# SURFACE-TO-AIR MISSILE PATH PLANNING USING GENETIC AND PSO ALGORITHMS

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Optimization algorithms use various mathematical and logical methods to find optimal points. Given the complexity of models and design levels, this paper proposes a heuristic optimization model for surface-to-air missile path planning in order to achieve the maximum range and optimal height based on 3DOF simulation. The proposed optimization model involves design variables based on the pitch programming and initial pitch angle (boost angle). In this optimization model, we used genetic and particle swarm optimization (PSO) algorithms. Simulation results indicated that the genetic algorithm was closer to reality but took longer computation time. PSO algorithm offered acceptable results and shorter computation time, so it was found to be more efficient in the surface-to-air missile path planning.

*Keywords:* path planning, genetic algorithm, PSO algorithm, surface-to-air missile, 3DOF simulation

## 1. Introduction

System optimization means minimizing or maximizing system functions to improve its efficiency. Several approaches have been proposed for designing acceptable answers under time limitation. These approaches involve some algorithms which do not guarantee an optimal answer but offer the best combination of quality and time based on evidences and records. These algorithms are called heuristic algorithms (Puchinger and Raidl, 2005). Normally, the air defense missile guidance system consists of three phases: boost phase, midcourse and terminal phase. The midcourse is the longest phase of the flight and aims to direct the missile towards the target and to move it through an optimal path in order to save energy and prevent it from being seen by the enemy. This paper aims to design the midcourse of a surface-to-air missile using genetic and PSO algorithms in order to achieve the maximum range for the missile. To do so, we have to determine the initial boost angle and pitch angle over the path in the vertical sheet. This is normally a difficult job and entails real-time trial and error, which in turn imposes heavy cost, long time and real-time modeling. Path planning using heuristic algorithms helps to achieve the maximum height and range.

In 2001, a study was conducted under the title of "designing guidable interceptor missile using genetic algorithm" with a view to minimizing the contact error, interception time and takeoff weight (Anderson *et al.*, 2001). In 2004, a research was conducted under the title of "finding path for tactical missiles using genetic algorithm", in which the application of genetic algorithm in path planning was investigated. The objectives were to increase speed, range and flight time (Cribbs, 2004). In 2006, a research was conducted under the title of "path optimization using genetic algorithm simulation", in which path data used in optimization process were produced by simulation of the equation of motion. This paper examines a moving hypersonic missile using a path optimization technique (Farooq and Limebeer, 2002). The results indicated that the genetic algorithm was an efficient method in path planning. In 2007, a study was conducted

under the title of "path planning, optimization and guidance of boost vehicles in terminal phase of flight". That PhD thesis proposed a method for path planning, optimization and guidance using 3DOF simulation, evaluated the paths planned for the terminal phase, and used them for the development of the guidance program (Chartres, 2007). Zhao and Fan (2009) dealt with optimal path planning for an anti-ship missile using MAKLINK graph method. In this method, genetic algorithm was used to find optimal points with an emphasis on the points which satisfied all problem constraints. Shu *et al.* (2010) optimized path height for cruise missile using the improved PSO algorithm and simulated the annealing algorithm. Peibei and Jun (2010) compared the Voronoi algorithm, grid method and visual graph for multi missile path planning. Wang *et al.* (2011) proposed a real-time path planning for UAV (Unmanned Air Vehicle) based on PSO algorithm improved by modification of inertia weight and self-adaption. Huang *et al.* (2012) proposed a method for cruise missile path planning based on the voronoi diagram and biogeography-based optimization. Liu *et al.* (2015) proposed an algorithm for path planning based on a series of geometrical constraints and rules using multi-attribute fuzzy optimization (MAFO), which produced successful results for real-time functions.

Some of the above-mentioned papers focused only on the optimization method and solved the problems using heuristic methods to increase convergence speed, reduce the number of assessments, reduce optimization time, reduce computation volume, and combine the optimization methods. They have also compared their methods with other optimization methods. Others focused on optimization results and interpreted them based on bird dynamics and the objective function by changing design variables and comparing the results with empirical methods. This paper deals with surface-to-air missile path planning based on pitch programming in order to achieve the maximum range and optimal height. In this optimization model, we used genetic and PSO algorithms and compared them in a specific problem.

## 2. Exploration algorithm

Generally, heuristic algorithms can be divided into three groups:
- Algorithms which focus on structural features of the problem to define a producer algorithm or local search.
- Algorithms which focus on heuristic guidance of a producer algorithm or local search so that the algorithm can overcome sensitive conditions (e.g. optimal local escape).
- Algorithms which focus on a heuristic framework or concept using mathematical programming (usually by precise methods).

The first group may perform the job very well (sometimes in optimal level) but is trapped in low quality answers. These algorithms were improved by new approaches, including algorithms which explicitly or implicitly managed the relationship between search diversity (where there are symptoms that the search is going towards bad regions of search space) and search intensification (with a view to find the best answer in the studied region). Among such algorithms, we can mention simulated annealing, particle swarm optimization, and colony optimization and neural network. The most famous and efficient algorithms are those which provide problem solving models using genetic evolution patterns. These algorithms develop an effective search method in large spaces which finally lead to finding the optimal answers. In this part, we first introduce the heuristic algorithms and then explain how to find the answer (Puchinger and Raidl, 2005).

### 2.1. Genetic algorithm

The idea of evolutionary algorithms was coined by Richenberg in 1960. According to Darwin's Theory of Evolution, those natural traits which adapt more to natural laws have more chance of

survival. Based on the natural selection law, those species of a population which possess the best traits continue their generation and those which lack such traits are gradually destroyed over time. Therefore, natural selection may be considered as a competition for preserving superior traits. Genetic algorithms are evolutionary algorithms inspired by biological sciences such as genetics, mutation, natural selection and combination.

Important parameters in a genetic algorithm are encoding, population size, initial population, chromosome rating (fitness function scale), parent selection mechanism, crossover rate, genetic operators, replacement, and algorithm stoppage parameters (Holland, 1975).

Evolution begins from the initial population and is repeated in the next generations. Figure 1 illustrates the steps of a genetic algorithm. The important point in a genetic algorithm is to select the most appropriate members of each generation, not the best ones (Puchinger and Raidl, 2005; Jarvis and Goodacre, 2005).
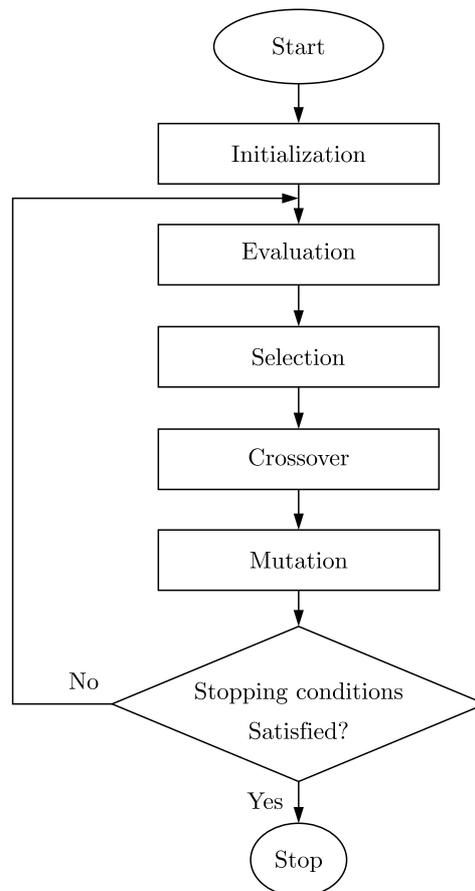


Fig. 1. Genetic algorithm

### 2.2. Particle swarm optimization algorithm

Birds show certain social behaviors. To better understand this technique, we will explain a scenario in the next paragraphs.

A group of birds are randomly seeking food in a specific area. In this area, only a piece of food exists and the birds are unaware of its exact location. However, they know their distance from the food in any moment. In such circumstances, a good strategy to find the exact location of food is to follow the bird that is closest to the food.

In fact, each bird in PSO algorithm is a solution to the problem. Every answer has a fitness value which is obtained from the fitness function of the problem. This technique aims to find the

location with the best fitness value in the problem space. The fitness value directly affects the direction and speed of bird movement (problem answers) towards food location (optimal answer). This algorithm starts to work with a number of initial answers and searches for the optimal answer by moving the answers during frequent repetitions. In each repetition, the location of best fitness value for each particle (pBest) and the location of the best particle in the current population (gBest) are specified (Fan and Shi, 2001). Figure 2 illustrates the steps of PSO algorithm.
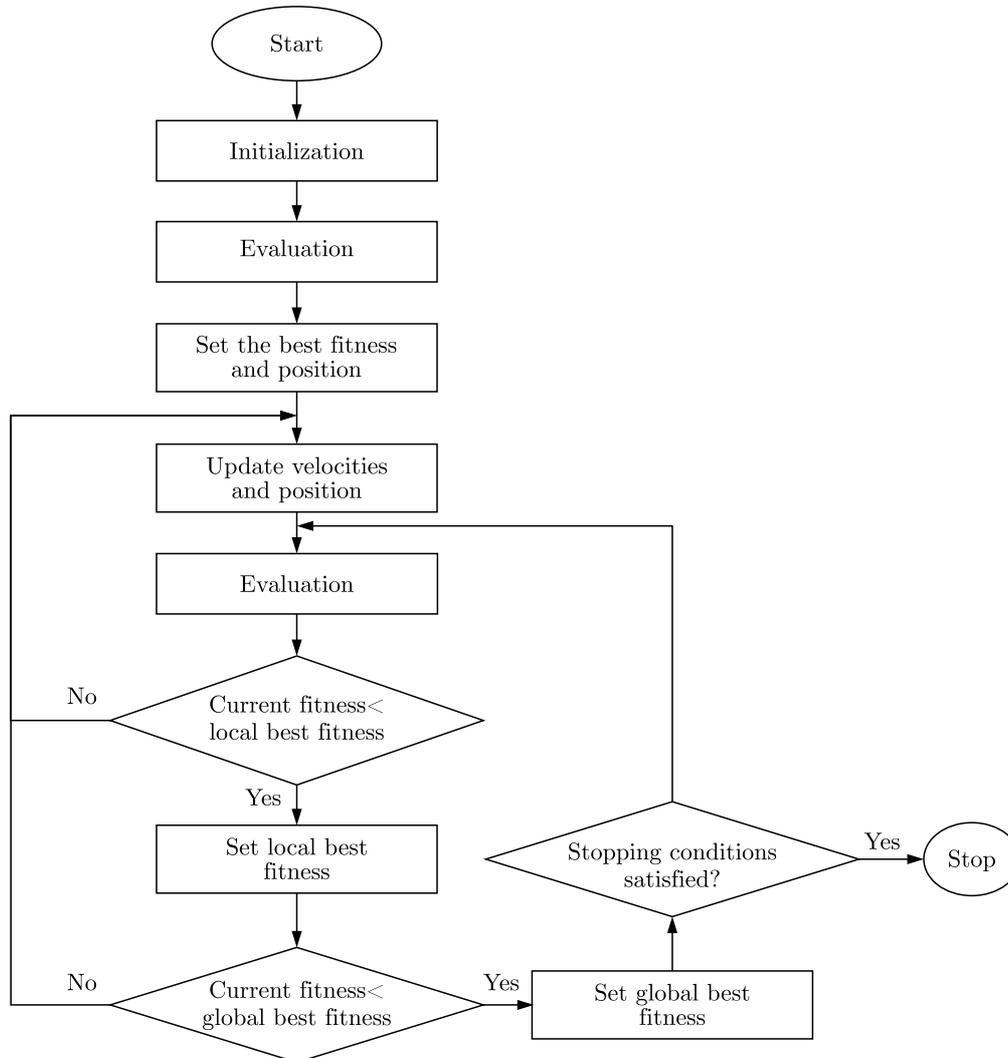


Fig. 2. Particle swarm optimization algorithm

## 3.  3DOF simulation

Designing and testing guidance and control systems of aerospace vehicles requires path simulation based on the system model. The advances in computer science, the increased processing power, and the efforts to model subsystems and other associated items have led to the improved planning process. On the other hand, special attention has been paid to the application of simulation in multithreaded optimization, and efforts have been made to perform simulation with high accuracy and speed. Generally, simulation of flight dynamics is divided into five parts:

1. Simplification
2. Selection of reference coordinates

3. Extraction of subsystem equations and modeling

4. Simulation of motion equations in a computer program

5. Authentication of the simulation

The simplification refers to the assumptions used to simplify the study of vehicle dynamics. Since the mass center path of the vehicle is more important than its rotation, 3DOF simulation greatly contributes to the estimation of vehicle performance and investigation of the path. In contrast to 6DOF simulation, 3DOF simulation does not use Euler laws and does not need to compute body rates, so there is no need to aerodynamic and propulsion moments. One of the subjects in each simulation is the selection and conversion of coordinates. In many parts of the simulation, we need to convert coordinates of the parameters so that we can use their values in other coordinates (Zipfel, 2007).

Body coordinates (Fig. 3) are one of the most important coordinates because they make many measurements and computations. For example, accelerations are measured by accelerometers installed in body coordinates.
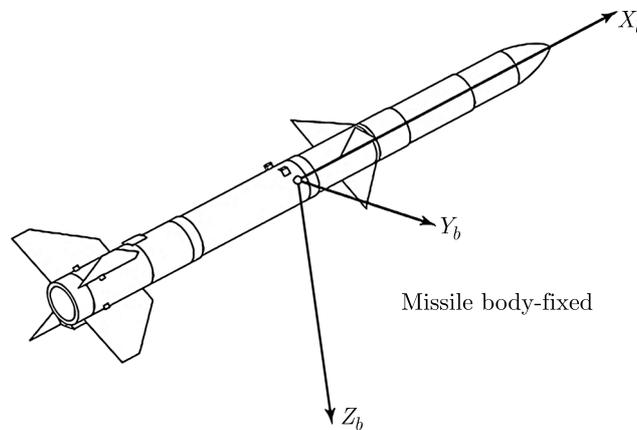


Missile body-fixed

Fig. 3. Body coordinates

In missiles, all $X_b$ and $Y_b$ directions are the main axes due to rotational symmetry, so geometrical signs are used to locate unit vectors. As one can see, the origin of coordinates is on the boost point of the ground, the axis $x$ is in the boost direction, the axis $z$ is perpendicular to the ground (towards the ground), and the axis $y$ makes the coordinate (Zipfel, 2007), see Fig. 4.

If Missile DATCOM (MD) software is used in simulation to compute aerodynamic coefficients, it is necessary to pay attention to the body coordinate and the positive directions of its axis in the software (Fig. 5).

### 3.1. Gravity model

In any simulation, a gravity model must be selected with the required accuracy. Distribution of non-spherical mass of the Earth affects the size and direction of gravity on the missile, but these components are so small that they are omitted in surface-to-air missile programs. According to equation (3.1), gravity acceleration depends on vehicle height in each moment and decreases with the increased height (Tewari, 2007)

$$g = g_0 \Big( \frac{R_e}{R_e + H} \Big)^2 \tag{3.1}$$

where $g$ is gravity acceleration, $H$ is height, $R_e$ is ground radius (6378140 m), and gravity acceleration at sea level is 9.80665 m/s$^2$.
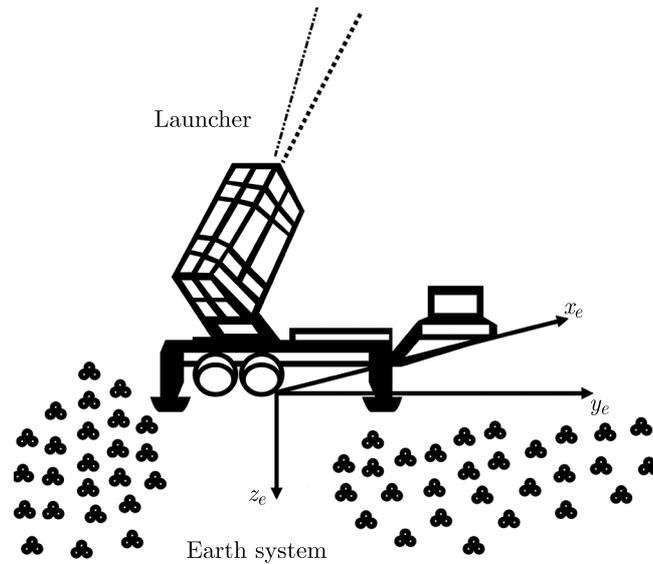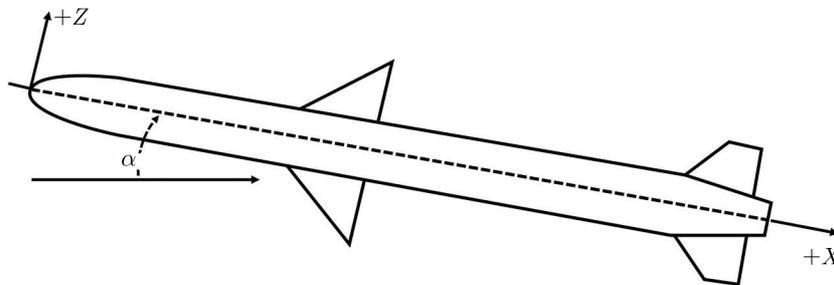
Fig. 4. Ground coordinates



Fig. 5. MD software coordinates

### 3.2. Standard atmosphere model

Investigation of aerospace vehicle flight has two parts: atmosphere flight mechanics and space flight mechanics. The standard atmosphere is modeled in the form of frequent layers with different temperature rates based on height $T(h)$. The objective is to provide and develop a 21-layer standard atmosphere model for the ground to be used in simulation of atmosphere paths and in determination of dimensionless aerodynamic parameters for aerodynamic force modeling. To do so, two standard atmosphere models of 1976 and 1962 are used. These two models have negligible difference until the height of $0 \leqslant h \leqslant 86$, but the difference becomes noticeable in the exosphere layer (Tewari, 2007).

### 3.3. Point mass 3DOF equations

The most important step before modeling is the selection of inertia reference coordinates. For example, in aerospace vehicles flying near the Earth (such as the satellites rotating in lower orbits), circular or elliptical inertia reference is used. This may be accompanied with circular or elliptical models. The flat ground model is used for airplanes and tactical missiles. First, using Newton's second law, we write transmission equations for an aerospace vehicle exposed to aerodynamic forces and gravity

$$\dot{v}_{x_{body}} = \left(\frac{1}{M}\right)(T + F_{x_{aero}} + F_{x_{gravity_{body}}}) - (qv_{z_{body}} - rv_{y_{body}})$$

$$\dot{v}_{y_{body}} = \left(\frac{1}{M}\right)(F_{y_{aero}} + F_{y_{gravity_{body}}}) - (rv_{x_{body}} - pv_{z_{body}}) \qquad (3.2)$$

$$\dot{v}_{z_{body}} = \left(\frac{1}{M}\right)(F_{z_{aero}} + F_{z_{gravity_{body}}}) - (pv_{y_{body}} - qv_{x_{body}})$$

where $M$ is vehicle mass and $\mathbf{v}_{body} = [v_{x_{body}}, v_{y_{body}}, v_{z_{body}}]$ is body mass center speed of the vehicle. $T$ describes the force produced by thrust. $\mathbf{F}_{aero} = [F_{x_{aero}}, F_{y_{aero}}, F_{z_{aero}}]$ and $\mathbf{F}_{gravity} = [F_{x_{gravity_{body}}}, F_{y_{gravity_{body}}}, F_{z_{gravity_{body}}}]$ denote the aerodynamic force and gravity force, respectively. $p$, $q$ and $r$ denote the angular velocity about $X_B$, $Y_B$ and $Z_B$ directions in the body coordinates. The left side of the above equations can be easily computed in body coordinates, through which vehicle acceleration components in body coordinates will be determined. By integration of the above equations based on initial zero conditions, speed components in body coordinate will be determined (Handbook MIL, 1995).

### 3.4. Aerodynamic forces and torques

Atmosphere path of aerospace vehicles is under the influence of aerodynamic forces and moments. Aerodynamic forces are developed by the interaction between particles and vehicle body during movement in atmosphere. An influential factor in vehicle aerodynamics is the general configuration of the vehicle. On the other hand, the constituent parts of these forces and torques include aerodynamic factors. Identification of importance and accuracy of these factors has a determining role in the design, control and planning the path and in the analysis of vehicle stability. Assuming that wind speed is zero and angular speed of the missile is negligible, aerodynamic forces and moments relate only to dimensions, geometry, speed and parameters of atmosphere.
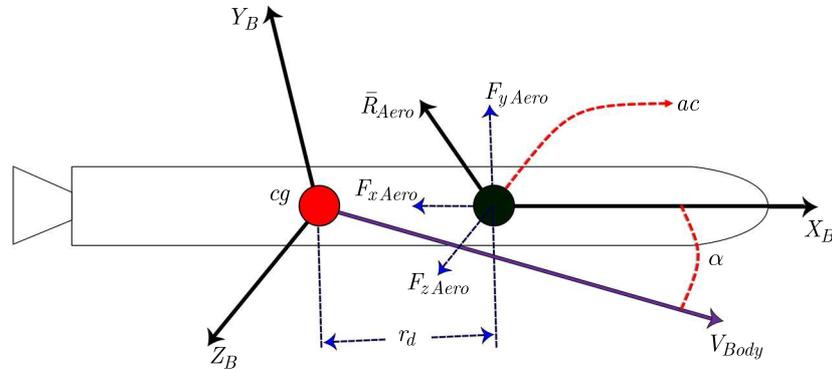


Fig. 6. Aerodynamic forces on missile

According to Fig. 6, the aerodynamic forces are determined by

$$F_{x_{aero}} = \frac{1}{2}\rho V^2 S C_A \qquad F_{y_{aero}} = \frac{1}{2}\rho V^2 S C_y \qquad F_{z_{aero}} = \frac{1}{2}\rho V^2 S C_N \qquad (3.3)$$

where $S$ is surface, $\rho$ is density, $C_A$, $C_y$ and $C_N$ are coefficients of axial, lateral and normal forces, respectively, and $V$ is mass center speed of the vehicle in the body coordinates (Tewari, 2007).

## 4. Numerical results

To achieve optimal planning, a code has been codified in MATLAB environment for genetic and PSO algorithms. In this program which is connected to MATLAB Simulink, first the parame-

ters of each algorithm are adjusted by the user. The cost function is optimized according to the adjusted parameters and simulation results (which exist in the algorithm). In this specific case, the optimization problem includes the cost function in the form of equation (4.1) for the achievement of optimal height and the desired maximum

$$f = (H - H_t)^2 + (R^2 - R_t)^2 \tag{4.1}$$

where $H$ and $R$ are height and range requested by the designer, $H_t$ and $R_t$ are height and range of the vehicle in each moment of flight. Tables 1 and 2 contain the parameters of genetic and PSO algorithms.

**Table 1.** Parameters of genetic algorithm

| Parameter | Value |
|---|---|
| Generation number | 100 |
| Population number | 50 |
| Mutation rate | 0.1 |
| Selection rate | 0.5 |

**Table 2.** Parameters of PSO algorithm

| Parameter | Value |
|---|---|
| Particle number | 100 |
| Local optimal coefficient | 2 |
| Comprehensive optimal coefficient | 2 |
| Speed contraction coefficient | 0.5 |

Table 3 represents the system parameters needed for 3DOF simulation of a surface-to-air missile.

**Table 3.** System parameters needed for simulation

| Parameter | Value |
|---|---|
| Total mass in boost time | 237.777 kg |
| Total mass of booster | 45 kg |
| Main engine trust | 35585.766 N |
| Booster trust | 60453 N |
| Main engine burn time | 2.9 s |
| Booster burn time | 2 s |
| Pressure behind the nozzle | 70000 Pa |

To guide the vehicle, we used pitch programming in the simulation problem. For this purpose, we designed a boost angle and angular rate schedule and used them as the simulation input. The preset pitch rate command is generated by

$$\dot{\theta} = \begin{cases} 0 & \text{for} \quad t < t_1 \\ \dfrac{a(t - t_1)}{t_2 - t_1} & \text{for} \quad t_1 \leqslant t < t_2 \\ a & \text{for} \quad t_2 \leqslant t < t_3 \\ a e^{b(t - t_3)} & \text{for} \quad t \geqslant t_3 \end{cases} \tag{4.2}$$

where $\dot{\theta}$ isthe pitch rate command used as pitch programming in the simulation problem, $a$ and $b$ schedule the angular rate, $t$ denotes the simulation time, $t_1$ is defines the engine start time, $t_2$ and $t_3$ are 0.1 s and 0.3 s after starting main engine, respectively.

Geometrical parameters of the vehicle, aerodynamic coefficients tables and angular schedule were recalled by the input file at the beginning of the program, and the related parameters were initialized.

To evaluate the vehicle performance, we had to determine the range that the vehicle would achieve if it reached the intended height. To do so, we planned the path in two scenarios: 1) the ability to achieve flight height of 10 000 m, and 2) reaching the height of 6000 m as the most common altitude in the path planning strategies. These two scenarios were investigated to reach maximum range as well as achieving altitudes in the two case studies. Optimization results of the algorithms will be represented in the following Sections.

According to the boost conditions, optimization algorithms modified speed, acceleration and height. These modifications affected aerodynamic coefficients and dynamic pressure. For this reason, the force coefficients are calculated for seven particular Mach numbers ranging from 0.3 to 3, at five angles of attack $\alpha$ for each Much number in the range of 0° to +15°. The outputs of Missile DATCOM are shown in Figs. 7 and 8. These results are set as a lookup table in SIMULINK and the interpolated based Mach number, altitude and angle of attack in the flight simulation process.
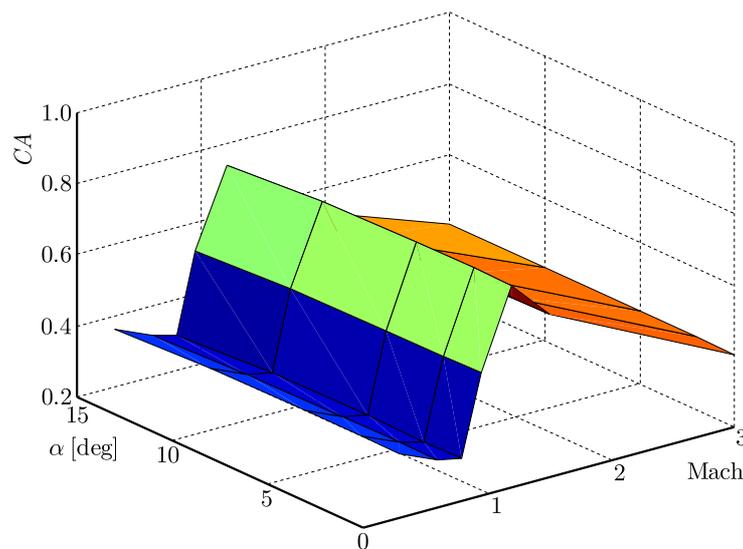


Fig. 7. Axial force coefficient with respect to Mach number and angle of attack

The average execution time of both algorithms is measured and given in Table 4, using a specific computer, characterized by Intel(R) Core(TM) i3 CPU M370 at 240GH. Given the performance of optimization algorithms in this specific problem, we found that the genetic algorithm had a relatively good performance and its optimal solutions were closer to reality. However, it had higher computation cost.

**Table 4.** Average execution time of PSO and GA in the path planning problem

| Algorithm | Average execution time [s] | |
| --- | --- | --- |
|  | First scenario | Second scenario |
| PSO | 510.2672 | 305.7908 |
| GA | 785.2714 | 486.6852 |

Figures 9 and 10 illustrate some of these modifications for both scenarios.
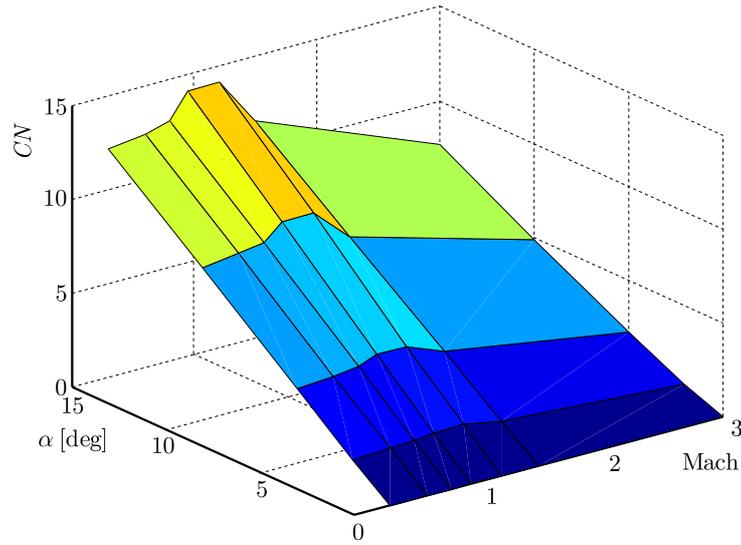
Fig. 8. Normal force coefficient with respect to Mach number and angle of attack
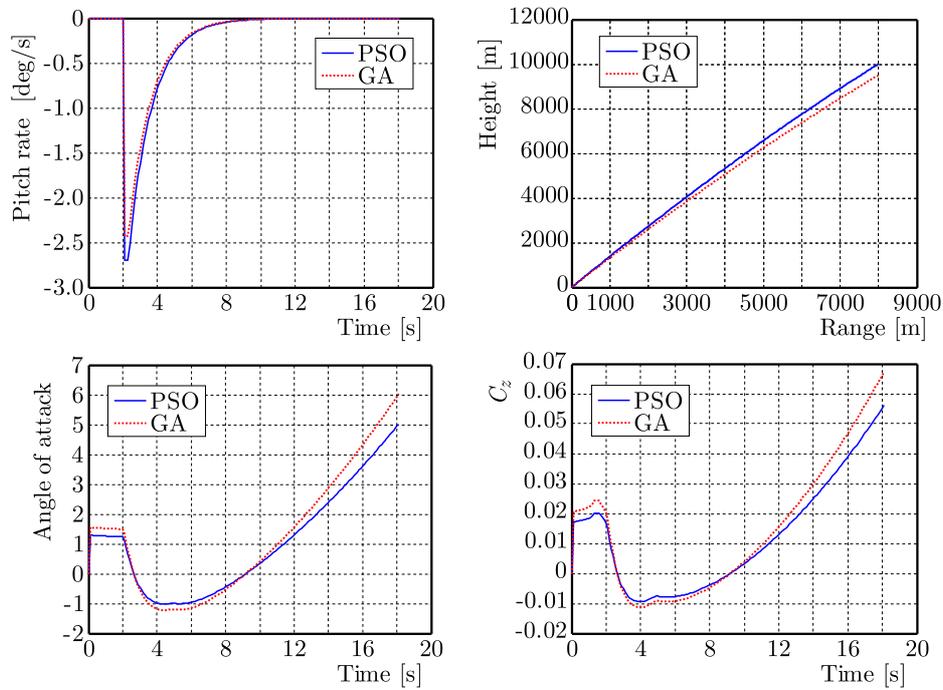


Fig. 9. Functional changes of the vehicle in the first scenario

As you can see in Figs. 9 and 10, vehicle performance in reaching the specified height is similar in both scenarios. Therefore, the changes have similar functional parameters but varied in numerical values. Tables 5 and 6 summarize the optimization results.

## 5. Conclusion

In this paper, we optimized a surface-to-air missile path using genetic and PSO algorithms in order to achieve the maximum range and optimal height based on 3DOF simulation. In this optimization model, design variables are based on the pitch programming, initial pitch angle and pitch variations rate slope. According to 3DOF simulation results, vehicle performance
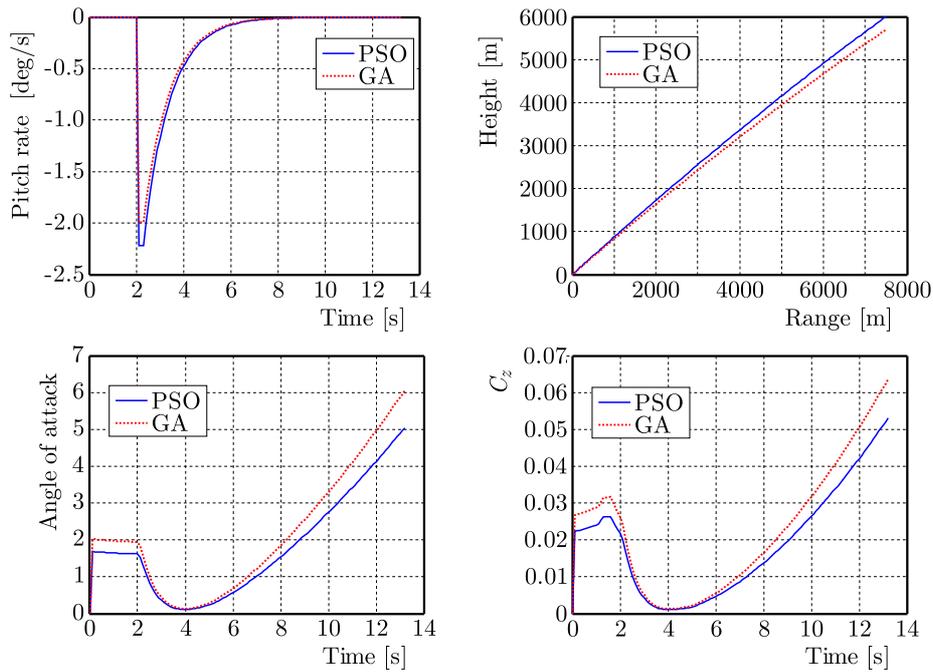
Fig. 10. Functional changes of the vehicle in the second scenario

**Table 5.** Comparison of algorithms for the height of 10 000 m

| Parameter | Algorithm | |
|---|---|---|
| | PSO | GA |
| Initial boost angle [deg] | 55.3114 | 53.4212 |
| $a$ | $-0.0492$ | $-0.0558$ |
| $b$ | $-0.7135$ | $-0.8228$ |
| Operating range [m] | 8250 | 8125 |

**Table 6.** Comparison of algorithms for the height of 6000 m

| Parameter | Algorithm | |
|---|---|---|
| | PSO | GA |
| Initial boost angle [deg] | 43.8424 | 42.4933 |
| $a$ | $-0.0784$ | $-0.0622$ |
| $b$ | $-0.3855$ | $-0.4295$ |
| Operating range [m] | 7500 | 7225 |

did not differ in the mentioned optimization algorithms. The difference lied only in the type of algorithm. In this specific case, the genetic algorithm was closer to reality but took longer computation time. PSO algorithm offered acceptable results and shorter computation time, so it was found to be more efficient in the surface-to-air missile path planning.

# References

1. Anderson M.B., Burkhalter J.E., Jenkins R.M., 2001, Design of a guided missile interceptor using a genetic algorithm, *Journal of Spacecraft and Rockets*, **38**, 1, 28-35

2. Chartres J.T.A., 2007, Trajectory design, optimisation and guidance for reusable launch vehicles during the terminal area flight phase, Diss. Universität Stuttgart

3. CRIBBS H.B., 2004, Genetics-based trajectory discovery for tactical missiles, *AIAA 1st Intelligent Systems Technical Conference*, 1-6

4. FAN H., SHI Y., 2001, Study on $V_{max}$ of particle swarm optimization, *Proceedings of Workshop on Particle Swarm Optimization*, Purdue School of Engineering and Technology, Indianapolis, IN, USA

5. FAROOQ A., LIMEBEER D.J.N., 2002, Trajectory optimization for air-to-surface missiles with imaging radars, *Journal of Guidance, Control and Dynamics*, **25**, 5, 876-887

6. Handbook, 1995, *Military. Missile Flight Simulation*, Part One: *Surface-to-Air Missiles*, MIL--HDBK-1211 (MI)

7. HOLLAND J.H., 1975, *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control and Artificial Intelligence*, U Michigan Press

8. HUANG N., LIU G., HE B., 2012, Path planning based on Voronoi diagram and biogeographybased optimization, *Advances in Swarm Intelligence*, Springer Berlin Heidelberg, 225-232

9. JARVIS R.M., GOODACRE R., 2005, Genetic algorithm optimization for pre-processing and variable selection of spectroscopic data, *Bioinformatics*, **21**, 860-868

10. LIU G., LAO S.-Y., HOU L.-L., LI Y., TAN D.-F., 2015, OARPER-MAFO algorithm for anti-ship missile path planning, *Aerospace Science and Technology*, **47**, 135-145

11. PEIBEI M.A., JUN J.I., 2010, Comparison of three algorithms for multi missile path planning (in Chinese), *Electronics Optics and Control*, **10**, 007.

12. PUCHINGER J., RAIDL G.R., 2005, Combining metaheuristics and exact algorithms in combinatorial optimization: A survey and classification, *International Work-Conference on the Interplay Between Natural and Artificial Computation*, Springer Berlin Heidelberg, 41-53

13. SHU J., WU J., ZHAO J., WANG X., WANG S., 2010, Cruise height optimization based on improved PSO algorithm (in Chinese), *Electronics Optics and Control*, **2**, 004

14. TEWARI A., 2007, *Atmospheric and Space Flight Dynamics*, Birkhaüser Boston

15. WANG X.-Z., ET AL., 2011, Real-time route planning for UAV based on improved PSO algorithm (in Chinese), *Microelectronics and Computer*, **4**, 023

16. ZHAO X., FAN X., 2009, A method based on genetic algorithm for anti-ship missile path planning, *IEEE, International Joint Conference on Computational Sciences and Optimization, CSO 2009*, **2**

17. ZIPFEL P.H., 2007, *Modeling and Simulation of Aerospace Vehicle Dynamics*, American Institute of Aeronautics and Astronautics, AIAA Education Series