{pt}}. \tag{47}$$

Low-frequency value $Y_{d0}$ (BOOST) in (47) is:

$$Y_{d0}(BOOST) = G_A \cdot \frac{G_A + G \cdot M_{VT}^2}{G_A + G \cdot (M_{VT} - 1)^2}. \tag{48}$$

$M_{VT}$ in the above equations (D.C. voltage transmittance of BOOST in DCM) is given by:

$$M_{VT} = \frac{1}{2} \cdot \left(1 + \sqrt{1 + 4G_A / G}\right). \tag{49}$$

Angular frequencies $\omega_{zt}$ and $\omega_{pt}$ are as follows:

$$\omega_{zt} = \frac{G_A + G \cdot M_{VT}^2}{C \cdot M_{VT}^2}, \tag{50}$$

$$\omega_{pt} = \omega_p = \frac{G_A + G \cdot (M_{VT} - 1)^2}{C \cdot (M_{VT} - 1)^2}. \tag{51}$$

Figure 4c corresponds to the condition $V_g$=0, and in this case the following equations are obtained:

$$I_g = \alpha_{gT1} \cdot \theta + \alpha_{gT3} \cdot V_o, \tag{52}$$

$$V_o = \alpha_{d1} \cdot \theta \cdot \frac{1}{G_d + G + sC}. \tag{53}$$

After excluding output voltage $V_o$, one obtains the dependence of $I_g$ on θ (for $V_o$=0). In this case $\Gamma_d$ (BOOST) is:

$$\Gamma_d(BOOST) = \alpha_{gT1} \cdot \frac{\alpha_{gT3} \cdot \alpha_{d1}}{G_d + G + sC}. \tag{54}$$

Quantities $\alpha_{gT1}$ and $\alpha_{d1}$ are:

$$\alpha_{gT1} = 2 \cdot \frac{G_A}{D_A} \cdot \frac{V_G \cdot V_O}{V_O - V_G}, \tag{55}$$

$$\alpha_{d1} = 2 \cdot \frac{G_A}{D_A} \cdot \frac{V_G^2}{V_O - V_G}. \tag{56}$$

After substituting expressions for coefficients α, $G_d$ into (54), we get the following equation:

$$\Gamma_d(BOOST) = \Gamma_{d0}(BOOST) \cdot \frac{1 + s/\omega_{z1}}{1 + s/\omega_p}, \tag{57}$$

where low-frequency value $\Gamma_{d0}$ (BOOST) in (57) is:

$$\Gamma_{d0}(BOOST) = \frac{2G_A}{D_A} \cdot V_G \cdot \frac{G \cdot (M_{VT} - 1) \cdot M_{VT} + G_A}{G \cdot (M_{VT} - 1)^2 + G_A}. \tag{58}$$

Angular frequency $\omega_p$ in (57) is equal to pole $\omega_{pt}$ in (51). Zero in equation (57) corresponds to angular frequency:

$$\omega_{z1} = \frac{G \cdot (M_{VT} - 1) \cdot M_{VT} + G_A}{C \cdot (M_{VT} - 1) \cdot M_{VT}}. \tag{59}$$

## 4. Characteristics in time domain and numerical examples for DCM

Frequency dependencies of quantities $Y_d$ and $\Gamma_d$ for discontinuous conduction mode have general form:

$$H(s) = H_o \cdot \frac{1 + s/\omega_z}{1 + s/\omega_p}. \tag{60}$$

It may be written as:

$$H(s) = H_o \cdot \left(\frac{\omega_p}{\omega_z} + \frac{1 - \omega_p/\omega_z}{1 + s/\omega_p}\right). \tag{61}$$

The equivalent description in time-domain is the time response h (t) to step pulse excitation:

$$h(t) = H_o \cdot \left(\frac{\omega_p}{\omega_z} - \left(\frac{\omega_p}{\omega_z} - 1\right) \cdot \left(1 - e^{-\omega_p \cdot t}\right)\right). \tag{62}$$

The waveforms of the time-domain response of the input current of BUCK and BOOST converters in DCM for the step pulse of input voltage $v_G$ or duty ratio $d_A$ correspond to the above description of h (t).

In the laboratory model of BOOST converter, the following set of parameters have been used: $V_G$ = 3V; $D_A$ = 0.2; C = 560μF; L = 5μF; $T_S$ = 10μs; G = 0.02S. The corresponding values of $G_A$ and $M_{VT}$ are: $G_A$ = 0.04S; $M_{VT}$ = 2. From (47–59) it is obtained: $Y_{d0}$ (BOOST) = 0.08S; $\Gamma_{d0}$ (BOOST) = 1.6A; $\omega_{zt}$ = 52.632rad/s; $\omega_{pt}$ = 105.26rad/s; $\omega_{z1}$ = 70.175rad/s.

The time-domain response of input current on the step pulse excitation of $v_G$ (from (3V to 4V) and $d_A$ (from 0.2 to 0.3) have been measured and calculated according to formula (62). The comparison of the results of measurements and calculations is presented in Figs. 5 and 6.

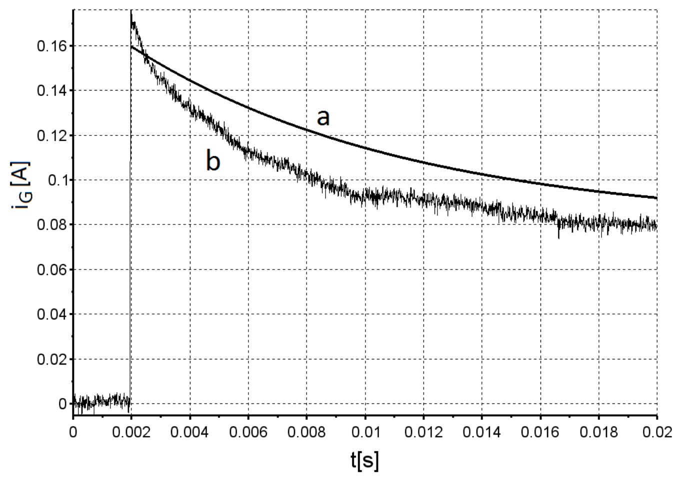


Fig. 5. Time-domain response of input current on the step pulse excitation of input voltage: a) calculation; b) measurement

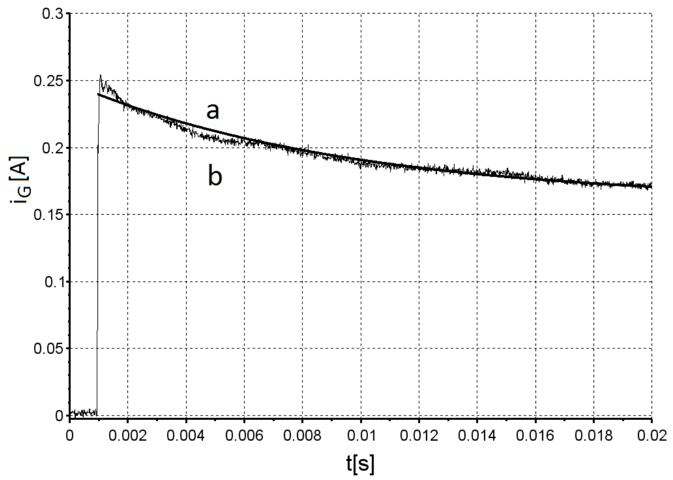


Fig. 6. Time-domain response of input current on the step pulse excitation of duty cycle: a) calculation; b) measurement

## 5. Conclusions

Small-signal input characteristics of DC–DC pulse converters may be useful in the designing process of some power conversion systems, for example in power factor corrections or maximum power point tracing in photovoltaic systems. Small-signal input characteristics may be derived from the general form of converter averaged models presented in the literature. Such characteristics for step-down and step-up DC–DC converters working in continuous conduction or discontinuous conduction mode are presented in the paper. Special attention is paid to characteristics for DCM, which have not been published previously.

According to formulas derived in the paper, input admittances and Γ parameters describing the dependence of input current on input voltage and control signal in DCM are frequency-dependent in a relatively narrow region of low frequencies. These frequency-dependencies are determined mainly by the value of converter output capacitance.

From the small-signal characteristics in the frequency domain, the time-domain responses of input current to step-pulse excitation of input voltage or duty ratio may be easily derived. The exemplary time-domain characteristics are presented in the paper. The results of calculations performed with the use of presented formulas are consistent with measurements of the laboratory model of BOOST converter.

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